6. Friction

A) Overview

This unit is devoted to the introduction of a single force, friction. Frictional forces are omnipresent in our world. Indeed, pretty much all science prior to Galileo, focused on what was directly observed, much of that being dominated by friction. Galileo, in his description of free fall, and Newton in his first law, took as fundamental the more idealized description of motions in the *absence* of friction. The effects of friction then can be "added back in". We will learn in this unit exactly how to account for these effects of friction within Newton's framework.

In particular, we will adopt a simple model for frictional forces that is specified by two constants for each pair of surfaces that are in contact with each other. These constants are the coefficients of static and kinetic friction. We will then address specific examples to demonstrate how we can use these constants to account for frictional forces within Newton's framework.

B) Friction

In the last unit we introduced several new forces. The gravitational force is a fundamental force of nature that exists between any two objects of any size that have mass. In this unit, we will introduce a force, *friction*, that is more similar to the contact forces we discussed in the last unit.

In general the force between two surfaces that are in contact has components both perpendicular and parallel to the surfaces. The perpendicular component is the *normal force* we discussed last time. The parallel component is called the *friction force*.

The direction of the friction force is *always* such that it opposes any relative motion of the two surfaces. We distinguish between two kinds of friction forces. *Kinetic friction* refers to cases in which one surface moves relative to the other one, such as when a box slides across the floor, while *static friction* refers to cases in which the surfaces do not move relative to each other, such as when a person is pushing on a stationary heavy box.

The microscopic origins of these friction forces are complicated; they arise from the interactions of atoms on the surface of materials. We will not try to understand these interactions in this course. We will only be concerned with characterizing the friction forces on macroscopic objects using a simple model.

C) Kinetic Friction

When one object slides across another the frictional force between them is found to depend linearly on the perpendicular force between them. In other words, the frictional force is proportional to the normal force. The constant of proportionality, μ_K, is called the *coefficient of kinetic friction*. This constant depends only on the properties of the two

$$f_K = \mu_K N$$

surfaces and not on the size or weight of the objects.

We'll now do an example to see how this works. Suppose a box is given an initial shove after which it slides on a horizontal floor. If the coefficient of kinetic friction between the floor and the box is μ_K, what is the acceleration of the box as it slows to a stop?

We will follow the problem solving procedure we developed last time. We first draw the free body diagram for the box as shown in Figure 6.1. The forces acting on the
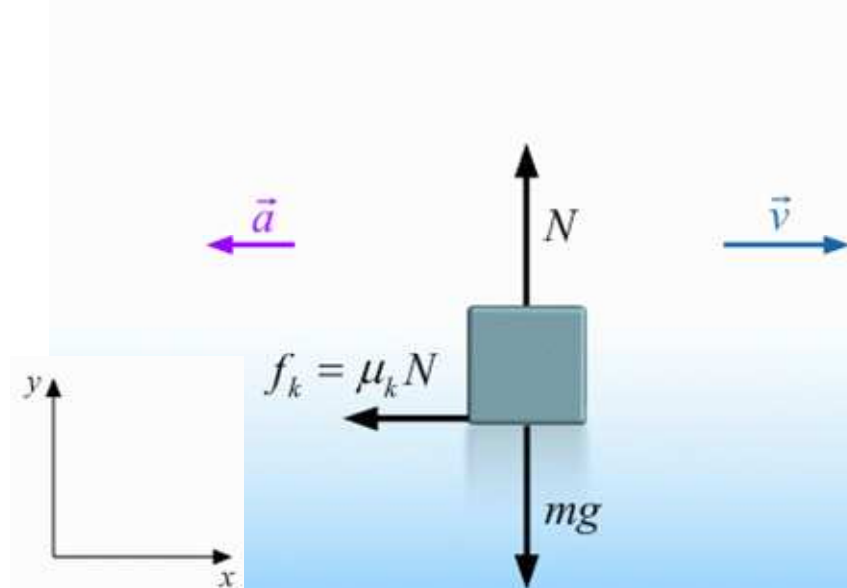

Figure 6.1
The free-body diagram for a box of mass m sliding across a
floor characterized by a coefficient fo sliding friction μ_K
between th box and the floor.

box once it is in motion are the weight of the box, the normal force exerted by the ground on the box, and the kinetic friction force. The next step is to write down Newton's second law for both the x and y directions.

$$-\mu_K N = ma_x$$
$$N - mg = ma_y = 0$$

The acceleration in the y direction is zero; therefore, the magnitude of the normal force is just equal to the weight of the box. Turning now to the x direction, we see that we can determine the acceleration since we now know the magnitude of the kinetic friction force. Namely, the acceleration in the x direction is just equal to the coefficient of kinetic friction times the acceleration due to gravity. The minus sign indicates that the direction of the acceleration is to the left.

$$a_x = -\mu_K g$$

Note that the acceleration does not depend upon the mass of the box! How does this come about? The reason is that the net force here is just the kinetic friction force which, by its definition, is proportional to the normal force. But the normal force here is equal to the weight of the box; therefore, we see that the kinetic friction force is proportional to

the mass of the box. Consequently, when we write down Newton's second law, we see the mass cancels. Therefore, we see that the box slows down with a constant acceleration whose magnitude depends only on the coefficient of kinetic friction.

