Physics 211 **Week 1**

One-Dimensional Kinematics: Bonnie and Clyde (Solutions)

In your new job, you are the technical advisor for the writers of a gangster movie about Bonnie and Clyde. In one scene Bonnie and Clyde try to flee from one state to another. If they get across the state line, they could evade capture, at least for a while until they become Federal fugitives. In the script, Bonnie is driving down the highway at 108 km/hr and passes a concealed police car that is 1 km from the state line. The instant Bonnie and Clyde pass the patrol car, the cop pulls onto the highway and accelerates at a constant rate of 2 m/s^2. The writers want to know if they make it across the state line before the pursuing cop catches up with them.

Conceptual Analysis:

- Since Bonnie and Clyde are traveling at a constant velocity and the cop is accelerating, at a certain place and time, the cop will overtake Bonnie and Clyde.

Strategic Analysis:

- Create two equations- one describing the motion of the accelerating cop and the other describing the motion of the constant velocity of Bonnie and Clyde.
- Pick one of two strategies to solve the problem. You can either
  a) find the place where they meet to see if it is within the state line or after it, or
  b) find the time it takes for each to reach the state line to see who reaches it first.
- Solve the two equations to find the place where the cop overtakes Bonnie and Clyde or who crosses the state line first, depending on which method you choose to use.

Quantitative Analysis:

Begin by labeling the given values:

v_{BC} speed of Bonnie and Clyde

d_{bord} distance from cop to border

a_c acceleration of cop

We will be looking for either

x_{BC} distance Bonnie and Clyde travel, and

x_c distance the cop travels, which are equal where the two cars meet

or, using the other method, we will find

t_{BC} the time it takes Bonnie and Clyde to reach the state line, and

t_c the time it takes the cop to reach the state line

Next we can create equations to describe the motion of Bonnie and Clyde and the cop.
If we choose the initial time and position of the cop to be $t_o = 0$ and $x_o = 0$, then we have the position of Bonnie and Clyde and the position of the cop after time t to be

BC: $x_{BC} = v_{BC}*t$

Cop: $x_c = .5*a_c*t^2$

- Now you can choose between the two methods as you solve the equations.
- Let's start by solving for the place that they meet. When they meet $x_{BC} = x_c$. Knowing this we can solve:

$$v_{BC}*t = .5*a_c*t^2$$

The time that they are in the same place, t_{same}, then, is

$$t_{same} = 2*v_{BC} / a_c$$

Now we can find the place where they are at this time by plugging the time back into either of the position equations:

$$x_c = x_{BC} = 2*v^2_{BC} / a_c$$

- The other method is to find the time it takes each party to reach the state line.

BC: $d_{bord} = v_{BC}*t_{BC}$

Cop: $d_{bord} = \_*a_c*t^2_c$

Then

$t_{BC} = d_{bord} / v_{BC}$

$t_c = sqrt\ (2*d_{bord} / a_c)$

- We now have all of our solutions in terms of the given quantities. Let's insert the numbers so we can see whether or not Bonnie and Clyde escape.
  (Make sure that your units match in the equations as you solve them (km/hr or m/s).)

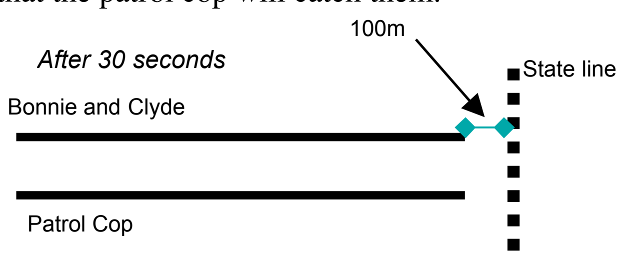
$$108\left(\frac{km}{h}\right)\times 1000\left(\frac{m}{km}\right)\times\left[\frac{1}{3600\left(\frac{s}{h}\right)}\right] = 30m/s$$

(Bonnie and Clyde's speed)

- Solving for the place where they meet:

$$x_c = x_{BC} = 2*v^2_{BC} / a_c = [2*(30^2 m/s)] / 2\ m/s^2 = 900m$$

You will find that the cop reaches Bonnie and Clyde 0.1*km* before the state line. Tell the writers that the patrol cop will catch them.



