Two-Dimensional Kinematics: Heading North (Solutions)

You are the navigator of a TWA flight scheduled to fly from New Orleans due north to St. Louis, a distance of 673 miles. Your instruments tell you that there is a steady wind from the northwest with a speed of 105 mph. The pilot sets the air speed at 575 mph and asks you to find the estimated flying time. What do you tell her?

Conceptual Analysis:

- The velocity of the plane with respect to the air is not the same as the velocity of the plane with respect to the ground.
- The wind from the northwest can be assumed to have equal easterly and southerly components.
- To avoid being blown off course, the pilot will need to head the plane in the westerly direction so that her resultant path with respect to the ground will be due north as opposed to being pushed off course to the east by the wind.

Strategic Analysis:

- Find the easterly component of the wind to determine the westerly component of the pilot's path.
- Use the given air speed and westerly component to determine the northerly component of the plane's air speed.
- Find the southerly component of the wind to determine the northerly component of the pilot's speed with respect to the ground.
- Use the ground speed and distance traveled to estimate the flying time.

Quantitative Analysis:

- Begin by labeling the given quantities:
  - d distance from New Orleans to St. Louis
  - v_w speed of the wind
  - v_a air speed of the plane

  We are looking for
  - t the estimated flying time
- Let's draw a picture to visualize the problem.

- The wind can be broken down into x and y components.

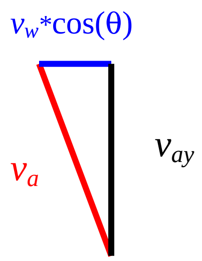
  $v_{wx} = v_w*\cos(\theta)$ (westerly component of the wind)

  $v_{wy} = v_w*\sin(\theta)$ (southerly component of the wind)

- The air speed of the plane is made up of a westerly component and northerly component. Since the westerly component of the plane's speed must equal the easterly component of the wind, we can set the two equal to each other.

$$v_{wx} = v_{ax} = v_w*\cos(\theta)$$

- Next, we can use vector addition to determine the northerly component of the speed of the plane in the air.



$$v_a^2 - v_w^2*\cos^2(\theta) = v_{ay}^2$$

$$v_{ay} = \text{sqrt}[v_a^2 - v_w^2*\cos^2(\theta)]$$

- Knowing v_{ay} and using the southerly component of the wind, we can find the ground speed of the plane.

$$v_{ay} - v_w*\sin(\theta) = v_g$$

- Now we can use the ground speed and the total distance to find the time.

$$d/ v_g = t$$

- Inserting the values and solving:

$$t = d /(\text{sqrt}[v_a^2 - v_w^2*\cos^2(\theta)] - v_w\sin\theta)$$

$$t = 673\ \textit{miles} /(\text{sqrt}[(575\ \textit{mph})^2 - (105\ \textit{mph})^2*\cos^2(45^o)]-(105\ \textit{mph}*\sin 45^o))$$

$$t = 1.36\ \textit{hr}$$

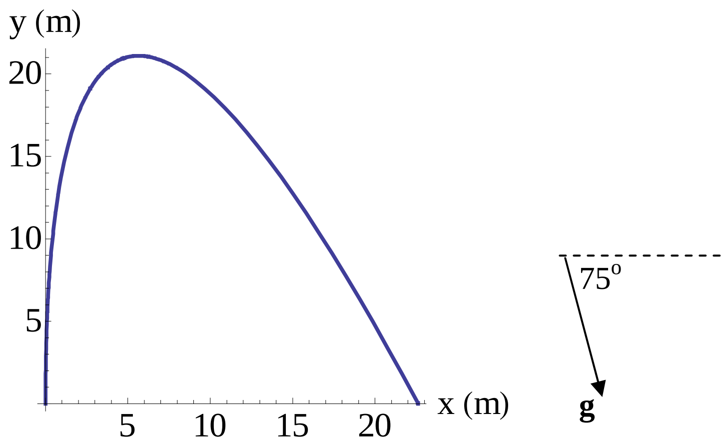
- The estimated flying time is 1.36 hours.

Physics 211 - Week 2
Two-Dimensional Kinematics: Which Way Is Up? (Solutions)

Tilted Land is flat, much like central Illinois, but in Tilted Land gravity does not point straight down! It points down at an angle of 75 degrees with respect to the ground. (In central Illinois gravity points straight down at an angle of 90 degrees with respect to the ground.) This circumstance causes major changes in everyday life, but people manage. Suppose that a person in Tilted Land throws a ball straight up at 20 m/s. Where does the ball hit the ground?

