Introduction to Random Uncertainty

Scientists routinely take many measurements of the same quantity, each measurement giving a slightly different result due to small random changes between experiments.

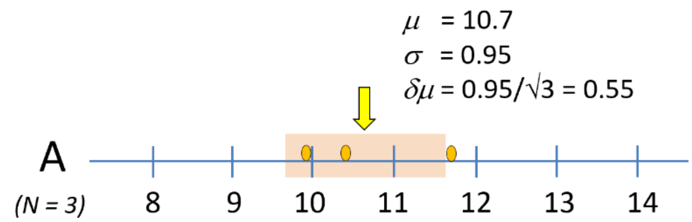
The **mean** of these measurements (often denoted by the symbol μ) is usually assumed to be the best estimate of the measured quantity. The mean is just the sum of the measurements divided by the number of measurements.

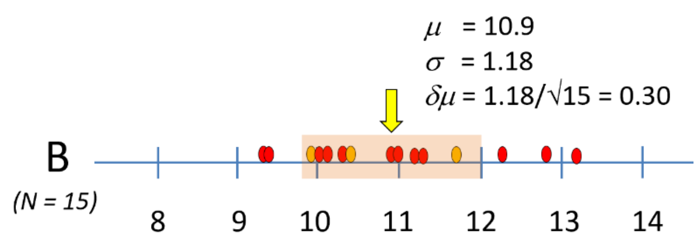
The **standard deviation** of the measurements (often denoted by the symbol σ) tells us how spread out the individual measurements are. For a normal random distribution of measurements, about two thirds of the measurements will lie within one standard deviation of the mean value (in other words, in the range between $(\mu - \sigma)$ and $(\mu + \sigma)$). We will show you how to estimate σ on the next page.

The **standard error** (which we will denote by $\delta\mu$) is the best estimate of the uncertainty on the mean. For a normal random distribution of measurements, there is about a two thirds chance that true mean will lie within one standard error of the measured mean (in other words, in the range between $(\mu - \delta\mu)$ and $(\mu + \delta\mu)$). For a normal random distribution of measurements there is a simple relationship between the standard error and the standard deviation: by $\boldsymbol{\delta\mu = \sigma/\sqrt{N}}$, where N is the number of measurements you have made. In other words, the more measurements you make the smaller the uncertainty on the mean will be, which seems sensible.

The figures on the right illustrate these quantities. The oval dots on the number-line represent individual measurements of some quantity (perhaps the time in seconds it takes a ball dropped from a tall building to hit the ground.)

The top diagram (A) shows the first 3 measurements (10.38, 9.91, 11.74), and the bottom diagram (B) shows what happens when 12 more measurements are added (13.22, 11.28, 12.26,10.88, 10.96, 12.79, 10.34, 11.35, 9.34, 9.39, 10.00, 10.09).





In both diagrams the yellow arrow shows the mean of the measurements μ, and the shaded orange region shows the range between $(\mu - \sigma)$ and $(\mu + \sigma)$, where σ for each set of numbers was calculated with Excel using the STDEV.S function (we will show you an easier way to find σ on the next page). In both cases $\delta\mu$ is calculated, and as you can see it gets smaller when N is increased.

In this example the ***true*** mean and standard deviation used to create the simulated measurements was $\boldsymbol{\mu = 11.0}$ and $\boldsymbol{\sigma = 1.0}$, so we see that we can get reasonable estimates of these "true values" with a rather small number of measurements.

Estimating Standard Deviation

You are welcome to use any formulas or web pages or other tools you may be familiar with whenever you need to calculate the standard deviation σ of a set of measurements.

In this section we give you a simple way to estimate σ on your own without doing any fancy math. You should use whichever method you like the most – just be sure to describe what you did when you write your lab report.

a) If you have small number of measurements (like 3 or 4) you can estimate the standard deviation by finding the range of the measurements and dividing by 2. The range of a set of numbers is just the difference between the smallest and largest numbers in the set.
   As an example, using this method for case A we get $\sigma = (11.74 - 9.91)/2 = 0.91$, which is close to the value obtained with Excel. **This is a great method to use in Physics 211 this semester.**

b) If you have more than 3 or 4 measurements to deal with you can estimate the standard deviation to be half the range of the 2/3 of the measurements that are closest to the average. This method is based on the assumption that about 2/3 of the measurements in range between $(\mu-\sigma)$ and $(\mu+\sigma)$ for a normal distribution. Let's try this method for case B. Finding the 2/3 of the measurements closest to the mean is the same as ignoring the 1/3 that are furthest from it. A third of 15 is 5, so we want to identify the 5 measurements that are furthest from the average. Looking at the diagram we see that the two left-most dots (representing 9.34 and 9.39) and three right-most (representing 13.22, 12.79 and 12.26) are further away from the average than the remaining dots. If we ignore these five measurements then half the range of the remaining numbers gives us $\sigma = (11.74 - 9.91)/2 = 0.91$, which is again quite close to the value obtained with Excel.

If you have a lot of measurements to deal with it may be easiest just to cut and paste these into a spreadsheet or to go to any of the many websites that can help you calculate average and standard deviation. **It doesn't matter how you do it in your lab reports as long as you tell us what you did.**

Always quote an uncertainty on any number you measure.

In Physics 211 you will often make repeated measurements of a quantity that you assume is constant and will find the mean μ and uncertainty on the mean $\delta\mu$ using your measurements.
<u>The proper way to present your result is in the form $\mu \pm \delta\mu$.</u>

For example, in example A shown on the previous page your lab report should say something like:
"The best estimate of the time taken for the ball to hit the ground is ***10.7 ± 0.55*** seconds."

And in example B you it should say something like:
"The best estimate of the time taken for the ball to hit the ground is ***10.9 ± 0.30*** seconds."