5. Forces and Free-Body Diagrams

A) Overview

We will begin by introducing the bulk of the new forces we will use in this course. We will start with the weight of an object, the gravitational force near the surface of the Earth, and then move on to discuss the normal force, the force perpendicular to the surface that two objects in contact exert on each other, and the tension force, the force exerted by a taut string. Finally, we will introduce Newton's universal law of gravitation that describes the forces between any two objects that have mass. We will close by introducing free body diagrams which we will then use in the solution of a Newton's second law problem.

B) Weight

In order to apply Newton's Second law in physical situations, we will need to increase our inventory of forces. We will start with the gravitational force near the surface of the Earth. We have already seen that an object in free fall near the surface of the Earth has a constant acceleration whose direction is down and whose magnitude is equal to the constant g, which is equal to about 9.8 m/s^2. From this description of the motion, we can use Newton's second law to conclude that there must be a force in the downward direction acting on the object and that the magnitude of this force must be equal to the product of the mass of the object and the constant g.

$$\vec{W} = m\vec{g}$$

We call this force the weight of the object. It is important to realize the weight of an object is ***NOT*** the same thing as its mass! Mass is an intrinsic property of the object; its value determines how hard it is to change its velocity. Mass does ***NOT*** depend on the location of that object or on its surroundings. Weight, on the other hand, just tells us the magnitude of the gravitational force that is acting on the object. We will investigate the nature of this gravitational force more fully after we first introduce a few more straightforward forces.

C) Support forces: The Normal Force and Tension

We can use our knowledge of the weight force from the last section to motivate the need for two more forces. First, consider the incredibly mundane situation of a heavy box sitting on a floor as shown in Figure 4.1. What forces are acting on this box? Well, certainly the weight of the box is acting, supplying a force vertically downward. This can't be the only force on the box, though, since if it were, Newton's Second law would tell us that the box should be accelerating downward with constant acceleration equal to g. Therefore, to obtain the needed zero acceleration, there must be another force that acts vertically upward with the same magnitude. Note that this force is ***NOT*** the Newton's third law pair to the weight since both forces act on the same object, the box. This force is the force exerted by the floor on the box and is usually called the *normal force,* since its direction is perpendicular to the surface.

What determines the magnitude of this force, in general? Well, to determine the magnitude of the normal force in any particular case, we do just as we did here; we apply Newton's law. *The normal force is simply what is has to be to do what it does!* What it does is to supply a supporting force for objects!

The total force exerted by any surface in contact with another surface will always have a normal component, but it may also have a component parallel to the surfaces called the frictional force. We will discuss the nature of frictional forces in the next unit.

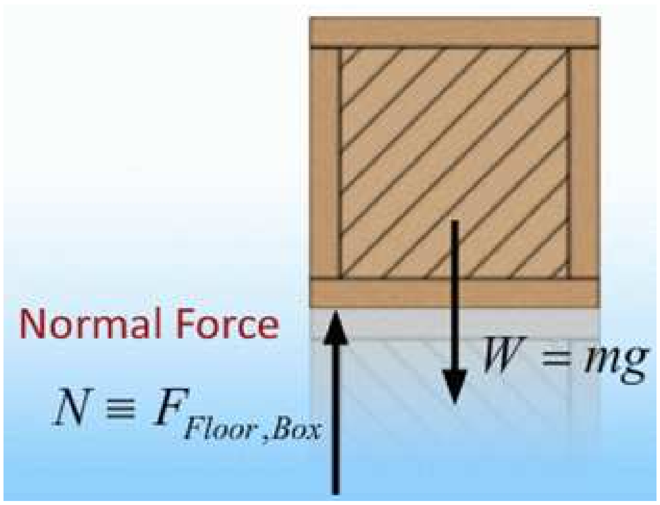


Figure 5.1
Two forces act on a box that is at rest on the floor: the weight W, the gravitational force exerted by the Earth, and N, the normal force exerted by the floor

A force similar to the normal force between surfaces in contact with each other is the *tension force* in strings, wires and ropes. Figure 5.2 shows a ball hanging from a string. What forces are acting on the ball? Clearly the weight force acts vertically downward. In order to obtain the needed zero acceleration, the string must also be exerting a force on the ball. We call this force the *tension force*; it exists whenever the string is taut and its direction is along the string, in this case, vertically upward. From Newton's second law, we can determine that the magnitude of this force must be equal to the weight of the ball in order to provide the observed zero acceleration. Just as was the case

