Name ______________________________

Section ____________ Date ___________

Physics 211 PreLab #6: Moments of Inertia

We're Rollin' Now

You have studied the equations describing translational motion (falling, sliding, bouncing, etc.). In this lab, you will study in greater detail the kinematics and dynamics of rotating bodies. The equations you will use are straightforward extensions of the equations you already know for translational motion.

The new idea you will study in this laboratory is that of the *moment of inertia* of an object. Mathematically, the moment of inertia I relates an applied torque τ to the resulting angular acceleration α through the equation,

$$\tau = I\alpha \qquad \text{(Eq. 1)}$$

The moment of inertia of an object provides a measure of how hard it is to change that object's rotational velocity. Thus, the moment of inertia is to rotational motion what the mass of an object is to translational motion.

The moment of inertia depends in general about *which* axis the object is rotated. Moments of inertia for spheres and cylinders (about the principal axes) can be written $I = \eta MR^2$, where η is a constant which is dependent upon the object's mass distribution, M is the object's mass and R is the object's radius. This dependence on R tells you that the farther away from the axis of rotation the mass of an object is located, the harder it is to alter the object's rotational velocity. Figure 1 provides a list of moments of inertia for several standard shapes in the following instances: *Spheres* - for rotation about an axis through the center of the sphere; *Cylinders* - for rotation about the long axis of the cylinder.

By rolling objects of various shapes down an incline in Investigation 1 (see Figure 1), you will study how various properties of the object influence the object's acceleration.

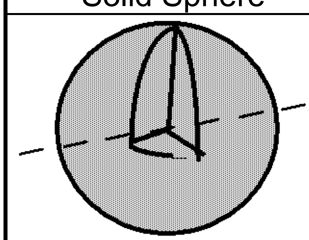
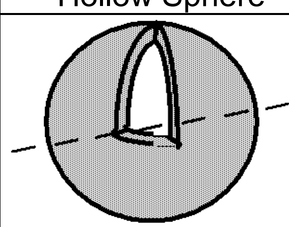
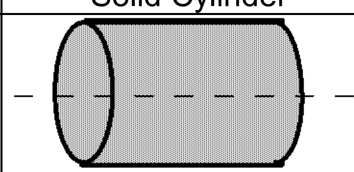
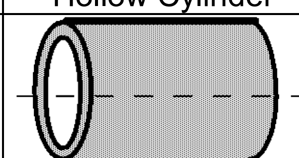
Solid Sphere	Hollow Sphere	Solid Cylinder	Hollow Cylinder
			
$\frac{2}{5}MR^2$	$\frac{2}{3}MR^2$	$\frac{1}{2}MR^2$	MR^2

Figure 1. Moments of inertia for spherical and cylindrical objects

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Answer the following question concerning Investigation 1.

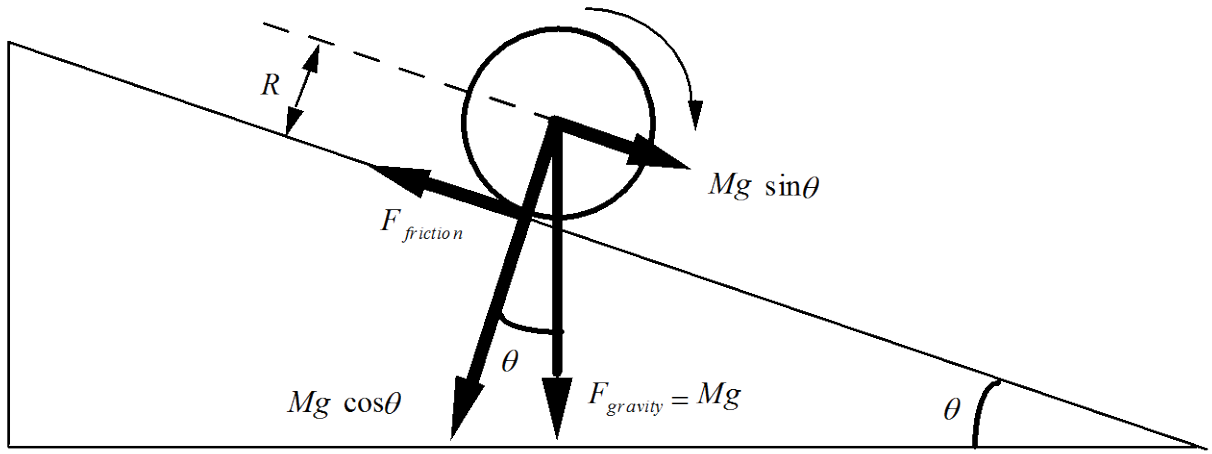

Figure 2. A rolling object of mass *M*, radius *R*, and moment of inertia *I*

Consider an object rolling down an incline of angle θ as shown in Figure 2. The following equations apply (make sure you can derive these):

(1) The sum of the forces yielding the object's translational acceleration *a* along the ramp is given by

$$\Box F = Mg\ \sin\theta\ - F_{friction} = Ma$$

(2) The sum of the *torques* providing the object's rotational acceleration α about its center of mass can be written:

$$\sum \tau = F_{friction} R = I\alpha$$

(3) Because the object rolls without slipping, one also has the following relationship between the translational and rotational accelerations

$$a = R\alpha$$

Q1 - Using the information above, derive a relationship for the translational acceleration of the object as it rolls down the incline, in terms of g, θ, I, M and R.

