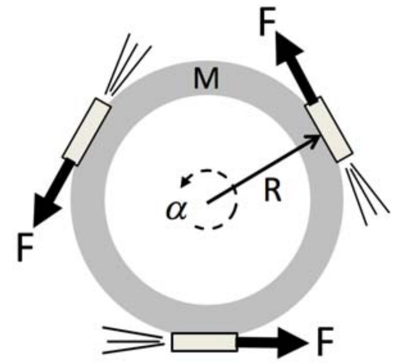
The next four questions pertain to the situation described below.

A hoop-shaped space station is initially at rest in outer space. The mass of the space station is $M = 5\times10^5\ kg$ and its radius is $R = 70\ m$. The station's chief engineer decides to generate artificial gravity to make life easier for the crew. She starts the rotation of the space station by firing three rockets evenly spaced on the outer edge of the hoop, as shown. Each rocket provides a tangential force of F, causing the space station to start rotating around an axis through its center with a constant angular acceleration $\alpha = 7\times10^{-4}\ rad/s^2$.



1) What is the magnitude of the force provided by each of the rockets?

a. 1.715×10^6 N
b. 4083 N
c. 571667 N
d. 8167 N
e. 24500 N

2) What is the angular velocity of the space station after it has made one complete revolution ?

a. 0.066 rad/s
b. 0.094 rad/s
c. 0.026 rad/s
d. 0.047 rad/s
e. 0.009 rad/s

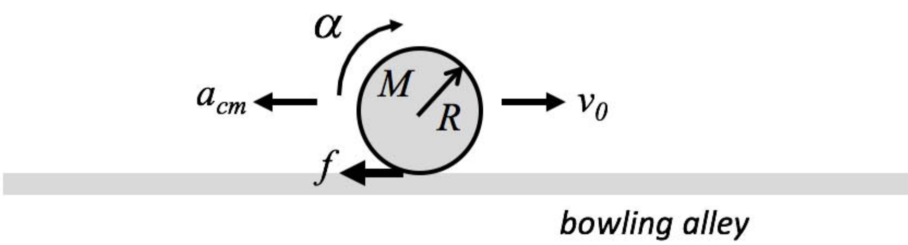
3) From the time that the rockets start pushing, how long does it take until the centripetal acceleration inside the space station (a distance R from the rotation axis) is equal to g, the acceleration of gravity on the surface of the Earth?

a. 11952 seconds
b. 756 seconds
c. 14 seconds
d. 535 seconds
e. 926 seconds

4) Suppose the answer to the previous questions is t_g. If we had instead wanted the centripetal acceleration inside the space station to be half as big, $g/2$, how long would that have taken?

a. $t_g/\sqrt{2}$
b. $t_g/4$
c. $t_g/2$

The next three questions pertain to the situation described below.



A bowling ball has mass M, radius R, and moment of inertia $I = \frac{2}{5}MR^2$ about an axis through its center of mass. The ball is thrown down a horizontal bowling alley in such a way that the initial speed of the center of mass of the ball is v_0 and the initial angular velocity of the ball around its center of mass is zero. As the ball slides down the bowling alley the magnitude of the force of kinetic friction between the ball and the floor is f, the magnitude of the angular acceleration of the ball is α, and the magnitude of the acceleration of center of mass of the ball is a_{cm}.

5) While the ball is sliding, but before it starts to roll without slipping, what is the magnitude of the angular acceleration of the ball around an axis through its center of mass?

a. $\alpha = a_{cm}/R$
b. $\alpha = 5f/(2MR)$
c. $\alpha = 7f/(2MR)$
d. $\alpha = 7f/(5MR)$
e. $\alpha = 7f/(5MR^2)$

6) While the ball is sliding, but before it starts to roll without slipping, which of the following statements best describes the relationship between the magnitude of the velocity of the center of mass V and the magnitude of the angular velocity of ball around an axis through its center of mass ω?

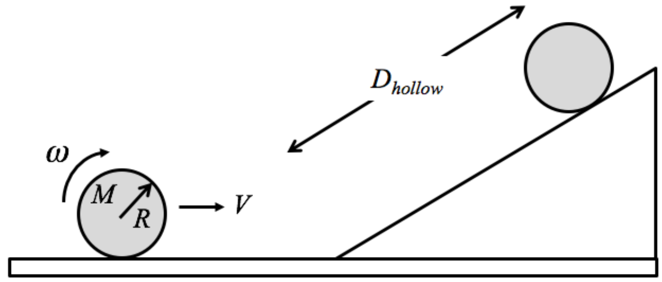
a. $V < \omega R$
b. $V = \omega R$
c. $V > \omega R$

7) Eventually the ball starts to roll without slipping. Which of the following statements best describes the total kinetic energy of the ball as it rolls?

a. The total kinetic energy of the ball when it is rolling is lower than its initial kinetic energy because friction did negative work on the ball.
b. The total kinetic energy of the ball when it is rolling is higher than its initial kinetic energy because it now has translational kinetic energy as well as rotational kinetic energy.
c. The total kinetic energy of the ball when it is rolling is the same as its initial kinetic energy, but is now distributed between translational and rotational kinetic energy.

