4. Newton's Laws

A) Overview

The last unit marked the completion of our study of kinematics, the description of motions in terms of displacements, velocities and accelerations. We will now begin our study of dynamics, an account of what causes these motions.

The framework we will use was first introduced by Isaac Newton in 1687 in his major work, *Principia Mathematica*. We will use what are commonly called Newton's three laws of motion to develop this framework. We will begin with Newton's Second Law which relates two new physics concepts, *force* and *mass*, that are needed to determine the motion of an object in any given physical situation. We will then recast Newton's Second Law in terms of another important physics concept, momentum.

We will then introduce Newton's First Law which establishes the concept of inertial reference frames. We will conclude by introducing Newton's Third Law, which states that all forces come in pairs: that for every action, there is an equal and opposite reaction.

B) Two Concepts

Up to this point our focus has been *kinematics*, the description of motion. We now want to shift our focus to *dynamics*, the causes of motion. In order to discuss the causes of motion, Newton introduced two new concepts that were not needed in our study of kinematics.

These two new concepts are called *mass* and *force*. Although these words are in common use in the English language, in physics, these words have very specific meanings. We will need to be very careful when we use these words to make sure that we are referring to their physics meanings, which are much more restricted than their ordinary language meanings.

We will start with mass. *Mass is the property of an object that determines how hard it is to change its velocity*. Objects that are difficult to accelerate have large mass, while objects that are easy to accelerate have small mass.

The second concept is force. *Force is the thing that is responsible for an object's change in velocity*. It's common to think of forces in terms of pushes and pulls, but we will find that we must extend our notion of force beyond that of the exertion made by people to move objects. We will need to think of forces simply in terms of producing a change in an object's velocity.

The most important thing to notice in these definitions is that *both* mass and force refer to changing an object's velocity, that is, to its *acceleration*. Acceleration is the kinematic concept that *links* these two dynamic concepts. This link is formalized in Newton's Second Law which we will now introduce.

C) Newton's Second Law

Newton's insight, which is captured in his Second Law, is that when a force acts on an object it causes that object to accelerate in the same direction that the force acts, and that the magnitude of this acceleration is proportional to the magnitude of the force.

$$\vec{a} = \frac{\vec{F}}{m}$$

Note that the constant of proportionality in this equation is 1/mass. It is the mass of an object that determines the acceleration for a given applied force. The bigger the mass, the smaller the acceleration for a given force.

We have introduced the form of Newton's Second law as $\boldsymbol{a} = \boldsymbol{F}/m$ rather than the more familiar (and equivalent) $\boldsymbol{F} = \mathrm{m}\boldsymbol{a}$, in order to stress that it is the force that causes the acceleration, and not vice versa. Often students want to invoke an "*ma*" force in their problem solutions. By writing $\boldsymbol{a} = \boldsymbol{F}/m$, we are encouraging you to think of the forces as being primary and the acceleration as the result of applying the forces to the mass.

Finally, we want to stress that this equation is a *vector* equation. The importance of the vector nature of this equation can be demonstrated by two observations. First, the direction of the acceleration is the same as the direction of the force. Second, we exploit the vector nature of forces to determine what happens when more than one force acts on an object. Namely, if more than one force acts on an object, the object's acceleration is determined by applying Newton's Second law using the total force acting on the object which is defined to be the vector sum of all the individual forces acting on the object.

$$\vec{F}_{net} = \sum_i \vec{F}_i$$

D) Units

We've just introduced the two new concepts we need to describe dynamics (namely, force and mass) and the fundamental law (Newton's Second law) that links them. We now need to address the issue of units.

So far, we have used two basic units, one for space (meters) and one for time (seconds) to describe kinematics. To accommodate the two new concepts needed to describe dynamics, we will need to introduce a new unit. We choose this new unit, the *kilogram*, to be the *SI* unit for mass. Using Newton's second law, we see that the *SI* unit for force is kg-m/s^2. It is common to then define the Newton, a unit of force, to be equal to 1 kg-m/s^2.

