PHYSICS 211

LAB #8: Periodic Motion

A Lab Consisting of 6 Activities

Name: ______________________________

Section: ______________________________

TA: ______________________________

Date: ______________________________

Lab Partners: ______________________________

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Circle the name of the person to whose report your group printouts will be attached. Individual printouts should be attached to your own report.

Physics lab 211-8

Equipment list
Motion detector
Force probe
Rods and clamps to elevate force probe
Red (with orange stripe), white, and blue extension springs
Masses:
- 50 gram mass hanger (1)
- 50 gram mass (1)
- 100 g mass (4)

Tape to secure masses

Computer file list
MacMotion file "211-08 oscillating"
MacMotion file "211-08 data entry 1"
MacMotion file "211-08 data entry 2"
MacMotion file "211-08 oscillator energy"

Investigation 1: Motion of a Mass Hanging From a Spring

Goals: • To study the characteristics of periodic motion.

Activity 1: Getting A Sense of Déjà Vu

Introduction: Periodic motion is motion that repeats itself. As you will see in this activity, periodic motion of an object is evident not only in the position versus time of the object, but also in the time-dependence of the object's velocity and acceleration.

In this activity you will examine the motion of a mass oscillating at the end of a spring, and study the relationship between the mass' position, velocity, acceleration, and the spring force.

Procedure: 1. Set up the computer.

- If necessary, launch the *MacMotion* program by double-clicking on the *MacMotion* icon on the Desktop.
- Pull down the **File** menu and select **Open.** Open the file ***Oscillating*** in the **Lab 8** folder. A graph like that shown in Figure 1 should appear.

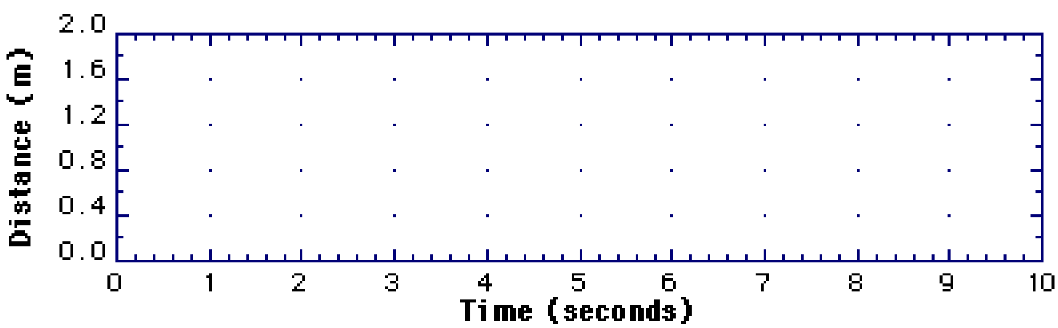

Figure 1. Oscillating graph format for Activity 1

Procedure: (continued)

2. Set up the detector, force probe, mass, and spring as shown in Figure 2.
   - Hang the "red" spring (k = 6.0 N/m) from the support and attach a 0.40 kg mass to the end of the spring. Be sure to secure the loose mass with tape.
   - Place the motion detector face up underneath the mass, but far enough away so that the mass never comes closer than 0.5 m to the detector while oscillating. The motion detector should be directly beneath the mass.

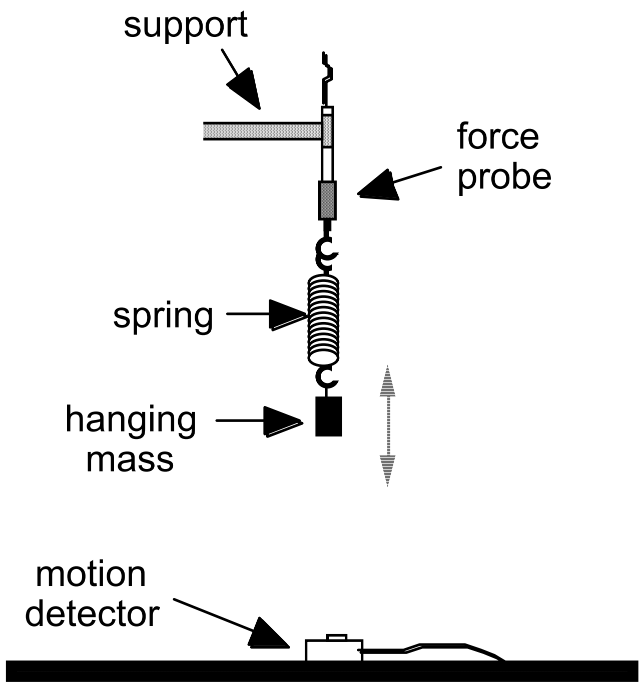

Figure 2. Experimental arrangement for Activity 1

Prediction:

- If you pull or push the hanging mass in Figure 2 away from its stationary ("equilibrium") position and release it, sketch in Figure 3 your prediction for the resulting position (x) of the mass as a function of time.