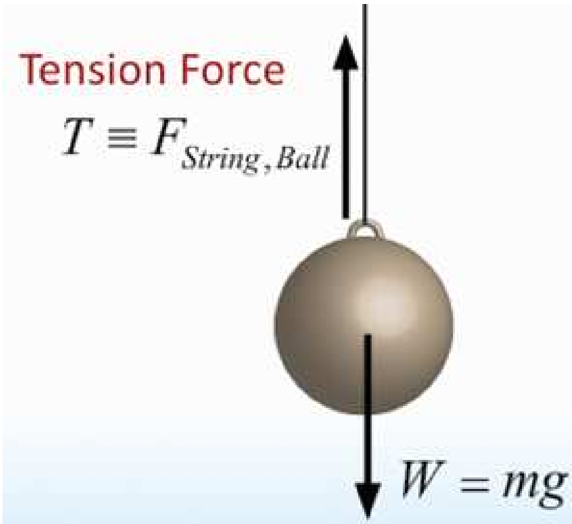


Figure 5.2
Two forces act on a ball that is suspended by a string: the weight ***W***, the gravitational force exerted by the Earth, and ***T***, the tension force exerted by the string.

for the normal force, *the tension force is simply what is has to be to do what it does!* In this case, the sting is just holding up the ball; strings can also be used to pull objects across a surface. In either case, the magnitude of the tension must be determined from Newton's second law.

D) Springs

Figure 5.3 shows a ball hanging from a spring. We can use Newton's second law to determine that the spring must be exerting a force on the ball that is equal to its weight. However, if we were to replace the first ball with a new ball that has twice the mass, we would see that the spring would be stretched more. In this new situation, we know the spring would be exerting twice the force, but its length would be increased. In fact, the amount by which the length changes tells us the magnitude of the force! The key concept here is that every spring has an equilibrium length, and if it is stretched or compressed by some amount *Δx* from this length, it will exert a restoring force that opposes this change. The magnitude of this force is proportional to *Δx*, the extension or compression of the spring from its equilibrium position.

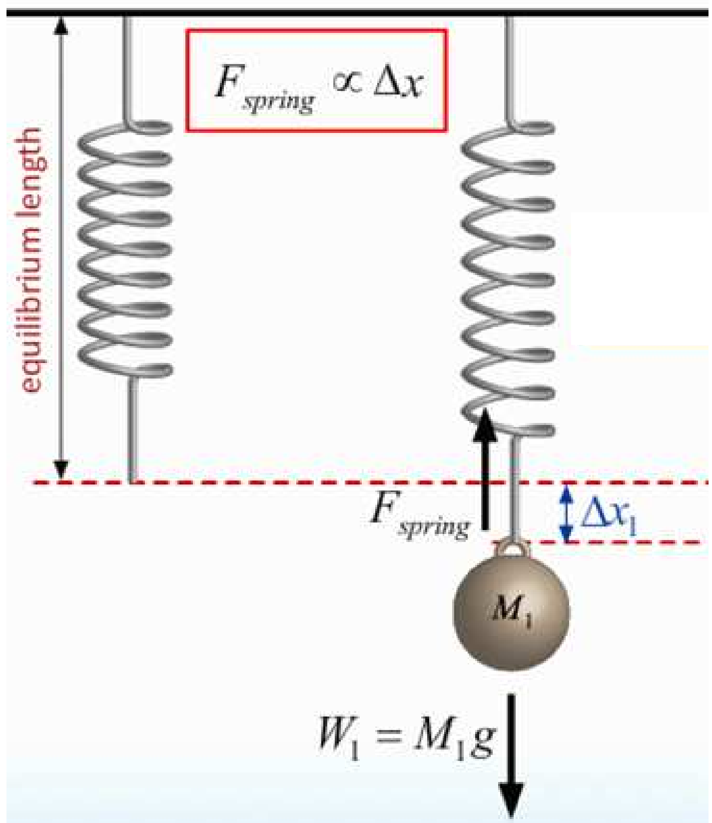


Figure 5.3
Two forces act on a ball that is suspended by a spring: the weight ***W***, the gravitational force exerted by the Earth, and $\boldsymbol{F_{spring}}$, the force exerted by the spring. The magnitude of the spring force is proportional to the extension (or compression) fromits equilibrium position.