The next three questions pertain to the situation described below.

A hollow sphere has mass M, radius R, and moment of inertia $I_{hs} = \frac{2}{3}MR^2$ about an axis through its center of mass. It is initially rolling on a horizontal plane with center of mass velocity V and angular velocity ω. For all of the questions on this page you should assume that there is no slipping.



8) What is the total kinetic energy of the sphere as it rolls on the horizontal plane?

a. $K_{total} = \frac{7}{10}MV^2$
b. $K_{total} = \frac{2}{3}MR^2\omega^2$
c. $K_{total} = \frac{1}{2}MV^2$
d. $K_{total} = \frac{5}{6}MR^2\omega^2$
e. $K_{total} = \frac{1}{2}MV^2 + \frac{1}{2}MR^2\omega^2$

9) The hollow sphere encounters a ramp and is observed to roll a maximum distance D_{hollow} up the ramp before rolling back down. As the sphere rolls on the ramp, which of the following statements best describes the static force of friction that the ramp exerts on the sphere ?

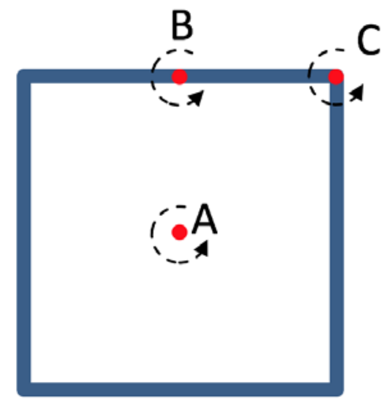
a. It points down the ramp when the sphere is rolling up, and up the ramp when the sphere is rolling back down.
b. It points up the ramp the whole time the sphere is rolling on the ramp.
c. It points down the ramp the whole time the sphere is rolling on the ramp.
d. It is zero.
e. It points up the ramp when the sphere is rolling up, and down the ramp when the sphere is rolling back down.

10) Suppose the experiment is repeated using a **solid** sphere ($I_{ss} = \frac{2}{5}MR^2$) that has the same mass, radius, and initial velocity as the hollow sphere described above. The solid sphere is observed to roll a maximum distance D_{solid} up the ramp before rolling back down. How does D_{solid} compare to the maximum distance reached by the hollow sphere D_{hollow}?

a. $D_{solid} = D_{hollow}$
b. $D_{solid} > D_{hollow}$
c. $D_{solid} < D_{hollow}$

The next three questions pertain to the situation described below.

A square frame is constructed of 4 thin rods, each having the same mass M and the same length L, as shown in the figure to the right. All 4 rods are in the same plane.



11) What is the moment of inertia for rotations about a perpendicular axis through the center of the square. In other words, an axis through point A in the picture, perpendicular to the plane of the page.

a. $I_A = 4ML^2$
b. $I_A = \frac{4}{3}ML^2$
c. $I_A = \frac{1}{3}ML^2$
d. $I_A = 2ML^2$
e. $I_A = \frac{2}{3}ML^2$

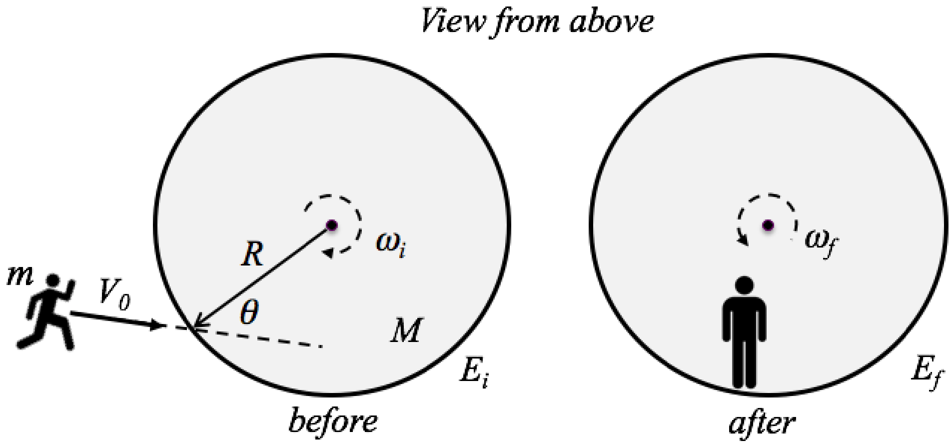
12) Let I_A, I_B, I_C be the moment of inertia of the square frame for rotations about axes that are perpendicular to the page and passing through points A, B, and C respectively. Rank the magnitudes of I_A, I_B and I_C.

