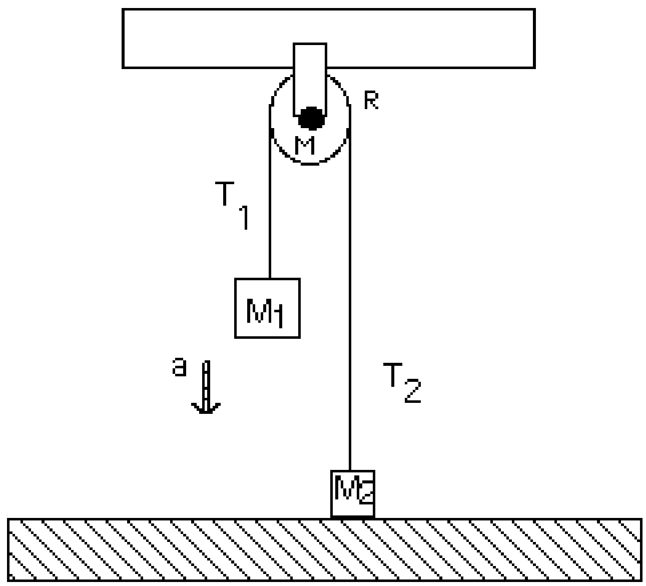
Physics 211 Week 9

Rotational Dynamics: Atwood's Machine Revisited

Consider a realistic Atwood's machine where the pulley is not massless. Instead it is a disk of radius 0.1 m and mass M=3 kg. The heavier weight has mass M_1=5 kg and the lighter weight has mass M_2= 2 kg. The system is released from rest when the lighter mass is on the floor and the heavier mass is 1.8 m above the floor. How long does it take the heavier mass to hit the floor?



Conceptual Analysis:

- The system accelerates so that mass 1 will move down, mass 2 will move up, and the pulley will rotate counter clockwise.
- The acceleration is the same for each object in the system.
- The tensions in the ropes are not the same, if they were, the pulley would not rotate.

Strategic Analysis:

- Draw a free body diagram for each object and create net force and torque equations for each.
- Solve the equations for the acceleration.
- Use the acceleration and given height to find the time it takes for the mass to fall.

Quantitative Analysis:

- Begin by labeling the given values:

  M_1 mass 1
  M_2 mass 2
  M mass of the pulley
  h height of mass 1 above the floor before the system is released
  R radius of the pulley

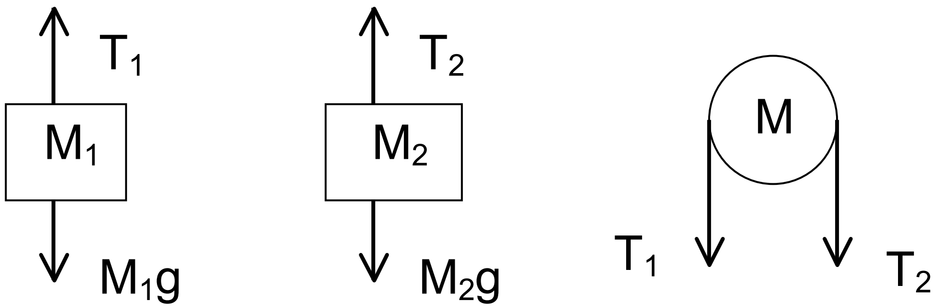
  we will also use

  T_1 the tension in the string connecting mass 1 and the pulley
  T_2 the tension in the string connecting mass 2 and the pulley
  a the acceleration of the system

  We are looking for

  t the time it takes for mass 1 to hit the floor

- We'll begin by drawing a free body diagram for each mass.



- Next, we'll write net force equations for masses 1 and 2 and a net torque equation for the pulley.

For mass 1:

$$M_1 g - T_1 = M_1 a$$

For mass 2:

$$T_2 - M_2 g = M_2 a$$

For the pulley:

$$T_1 R - T_2 R = I\alpha = \frac{Ia}{R} = \frac{1}{2} MaR$$

$1/2\ Ma = T_1 - T_2$

- We can combine all three of the equations we just found and solve for the acceleration.

$$M_1 g - M_2 g = a\left(M_1 + M_2 + \frac{1}{2} M\right)$$

$$a = \frac{(M_1 - M_2)g}{M_1 + M_2 + \frac{M}{2}}$$

- The time can be found using our old kinematics equations

$$y = \frac{1}{2} at \Rightarrow t = \sqrt{\frac{2y}{a}}$$

- Plugging in our value for acceleration that we solved for, we obtain

$$t = \sqrt{\frac{2h \ \ M_1 + M_2 + \frac{M}{2}}{(M_1 - M_2)g}}$$

- Then plugging in our values:

$$t = \sqrt{\frac{2^* (1.8) ^*[5+2+(3/2)]}{9.81^*(5-2)}} = 1.02 \text{ seconds}$$

So the heavier mass would reach the ground in 1.02 seconds.

