Course Overview

The framework we will adopt in this course will be that introduced by Isaac Newton in the 17th century. This framework remained the standard in science until the 20th century when fundamental changes were needed to describe the complete nature of space, time, and matter. In particular, the theory of *special relativity* proposed a constant speed of light that led to a reformulation of the nature of space and time. The theory of *quantum mechanics* was created to describe the interactions of elementary particles, such as electrons and photons, leading to a description of matter that included both particle and wavelike aspects. In this course, we will restrict ourselves to describing macroscopic objects moving at relative velocities that are small with respect to the speed of light, so that Newtonian mechanics is *all* we need to accurately describe the physics.

In particular, we will present Newton's laws which introduce the new concepts of force and mass that are needed to describe the actions of macroscopic objects. Indeed, Newton's laws establish the mechanical world view that forms the basis for the scientific revolution of the 17th century. In particular, he introduced a universal force of gravitation that he claimed applied to all objects having mass. He then demonstrated that he could relate the motion of the Moon in its orbit about the Earth to the falling of an apple to the ground here on Earth. The deep significance of this demonstration was that, for the first time, a connection was made between the motions of ordinary things on the Earth and the motions of heavenly bodies. Prior to Newton, the Heavens and the Earth were treated entirely separately and differently. Newton showed that the laws that apply here on Earth extend to the Heavens.

We will then introduce other important quantities such as energy, momentum and angular momentum that are commonly conserved in a variety of situations. We'll close with a brief study of oscillatory motions, wave motions, and fluids.

1. One Dimensional Kinematics

A) Overview

This course is concerned with classical mechanics, the study of the forces and motions of macroscopic objects. We will begin with a study of *kinematics*, the description of motion, without regard to its cause. In particular, we will define the concepts of *displacement*, *velocity* and *acceleration* that are needed to describe motion. We will initially restrict ourselves to motions in one dimension. We will use these definitions to demonstrate how to obtain the *change in position* from the velocity and the *change in velocity* from the acceleration. We will close this unit with a discussion of an example of a particular motion, that of constant acceleration.

B) Displacement and Average Velocity

To discuss motion in classical physics, we begin with two quantities, *displacement* and *velocity*. These quantities are not unfamiliar to you; I'm sure you already have a working knowledge of the relationship between displacement and velocity. If it takes you three hours to walk six miles you can figure out that your average velocity during that walk was 2 miles/hr by simply dividing the distance you walked by the time it took.

In this course, you will find that many of the words that represent the quantities of physics will be very familiar words. It is important to note, however, that these words all have very precise meanings in physics, whereas in everyday language, these words are often used to mean many related, but different, things. Therefore, it is important that we start right away with careful definitions of our terms. Most often, these definitions will obtain their precision through their expression in terms of mathematics.

To illustrate this point, we introduce an arguments made by the Greek philosopher Zeno to prove that it is impossible to move from some point *A* to another point *B*. His argument goes as follows: clearly before we can move to point *B*, we need first to move to point *C* which is halfway between points *A* and *B*. Sounds true enough, however, this argument can be repeated *ad infinitum*. *i.e.*, once at *C*, we would need to move first to point *D* which is halfway between points *C* and *B*. You get the drift, I'm sure. We will need to make an infinite number of moves to get to point *B*.

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What is Zeno's point? It certainly is *not* to prove that motion is impossible; we all know that is not true. In fact, the reason that these arguments are called "paradoxes" is that what seems to be a reasonable argument leads to a conclusion that we know is false. Zeno initiated these arguments as ways to investigate the nature of space and time.

How do we resolve these paradoxes? Clearly the problem lies with the notion of infinity. Mathematics can help us. We know, for example, that an infinite series can have a well-defined sum. This sum is defined in terms of a limit which is the key concept of calculus. Indeed, we will soon find that the use of calculus will be central to the definition of velocity. For now, though, we'll begin by defining the *average velocity* of an object within some time interval Δt to be equal to Δx, the distance it has travelled during that period divided by the time it takes.

$$v \equiv \frac{\Delta x}{\Delta t}$$

We can represent this definition graphically as shown in Figure 1.1. On the vertical axis we plot the displacement x, which is defined to be the distance travelled from some fixed origin, while on the horizontal axis, we plot the time t, from some fixed time defined to be $t = 0$. If we choose some time interval defined by the times t_i and t_f, we see the corresponding displacements x_i and x_f and that the average velocity is just the slope of the line connecting the initial and final points on the graph.