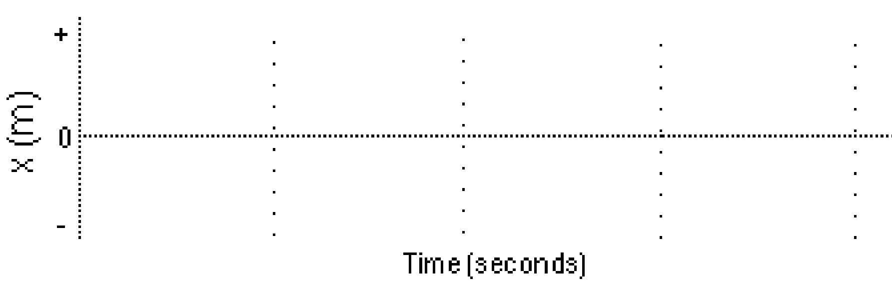

Figure 3. Prediction for Activity 1

Procedure: (continued)

3. Test your prediction by recording the motion of the hanging mass.
   - **Zero** the force probe with the mass and spring suspended from it. This will allow you to measure only the "restoring" force exerted when the mass is moved away from equilibrium.
   - Gently push the mass up about 20 cm and let go. After the mass is oscillating freely, click on **Start** to begin graphing.
4. Make a record of your experiment.
   - Set **Graph Title...** to ***Periodic Motion***, and add your group's names. **Print…** out one copy for each group member.

An Important Aside:

The periodic motion of the hanging mass you just observed in this activity is an example of simple harmonic motion, which results when an object is subject to a force, F, that is proportional to the object's *displacement from its equilibrium position*, x. Such a "restoring" force has the general form F = - kx, where k is a proportionality constant.

In order to quantitatively describe the motion you just observed, we need to apply Newton's second law to the mass+spring system. Neglecting frictional forces, the only external forces acting on the hanging mass are the spring and gravitational forces,

$$\sum_i F_i = mg - kx \qquad \text{(Eq. 1)}$$

where x is the displacement from the spring's equilibrium position, x_o. Now, you should have noticed that the hanging mass in your experiment does not oscillate about the spring's equilibrium position, x_o, but rather about a new equilibrium position, x_1, which is the position to which the spring stretches so that the spring force exactly balances the gravitational force, i.e.,

$$mg - kx_1 = 0 \;\longrightarrow\; x_1 = \frac{mg}{k} \qquad \text{(Eq. 2)}$$

So, the net force acting on the hanging mass (Eq. 1) can be written:

$$\sum_i F_i = mg - kx = mg - k(x_1 + \tilde{x}) \qquad \text{(Eq. 3)}$$

where x_1 is the new equilibrium position of the spring + hanging mass, and $\tilde{x}$ is the displacement of the mass from the new equilibrium position. Plugging Eq. 2 into Eq. 3, and applying Newton's second law, one obtains an equation describing the position of the hanging mass away from its new equilibrium position, x_1, as a function of time:

$$m\frac{d^2\tilde{x}}{dt^2} = \sum_i F_i = mg - k\left(\frac{mg}{k} + \tilde{x}\right) = -k\tilde{x}$$

$$\longrightarrow \boxed{m\frac{d^2\tilde{x}}{dt^2} = -k\tilde{x}} \qquad \text{(Eq. 4)}$$

Eq. 4 has the general time-dependent solution,

$$\tilde{x}(t) = A\cos(\omega t + \delta) \qquad \text{(Eq. 5)}$$

where A is the amplitude of the oscillation (see Figure 4), $\omega = 2\pi/T$ is the angular frequency of the oscillation, T is the period of the oscillation, and δ is the phase shift. (You can verify that this solution works by simply "plugging" it into Eq. 4.)