a. $I_A < I_B = I_C$
b. $I_A = I_B < I_C$
c. $I_A < I_B < I_C$

13) Now suppose we have two shapes: A square frame composed of four thin rods (as above), and a solid square sheet. Both objects have the same total mass and have sides of the same length. Which object has the larger moment of inertia for rotations about a perpendicular axis through its center?

a. The frame.
b. The square.
c. They have the same moment of inertia.

The next three questions pertain to the situation described below.



As viewed from above, a merry-go-round is initially spinning clockwise around a frictionless vertical axis through its center with a constant angular velocity of magnitude $\omega_i = 0.74\ rad/s$. The outer radius of the merry-go-round is $R = 2.3\ m$, and its moment of inertia around the rotation axis is $I_{mgr} = 370\ kg\ m^2$. A boy of mass $m = 84\ kg$ runs toward the merry-go-round with a speed V_0 along a straight line that makes an angle $\theta = 65°$ with the radius that connects the center of the merry-go-round to the spot where he jumps on, as shown in the picture. After the boy jumps on, he stands at the edge of the merry-go-round, which now is seen to rotate in the **other** direction (counter clockwise) with a final angular velocity of magnitude $\omega_f = 0.37\ rad/s$.

14) What was the speed of the boy just before he jumped on to the merry-go-round

a. V_0 = 2.35 m/s
b. V_0 = 2.98 m/s
c. V_0 = 1.72 m/s
d. V_0 = 2.13 m/s
e. V_0 = 3.28 m/s

15) Suppose E_i and E_f are the initial and final total mechanical energies of the boy plus the merry-go-round. What is the change in the total mechanical energy of the system $\Delta E = E_f - E_i$?

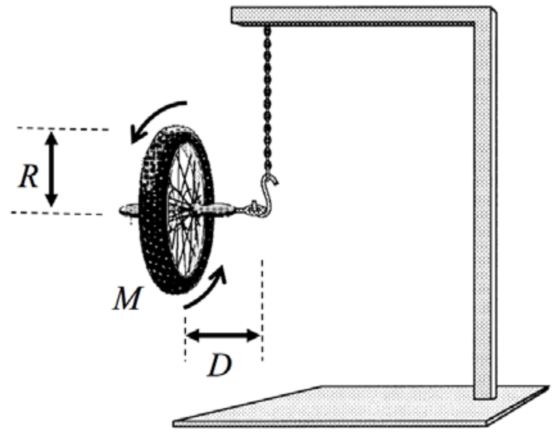
a. $\Delta E = \frac{1}{2}(I_{mgr} + mR^2)\omega_f^2 - \frac{1}{2}mV_0^2 - \frac{1}{2}I_{mgr}\omega_i^2$.
b. $\Delta E = 0$
c. $\Delta E = \frac{1}{2}I_{mgr}\omega_f^2 - \frac{1}{2}I_{mgr}\omega_i^2$.

16) Suppose that the boy stands on the edge of the merry-go-round facing outward, and suppose that he takes a step forward, away from the center of the merry-go-round, landing on the ground. How does the magnitude of the angular velocity of the merry-go-round change as he does this?

a. It decreases.
b. It increases.
c. It stays the same.

The next three questions pertain to the situation described below.

A gyroscope made from a bike wheel mass $M = 5.3\ kg$ and radius $R = 0.44\ m$ hangs from a stationary rope as shown. The wheel spins around a horizontal axle through its center in such a way that the top of the wheel is moving out of the page and the bottom of the wheel is moving into the page. The rope is attached to one end of the axle at a distance $D = 1.4\ m$ from the wheel. The magnitude of the angular momentum of the wheel is $L = 80\ kg\, m^2/s$.



17) At the instant shown, which way is the center of the wheel moving ?

a. Into the page.
b. Out of the page.
c. There is not enough information given to determine this.

