Work and Kinetic Energy: Block on Incline (Solutions)

A block of mass 3 kg is moved up an incline that makes an angle of 37° with the horizontal under the action of a constant horizontal force of 40 N. The coefficient of kinetic friction between the block and the incline is 0.1. The block is initially at rest. What is the kinetic energy of the block after it has been displaced 2 m along the incline?

Conceptual Analysis:
- The block begins with zero kinetic energy; whatever energy it obtains must be equal to the work done on the block.
- Three forces do work on the block: horizontal force, gravitational force, and frictional force.
- Work is the net force on the block times the distance it is displaced.

Strategic Analysis:
- Find the net force on the block.
- Multiply the net force by the distance to obtain the net work done on the block, and thereby obtain the change in kinetic energy.

Quantitative Analysis:
- Begin by labeling the given quantities
    - θ angle of incline
    - m mass of block
    - d displacement of block
    - μ coefficient of kinetic friction
    - F_H horizontal force
  We'll also use
    - F_f frictional force
    - F_g gravitational force
    - F_N normal force
    - F_{net} net force on the block
  We are looking for
    - KE kinetic energy of the block after it has been displaced.
- Since the block remains on the incline, we know that the net force in the direction perpendicular to the incline must be zero. The forces perpendicular to the incline are F_N and a component of both F_g and F_H acting perpendicular to the plane. We can sum the forces to zero to obtain an expression for the normal force.
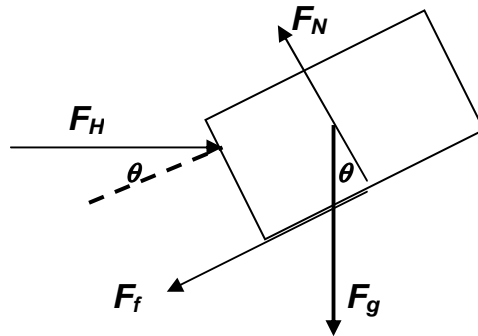$$F_N = F_g \cos\theta + F_H \sin\theta$$
- Next, we look at the net force parallel to the plane. There are three forces that act parallel to the plane. Begin by determining which component of each force acts parallel to the plane.
$$F_H : F_H \cos\theta$$
$$F_g : F_g \sin\theta$$

$F_f : F_f$

- The force of friction is just $F_f = \mu F_N = \mu(F_g \cos\theta + F_H \sin\theta)$
- Combining the three forces, we obtain a net force equation:
  $$F_{net} = [F_H \cos\theta - F_g \sin\theta - \mu(F_g \cos\theta + F_H \sin\theta)]$$
- We can multiply the net force equation by the net distance moved to find the net work.
  $$W_{net} = F_{net} d = [F_H \cos\theta - mg \sin\theta - \mu(mg \cos\theta + F_H \sin\theta)] d$$
- We also know that $\Delta KE = W_{net} = F_{net} d$
- So, inserting our values:
  $$\Delta KE = [40 \ N \cos 37^\circ - 3kg * 9.81m/s^2 * \sin 37^\circ$$
  $$- 0.1 (3kg * 9.81m/s^2 * \cos 37^\circ + 40 \ N \sin 37^\circ)] \ 2 \ m$$
 $$\Delta KE = 18.95 \ J$$

Since the block began with zero *KE*, it now must have **18.95 J.**

(This problem can also be solved by finding the work done individually by each of the three parallel force components and then combining them to obtain the result of 18.95 J.

Physics 211 **Week 4**

Work and Kinetic Energy: Loop (Solutions)

You are designing a new roller-coaster. The main feature of this particular design is to be a vertical circular loop-the-loop where riders will feel like they are being squished into their seats even when they are in fact upside-down (at the top of the loop).

The coaster start at rest a height of 80m above the ground, speeds up as it descends to ground level, and then enters the loop which has a radius of 20m.   Suppose a rider is sitting on a bathroom scale that initially reads W (when the coaster is horizontal and at rest). What will the scale read when the coaster is moving past the top of the loop?
(You can assume that the coaster rolls on the track without friction).

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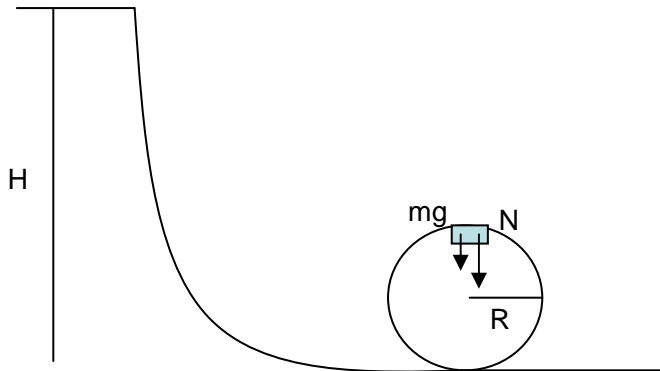
You can use conservation of energy to find the velocity of the coaster at the top of the loop. The coaster will begin with gravitational potential energy, which will change to kinetic energy as the coaster begins moving. You can use the value of the kinetic energy to determine an expression for the velocity in terms of gravity, the initial height, and the radius of the loop. With the values given in the problem, you know that the initial height is 4 times the radius of the loop. You can use this relationship to simplify the velocity expression to be only in terms of the acceleration due to gravity and the radius. Next, you can use Newton's second law to determine the normal force on the coaster at the top of the loop. This can be done by combining the normal force and force of gravity to obtain an expression for the mass times the centripetal acceleration. You then use your velocity from the conservation of energy and insert it into the centripetal acceleration equation. If you solve the expression for the normal force, you should obtain the value N = 3mg. The normal force is the value that the scale will read. A scale would usually read mg, so the rider experiences what we would call 3g's.