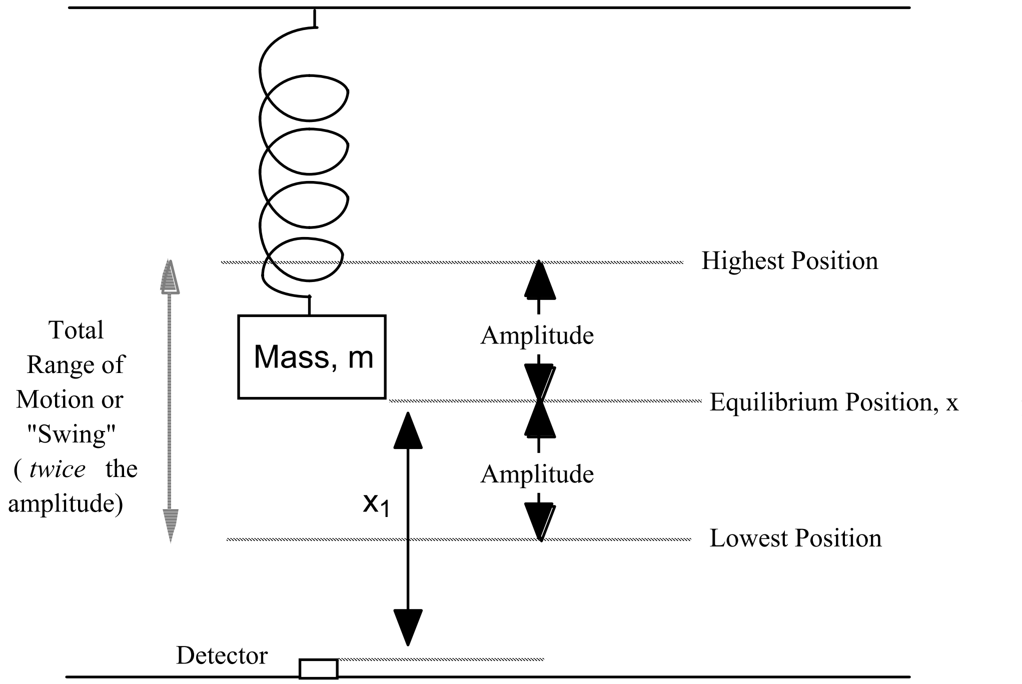

Figure 4. Amplitude and equilibrium position of the hanging mass

Procedure: (continued)

5. On the graph you just printed out, label the following quantities:
   - The equilibrium position, x_1
   - The amplitude A of the oscillation
   - The period T of the oscillation

6. Move your data to **Data B** by selecting **Data A-->Data B** from the **Data** menu. This will ensure that you do not accidentally erase your data, since you will need it for this and other activities!!

7. Measure the amplitude A and period T of your distance vs. time plot.
   - Select **Analyze Data B...** under the **Analyze** menu, then use the cursor to measure the quantities listed above.
   - Record your measured values in the appropriate columns of the “Displacement, x” row of Table 1.

Predictions:

- Consider a single cycle of the motion you observed in your last experiment, one example of which is reproduced in Figure 5. For this cycle:
  - Sketch your predictions for the velocity and acceleration of the hanging mass while the mass is oscillating.
  - Sketch your prediction for the force measured by the force probe while the mass is oscillating.

  In your sketches, pay particular attention to the phase relationship between these curves.

- Next to each graph, write the mathematical relationship describing each of the curves you've drawn in terms of the oscillatory amplitude, A, angular frequency $\omega = 2\pi/T$, phase shift δ, and other relevant parameters (see Eq. 5).