The truly amazing thing here is that during this course in mechanics, we will introduce many new concepts, but we will need to introduce *no more fundamental units*!! Indeed, we need just two kinematics units (meters and seconds) and one dynamic unit (mass) to completely specify all of the concepts we will introduce in mechanics. Next term, when we study electricity & magnetism, we will need to introduce only one more unit, the Coulomb, the unit for electric charge.

E) Momentum

Before moving on to do an example that illustrates Newton's second law, we first want to make a natural connection between forces and another physics quantity of fundamental importance: *momentum*.

Recalling that the acceleration is defined to be the time derivative of the velocity, we can rewrite Newton's Second law in terms of this derivative. Now, as long as the mass is a constant, we can bring it inside the derivative to obtain the expression shown.

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

The expression inside the parentheses (the product of the object's mass and its velocity) is defined to be the *momentum* of the object, usually denoted by the symbol *p*. With this definition, we see that Newton's 2nd law is equivalent to the statement that the time rate of change of an object's momentum is equal to the total force acting on the object. In fact, when Newton first introduced this law in his Principia in 1687, he used this formulation in terms of the change in momentum. One advantage of this formulation is that it can be used for processes in which the mass of the system is changing in time.

There are two simple, yet very interesting conclusions we can draw from this formulation of Newton's second law in terms of the change in momentum.

First, if there are no forces acting on an object then its momentum cannot change. Indeed, since Newton's law is a vector equation, this statement must be individually true for all components of the momentum.

Second, to change an object's momentum, we need to have a non-zero force acting on the object for a finite period of time.

We will revisit both of these concepts later in this course when we discuss collisions.

F) Example: Spaceship

We will now return to the usual formulation of Newton's law to do a simple example that illustrates some of its main features. Consider a spaceship having a constant mass *m* far out in space. Suppose the engines on the spaceship are turned on at $t = 0$ and are then turned off 50 seconds later, and that during the time the engines are on they exert a force $\boldsymbol{F}$ on the spaceship. Figure 4.1 shows the plots of the displacement, velocity and acceleration of the spaceship as a function of time.

While the engines are on, the spaceship will have a constant acceleration equal to $\boldsymbol{F}/m$. The acceleration is just equal to $\boldsymbol{F}/m$ between 0 and 50 seconds, and zero thereafter. While the spaceship has a constant non-zero acceleration, the velocity increases linearly with time as shown. During this time the displacement increases quadratically as shown. In fact, these plots are just the plots of motion at constant acceleration that you are familiar with from kinematics. The only new thing here is that we *know* that the acceleration is constant, because the force (and mass) are constant.