- Solving for the time it takes each to reach the state line:

$$t_{BC} = d_{bord} / v_{BC} = 1000m / (30m/s) = 33.3\ s$$

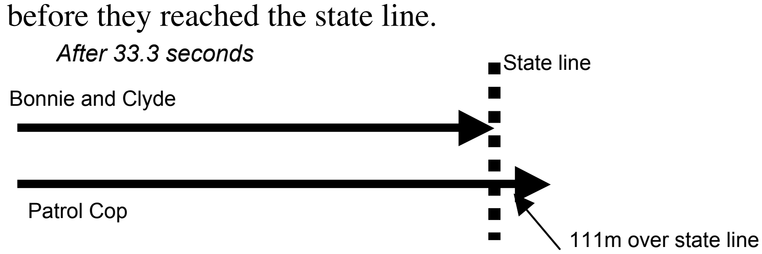
$$t_c = sqrt\ (2*d_{bord} / a_c) = sqrt\ [2*1000m / (2m/s^2)] = 31.6\ s$$

You can see that the cop takes less time to reach the border, which means that he will overtake Bonnie and Clyde on this side of the state line.

Another way to solve the problem is to see where the cop is when Bonnie and Clyde cross the state line. You can insert their time into the cop's distance equation. If he is still on this side of the state line, they make it.

$$x_c = \_*a_c*t^2 = \_*\ (2\ m/s^2)*(33.3\ s)^2 = 1111m$$

You will find that the cop goes 1111*m* in the same time that it takes Bonnie and Clyde to go 1000*m* to reach the state line; therefore, we know the cop must have passed Bonnie and Clyde some time before they reached the state line.



Physics 211 Week 1
One-Dimensional Kinematics: Catching the Train (Solutions)

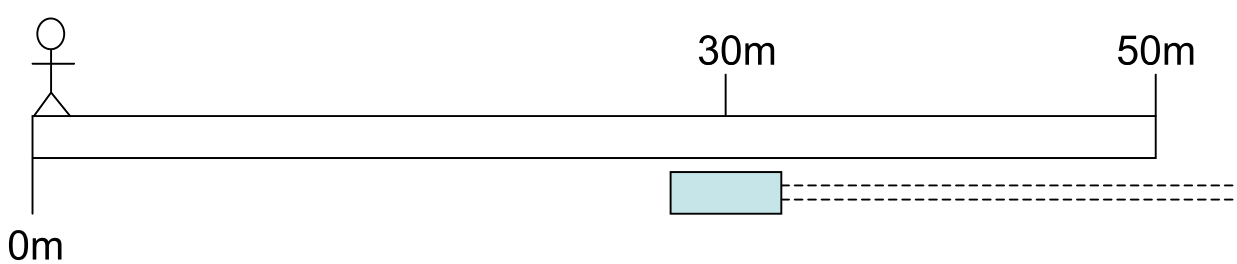
You are going to Chicago for the weekend and you decide to go first-class by taking the AmTrak train. Unfortunately, you are late finishing your mathematics exam, so you arrive late at the train station. You run as fast as you can, but just as you reach the platform your train departs, 30 meters ahead of you down the platform. You can run at a maximum speed of 8 m/s and the train is accelerating at 0.8 m/s^2. You can run along the platform for 50 meters before a barrier prevents you from going further. Will you catch your train?

Conceptual Analysis:
- The train moves with constant acceleration while you are running with a constant velocity.
- You will catch the train if you are at the same place at the same time before you reach the barrier.

Strategic Analysis:
- You want to the find the time and the place that you and the train meet each other.
- We can use kinematics to find equations describing the motion of you and of the train.
- You will need to set up a way of measuring distance that is consistent for the platform, you, and the train.

Quantitative Analysis:
- Label the given quantities:
  - v_y your speed
  - x_t initial position of the train
  - a_t acceleration of the train
  - x_p length of platform

  We are looking for
  - x the position that you and the train occupy at the same time
- Set up distances. The following diagram is one way.