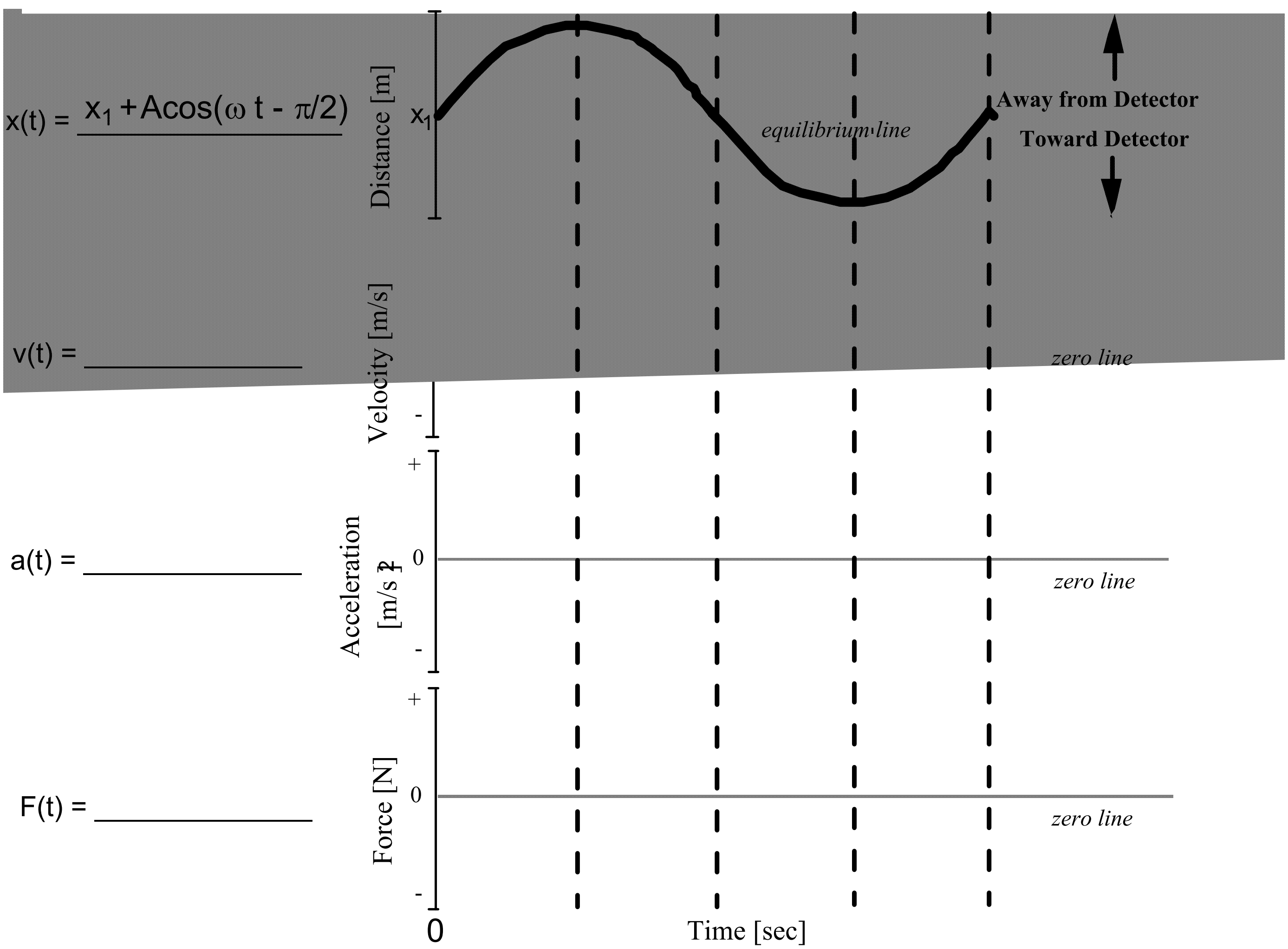

Figure 5. Predictions for Activity 1

Predictions: (continued)

- Using your earlier results for the distance versus time of the oscillating mass, record your predictions for the following quantities in the appropriate rows and columns of Table 1:
  - The *amplitudes* of the velocity and acceleration vs. time graphs in your last experiment.
  - The amplitude the force measured by the force probe in your last experiment.
  - The phase shifts of the velocity, acceleration, and force graphs *relative to the distance versus time graph* you measured earlier.
- Do you predict that the oscillatory period T exhibited by the velocity, acceleration, and force graphs will be the same as, or different from, the oscillatory period of the distance versus time graph? Why?

Quantity	Predicted Amplitude of Quantity	Measured Amplitude of Quantity	% Difference	Measured Period T [sec.]	Predicted Phase Shift Relative to x(t) [in multiples of π]	Observed Phase Shift Relative to x(t) [in multiples of π]
Displacement, x	----------		-----------		-----------	0
Velocity, v						
Acceleration, a						
Force, F						

Table 1. Results and predictions for Activity 1

TA Discussion Checkbox

Procedure: (continued)

8. Test your predictions.
   - Select **Four Graphs** under the **Display** menu. By click-dragging on the y-axis label of each graph, change each of the four graphs as necessary so that the graphs shown in Figure 6 are displayed.
   - Select **Analyze Data B** under the **Analyze** menu. Measure the amplitudes of each of the quantities listed in Table 1, and record your measurements in Table 1. Compute the percent differences between your measurements and predictions.
   - Record the measured periods of each of the quantities in Table 1, and record your estimate of the phase shifts of these quantities relative to the distance versus time graph.

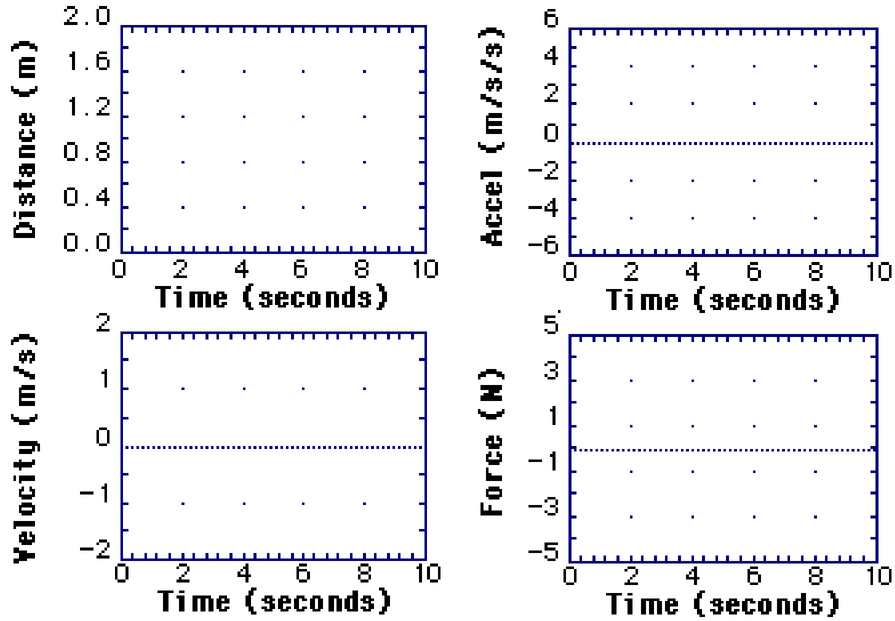

Figure 6. Results and predictions for Activity 1

Questions:

- By what numerical factors do each of the following properties of a simple harmonic oscillator change after doubling the displacement amplitude: period, spring constant, total energy, maximum velocity, and maximum acceleration? Assume that the potential energy of the oscillator is zero when it is in equilibrium.

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- Do position, velocity, acceleration, and the spring force have the same oscillatory period during simple harmonic motion according to your results?

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- Do position, velocity, acceleration, and the spring force have the same phase? Did you predict this?

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Activity 2: What's "the Shove" Got to Do With It?

Introduction: Does the oscillatory period during simple harmonic motion depend on the amplitude of vibrations? You tell us.

Prediction: • Do you predict that the amplitude of vibrations during simple harmonic motion will influence the period of oscillation? Why?