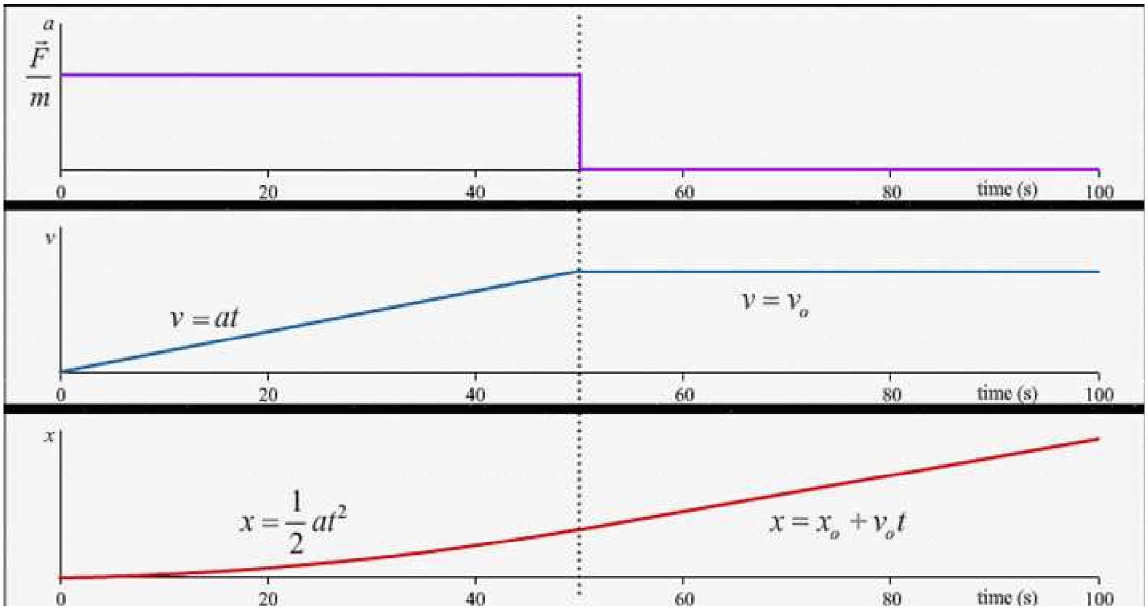

Figure 4.1
Plots of the acceleration, velocity, and displacement of a spaceship of mass *m* that experiences a constant net force ***F*** for a fixed time (50 seconds).

After the engines shut off at t=50s, the acceleration goes to zero. Therefore, the motion for t > 50 s is one of constant velocity. Consequently, the displacement, for t > 50s, just increases linearly.

G) Newton's First Law

Newton's Second Law is, in some sense, the "working equation" of this course. You will use it time and time again to solve problems. In his famous book, the Principia, Newton introduced two other laws, though, that we want to discuss at this time.

Newton's First Law states that "an object subject to no external forces is at rest or moves with constant velocity if viewed from an inertial reference frame" There are two features here that we want to address. First, since we know that if an object is at rest or moving with constant velocity, its acceleration is zero. From Newton's Second Law, we already know that if an object's acceleration is zero, the total force acting on it must be zero also. So what's new here? Well, what's new here is the second point I want to make. Namely, the concept of an inertial reference frame is explicitly introduced in this law.

In particular, Newton's first law actually serves to define an inertial reference frame as a reference frame in which Newton's laws hold good. We know Newton's laws do not appear to hold in all reference frames. For example, we have noted that objects appear to behave strangely in accelerating frames, as though forces were acting on them, even though we could not identify the agent that produced these forces. If you turn a tight corner in your car, the cell phone on your dashboard may slide from one side to the other, clearly accelerating, even though there aren't any sudden new forces acting on it that we could identify to be the cause of this acceleration.

For our current purposes, we can identify inertial frames as non-accelerating frames. We want to relax this identification a bit, though, as we have already seen that the earth's acceleration as it spins on its axis is not zero, but it is sufficiently small that we may, for practical purposes, consider the earth to be an inertial reference frame.

We will close this discussion by noting that Newton's First Law implies that once we have found one inertial reference frame, then any other reference frame moving with a constant velocity with respect to the first one is also an inertial reference frame. Therefore, accepting the Earth as an inertial reference frame implies that any frame moving at constant velocity with respect to the Earth is also an inertial reference frame.

H) Newton's Third Law

Newton's Third Law states that "for every action there is an equal and opposite reaction." This sentence can be misleading, but its meaning is succinctly and completely captured in the vector equation shown.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Namely, that the force that object A exerts on object B is equal to minus the force that object B exerts on object A.

The important point to realize here is that Newton's Third Law implies that all forces come in pairs! There is no such thing as an isolated force. If I exert a force on the wall by pushing on it, the wall exerts a force on me by pushing back with a force of exactly the same magnitude, but in the exact opposite direction. Figure 4.2 shows the forces that are exerted in this situation. Our notation for force labels uses subscripts to demote what is causing the force and what the force is pushing against. $\boldsymbol{F}_{ManWall}$ therefore refers to the force exerted *by* the man *on* the wall, and $\boldsymbol{F}_{FloorMan}$ refers to the force exerted *by* the floor *on* the man. With this notation, it is clear which forces are the third-law "pairs". For every $\boldsymbol{F}_{AB}$ there must be an equal and opposite $\boldsymbol{F}_{BA}$.