The force law for springs that quantifies this relation is given by:

$$\vec{F}_{spring} = -k(\vec{x} - \vec{x}_o)$$

where the vector $\boldsymbol{x_o}$ represents the equilibrium length of the spring, while the vector ***x*** represents the final length of the spring. The vector difference $\boldsymbol{x - x_o}$ represents the amount by which the spring is either stretched or compressed. The minus sign in the equation illustrates that the force is always in the opposite direction of the vector $\boldsymbol{x - x_o}$, the extension or compression of the spring. If the spring is stretched, $\boldsymbol{x - x_o}$ points away from the equilibrium position; therefore the force is directed back towards the equilibrium position. If the spring is compressed, $\boldsymbol{x - x_o}$ once again points away from the equilibrium position; therefore, the force is again directed back towards the equilibrium position. Since the force exerted by the spring is always directed towards its equilibrium position, we call this force a restoring force. The directions of these vectors are illustrated in Figure 5.4.

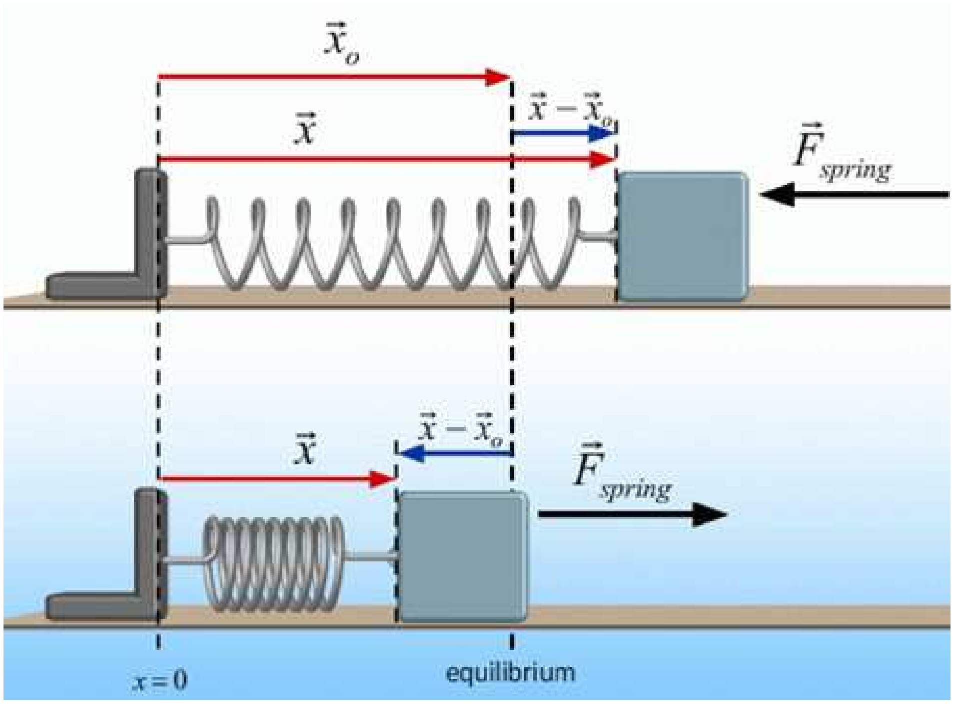


Figure 5.4
The spring force is a restoring force. The force vector ($\boldsymbol{F} = -k(\boldsymbol{x} - \boldsymbol{x_o})$) always points back towards the equilibrium position as illustrated in extension (top) or compression (bottom).

If we define the origin of our coordinate system to be at the equilibrium position, then our force equation simplifies to $\boldsymbol{F} = -k\boldsymbol{x}$.

The symbol *k* stands for the *spring constant* of the particular spring and is a measure of its stiffness. The units of *k* are Newtons per meter; a large value of *k* means that a small deformation results in a big force.

E) Universal Gravitation

We now want to generalize our discussion of the force we called the weight earlier. You know that the moon orbits the Earth with a period of about a month and the Earth orbits the Sun with a period of a year. To a good approximation these orbits are examples of uniform circular motion and therefore we know that each orbiting body experiences a centripetal acceleration. Therefore, in Newton's framework, there must be a real force being exerted on the orbiting body that is responsible for this acceleration. Newton proposed that this force was a universal gravitational force that exists between any two objects that have mass. .