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Procedure:

1. Test your prediction. Select **One Graph** under the **Display** menu so that you are only plotting the distance versus time graph.

2. Record in "Run 1" of Table 2 the results of the oscillatory period and amplitude from the distance versus time graph you obtained in Activity 1. If you wish, you can repeat this measurement.
   - For better accuracy, you may want to obtain the period by measuring the total time duration of as many complete cycles as possible, then dividing that time by the number of cycles.

3. Repeat the measurement in Activity 1 using an amplitude *less than half* that used in Activity 1 (Run 1), and the same mass (m = 0.40 kg) and spring ("red") as in Activity 1.

4. Use **Analyze Data A** to determine both the amplitude and period. Record your results in the Run 2 row of Table 2.

Run	Measured Amplitude [m]	Ratio of Amplitudes $A_{run\ 2}/A_{run1}$	Total Time Measured [sec]	Number of Cycles Counted	Period T [sec]	Ratio of Periods $T_{run\ 2}/T_{run\ 1}$
1		----------				---------
2						

Table 2. Results of amplitude dependence for Activity 2

Question: • Compare the ratio of amplitudes to the ratio of periods you measured. Is there evidence that the oscillatory period depends on amplitude? Explain.

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Activity 3: A Force to Be Reckoned With

Introduction: In this activity, you will study the dependence of oscillatory period T on the spring constant k during simple harmonic motion.

Prediction:

- Do you expect that the oscillatory period of an object will depend on the spring constant? If your answer is "yes," what mathematical expression describes this dependence? If your answer is "no," explain your reasoning.

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Procedure:

1. Test your prediction. Using a hanging mass of m = 0.30 kg, carefully measure the oscillatory period for each of the "red," "white," and "blue" springs.
   - Use **Analyze Data A** to measure the total time over many complete oscillatory cycles, and count the number of cycles. Record your results in Table 3.

Spring Color	Spring Constant, k [N/m]	Total Time Measured [sec]	Number of Cycles Counted	Period T [sec]
red	6.0			
white	10.0			
blue	17.5			

Table 3. Results of spring constant dependence (m = 0.30 kg) for Activity 3

2. Make a graph of period versus spring constant.
   - Open the file ***Data Entry 1***.
   - Enter your spring constant and period data from Table 3 into the ***Data A Table***, then click on the graph window to display the plot.

Question:

- Is there evidence that the oscillatory period depends on the spring constant, k? If your answer is "yes," does the period increase or decrease as k increases?

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Procedure: (continued)

3. Determine the mathematical relationship between the *period T* and the *spring constant k*.
   - Try to fit your data with a “Power Law” fit, i.e., a fit of the general form

     $$T = b_o k^{b_1}$$

     To do this, select **Fit...** under the **Analyze** menu. A window like that shown in Figure 7 should appear.
   - Select **Power**, then click on **Maintain Fit** (see Figure 7).

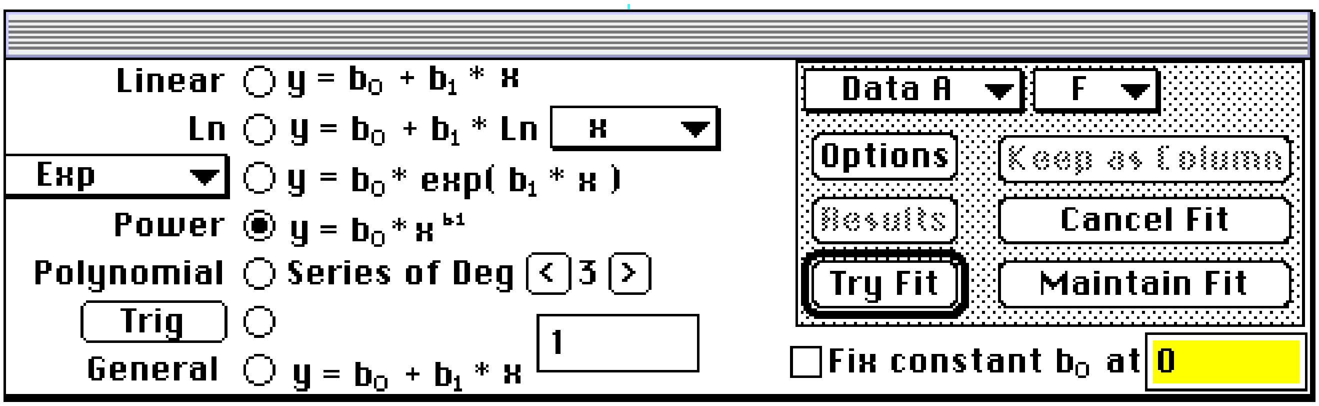

Figure 7. Window for defining fitting function in Activity 3

4. Record below the computer’s formula for the “best fit” to your period T versus spring constant k data.

   T = _______

5. Make a record of this experiment.
   - Set **Graph Title...** to ***Period vs. K***, and add your group’s names.
   - **Print...** out a single copy for your group.

☐ TA Discussion Checkbox

Question: • Based on your results, what is the mathematical relationship between the period T and spring constant k?

