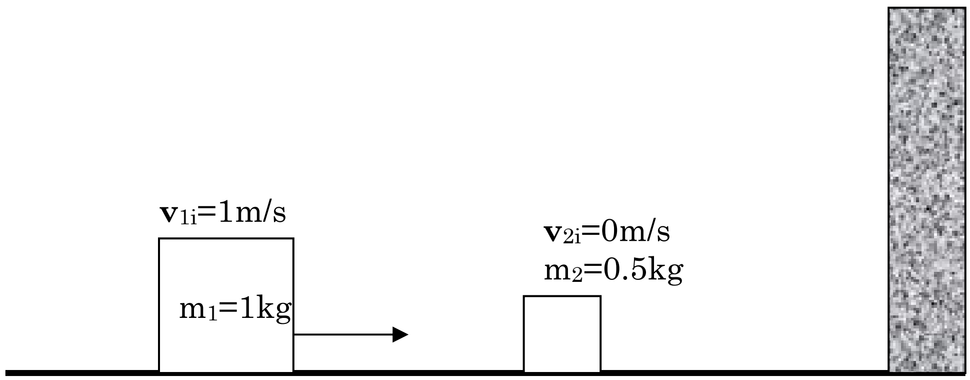
Physics 211 **Week 7**

Momentum: Elastic Collisions

Initially, Block 1 with mass m_1=1kg is moving on a frictionless table with velocity v_1=1m/s and block 2 with mass m_2=0.5kg is at rest. Block 1 collides **elastically** with block 2.



Find the velocities of block 1 and block 2 after the collision. Afterwards, block 2 collides elastically with a wall. What impulse does the wall give to block 2?

To find the final velocities, we can follow the four-step program for elastic collisions:

1. Find the velocity of the center of mass of the system of blocks.
2. Find initial velocity of each block in the center of mass frame.
3. Find the final velocity of each block in the center of mass frame remembering that in an elastic collision, in the center of mass frame the velocity before the collision is equal to the velocity of the object after the collision.
4. Find the final velocity of each block in the lab frame.

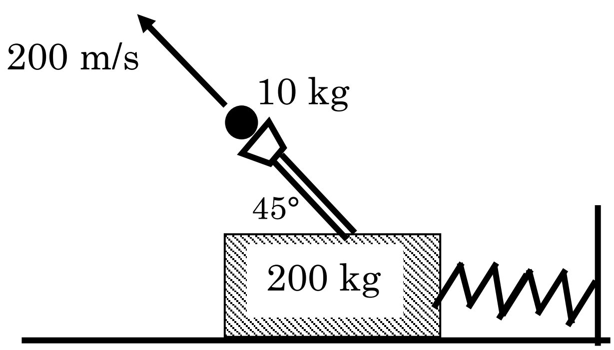
Elastic collisions conserve both kinetic energy and momentum, so you can check the initial and final velocities and masses in the lab frame to see if energy and momentum are conserved before and after the collision. You will find both conservation equations hold true. The velocities that you should obtain are 1/3 *m/s* for the first block and 4/3 *m/s* for the second block, both moving towards the wall.

For the next part, we know that the impulse is equal to the change in momentum. We can assume the wall has infinite mass, which means that the lab frame and center of mass frame are identical in this case. Remembering that the final velocity is simply the opposite sign of the initial velocity (in the center of mass frame of an elastic collision), you know that block 2 will rebound off the wall with the same velocity that it hits the wall. Therefore, the final momentum will have the same magnitude as, but be in the opposite direction to, the initial momentum. The impulse, or change in momentum, when block two collides with the wall is 4/3 *N-s.* Note that block 2 carries the same magnitude momentum before and after its collision with the wall. The wall delivers a nonzero impulse to block 2, which appears as a change in direction of the momentum vector.

Physics 211 **Week 7**

Momentum: Gun Cushion **(solutions)**

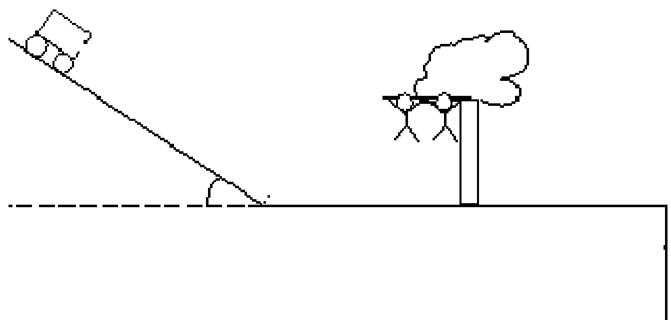
A cannon fires a 10 kg projectile with a velocity of 200 m/s at an angle of 45° with respect to horizontal. The cannon, which has a mass of 200kg, is cushioned by a relaxed, horizontal spring that is designed to absorb the recoil. Assume that the gun stands on a frictionless surface. Compute the minimum spring constant required so that the spring compresses by less than 20 centimeters.



The cannon and projectile can be treated as an explosion where momentum is conserved. This means that you can use the momentum in the horizontal direction of the projectile to find the recoiling momentum of the cannon. Since you know the mass of the cannon, you can find the recoiling speed and then the kinetic energy of the cannon as it recoils. The kinetic energy of the cannon becomes spring potential energy as the spring is compressed and the motion of the cannon is stopped. Setting up an equation for the conservation of energy, you can solve for the spring constant using the given mass and maximum compression as well as the velocity you found from the conservation of momentum. You should obtain a spring constant of 250000 *N/m*.