Conceptual Analysis:

- This problem is different from our usual projectile problems.
- There are components of acceleration due to gravity in both the x and y directions.
- The motion of the projectile can be described by a tilted parabola as shown below.



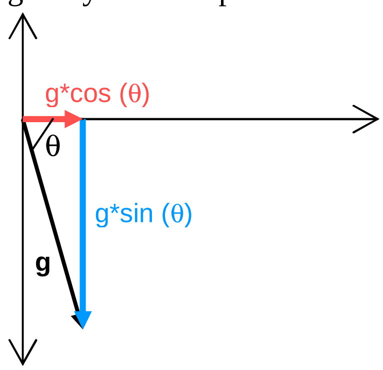
Strategic Analysis:

- This problem can be solved in the same manner as a normal projectile problem you just need to account for the acceleration in the x-direction.
- Find the components of acceleration in the x and y directions.
- Find the time in the air from the information about the y-direction.
- Use the total time in the air to determine the distance traveled in the x-direction.

Quantitative Analysis:

- We can begin by assigning labels to the given quantities:
  - θ angle gravity points down at with respect to the ground
  - v_o initial velocity of the ball
  - g acceleration due to gravity

  We are looking for
  - x the horizontal distance traveled by the ball
- Next we can find the acceleration in the x and y directions by breaking the tilted gravity into components.



y: $a_y = g*\sin(\theta)$

x: $a_x = g*\cos(\theta)$

- Find the time in the air using

$$y = y_o + v_o*t + ½*a*t^2$$

the given values, and your knowledge that the ball will begin and end on the ground (position y = 0):

$$0 = 0 + v_o*t + ½*g*\sin(\theta)*t^2$$

$$t = 2v_o / [g*\sin(\theta)]$$

(You could also find the amount of time it takes for the velocity to reach zero in the y-direction, the peak of the parabola. Even for a tilted parabola, the time to reach the peak is half the total time in the air.)

- Next, use the time in the air to find the distance the ball travels in the x-direction using

$$x = x_o + v_o*t + ½*a*t^2$$

the given values, and your knowledge that the initial velocity in the x direction is zero:

$$x = ½*g*\cos(\theta)*(t)^2$$

inserting your value for the time:

$$x = ½*g*\cos(\theta)*(2v_o / [g*\sin(\theta)])^2$$

$$x = ½*(9.81\ m/s^2)*\cos(75^o)*(2*(20\ m/s) / [(9.81\ m/s^2)*\sin(75^o)])^2$$

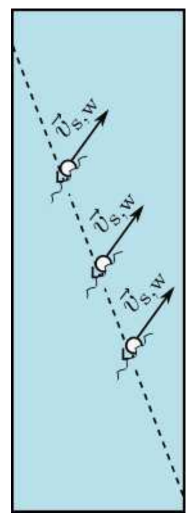
$$x = 22.6\ m$$

The ball will hit the ground 22.6m from where it was thrown in the direction that the x-component of gravity in Tilted Land points.

Against the Grain
(solution)

You are on the west bank of a river which flows due south and want to swim to the east bank. You have told your friends to meet you on the east bank directly opposite your starting point. Before starting out, you realize that, since the river is flowing swiftly at a speed of 12 ft/s and since your fastest swimming speed in still water is only 5 ft/s, you will inevitably be carried downstream. Nevertheless, you want to minimize the effort expended by your friends in walking downstream to meet you. Your guide book to the region tells you that the width of the river is 300 ft. After a quick calculation, you call your friends on your cellular phone and tell them to start walking to a new meeting point. How far downstream of the original meeting point should you tell them to walk?

Conceptual Analysis:
The river carries us downstream as we try to cross it, so we'll need some relative velocity relationships. If we head straight across the river, we minimize our time to cross it but have a large downstream velocity with respect to shore. If instead we point upstream, we have a smaller downstream velocity with respect to the shore but take longer to cross the river.



Strategy:
We need to find the angle of attack that minimizes the distance traveled downstream. We can determine the crossing time in terms of this angle and use it to minimize the distance traveled downstream. This is where our friends should meet us. We'll need to combine the effects of our swimming and the downstream current of the river.

We'll start by finding our velocity with respect to the ground using the relative velocity relationships. Our answer will contain the unknown angle of angle of attack at which we point. We'll then use distance-time relationships and some calculus to figure out the angle that minimizes the downstream drift distance. Once we know this angle, we can insert it back into the distance expression to find where our friends should meet us.