To illustrate the use of the *average velocity*, suppose someone calls your cell phone while you are driving and asks "what's up?". You might say "I left town half an hour ago and I'm heading east on the interstate". If you wanted to be more specific you might say "I am 35 miles east of town and I'm driving east at an average speed of 65 miles per hour." From this information your friend could do a simple mental calculation to predict that one hour from now you will be another 65 miles farther east, which would be about a hundred miles east of town.

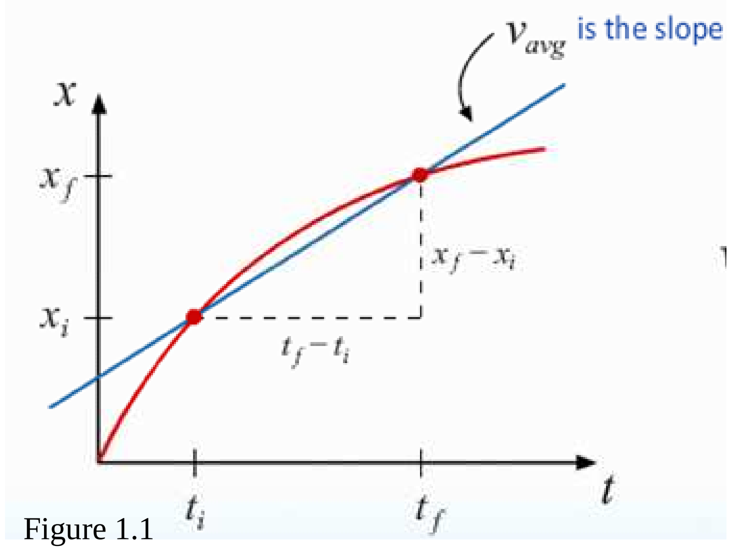


Figure 1.1
A plot of the displacement x as a function of time t. The ***average velocity*** for a time interval(t_i, t_f) is illustrated as the slope of the line connecting the points on the curve at those two times.

What your friend really did to make this estimate was to solve the following kinematic equation in her head.

$$x(\Delta t) = x_o + (v_{avg} \cdot \Delta t)$$

The translation of this equation into English is that your position at a time Δt after you start is just equal to your starting position, call it x_o, plus the additional distance you went during time Δt. This last piece is just your average velocity multiplied by the elapsed time. This last calculation assumes that your average velocity does not change in the next hour. We will discuss how to describe motion in which your velocity *does change* in the next section

Before we proceed any further, a remark about units is in order. Although the velocity used in this example was given in miles per hour, we will adopt for the most part in this course, the SI system of units in which velocity is measured in meters/second. Converting between these units is easy; we simply have to multiply by "one" until the units are right. For example in this case, we can multiply 65 miles/hour by 1609 meters/mile by (1/3600) hours/second to obtain the result that 65 mph is equivalent to 29 m/s.

C) Instantaneous Velocity

In the last example, we calculated the predicted distance the car would go during a specified period of time, assuming that the average velocity during that time did not change. You know this assumption is not always true; sometimes you may speed up to pass a car, resulting in an increased average velocity or you may have to slow down due to traffic, resulting in a decreased average velocity.

Therefore, to discuss all kinds of motion, we will need the ability to figure out both the displacement and the velocity for any instant in time, not just the average over some time interval.

We can visualize the procedure for finding the instantaneous velocity by starting with the displacement *vs* time plot shown in Figure 1.1 and then bringing the final and initial times closer and closer together until they are infinitesimally close together. As we do this the line connecting the points becomes the tangent to the curve as shown in Figure 1.2! In other words, the *instantaneous velocity* at some time t is just the slope of the tangent to the x *vs* t curve at that point. The slope of this tangent line is exactly equal to the derivative dx/dt at that time!