Work and Kinetic Energy: Colliding Binary (Solutions)

Two identical stars, each having a mass and radius equal to that of our sun (M $= 2 \times 10^{29}$ kg and R $= 7 \times 10^{8}$ m), are initially at rest in outer space. Their initial separation is the same as the distance between our sun and the earth, D $= 1.5 \times 10^{11}$ m. Their gravitational interaction causes the stars to be pulled toward one another. Find the speed of the stars just before they collide.
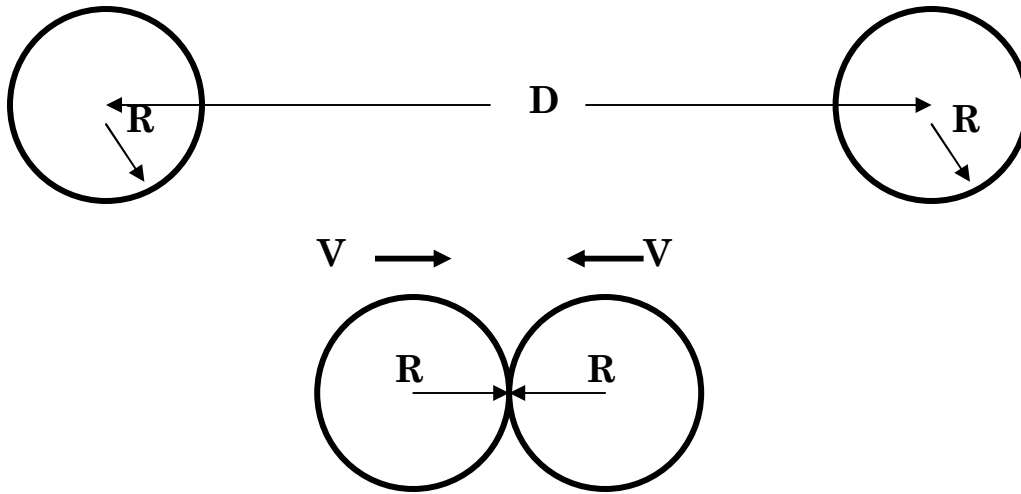

The pictures show the initial and final positions of the stars. You can approach this problem by noting that energy is conserved from initial to final positions. Initially, the stars have only gravitational potential energy. After they have been brought together, they have both gravitational potential energy (their centers are still separated by 2R) and kinetic energy. Remember that each star will have a kinetic energy that contributes to the final energy of the system. Also remember that gravitational potential energy has a negative value. Using the given values from the problem you should obtain a velocity of 97 *km/s*.

Newton's Laws: Noise-Maker (Solution)

After watching the movie "Crocodile Dundee", you and some friends decide to make a communications device invented by the Australian Aborigines. It consists of a noise-maker swung in a vertical circle on the end of a string. You are worried about whether the string you have will be strong enough, so you decide to calculate the tension in the string when the device is swung with constant speed. You and your friends can't agree whether the maximum tension will occur when the noise-maker is at the highest point in the circle, at the lowest point in the circle, or will always be the same. To settle the argument, you decide to calculate the tension at the highest point and at the lowest point and then compare them.

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The problem states that the device is swung with constant speed in a circle, which means it is swung in uniform circular motion. The centripetal acceleration needed to keep the device in a circle is provided by gravity and tension. The magnitude of the acceleration depends on the speed and radius, which both remain constant throughout the motion; therefore, the magnitude of the acceleration is constant. By drawing free body diagrams for the noise-maker in both the top and bottom positions, you can identify the direction of the tension, gravity, and resultant centripetal acceleration for each position. You will find that the gravity and tension combine at the highest point to both contribute positively to the centripetal acceleration. At the lowest point the gravity acts away from the center of the circle and so adds negatively to the tension in obtaining the resultant centripetal acceleration. For this reason, the tension must be greater at the bottom so that the centripetal acceleration is as great at the bottom where the gravity has a negative contribution as it is at the top where the gravity adds positively to the centripetal acceleration.
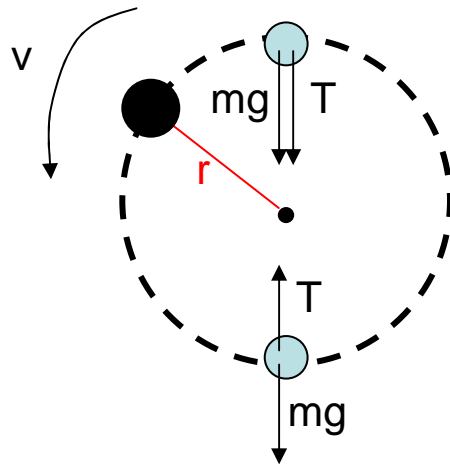
Force and Simple Dynamics: Carnival Ride **(Solutions)**