In particular, he said that any two objects with mass exert attractive forces on each other whose magnitude is proportional to the product of the masses divided by the square of the distance between them and whose direction lies along a line connecting them.

$$\vec{F}_{gravity} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

In this expression, G represents the universal gravitational constant and $\hat{r}_{12}$ represents the unit vector in the direction from m_1 to m_2. Figure 5.5 illustrates the application of this expression to the Earth-moon system. The symbols M_E and M_m refer to the masses of the Earth and moon respectively, while R_{Em} is the Earth-moon distance. We know the acceleration of the moon, a_m, is equal to the square of its speed divided by the Earth-moon distance. Applying Newton's second law, we can determine the acceleration of the moon.

$$a_m = \frac{v_m^2}{R_{Em}} = \frac{F_{Em}}{M_m} = G\frac{M_E}{R_{Em}^2}$$

All quantities in this expression were known to Newton except the universal gravitational constant and the mass of the Earth.

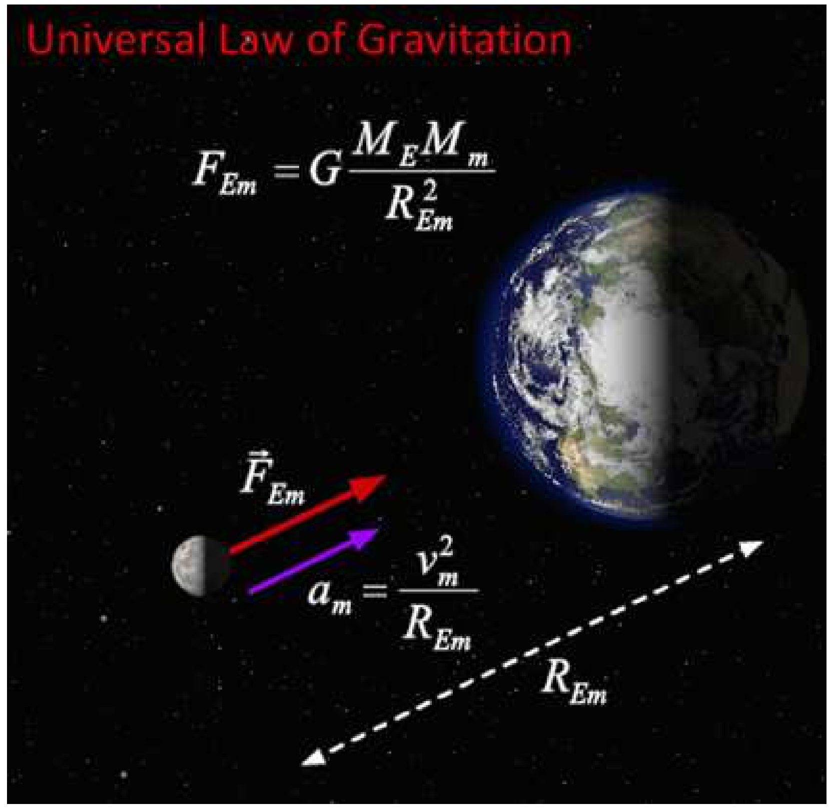


Figure 5.5
The universal gravitation force exerted by the Earth on the moon provides the necessary centripetal acceleration to keep the moon in its orbit about the Earth.

Newton, however, realized that the known acceleration due to gravity near the surface of the Earth, was also proportional to the product of these unknown quantities! In order to make this realization, though, he essentially had to invent the calculus to show that the force the Earth exerts on any object is equivalent to that obtained by simply placing all of the mass of the Earth at its center.

$$W_{apple} = m_{apple}g = G\frac{M_E m_{apple}}{R_E^2}$$

Given this result, we see that the acceleration due to gravity near the surface of the Earth is equal to the product of universal gravitational constant and the mass of the Earth divided by the square of the radius of the Earth.

$$g = G\frac{M_E}{R_E^2}$$

Therefore, we see that the ratio of the moon's acceleration to that of an apple in free fall near the surface of the Earth is predicted to be equal to the ratio of squares of the radius of the Earth to the Earth-moon distance.

$$\frac{a_m}{g} = \frac{R_E^2}{R_{Em}^2}$$

Now, the speed of the moon in its orbit is 1.02 km/s, while the Earth-moon distance is 3.844×10^5 km and the radius of the Earth is 6371 km. When we plug in these numbers for the known quantities, we find that this prediction is verified! This result is really amazing! It represents the first demonstration that the same physical laws that operate here on Earth also operate in the Heavens!