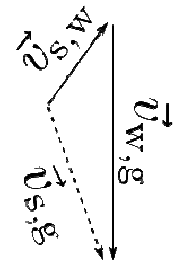
Quantitative Analysis:

We can use the relative velocity equation with the "domino rule" that the inner indices must match:

$$\vec{v}_{S,G} = \vec{v}_{S,W} + \vec{v}_{W,G}$$

where
$\vec{v}_{S,G} = $ velocity of the swimmer with respect to the ground
$\vec{v}_{S,W} = $ velocity of the swimmer with respect to the water
$\vec{v}_{W,G} = $ velocity of the water with respect to the ground

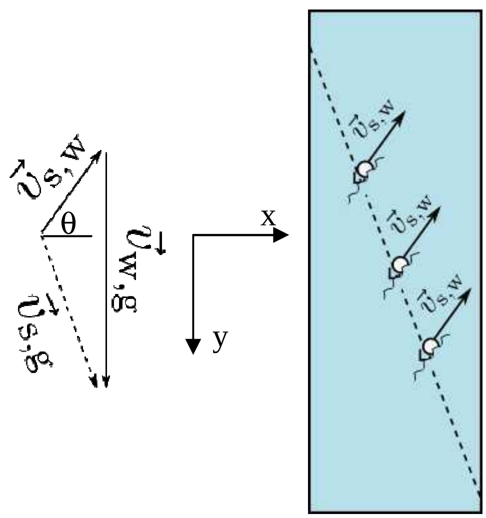
This makes more sense when expressed as a picture:



We'll call θ the angle between the east-west line and the velocity of the swimmer with respect to the water (i.e. the "angle of attack"). The velocities in the ground frame are then:

$$\begin{aligned} v_{SG,x} &= v_{SW,x} + v_{WG,x} \\ &= v_{SW}\cos\theta + 0 \\ &= v_{SW}\cos\theta \end{aligned}$$

$$\begin{aligned} v_{SG,y} &= v_{SW,y} + v_{WG,y} \\ &= -v_{SW}\sin\theta + v_{WG} \end{aligned}$$



Note that we've chosen to the right (East) and down (South) as positive.

Now we'll find the time taken to cross the river in terms of the angle of attack. We'll use W as the width of the river:

$$v_{SG,x}t = W$$

$$\left(v_{SW}\cos\theta\right)t = W$$

$$t = \frac{W}{v_{SW}\cos\theta} = \frac{300\text{ ft}}{5\,\frac{\text{ft}}{\text{s}}\cos\theta} = \frac{60\text{ s}}{\cos\theta}$$

Now we can use this time to find the distance traveled downstream in terms of the angle of attack. We'll call D the distance traveled downstream:

$$D = v_y t$$

$$D(\theta) = \left(v_{WG} - v_{\text{SW}}\sin\theta\right)t$$

$$D(\theta) = \left(12\,\tfrac{\text{ft}}{\text{s}} - 5\,\tfrac{\text{ft}}{\text{s}}\sin\theta\right)\left(\frac{60\text{ s}}{\cos\theta}\right)$$

$$D(\theta) = \left(720\text{ ft}\right)\sec\theta - \left(300\text{ ft}\right)\tan\theta$$

We need to minimize this distance, so we'll find the angle that makes the derivative of the distance function zero:

$$0 = \frac{dD}{d\theta}$$

$$0 = \left(720\text{ ft}\right)\frac{d}{d\theta}\left(\sec\theta\right) - \left(300\text{ ft}\right)\frac{d}{d\theta}\left(\tan\theta\right)$$

$$0 = \left(720\text{ ft}\right)\sec\theta_{\text{min}}\tan\theta_{\text{min}} - \left(300\text{ ft}\right)\sec^2\theta_{\text{min}}$$

$$0 = \left(720\text{ ft}\right)\frac{\sin\theta_{\text{min}}}{\cos^2\theta_{\text{min}}} - \left(300\text{ ft}\right)\frac{1}{\cos^2\theta_{\text{min}}}$$

$$\left(300\text{ ft}\right)\frac{1}{\cancel{\cos^2\theta_{\text{min}}}} = \left(720\text{ ft}\right)\frac{\sin\theta_{\text{min}}}{\cancel{\cos^2\theta_{\text{min}}}}$$

$$\sin\theta_{\text{min}} = \frac{300\text{ ft}}{720\text{ ft}} = 0.4167$$

$$\theta_{\text{min}} = 24.62°$$

Now we can insert this minimizing angle into our distance function to find where our friends should meet us:

$$D(\theta_{\text{min}}) = \left(720\text{ ft}\right)\sec\left(24.62°\right) - \left(300\text{ ft}\right)\tan\left(24.62°\right)$$

$$= 792\text{ ft} - 137.48\text{ ft}$$

$$= 654.5\text{ ft}$$

So our friends should meet us 654.5 feet downstream from our starting point.