- The motion of the train can be described by:

  $$x = x_o + v_o{*}t + \_{*}a{*}t^2$$

  inserting the values for the train:

$$x = x_t + \_*(a_t)*t^2$$

- Your motion can be described by:

$$x = x_o + v_o*t + \_*a*t^2$$

inserting your values:

$$x = v_y*t$$

- You now have two equations and two variables. You can utilize the quadratic formula to find two solutions.

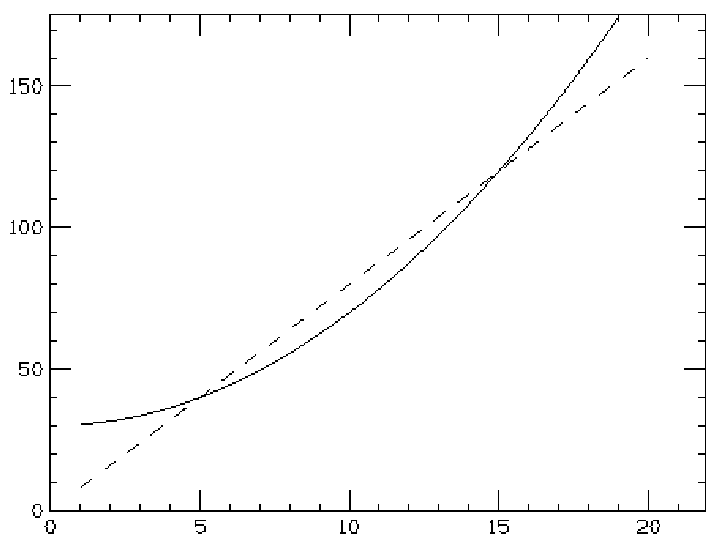
$$t = x/ v_y$$
$$x = x_t + \_*(a_t)*(x/ v_y)^2$$
$$0 = [\_*(a_t/ v_y^2)]* x^2 - x + x_t$$

$$X = \frac{1 \pm \sqrt{(-1)^2 - 4*1/2*a_t/v_y^{2}*x_t}}{2*1/2*a_t/v_y^2}$$

$$X = \frac{1 \pm \sqrt{(-1)^2 - 4*1/2*(0.8\ m/s^2)/(8m/s)^2*30m}}{2*1/2*(0.8\ m/s^2)/(8m/s)^2}$$

$x = 120m$ or $40m$

- You can determine the amount of time it takes to reach the train by inserting your new distance back into one of your motion equations. You will catch the train after 5 seconds at 40*m*. If there were no barrier and the train continued to accelerate at a constant rate, it would pass you after 15*s* when you had run 120*m*.
- Because the motion of an accelerating object on a distance versus time graph is given by a quadratic and the constant velocity motion has a linear representation on a distance versus time graph, you and the train cross paths twice: once before the barrier and once after, as shown by the graph.



We are only concerned that you catch the train before the 50m barrier; we are not concerned with the solution at 15s, 120m.

You will catch the train after 5 seconds; 40m from your starting position (10m before the barrier).

Physics 211 - Week 1

One-Dimensional Kinematics: Falling Brick (Solutions)

As you are cycling to classes one day, you pass a construction site on Green Street for a new building and stop to watch for a few minutes. A crane is lifting a batch of bricks on a pallet to an upper floor of the building. Suddenly, a brick falls off the rising pallet. You clock the time it takes the brick to hit the ground at 2.5 seconds. The crane, fortunately, has height markings, and you see the brick fall off the pallet at a height of 22 meters above the ground. A falling brick, as we all know, can be dangerous, and you wonder how fast the brick was going when it hit the ground. Since you are taking physics, you quickly calculate the answer.

A strategy: Using the 22 m height and 2.5 sec fall time, compute the initial velocity of the brick using the expression for position versus time under uniform acceleration. Using the expression for velocity as a function of time for constant acceleration, and your expression for the initial velocity, to calculate the velocity after 2.5 seconds.

