Physics 211 Week 6 Momentum: Along Came a Spider

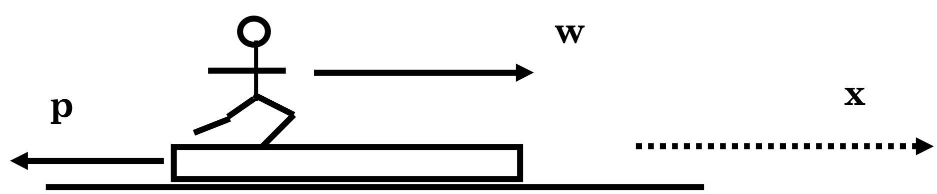
A man and a woman are sitting in a sleigh that is at rest on frictionless ice. The mass of the man is 80 kg, that of the woman is 60 kg, and that of the sleigh is 120 kg. The people suddenly see a poisonous spider on the floor of the sleigh and jump out. The man jumps to the left with a velocity of 5.0 m/s at 30^{o} above the horizontal. The woman jumps to the right at 9.0 m/s at 37^{o} above the horizontal. What is the velocity (magnitude and direction) of the sleigh after they have both jumped out of it?

The system of the sleigh, man, and woman initially has zero momentum. Because there are no external forces exerted on the system, the final momentum will also be zero. We assume that the sleigh does not move in the vertical direction, and we focus on horizontal motion. The man and woman's jumps each have an assigned velocity and direction. You can break the jump into components to analyze the horizontal motion. Knowing that the final momentum must be zero, you can add the known components of momentum of the man and woman to the momentum of the sleigh. Solving for the sleighs momentum, you can then use the given mass of the sleigh to find the sleigh's velocity. The sleigh will move 0.707 *m/s* to the left.

Physics 211 Week 6

Momentum: Run the Plank (solutions)

In frozen Minnesota the Winter Sports Carnival includes some unusual events. Since it is dangerous to run on ice, each runner runs on a heavy (240 kg) and long (40 m) wooden plank, which itself rests on the smooth and horizontal ice. One of the competitors is a 60 kg woman who runs the length of the plank in 4.4 seconds, quite an impressive time. Her performance is viewed by a crowd huddled on the ice. The performance that they see is less impressive. With what speed do they see her moving?



The center of mass will remain stationary throughout the motion of the woman; therefore, as she runs to the end of the plank, the plank will move under her in the opposite direction. To the audience, it will appear as though she has not traveled as far in the same time, so they will see her as moving slower than her actual speed. The speed they will observe is the distance they see her move in their frame divided by the time it takes her to run to the end. The distance they see her move is the length of the plank minus the displacement of the plank. To find the amount the plank is displaced, you can create equations for the center of mass of the plank both before and after she runs. Solve for the shift, subtract it from the total length of the plank, and divide by time. You should obtain 7.27 *m/s*, which is less than her actual speed of 9.09 *m/s*.

Physics 211 Week 6

Momentum: Space Shuttle Emergency (Solutions)

You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the space shuttle. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space shuttle. She checks and finds that her thruster pack has also been damaged and no longer works. She is 200 m from the shuttle and moving with it (i.e., she is not moving with respect to the shuttle). She is drifting in space with only 4 minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10-kg tool kit and throw it away with all her strength, so that it has a speed of 8 m/s relative to her. In the script, she survives, but is this correct? Her mass, including her space suit, is 80 kg.

Conceptual Analysis:

- Pushing the pack away from her body results in the astronaut recoiling towards the space shuttle.
- In the frame of the shuttle, momentum is conserved.
- The relative speed of the astronaut to the pack is not the same speed that the astronaut will move with towards the shuttle.
- The astronaut survives if her time to reach the shuttle is less than the time of air she has left.

Strategic Analysis:

- Set up a conservation of momentum equation.
- Make substitutions with the given relative speed to eliminate an unknown velocity in your momentum equation.
- Use the speed of the astronaut and her distance from the shuttle to find the time it takes her to reach the shuttle.
- Compare the time it takes the astronaut to reach the shuttle to the time of air she has remaining.

Quantitative Analysis:

- Begin by labeling the given quantities
  m_a mass of the astronaut
  m_p mass of the pack
  v_{pa} relative speed of the pack and astronaut
  d distance to the shuttle
  t_a time of air remaining
  and also the two values we'll use intermediately
  v_{ps} speed of the pack relative to the shuttle
  v_{as} speed of the astronaut relative to the shuttle

  We are looking for:
  t the time it takes the astronaut to reach the shuttle

- In the frame of the shuttle, the conservation of momentum looks like:

$m_a v_{as} = m_p v_{ps}$

However, we do not have the speed of the pack relative to the shuttle, so we will have to substitute this value with values we do know.

- The velocity of the pack relative to the shuttle is equal to the sum of the velocities of the pack relative to the astronaut and the astronaut relative to the shuttle.

  $\mathbf{v}_{ps} = \mathbf{v}_{pa} + \mathbf{v}_{as}$

  Since the velocity of the astronaut is in the opposite direction of the pack, we can write the relationship among the speeds as

  $v_{ps} = v_{pa} - v_{as}$

- Substituting:

  $m_a v_{as} = m_p v_{ps}$

  $m_a v_{as} = m_p(v_{pa} - v_{as}) = m_p v_{pa} - m_p v_{as}$

  and solving for the velocity of the astronaut relative to the shuttle:

  $v_{as} = m_p v_{pa} / (m_a + m_p)$

- Use the velocity and distance to find the time.

  $t = d / v_{as}$

  $t = d / [m_p v_{pa} / (m_a + m_p)]$

  $t = 200m / [\ 8m/s * 10kg / (80kg + 10kg)$

  $t = 225\ s$

- It will take her 225 seconds to reach the shuttle. How much air does she have remaining?

  $t_a = 4\ min = 4*60s = 240s$

- She has 240 seconds to reach the shuttle but only needs 225 seconds. The time it takes her to reach the shuttle is less than the amount of air she has remaining, so she will survive; the script is correct.

Physics 211 Week 6

Work and Kinetic Energy: Bungee Jumpin'

In order to raise money for a scholarship fund, you convince the Physics 211 lecturer to bungee jump from a crane. To add some interest, the jump will be made from 44 m above a 2.5m deep pool of Jello. A 30m long bungee cord would be attached to his ankle. First you must convince him that your plan is safe for a person of his mass, 70kg. He knows that as the bungee cord begins to stretch, it will exert a force that has the same properties as the force exerted by a spring. Your plan has him stepping off a platform and being in free fall for the 30 m before the cord begins to stretch. You must determine the elastic constant of the bungee cord so that it stretches exactly 12 m, which will just keep his head out of the Jello.

One strategy to solve this problem is to use the work energy theorem. Your lecturer begins and ends with zero kinetic energy, but both gravity and the bungee chord do work during the jump. Since there is no change in kinetic energy, the sum of the two works must be zero. You can solve for the elastic constant by setting the work done by the chord equal to the work done by gravity. Remember to carefully determine the total height the jumper falls, the distance over which the force of gravity acts. The total fall includes both the free fall part and the stretch since there is a change in potential energy over that entire distance. Solving for the spring constant, you should obtain the value 400.575 N/m^2.