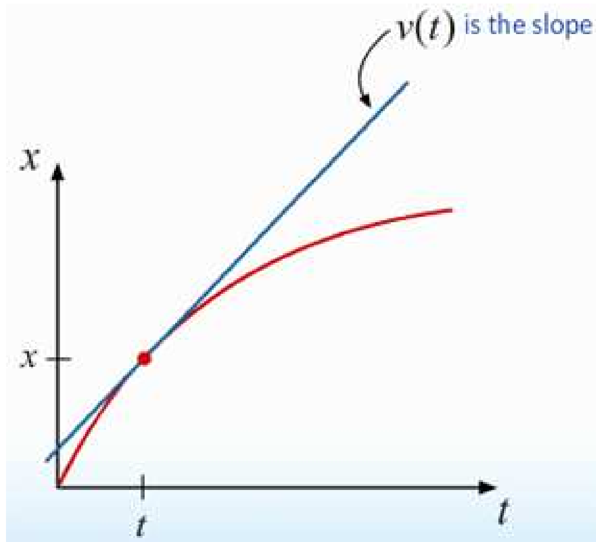


Figure 1.2
A plot of the displacement x as a function of time t. The ***instantaneous velocity*** at time t is illustrated as the slope of the tangent to the curve at time t.

We can now see the simple relationship between *displacement* and *velocity*: The instantaneous velocity at a particular time t is defined to be the time derivative of the displacement at that time.

$$v \equiv \frac{dx}{dt}$$

We can construct a graph of the instantaneous velocity as a function of time by finding the slope of the corresponding x vs t graph at each time t as shown in Figure 1.3.

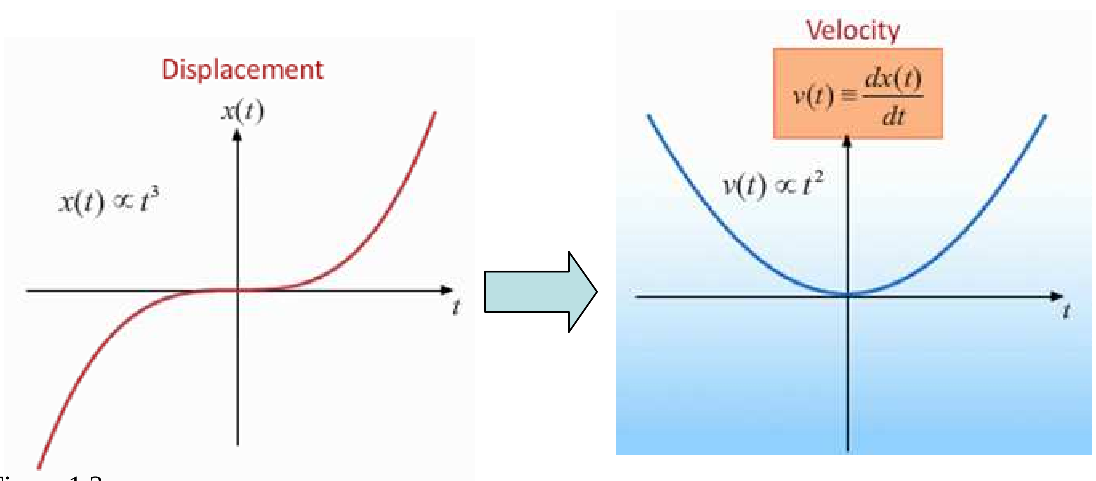


Figure 1.3
The instantaneous velocity at any time is obtained by differentiating the displacement at that time.

This relationship is *always* true, no-matter how strangely the displacement may be changing with time. This relationship is the definition of instantaneous velocity. We were led to this definition of the instantaneous velocity by a natural refinement of the concept of the average velocity and we have discovered, as predicted, that the calculus (in this case, the derivative) is needed to carefully define this kinematic quantity.

D) Position from Velocity

We've just defined the instantaneous velocity at time t as the time derivative of the displacement at time t. Therefore, if we know the displacement as a function of time for some object, we can calculate its velocity at any time by simply evaluating the derivative of the displacement function at that time.

Suppose, on the other hand, that we know the velocity as a function of time; what can we say then about the displacement at any time? It seems like we should be able to use the inverse operation to go the other way – that we should be able to evaluate the integral of the *v vs t* graph to find the displacement as a function of time.

We know the integral can be represented graphically as the area under the curve. Therefore, we expect the displacement to be related to the area under the *v vs t* graph. We can verify this expectation for the special case of motion with a constant velocity as shown in Figure 1.4. In this case the area under the curve from 0 to time t is simply equal to the magnitude of the velocity times the time t. Note that the integral needed to find the displacement at time t is the definite integral from t =0 to $t = t$.