18) What is the period of the precession of the gyroscope (in other words, the time needed for the gyroscope to make one complete revolution in the horizontal plane) ?

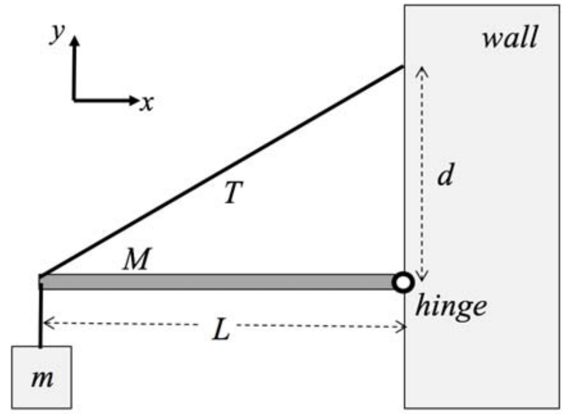
a. T = 3.45 s
b. T = 1.1 s
c. T = 0.91 s
d. T = 21.97 s
e. T = 6.91 s

19) Suppose the answer to the above problem is T. If the identical setup is moved to the surface of a new planet where the acceleration of gravity on the surface is larger than it is on Earth, how would the new precession period T_{new} compare to T.?

a. $T_{new} > T$
b. $T_{new} < T$
c. $T_{new} = T$

The next two questions pertain to the situation described below.

A beam of mass $M = 8\ kg$ and length $L = 2.3\ m$ is attached to a vertical wall by a hinge. A box of mass $m = 3\ kg$ is hung from the end of the beam. A wire is used to keep the beam horizontal. One end of the wire is attached to the outer end of the beam, and the other end is attached to the wall a distance $d = 1.4\ m$ above the hinge.



20) What is the tension in the wire ?

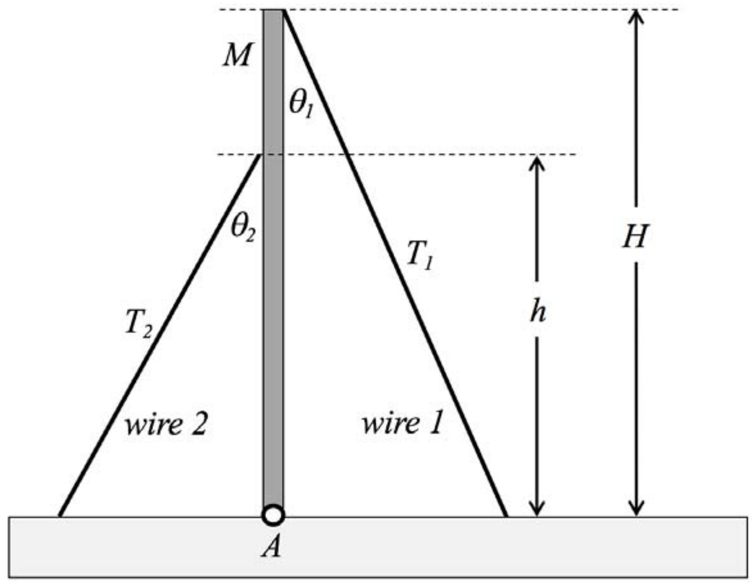
a. 56.6 N
b. 80.4 N
c. 112.8 N
d. 75.5 N
e. 132.1 N

21) Suppose a longer wire is used so that d is increased while keeping the beam horizontal. How does the tension in the wire change?

a. It stays the same.
b. It decreases.
c. It increases.

The next two questions pertain to the situation described below.

A vertical telephone pole of mass $M = 170\ kg$ and height $H = 17\ m$ is braced with two wires that run between the pole and the ground as shown in the figure (the wires and the pole lie in the same plane). Wire 1 makes an angle $\theta_1 = 34°$ with the pole and is attached at the top. Wire 2 makes an angle $\theta_2 = 24°$ with the pole and is attached a distance $h = 12\ m$ above the ground. The tension in wire 1 is $T_1 = 1500\ N$.



22) What is the direction of the torque exerted on the telephone pole by Wire 1 about an axis perpendicular to the page that goes through point A at the bottom of the pole ?

a. Perpendicular to the plane of the page, pointing out of the page.
b. Perpendicular to the plane of the page, pointing into the page.
c. Along the direction of Wire 2, pointing up and to the left.

23) The tension T_2 in wire 2 is chosen so that the total torque around Point A is zero. What is the magnitude of the [vertical] force that the pole exerts on the ground at Point A?

a. 4795 N
b. 3912 N
c. 3128 N
d. 4673 N
e. 5580 N