$T \propto$ _______

Activity 4: A Massive Dependency

Introduction: At the risk of being repetitive, now you'll study the dependence of oscillatory period on the hanging mass during simple harmonic motion.

Prediction:

- Do you predict that the oscillatory period of an object will depend on the mass of the object? If your answer is "yes," what mathematical expression describes this dependence? If your answer is "no," explain your reasoning.

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Procedure:

1. Test your prediction.
   - **Open...** (**File** menu) the file ***Oscillating*** in the **Lab 8** folder again.
   - Using the "red" spring, carefully measure the oscillatory period for each of the hanging masses given in Table 4.

Hanging Mass [kg]	Total Time Measured [sec]	Number of Cycles Counted	Period T [sec]
0.30			
0.35			
0.40			
0.45			

Table 4. Results of hanging mass dependence (using "red" spring) for Activity 4

2. Make a graph of period versus hanging mass.
   - Open the file ***Data Entry 2***.
   - Enter your hanging mass and period data from Table 4 into the ***Data A Table***, then click on the graph window to display the plot.

Question:

- Is there evidence that the oscillatory period depends on the hanging mass, m? If your answer is "yes," does the period increase or decrease as m increases?

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Procedure: (continued)

3. Determine the mathematical relationship between the *period T* and the *hanging mass m*.
   - Try to fit your data with a "Power Law" fit,

     $$T = b_o m^{b_1}$$

     To do this, select **Fit...** under the **Analyze** menu. As in the previous activity, a window like that shown in Figure 7 should appear.
   - Select **Power**, then click on **Maintain Fit**.

4. Record below the computer's formula for the "best fit" to your period T versus hanging mass m data.

   T = _______

5. Make a record of this experiment.
   - Set **Graph Title...** to ***Period vs. m***, and add your group's names.
   - Print out a single copy for your group.

Questions:

- Based on your results, what is the mathematical relationship between the period T and mass m?

  $T \propto$ _______

- Using your results from both Activities 3 and 4, write a proportionality that relates the oscillatory period of a mass on a spring to relevant parameters of the system.

  $T \propto$ _______

- Will the oscillatory period of the mass+spring oscillator change if you perform the above experiment on the moon? Explain.

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- Will the oscillatory period of a simple pendulum (a child on a swing, for example) observed on the earth be any different on the moon? Explain why or why not.

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Investigation 2: Energetics of Simple Harmonic Motion

Goals: • To study the energetics associated with simple harmonic motion.

Activity 5: The Ups and Downs of Oscillator Energies

Procedure: 1. Set up the experimental arrangement shown in Figure 2 with the “red” spring (k = 6.0 N/m) and the 0.40 kg hanging mass.

Predictions:

- What are the contributions to the total energy of the spring + hanging mass system while the hanging mass in your experiment is oscillating? Write mathematical expressions if possible.

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- Sketch in Figure 8 your predictions for the time-dependent kinetic, potential, and total energies while the hanging mass is oscillating.