F) Free-Body Diagrams

In the last unit, we introduced Newton's three laws which supply the framework we will use to develop our understanding of dynamics. In particular, these laws will provide the basis for our understanding of the motion of any object in terms of the forces that act on it. In order to use these laws successfully, though, we need to keep careful track of the magnitudes and the directions of all forces acting on the object in question; we will use free-body diagrams to accomplish this task.

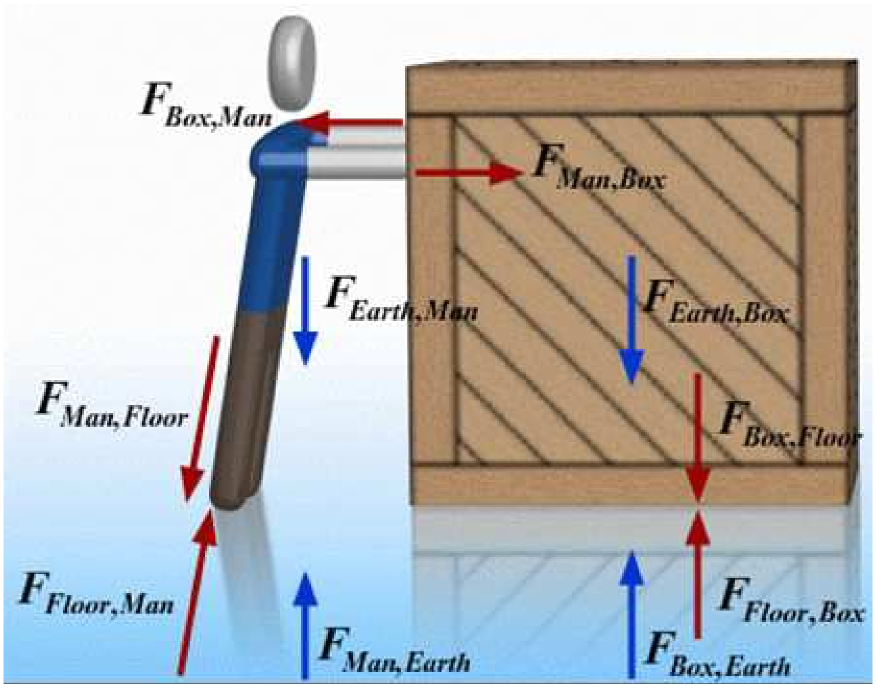


Figure 5.6
All forces that act when a man pushes a box across a smooth floor.

Figure 5.6 shows a man pushing a box across a smooth floor with a representation of all forces that are acting. Contact forces are shown in red and the gravitational forces are shown in blue. Note that all of forces come in pairs, as required by Newton's 3rd law. For example, the force exerted by the box on the man is equal and opposite to the force exerted by the man on the box.

We would like to calculate the acceleration of the box. How do we go about making this calculation? The key step here is to realize that the *only* forces which are relevant to this problem are the ones that act ***ON*** the box – all other forces can be ignored. A diagram showing only these forces is called a free-body Diagram for the box and is illustrated in Figure 5.7.