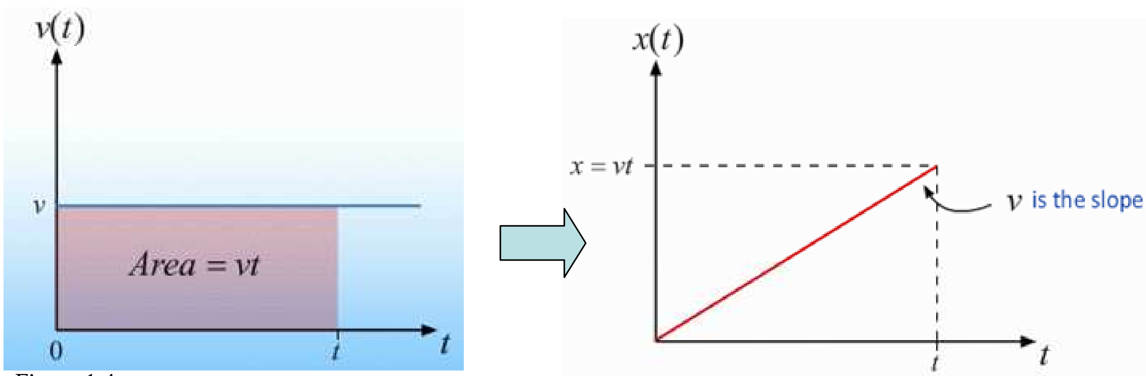


Figure 1.4
For motion at constant velocity, the displacement as a functin of time can be obtained by integrating the constant velocity over time to obtain a displacement that changes linearly with time..

To obtain the general equation that determines the displacement from the velocity, we actually need to be a little more careful. What we've really shown so far is that the *change* in displacement during a time interval (for example, t_i to t_f) is equal to the integral of the velocity between these initial and final times. We have no way of knowing where the particle was at any particular time, say at $t = 0$. The velocity tells us how the displacement changes; it can't tell us where it started from. That information must be given to us independently. Consequently, we write the general expression in terms of the definite integral of the velocity from t_i to t_f and the value of the velocity at t_i.

$$x(t_f) - x(t_i) = \int_{t_i}^{t_f} v(t)dt$$

This expression is completely general and will work for any velocity function

E) Acceleration

There is one more important kinematic quantity that we need to discuss. Namely, just as velocity tells us how fast the displacement is changing, *acceleration* tells us how fast the velocity is changing. In other words, acceleration is the time rate of change of velocity; acceleration is the measure of how many meters per second the velocity changes in a second. The units of acceleration are therefore meters per second per second.

$$a \equiv \frac{dv}{dt}$$

Figure 1.5 shows a plot of the velocity of an object as a function of time. The value of the acceleration at any given time is just equal to the slope of this curve at that time.

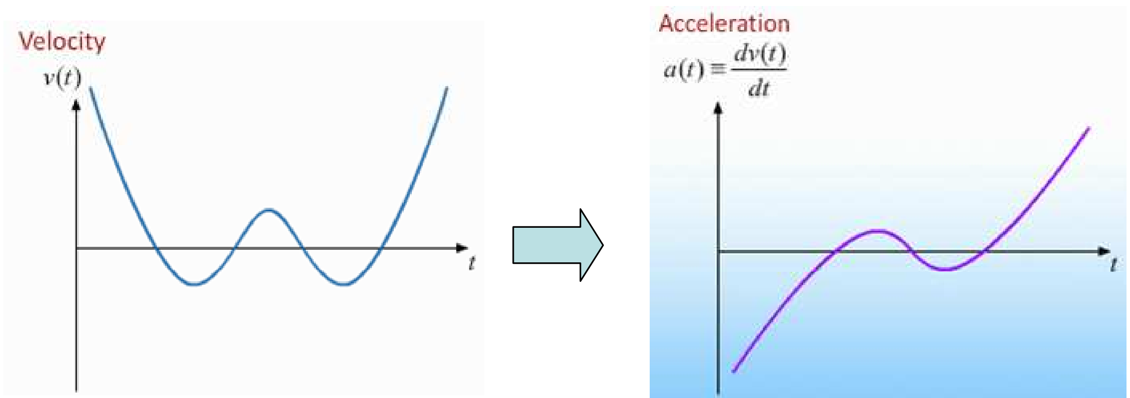


Figure 1.5
The acceleration of an object at any time is obtained by differentiating its velocity at that time.