- **Begin by computing the time it takes the brick to hit the ground assuming an instantaneous upward velocity of V_o and a downward acceleration of gravity.**

$$0 = -(1/2)\ g\ t\_ + V_o\, t + h \quad \text{(up is positive direction)}$$

Solving for V_o we have:

$$V_o = \frac{1/2\ g\ t^2 - h}{t}$$

Inserting these values:

g	t	h
9.81 m/s_	**2.5 s**	**22 m**

We obtain a velocity of V_o = 3.46 m/s

- **Next we calculate the velocity of the brick striking the ground**

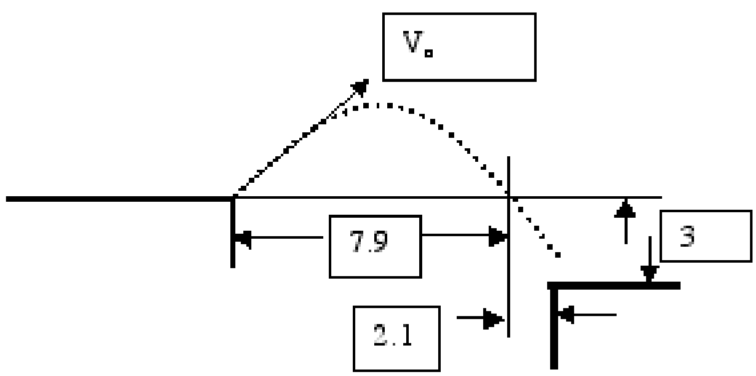
$$V(t=2.5\ s) = V_o - g\ t = 3.46\ m/s - 9.81\ m/s\_\ (2.5\ s) = -21.06\ m/s$$

Physics 211 - Week 1

Two-Dimensional Kinematics: Escape from Burning Building (Solution)

Your friend, a world-class long jumper, is trapped on the roof of a burning building. His only escape route is to jump to the roof of the next building. Fortunately for him, he is in telephone contact with you, a Physics 111 student, for advice on how to proceed. He has two options. He can jump to the next building by using the long-jump technique where he jumps at 45^o to the horizontal. Or, he can take his chances by staying where he is in the hopes that the fire department will rescue him. You learn from the building engineers that the next building is 10 m away horizontally and the roof is 3 m below the roof of the burning building. You also know that his best long-jump distance is 7.9 m . What do you advise him to do?

A strategy: Use the information on his best long-jump of 7.9 meters to compute his x component of velocity on a long-jump. Compute the extra flight time it will take him to sail the extra 10 - 7.9 = 2.1 meters to the next roof with the long-jump V_x. Compute the distance he will fall from height of the original roof during this extra flight time. If this distance exceeds 3 meters, he won't make the roof:



We begin by computing V_x which is the x component of the initial velocity. In the long-jump, described here, the initial velocity has equal x and y components ($V_x = V_y$). At maximum height, the y component of velocity is zero.

Thus $0 = V_y - g\ t$ which means $t = V_y/ g$.

In a normal long jump, the total flight time is 2 t and the range covered is :
Dist=7.9 = $(2V_y/ g) V_x = 2 V_x^2/g$ Hence

$$V_x = \sqrt{.5(d)g} = \sqrt{.5(7.9\,m)(9.8\,m/s^2)} = 6.22\,m/s$$

The extra time for the jumper to cover the remaining 2.1 meters is

$$\Delta t = 2.1 \text{ m}/6.22 \text{ m/s} = 0.338 \text{ s}$$

In that time, he will fall an additional distance of:

$$\Delta y = -(1/2)\ g\ \Delta t^2 - V_y \Delta t = -(1/2)\ 9.81 \text{ m/s } (0.338 \text{ s})^2 - (6.22 \text{ m/s}) (0.338 \text{ s}) = 2.663 \text{ m}$$

He thus will still have some distance to fall once he has cleared the additional 2.1 meter horizontal distance.

Physics 211 - Week 1

One-Dimensional Kinematics: Skydivers

The U of I Skydiving Club has asked you to plan a stunt for an air show. In this stunt, two skydivers will step out of opposite sides of a stationary hot air balloon 5,000 feet above the ground. The second skydiver will leave the balloon 20 seconds after the first skydiver, but you want them both to land on the ground at the same time. The show is planned for a day with no wind so you may assume all motion is vertical. To get a rough idea of the situation, assume that a skydiver will fall with a constant acceleration of 32 ft/sec^2 before the parachute opens. As soon as the parachute is opened, the skydiver falls with a constant speed of 10 ft/sec. If the first skydiver, Sue, waits 3 seconds after stepping out of the balloon before opening her parachute, how long must the second skydiver, Joe, wait after leaving the balloon before opening his parachute?