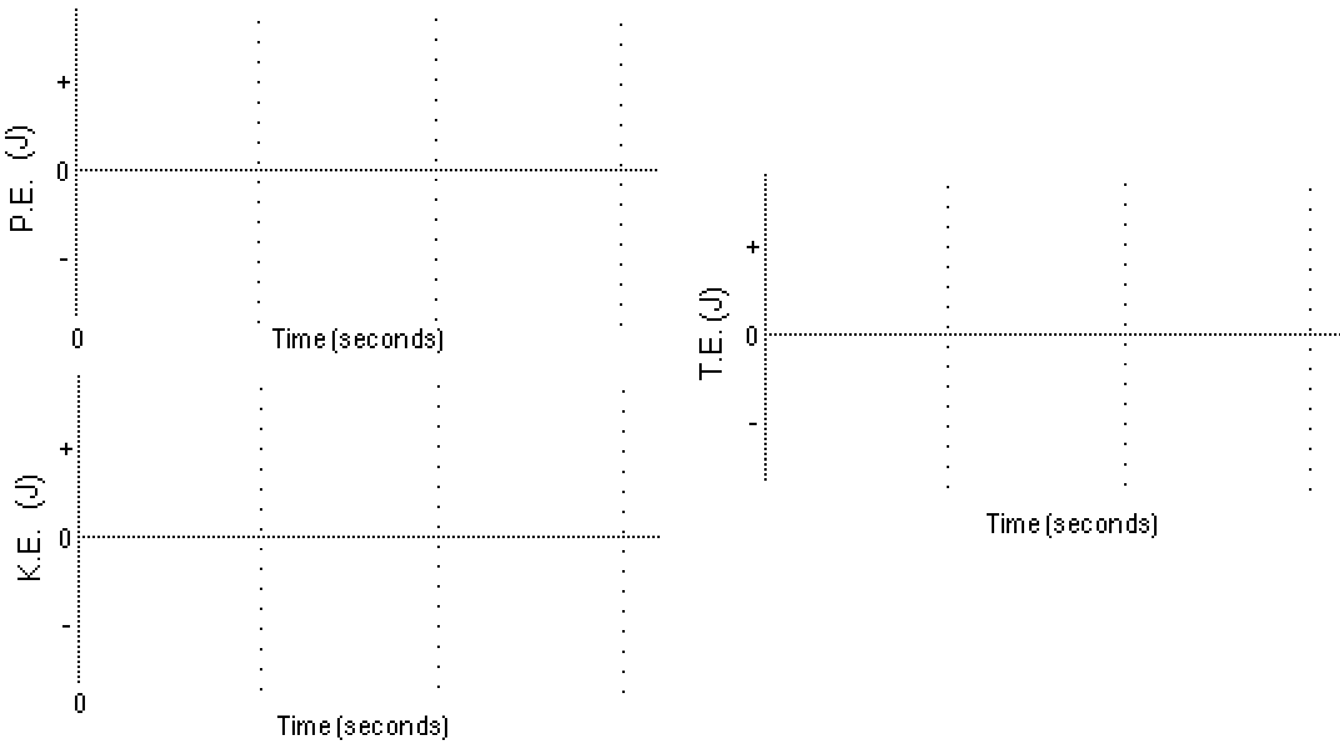

Figure 8. Predictions for Activity 5

- What are the mathematical relationships, if any, between the *total energy* and the *maximum kinetic* and *potential energies*?

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- Do you predict that the total mechanical energy will be conserved? Explain.

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Procedure: (continued)

2. Test your predictions. Pull down the **File** menu and select **Open.** Open the file ***Oscillator Energy*** in the **Lab 8** folder. A graph like that shown in Figure 9 should appear.

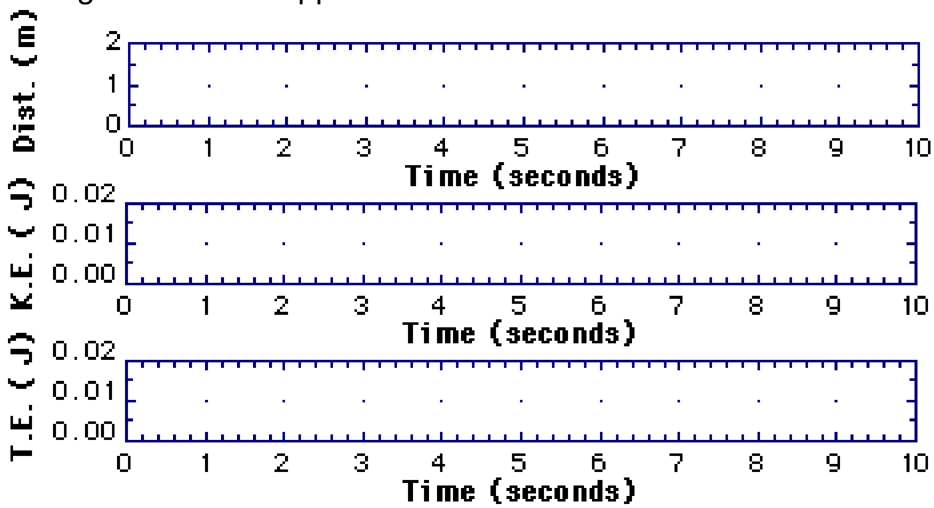

Figure 9. Oscillator Energy graph for Activity 5

3. **Modify** the total energy formula.
   - Select **Modify...** under the **Data** menu, then select **Total Energy**.
   - Replace the "**0**" in the "formula" space with the expected relationship for the total energy of the mass+spring system:

     "P.E." + "K.E."

4. **Modify** the kinetic energy formula.
   - Under the **Data** menu, first select **Modify...**, then select **K.E.**.
   - Replace the "**0**" in the "formula" space with the relationship for the kinetic energy of the oscillator, K.E. = $^1/_2 mv^2$:

     K.E. = 0.5 * (m) * "Vel"^2

     where m is the known mass value of the oscillating hanging mass in units of kg.

5. Determine the equilibrium position of the spring.
   - Hang the 0.40 kg mass at the end of the "red" spring.
   - With the mass hanging *completely stationary*, click **Start** to record the position of the mass.
   - Select **Analyze Data A**, then measure and record below the distance of the mass from the motion detector. This distance is the equilibrium position of the mass, $\mathbf{x_{eq}}$ (= x_1).

     $\mathbf{x_{eq}}$ = __________ [m]

Procedure: (continued)

6. **Modify** the potential energy formula.

   • Click on the top graph (“Distance”) to select it.

   • Under the **Data** menu, first select **Modify...**, then select **P.E.**

   • Replace the “**0**” in the “formula” space with the relationship for the oscillator’s potential energy, P.E. = $^1/_2 k(x-x_{eq})^2$:

   P.E. = 0.5 * (k) * (“Dist” - $\mathbf{x_{eq}}$)^2

   where k = 6.0 N/m is the approximate spring constant for the “red” spring, and $\mathbf{x_{eq}}$ (= x_1) is the equilibrium distance of the hanging mass that you recorded above.

7. Make the measurement.

   • First, make sure the **Vel/Accel.** averaging is set to **3**. (You get to the **Vel/Accel.** window via **Collect** → **Averaging** → **Vel/Accel**)

   • Next, gently push the mass up about 20 cm and let go. After the mass is oscillating freely, click on **Start** to begin graphing.