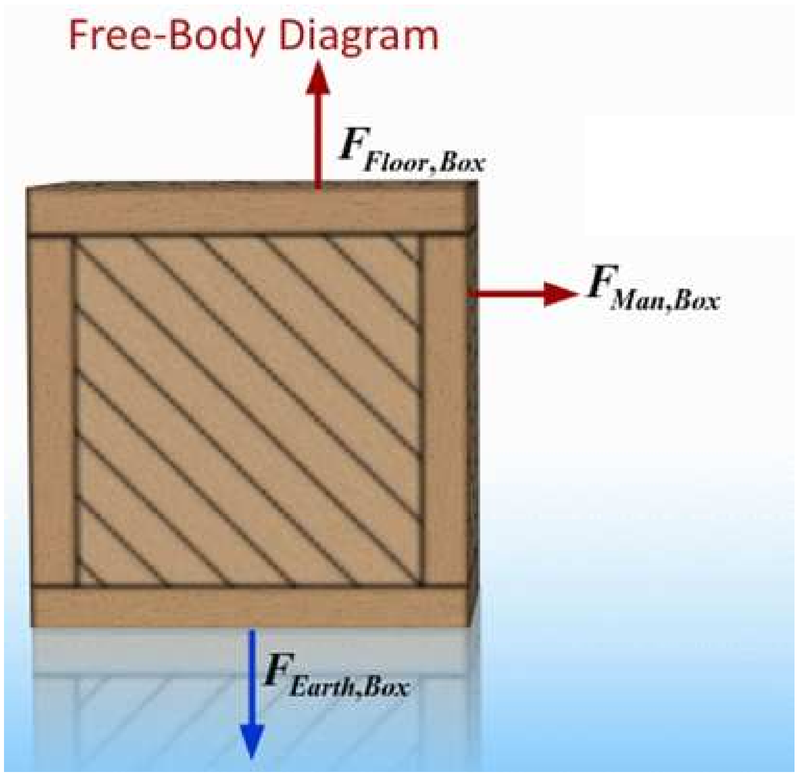


Figure 5.7
The free-body diagram for the box shown in Fig 5.6.

Applying Newton's second law to the box, we see that the acceleration of the box will be equal to the total force on the box divided by the mass of the box. To determine the total force on the box, we will need to add, as vectors, all of the forces shown in this free body diagram for the box. Usually, in order to add these vectors, we will want to decompose the forces into appropriate components and write Newton's second law equations for each component separately.

$$a_{horizontal} = \frac{F_{Man,Box}}{M_{box}}$$

$$a_{vertical} = F_{Floor,Box} - F_{Earth,Box} = 0$$

G) Example: Accelerating Elevator

We'll close this unit by considering a one-dimensional problem to illustrate the procedure to use when solving dynamics problems.

Figure 5.8 shows a box of mass *m* hanging by a rope from the ceiling of an elevator moving vertically with acceleration *a*. We want to calculate the tension in the rope for any value of this acceleration.