Physics 211 Week 7 Momentum: Lone Ranger

A wagon with two boxes of gold and having a total mass of 300 kg is cut loose from the horses by an outlaw when the wagon is at rest 50m up a 6^o slope. The outlaw plans to have the wagon roll down the slope and then across 50m of level ground before finally falling over a cliff into a canyon where his confederate waits. But, unknown to the outlaws, the Lone Ranger (mass of 80 kg) and Tonto (mass of 60kg) are waiting in a tree 40m from the cliff. They time their fall so that they drop vertically into the wagon just as the wagon passes beneath them. They require 5.0s to grab the gold and jump out of the wagon. Will they make it before the wagon goes over the cliff?



To know whether Tonto and the Lone Ranger make it out of the wagon before it goes over the cliff, we have to know the wagon's velocity. The velocity of the wagon changes after they fall into it because it is an inelastic collision and momentum is conserved. The velocity that the wagon has before the collision is a result of the conservation of energy from the top of the slope to the bottom. Beginning from the top of the slope, the wagon at rest has gravitational potential energy. All of the gravitational potential energy becomes kinetic energy by the time the wagon reaches the bottom of the slope. You can find the velocity of the wagon, then, using its mass, find its momentum. Use conservation of momentum to find the velocity of the wagon with Tonto and the Lone Ranger in it after the collision. The new velocity can be used with the known distance to the edge of the cliff to find the time available to Tonto and the Lone Ranger to grab the gold and jump out of the wagon. The wagon will go over the cliff 5.8 seconds after they jump in it. They only need 5 seconds, so they will have a little less than one second to spare!

Physics 211 Week 7 Momentum: Gone Fission

When a Uranium-235 nucleus decays, two lighter nuclei are ejected with high kinetic energy. It is this energy that is harnessed in reactors (and bombs). A pair of neutrons is typically also ejected, each of which can cause another ^{235}U nucleus to fission, leading to a chain reaction.

A fun way to model a ^{235}U nucleus is to say that it is really composed of two lighter nuclei, call them A and B. Say nucleus A has a mass of 90 amu (atomic mass units), and nucleus B has a mass of 145 amu, and that initially A and B are at rest and have a massless spring squeezed between them. We will also need to pretend that there is something holding the pair together, preventing them from being blown apart by the spring (a little latch, say). When a neutron happens to bump into this setup it releases the latch, allowing the spring to expand, sending A and B off in opposite directions at high speed. The total kinetic energy of A and B (the uranium fission products) after this happens is known to be about 200 MeV.

Use this simple model to figure out the following:

1. What are the speeds of both A and B after the ^{235}U nucleus fissions?
2. Assuming the spring was initially compressed 10 femtometers (the size of a nucleus), what is the spring constant?

Useful conversions: 1 amu = $1.7\text{x}10^{-27}$ kg
1 MeV = $1.6\text{x}10^{-13}$ J
1 femtometer = 10^{-15} m

Part 1 uses conservation of momentum. When A and B are released in opposite directions, the initial zero momentum of the system is conserved. The sum of the kinetic energies is also given. You can create a system of equations using the energy equation and momentum equation. Solve the system to obtain the speeds of A and B. A moves at $1.61\text{x}10^{7}$ *m/s* and B moves at $9.97\text{x}10^{6}$ *m/s*.

Part 2 uses conservation of energy. The total kinetic energy of A and B must have come from the energy stored in the spring before it was released. Set the spring potential energy equal to the given value for the total kinetic energy, solve for the spring constant, and you should obtain a spring constant of $6.4\text{x}10^{17}$ *N/m*.

Physics 211 Week 7 Momentum: Automobile Collision

Your friend has just been in a traffic accident and is trying to negotiate with the insurance company of the other driver to pay for fixing her car. She believes that the other car was speeding and therefore that the accident was the other driver's fault. She knows that you are taking Physics 111, so she asks you for help in proving her conjecture. She takes you out to the scene and describes what happened.

She was traveling north when she entered the fateful intersection. There was no stop sign, so she looked in both directions and did not see another car approaching. It was a bright, sunny, clear day. When she reached the center of the intersection, her car was struck by the other car which was traveling east. The two cars remained joined together after the collision and skidded to a stop. The speed limit on both roads entering the intersection was 80 km/hr. From the skid marks still visible on the street, you determine that after the collision the cars skidded 17m at an angle of 30° north of east before stopping. She has a copy of the police report which gives the year and make of each car. At the library you determine that the weight of her car was 1200kg and that of the other car was 1000kg, including the weight of the driver in each case. The coefficient of kinetic friction for a rubber tire skidding on dry pavement is 0.80. It is not enough to prove that the other driver was speeding to convince the insurance company. She must also show that she was under the speed limit. How do you advise her?

The first part of the problem is an inelastic collision: momentum is conserved. The initial momentum in each direction can be used to find the momentum components for the combined mass of the crashed cars. After the cars collide the force of friction slows and stops their motion. Here, the work energy theorem applies. Friction is the only force that does work on the system, so the kinetic energy must all be dissipated in the work done by friction. By setting the kinetic energy equal to the work done by friction and using the given distance and coefficient of friction, you should be able to find the velocity of the crashed cars immediately after the collision. Using this velocity, the given masses, and the components of the final momentum that you found, you can find the initial speed of each of the cars in the collision. The speeds you find will be in *m/s* while the speed limit is given in *km/hr*. Convert the units and you will find that the speed limit is 22.2 *m/s*, the other car was going 31.12 *m/s*, and your friend was going 14.97 *m/s*. In *km/hr*, your friend was going 53.89 *km/hr* and the other driver's speed was 112.03 *km/hr*. Whichever units you use, you find that the other driver was speeding and that your friend was not.