8. Check your results.

   • If you notice small oscillations in your total energy versus time graph, it is probably because the spring constant of your “red” spring is slightly different than the approximate value you entered in the P.E. formula, k = 6.0 N/m. In this case, the maximum potential energy will not quite equal the maximum kinetic energy, as it should. Try varying the spring constant value slightly in the P.E. formula until the Total Energy plot becomes “flatter.”

9. Make a record of your measurements.

   • **Set Graph Title...** to ***OSCILLATOR ENERGY*** and add your names.

   • **Print** one copy of this graph for your group.

TA Discussion Checkbox

Questions:

• Were your predicted graphs for the potential energy and kinetic energy of the oscillating mass confirmed? Explain any discrepancies.

• Is energy conserved? Why or why not?

Activity 6: Oscillations Be Damped!

Predictions: • What do you predict will happen to the total energy of the oscillating mass after a very long time? Sketch your prediction in Figure 10. If you predict that the energy will change over time, try to provide a mathematical expression for the time-dependence.

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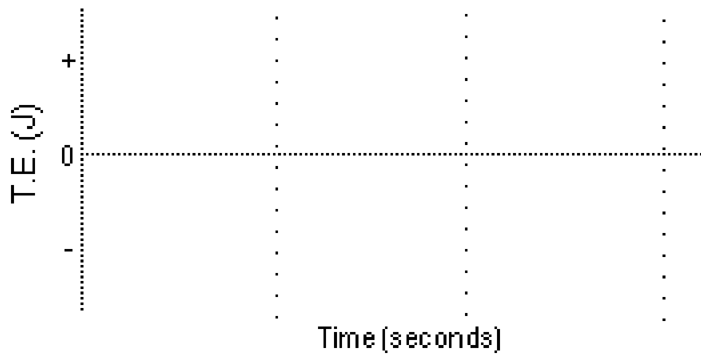

Figure 10. Prediction of the long-time behavior of the total energy

Procedure:

1. Test your prediction. First, change the graph format.
   - Change your present "three graph" display to a "two graph" display by selecting **Two Graphs** under the **Display** menu.
   - Change the top graph to plot *distance versus time*, and change the bottom graph to plot *total energy versus time*.
   - Change the time scale for both graphs to plot from 0 to 300 seconds by: (i) double-clicking on the top graph, (ii) changing the time-axis limits to 0 and 300 seconds, then (iii) clicking **Set All** then **OK**.
2. Select **Data Rate...** under the **Collect** menu, and change the Motion + Force probe collection rate from 20 points/second to *10 points/second*.
3. Make the measurement.
   - Keep the same mass and spring.
   - Gently push the mass up about 20 cm and let go. After the mass is oscillating freely, click on **Start** to begin graphing.
   - It will take several minutes to complete the experiment. Make sure you don't bump the table or otherwise interfere with the oscillating mass.

Questions: • Were your measurements of the total energy of the oscillator over a long time interval consistent with your predictions? Explain any discrepancies.

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Another Aside:

In real oscillating systems, there is always some drag force (or torque) present which will dissipate the total energy of the oscillator. Such drag forces are responsible for the loss of energy you just observed (presumably) in this activity. In many systems, the drag force is approximately proportional, and in the direction opposite, to the velocity,

$$F_{drag} = -bv = -b\frac{dx}{dt} \qquad \text{(Eq. 6)}$$

where b is a constant representing the strength of the drag force. The presence of this drag force modifies Newton's second law for the mass spring system written in Eq. 4 as:

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \qquad \text{(Eq. 7)}$$

The solution to this equation *for a small drag constant b* is

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \delta) \qquad \text{(Eq. 8)}$$

Notice that Eq. 8 describes sinusoidal motion, just as before, but with an amplitude that decreases exponentially with time. Notice also that because the total energy is proportional to the *square* of x(t) and v(t), Eq. 8 implies that the total energy should decrease (again, for small b) according to:

$$E(t) = E_{initial}e^{-bt/m} \qquad \text{(Eq. 9)}$$

Procedure: (continued)

4. Determine the mathematical relationship describing the total energy versus time.
   - Click on the energy versus time graph to select it for fitting.
   - In the spirit of Eq. 9, try to fit your data with an exponential fit

     $$E(t) = b_o e^{b_1 t}$$

     To do this, select **Fit...** under the **Analyze** menu.
   - Click **Exp**, then click on **Maintain Fit**.

5. Select **Fit Results...** under the **Analyze** menu to obtain the parameters associated with your exponential fit. Record these parameters below. Your results should have at least two significant figures.

   b_o = __________ b_1 = __________

6. Make a record of this experiment.
   - Set **Graph Title...** to ***Damped Oscillations***, and add your group's names.
   - Print out a single copy for your group.

Questions:

- How well is the total energy versus time for your simple harmonic oscillator described by an exponential decay, as described in Eq. 9?

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- Using the results of your fit, obtain a value for the drag parameter b in your experiment.

b = __________ [kg/s]

- What is the origin of the drag force(s) present in your experiment?

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