A neighbor's child wants to go to a carnival to experience the wild rides. The neighbor is worried about safety because one of the rides looks particularly dangerous. She knows that you have taken physics and so asks you for advice. The ride in question has a 4kg chair which hangs freely from a 10m long chain attached to a pivot on the top of a tall tower. When the child enters the ride, the chain is hanging straight down. The child is then attached to the chair with a seat belt and shoulder harness. When the ride starts up, the chain rotates about the tower. Soon the chain reaches its maximum speed and remains rotating at that speed, which corresponds to one rotation about the tower every 3 seconds. When you ask the operator, he says the ride is perfectly safe. He demonstrates this by sitting in the stationary chair. The chain creaks but holds, and he weighs 90kg. Has the operator shown that the ride is safe for a 25kg child?

Conceptual Analysis:
- The operator has shown that the ride is safe for the child if the demonstration tension is greater than the tension in the chain while the ride is operating with the child in the seat.
- The centripetal acceleration required to keep the child and swing rotating comes from the horizontal component of the tension in the chain.

Strategic Analysis:
- Find the demonstration tension in the chain by finding the weight of the operator and seat together.
- Find the tension in the chain that is present when the ride is operating with the girl in the seat.
- Compare the two tensions to see if the operator has shown the ride to be safe for the child.

Quantitative Analysis:
- Begin by labeling the given quantities:
  - m_s mass of the seat
  - m_c mass of the child
  - m_o mass of the operator
  - T period of the ride
  - L length of the chain
  - We are looking for a comparison between
  - F_T the tension in the chain when the girl in on the operating ride, and
  - F_T' the demonstration tension
- Find the demonstration tension by finding the weight of the operator and the seat.
  - Call his force of tension F_T'.
  $$F_T' = (m_s + m_o)*g$$
- Since we know the centripetal acceleration needed to keep the swing in motion is provided by the horizontal component of tension, F_{Tx}, we should find this component. We should begin by drawing a diagram of what the ride will look like while in

operation. We can also draw a free body diagram for the child and swing showing the forces of tension and gravity.

To break the tension into components, it is helpful to assign an angle, θ. According to our diagram, the component of the tension that is perpendicular to the tower is

$$F_{Tx} = F_T * \sin(\theta)$$

- Next we should find the centripetal acceleration, a_c, of the swinging child.
  We know that $a_c = \omega^2 * r$, where ω is the angular velocity and r is the radius from the swing to the tower.
  We can find ω by the relationship
  $$\omega = 2\pi/T$$
  r can be found by looking at our diagram. The radius will be the horizontal component of the length, L.
  $$r = L * \sin(\theta)$$
  Putting these two together, we find the centripetal acceleration:
  $$a_c = \omega^2 * r = L * \sin(\theta) * (2\pi/T)^2$$
- Next, we can multiply the centripetal acceleration by the mass of the child and swing and set it equal to the x-component of the tension in the chain.

$$(m_s + m_c) * a_c = F_{Tx}$$
$$(m_s + m_c) * L * \sin(\theta) * (2\pi/T)^2 = F_T * \sin(\theta)$$

  You can see that the quantity $\sin(\theta)$ cancels out of each side. This is good because we do not actually have a given value for the angle the chain makes with the rod during rotation.
- We now have enough information to solve for F_T, the value of the tension in the chain when the girl is riding.
$$F_T = (m_s + m_c) * L * (2\pi/T)^2$$
- Next we should compare F_T and F_T' (the demonstration tension) to see if the operator has proven the ride safe.
$$F_T < F_T' \ ?$$
$$(m_s + m_c) * L * (2\pi/T)^2 < (m_s + m_o) * g \ ?$$
$$(4 + 25) * 10 * (2\pi/3)^2 < (4 + 90) * (9.81) \ ?$$
$$1272 \ N < 922 \ N \ ??$$
$$NO!$$

- The demonstration tension is only 922 N; however, when the girl is on the ride, the tension in the chain will be 1272 N. Therefore, the operator has NOT demonstrated the ride to be safe.

- As another point of interest, let's say the tension in the cable *does* hold the child. What will the ride look like?
- Recall the angle between the chain and tower that we assigned for reference. Let's determine the angle for the given weight of the child.
- The horizontal component of the tension was related to the centripetal acceleration. We can use the vertical component of tension to find the angle by noting that the vertical component of tension must equal the force of gravity.

$$(m_s + m_c)*g = F_T*\cos(\theta)$$
$$\theta = \cos^{-1}[(m_s + m_c)*g/F_T]$$
$$\theta = \cos^{-1}[(4\ kg + 25\ kg)*9.81\ m/s^2 / 1272\ N]$$
$$\theta = 77.1^{\circ}$$

This angle indicates that the swing will be close to the horizontal position (90 degrees). Perhaps the neighbor should be more worried about her child falling out of the swing!