Q2 - A wheel of mass $M = 1.2$ kg and radius $R = 0.4$ m rolls down an incline having an angle $\theta = 10°$. The translational acceleration of the wheel is measured to be 0.9 m/s^2. What is the moment of inertia of the wheel?

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In Investigation 2 you will study the energetics of different objects rolling down an incline, in order to see how shape influences the rotational and translational contributions to an object's kinetic energy.

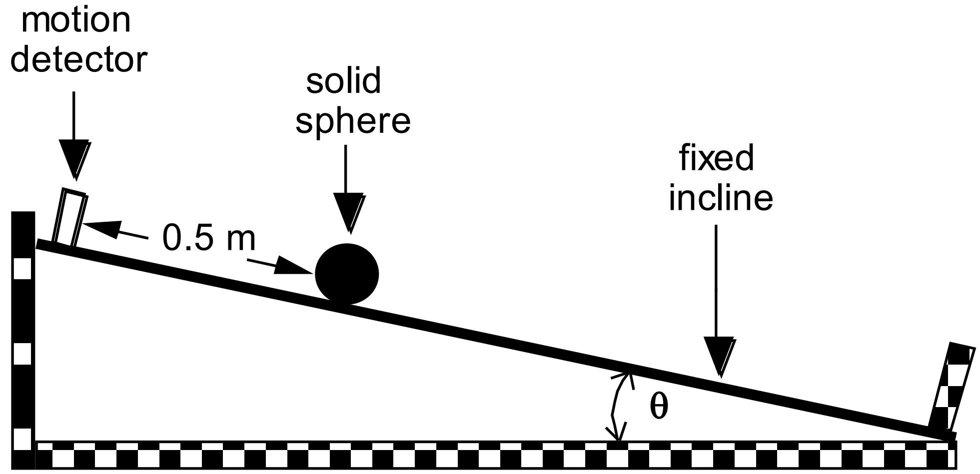

Figure 3. Experimental setup for Investigation 2

Answer the following questions regarding Investigation 2.

Q3 - Consider an object of mass m rolling down an incline of angle θ as shown in Figure 3. In terms of known parameters m, g, η, θ, and D, determine the velocity of the object after it has rolled a distance $D = d_2 - d_1$ meters down the incline from its stationary initial position. Assume that the object's potential energy is zero at the height of the detector.

HINT: The following facts apply (see Figure 4):

(1) Energy is conserved, $E_{initial} = E_{final}$.

$$\begin{array}{ccc} E_{init} & & E_{final} \\ -mgd_1\sin\theta & = & -mgd_2\sin\theta + {}^1/_2mv^2 + {}^1/_2I\omega^2 \end{array}$$

where $I = \eta mR^2$ is the moment of inertia of the object, $\eta = I/mR^2$ is the constant prefactor determined by the object's shape, v is the translational velocity of the object, and ω is the angular velocity.

(2) The rolling objects roll without slipping, $\omega = {}^v/_R$

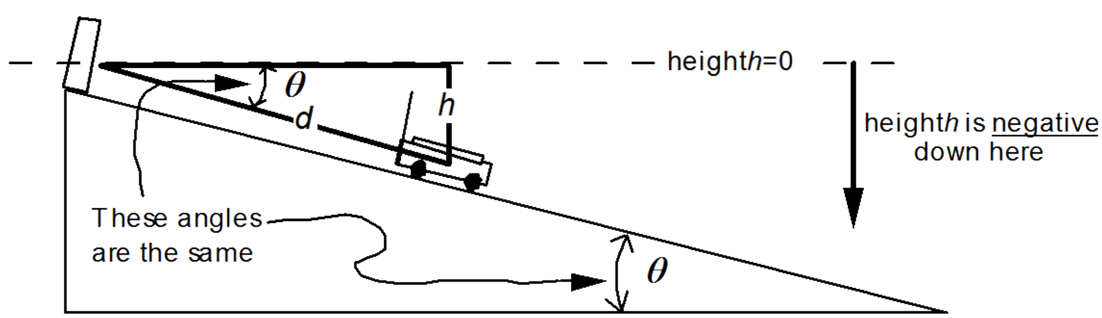

Figure 4. Experimental parameters for Question 3

Q4 - A solid sphere and hollow cylinder are rolled down an incline of angle θ (η = 0.4 and 1 for a solid sphere and hollow cylinder, respectively). What is the ratio of the velocities of the sphere and cylinder, $v_{sphere}/v_{cylinder}$, after they have rolled the same distance D down the incline?