In the same way that the change in displacement can be found by integrating the velocity, the change in velocity can be found by integrating the acceleration.

Before going any further, we must issue here a warning about a common confusion between everyday language and precise kinematic definitions. The evil word here is *deceleration*. I'm sure practically all of you associate decelerating with "slowing down". We have just defined acceleration as the time rate of change of velocity. The velocity is a *signed number* as is the acceleration. The sign of the velocity indicates its direction (either forwards (positive) or backwards (negative)). If a car is moving in the positive direction and slowing down, then its acceleration is negative. However, if the car is moving in the negative direction and slowing down, then its acceleration is *positive*! Therefore, the concept of "slowing down" is not the same as that of "negative acceleration". The safest way to proceed here is to just not use the word deceleration when dealing with kinematics problems. Accelerations are either positive or negative, depending on whether the velocity, a signed number, is increasing or decreasing.

F) Constant Acceleration

The equations we presented in the last section for the acceleration as the time rate of change of the velocity and the change in velocity as the integral of the acceleration are totally general; they are true always!

We'd like now to use these general equations to derive the specific equations that hold for a special, but important case, namely that of *motion at constant acceleration*. We start with the defining property, that the acceleration is a constant. We can integrate this constant acceleration to find the change in velocity. The result of this integration is that the velocity at any given time is simply equal to the initial velocity plus the acceleration multiplied by the elapsed time.

$$v(t_f) = v(t_i) + (a \cdot (t_f - t_i))$$

This equation is often written in a more compact form.

$$v = v_o + at$$

In writing this equation, we denote the velocity at the initial time by v_o, and the variable t really means the elapsed time $t_f - t_i$. We see that the velocity changes linearly with time, as it must since the acceleration is constant.

Now that we have the velocity as a function of time, we can integrate once again to find the displacement as a function of time.

$$x = x_o + \int_0^t (v_o + at)dt$$

In this case we see that the displacement changes quadratically with time.

$$x = x_o + v_o t + \frac{1}{2}at^2$$

We have now obtained expressions for the velocity and displacement as a function of time for the special case of motion at constant acceleration. We can eliminate the time from these equations, for example by solving for t in the velocity equation and substituting that expression back into the displacement equation to obtain a new expression that directly relates the velocity to the displacement.

$$2a(x - x_o) = v^2 - v_o^2$$

In particular, we see that the displacement increases as the square of the velocity.

Main Points

- **Definitions of Kinematic Quantities**

Displacement $x(t)$

Velocity is the time rate of change of displacement

Velocity $v(t) \equiv \dfrac{dx(t)}{dt}$

Acceleration is the time rate of change of velocity

Acceleration $a(t) \equiv \dfrac{dv(t)}{dt}$

- **Obtaining Displacement and Velocity from Acceleration**

Displacement is the integral of the velocity over time

Displacement $x(t_f) - x(t_i) = \displaystyle\int_{t_i}^{t_f} v(t)\,dt$

Velocity is the integral of the acceleration over time

Velocity $v(t_f) - v(t_i) = \displaystyle\int_{t_i}^{t_f} a(t)\,dt$

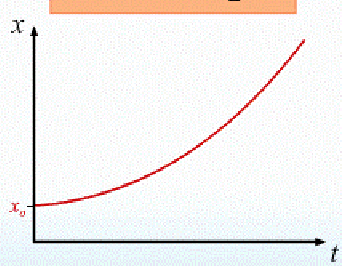
- **Special Case: Motion with Constant Acceleration**

The displacement is obtained by integrating the velocity over time

The velocity is obtained by integrating the constant acceleration over time

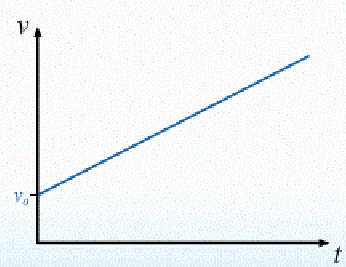
Displacement

$$\boxed{x = x_o + v_o t + \frac{1}{2}at^2}$$



Velocity

$$\boxed{v = v_o + at}$$



Acceleration

$a =$ constant