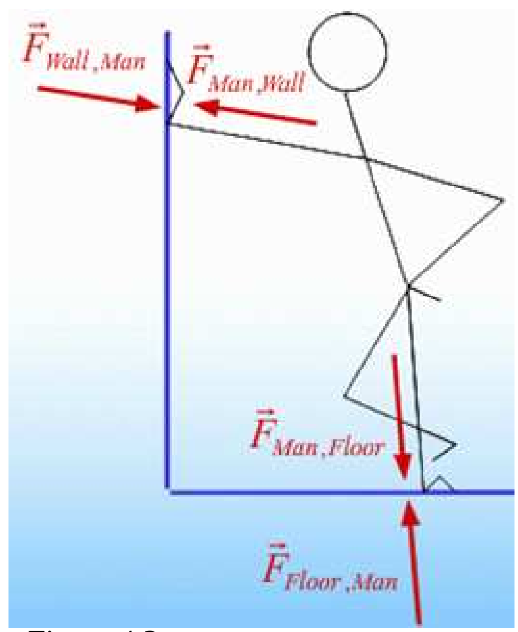

Figure 4.2
Newton's Thirld Law pairs of forces acting when a man stands on the floor and pushes against the wall.

Now consider the case of a man pushing on a box initially at rest on a smooth horizontal surface, causing it to accelerate to the left as shown in Figure 4.3. The man pushes on the box with a force $\boldsymbol{F}_{mb}$ and the box pushes back on the man with an equal and opposite force $\boldsymbol{F}_{bm}$. When we add together all of the horizontal forces they seem to cancel. But if that were true, the box would not accelerate, but we know it will! What's going on here? Well, the answer to our problem is simple: In order to understand the motion of the *box* we only need to consider the forces acting *on* the *box* – only $\boldsymbol{F}_{mb}$ needs to be considered

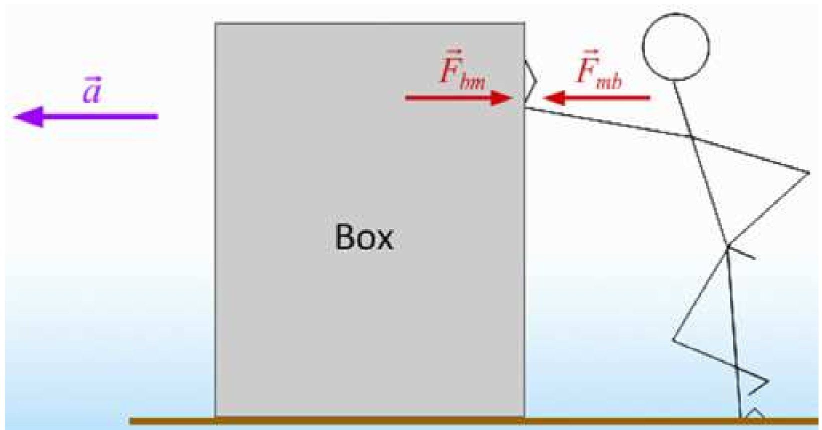

Figure 4.3
A man exerts a hrozontal force $\boldsymbol{F_{mb}}$ on a box, causing it to accelerate. The force $\boldsymbol{F_{bm}}$ acts on the *man*, *not the box*; therefore, it plays no role in the determination of the acceleration of the box.

when calculating the total force on the box, not $\boldsymbol{F_{bm}}$. This little exercise may seem more like bookkeeping than physics, but it is extremely important to get this picture correct at the outset. All forces come in pairs, but these force pairs are exerted on *different objects*. To determine the motion of an object, we only need to consider the forces acting on that object.

In the next unit we will develop the technique of free body diagrams that will formalize this notion and lay the groundwork for all problem solving using Newton's Second Law.

Main Points

- **Force, Mass & Newton's Second Law**
    - ***Mass*** ***is the property of an object that determines how hard it is to change its velocity.***
    - ***Force*** ***is the thing that is responsible for an object's change in velocit***

Newton's Second Law provides the link between these two new concepts

$$\boxed{\vec{a} = \frac{\vec{F}_{Net}}{m}}$$

- **Inertial Reference Frames and Newton's First Law**

Newton's First Law:
"An object subject to no external forces is at rest or moves with consstant velocity if viewed from an inertial reference frame."
serves to define "inertial reference frames"

- **Newton's Third Law**

Newton's Third Law:
"For every action there is an equal and opposite reaction"

All forces come in pairs:

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$