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Work and Kinetic Energy: Block Slide (Solutions)

In an effort to combine several aspects of her recent physics lectures, an enterprising student poses for herself the following question. An unstretched spring is attached to a 1.5 kg block on a ramp which makes an angle of 30° with respect to the horizontal. The other end of the spring is fixed. The mass is released and it slides down the ramp and stretches the spring. There is friction between the block and the ramp with a coefficient of 0.3. The spring has a constant of 30 N/m. Undaunted by the complexity of her problem, she computes the maximum distance that the block slides down the ramp. What is her answer?

Conceptual Analysis:

- This is a work-energy problem.
- The system is affected by gravitational potential energy, spring potential energy, and work done by friction.
- Since the block begins and ends with zero motion, there is no kinetic energy.
- The distance the spring stretches is the same as the distance over which the friction force acts. The distance over which the gravitational force acts is the vertical component of this distance.
- The gravitational potential energy becomes spring potential energy and work done by friction.
- The work done by the frictional force is negative in this case.

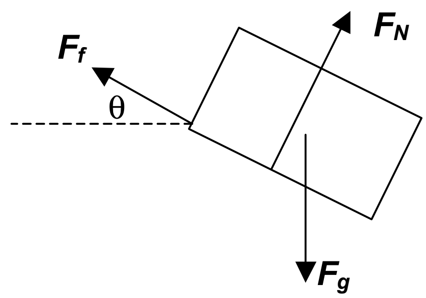
Strategic Analysis:

- Find the force of friction using Newton's second law.
- Find the work done by the non-conservative frictional force, in terms of the block's sliding distance.
- Find the change in gravitational potential energy when the block slides the maximum distance.
- Find the change in spring potential energy at the maximum distance the block slides down the incline.
- Use the relationship described in lecture: work done by non-conservative forces is equal to the sum of the changes in kinetic and potential energies.
- Solve the work- energy equation to determine the maximum distance the block slides down the incline.

Quantitative Analysis:

- Begin by labeling the given quantities:
  - m mass of block
  - θ angle of incline
  - μ coefficient of kinetic friction
  - k spring constant

  We are looking for
  - x the maximum distance that the block slides down the incline
- Draw a free body diagram to determine the force of friction.



- The force of friction is given by:

  $F_f = \mu F_N$

  By examining the free body diagram, we see that the normal force is equal to the vertical component of the gravitational force. Substituting this value into our frictional force equation we find

  $F_f = \mu F_N = \mu mg*\cos(\theta)$

- Find an expression for the work done by friction. The force of friction acts in the direction opposite to the slide of the block so the work done by friction is negative.

  $W_f = -F_f*d$

  $W_f = -\mu mg*\cos(\theta)*x$

  The work done by friction is the work done by non-conservative forces in this problem.

- Determine the change in gravitational potential energy. Since the block is sliding to a position closer to the ground, our gravitational potential energy will decrease.

  $\Delta GPE = -mgH$

  $H = x*\sin(\theta)$ (we want only the component perpendicular to the force of gravity)

  $\Delta GPE = -mgx*\sin(\theta)$

- Find the change in spring potential energy. The spring is extending beyond equilibrium and therefore gaining potential energy.

  $\Delta SPE = \_\ k*x^2$

- Using our equation from lecture:

  $W_{NC} = \Delta K + \Delta U$

  We know the change in kinetic energy is zero since the block begins and ends at rest. The frictional force does non-conservative work, and both the spring potential energy and gravitational potential energy change as the spring slides down the incline.

- Inserting the appropriate expressions, we have

  $-\mu mg*\cos(\theta)*x = \_\ k*x^2 - mgx*\sin(\theta)$

  The equation can be solved for the maximum distance, x:

  $x = [2mg/k]*[\sin(\theta) - \mu*\cos(\theta)]$

- Finally, insert the values given in the problem setup:

  $x = [2*(1.5\ kg)*(9.81\ m/s^2) / (30\ N/m)]*[\sin(30^o) - 0.3*\cos(30^o)]$

  $x = 0.24\ m$

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Rotational Kinematics and Energy: Two Blocks and a Pulley

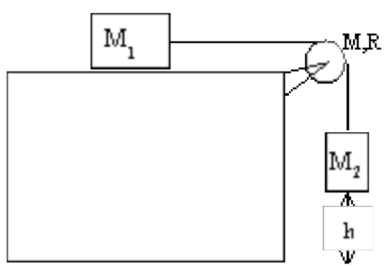
Block 1 (mass M_1) rests on a horizontal surface. A horizontal string is attached to the block, passing over a pulley to a hanging block having mass M_2 which hangs vertically a distance h from the floor. The pulley is a uniform cylinder of mass M and radius R. The string has negligible mass and the pulley has no friction. The string does not slip on the pulley. The coefficient of sliding friction between block 1 and the horizontal surface is m. The whole system is held in place, then released from rest. What is the speed of block 2 just before it hits the floor?