A strategy: Begin by computing the time it takes Sue to hit ground after pulling chute and add this time to 3 seconds for the total time Sue falls. Subtract 20 seconds from this time for the total time Joe falls. Call this time T_{JOE}, Write an equation for the time that Joe delays pulling chute based on accelerating for t' seconds and traveling with a constant velocity for T_{JOE} - t' seconds and traveling for a total fall of 5000 m. Solve this equation for t'.

Compute the time for Sue to hit the ground after pulling the chute at t' seconds. She accelerates from rest rate of 32 f/s_ for 3 seconds and then travels with a constant velocity for t seconds until she reaches a total fall distance of 5000 f.

$$5000 = \frac{32}{2}(3^2) + 10t \Rightarrow t = 485.6s \Rightarrow T_{Sue} = 485.6 + 3 = 488.6\ s$$

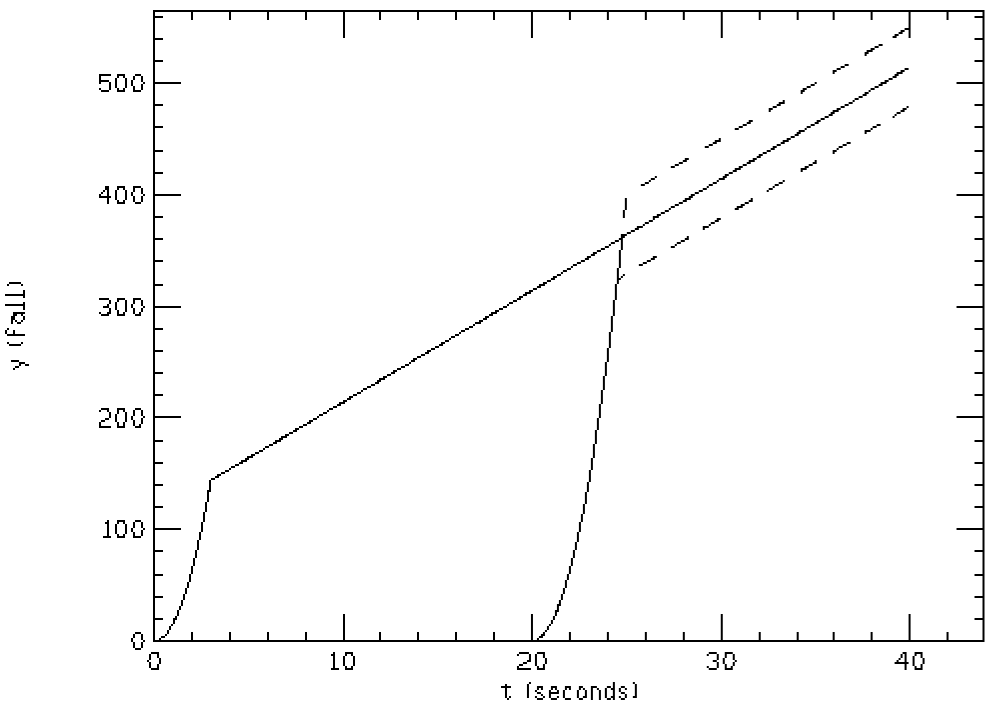
$$T_{JOE} = 488.6 - 20 = 468.6s$$

$$5000 = \frac{32}{2}(t'^2) + 10\,(468.6 - t')$$

We next subtract 20 seconds from this since Joe starts 20 seconds later.
Joe accelerates for t' seconds and moves with a uniform velocity for T_{JOE} - t' seconds to cover 5000 f.
Rearranging this equation to put it in the quadratic form and solving with the quadratic equation, we get:

$$16t'^2 - 10t' - 314 = 0 \Rightarrow t' = -4.1285, +4.7535$$

Joe should wait 4.754 seconds before deploying his chute.
Here is a plot of Sue and Joe's fall distance if he deploys his chute earlier , later, or at 4.754 seconds.



We note the 20 second delay. If Joe opens his chute on time he basically falls at the same position as Sue after roughly 25 seconds and they land together.