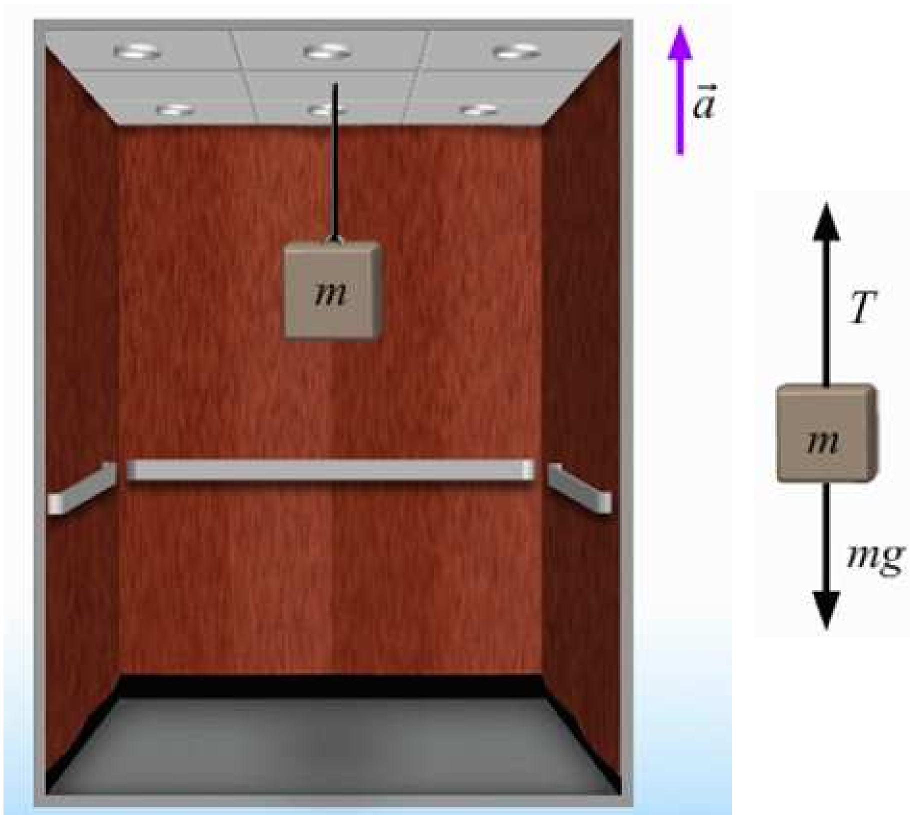


Figure 5.8
A box of mass *m* hangs by a rope in an elevator that is moving vertically with acceleration ***a***. What is the tension in the rope? To solve this problem, use the free-body diagram for the box that is shown.

The first step is to draw a picture and label all the forces acting on the object in question. In this case the object is the box, and the forces acting on it are the tension in the rope (*T*), which points upward, and the weight of the box (*mg*), which points downward, as shown

The second step is to choose a co-ordinate system. Any system will do, but you will soon discover that choosing one in which one of the axes is parallel to the acceleration will simplify the calculation.

The next step is to use your picture as a guide to write down the components of Newton's second law and solve for whatever variable you want to determine. In our case all of the forces act along a single direction so that we only have one equation to solve. The force on the box due to the rope is *T* in the +y direction and the force on the box due to gravity is *mg* in the –y direction; therefore the total force on the box in the +y direction is given by:

$$F_{net,y} = T - mg$$

Substituting this expression for the total force in Newton's Second law yields the result that the tension is equal to the weight of the box plus the product of the mass of the box and its acceleration.

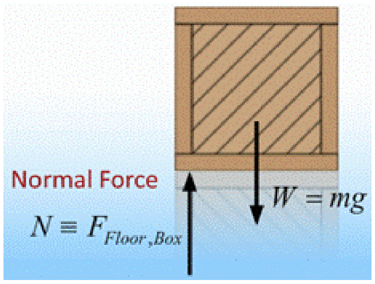
$$T = m(a + g)$$

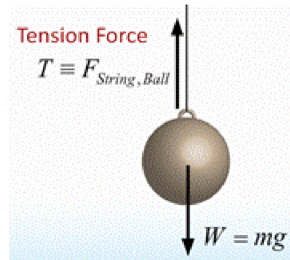
The final step is to check to see if your answer makes sense. In this case we just found that the tension in the rope is given by the weight of the box plus an extra part which is proportional to the acceleration. If we consider the case where the elevator is not accelerating we see that the tension in the rope is just equal to the weight of the box. If the elevator is accelerating upward, the tension is bigger than the weight, and if the elevator is accelerating downward the tension is less than the weight. All of these observations make sense.

Main Points

- **Support Forces**

Support forces, for example the normal force and the tension force, are what they have to be to do what they have to d. Magnitudes are determined by Newton's second law

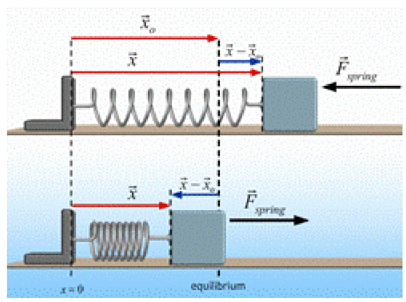




- **Spring force**

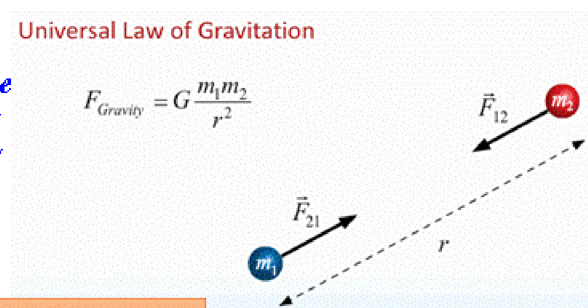
The spring force is a restoring force

$$\vec{F}_{spring} = -k(\vec{x} - \vec{x}_o)$$



- **Universal Gravitation**

Any two objects with mass exert attractive forces on each other whose magnitude is proportional to the product of the masses divided by the square of the distance between them and whose direction lies along a line connecting them.



$$g = G\frac{M_E}{R_E^2} = 9.8\,\tfrac{\text{m}}{\text{s}^2}$$