Conceptual Analysis:

- The work done on the system is equal to the change in kinetic energy.
- Work is done by friction and gravity.
- All three masses, the two blocks and pulley, have kinetic energy just before block 2 hits the floor.
- Since the string neither breaks nor slacks, the objects all must move the same distance, *h*, and have the same velocity.

Strategic Analysis:

- Find the work done by the force of friction and by the force of gravity.
- Create expressions for the kinetic energy of the each object in terms of the velocity.
- Solve the work-energy equation for the desired velocity.



Quantitative Analysis:

- Begin by labeling the given quantities:
  M_1 mass of block 1
  M_2 mass of block 2
  M mass of the pulley
  R radius of the pulley
  h vertical distance of block 2 from the floor
  μ coefficient of kinetic friction between the table and block 1

  We are looking for
  v the speed of block two just before it hits the ground
- The work done by friction is the force of friction times the distance over which it acts. Friction only acts on block 1. It does negative work since the force of friction points in the direction opposite to the motion of the block.

  W_f: $F_f * d = -\mu M_1 g\, h$

- The work done by gravity is found in the same way. The force of gravity will do work only on block 2 since the motion of block 1 is perpendicular to the force of gravity. Gravity does positive work because the force of gravity is in the same direction as the displacement of block 2.

  W_g: F_g*d = M_2g h
- The kinetic energy of block 1 is found by using the general expression for kinetic energy with the mass of block 1

  $KE_{M1} = 1/2\ M_1 * v^2$
- The kinetic energy of block 2, then is

  $KE_{M2} = 1/2\ M_2 * v^2$
- The kinetic energy of the pulley is found by using the pulley's values in the general expression for rotational kinetic energy

  $KE_{rot} = 1/2\ I*\omega^2$

  $KE_{pulley} = 1/2\ (1/2\ MR^2)*(v^2/R^2) = 1/4\ M*v^2$
- Now we can use our expressions to fill in and solve the work-energy equation.

  $\Delta K.E. = W_{total}$

  $KE_{M1} + KE_{M2} + KE_{pulley} = W_f + W_g$

  $1/2\ M_1 * v^2 + 1/2\ M_2 * v^2 + 1/4\ M * v^2 = -\mu M_1 g h + M_2 g h$

  $1/2\ v^2 (M_1 + M_2 + 1/2\ M) = gh (M_2 - \mu M_1)$

$$v^2 = \frac{2gh(M_2 - \mu M_1)}{M_1 + M_2 + \frac{1}{2}M}$$

$$v = \sqrt{\frac{2gh(M_2 - \mu M_1)}{M_1 + M_2 + \frac{1}{2}M}}$$

The speed block 2 will have just before it hits the ground is given by v.

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Rotational Kinematics and Energy: Bowling Ball

Two identical bowling balls are rolling on a horizontal floor without slipping. The initial speed of both balls is V = 9.9m/s. Ball A encounters a <u>frictionless</u> ramp, reaching a maximum vertical height H_A above the floor. Ball B on the other hand rolls up a regular ramp (i.e. <u>without</u> slipping), reaching a maximum vertical height H_B above the floor. Which ball goes higher, and by how much?

This problem can be solved using conservation of energy. Both balls begin with rotational and translational kinetic energy. At the top of the frictionless ramp (A), there has been no outside torque to stop the rotation of the ball, so Ball A has both rotational kinetic energy and gravitational kinetic energy at the height H_A. Ball B, however, has friction on the ramp to slow the rotation so that at height H_B, Ball B has only gravitational potential energy; all of the original kinetic energy, both rotational and translational, is now gravitational potential energy. Ball B, therefore, has more gravitational potential energy than Ball A has. Since gravitational potential energy is directly proportional to height, H_B must be greater than H_A; Ball B goes higher than Ball A. Write and solve appropriate conservation of energy equations for each ball and insert the given initial velocity of the balls; you will find that Ball B goes 2 *m* higher than Ball A.

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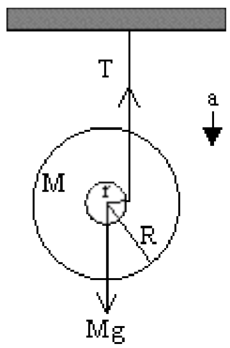
Rotational Statics and Dynamics: Yo^2

A yo-yo has mass M and outer radius R. The central stem has negligible mass and radius r and string of negligible mass is wrapped around it. The string is taut and held vertically and the yo-yo is released from rest. How long does it take for the yo-yo to hit the ground, which is a height h above the initial position?



The yo-yo will have both angular and translational acceleration. To find the translational acceleration, create a net force equation. You will also need a torque equation to relate the torque from the tension in the string to the angular acceleration of the yo-yo. The moment of inertia for the yo-yo can be found by treating it as a large cylinder since the central stem has negligible mass. The angular acceleration can be simply related to the translational acceleration by the inner radius r. Once you obtain the translational acceleration, with the given height, you can use kinematics to find the time it takes the yo-yo to fall. You should obtain a time of

$$\sqrt{\frac{2h}{g}\left(1+\frac{R^2}{2r^2}\right)}$$ t = seconds.