D) Static Friction

We'll now turn to another everyday example that illustrates how we account for static friction. Figure 6.2 shows a box is at rest on a horizontal floor being pulled gently by a rope attached to it. The box does not move. Pull a bit harder, and the box still does not move. If we keep pulling harder and harder, eventually the box does move. How do
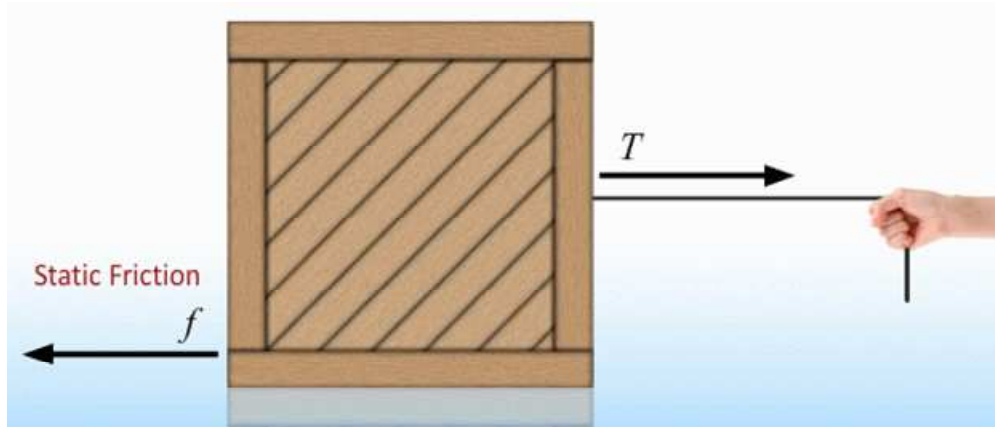

Figure 6.2

A rope exerts a force on a box and the floor exerts a friictional force on the box. The box is stationary, implying that the magnitude of the static frictional force is equal to the magnitude of the tension force.

we describe what happened in terms of Newton's laws? Clearly the acceleration of the box was zero until it began to move. We know then from Newton's second law, that until the box began to move, the total force on the box was zero! Since the rope was exerting a force on the box, directed to the right, there must be have been another force of the same magnitude that was directed to the left. This force is the static friction force f; it opposes the horizontal motion that the tension force would have produced.

The magnitude of the tension force reflects how hard we pull. The harder we pull, the bigger the tension gets, and therefore, the bigger the frictional force becomes, This dependence of the frictional force on the tension is demonstrated in the plot shown in Figure 6.3.

There is a limit to the force that friction can provide, though, and eventually we are able to move the box just by pulling harder. This maximum value of the force that static friction can provide is once again just proportional to the normal force. In this case the constant of proportionality is
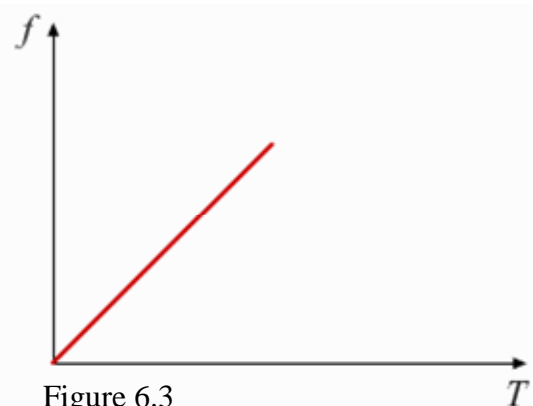

Figure 6.3

Aplot of the magnitude of the fritional force as a function of the tension of the rope used in the example in Fig 6.2

called the *coefficient of static friction,* μ_S, and depends only on the properties of the two surfaces. This behavior of the static friction force that we have described is captured in an inequality; namely, that the static friction force is *less than or equal to* the coefficient of static friction times the normal force. Note the important difference between kinetic

$$f_S \leq \mu_S N$$

and static friction: the kinetic force is always equal to $\mu_K N$, while the static force is *NOT* always equal to $\mu_S N$. In fact, the static force is only equal to $\mu_S N$ just before the surfaces began to move. In all other cases, the static force is less than this maximum force; just as it was for the normal force and for the tension force, the magnitude of the static frictional force must be determined from Newton's second law: *The static frictional force is simply what is has to be to do what it does!*

We can summarize what we have learned about friction so far by completing the plot we have created for static friction. As we increase T beyond the point at which the
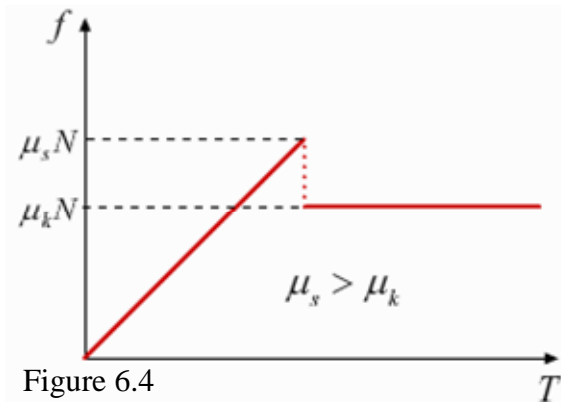

Figure 6.4
A plot of the magnitude of the fritional force as a function of the tension of the rope used in the example in Fig 6.2 The foprce is discontinuous at the point where the box begins to move.

box begins to move, we see that frictional force stays constant as T increases. The discontinuity at the point at which the box begins to move indicates that $\mu_S > \mu_K$.

E) Example: Box Sliding Down Ramp

We'll now do some examples that illustrate the use of these frictional forces within the framework of Newton's laws. We will start with the calculation of the acceleration of a box as it slides down a rough ramp.

Figure 6.5 shows a free-body diagram of all of the forces acting on the box. In the absence of friction between the ramp and the box, there are two forces acting on the box: its weight (mg), which points downward, and the normal force N that the ramp exerts on the box. The direction of this normal force is perpendicular to the ramp. Right away we can see that there is a net force directed down the plane which gives rise to the acceleration down the plane. If there is friction between the box and the ramp, there will

be an additional force that opposes this motion. Therefore, this frictional force is directed up the plane and has a magnitude equal to $\mu_K N$, when the box is sliding.
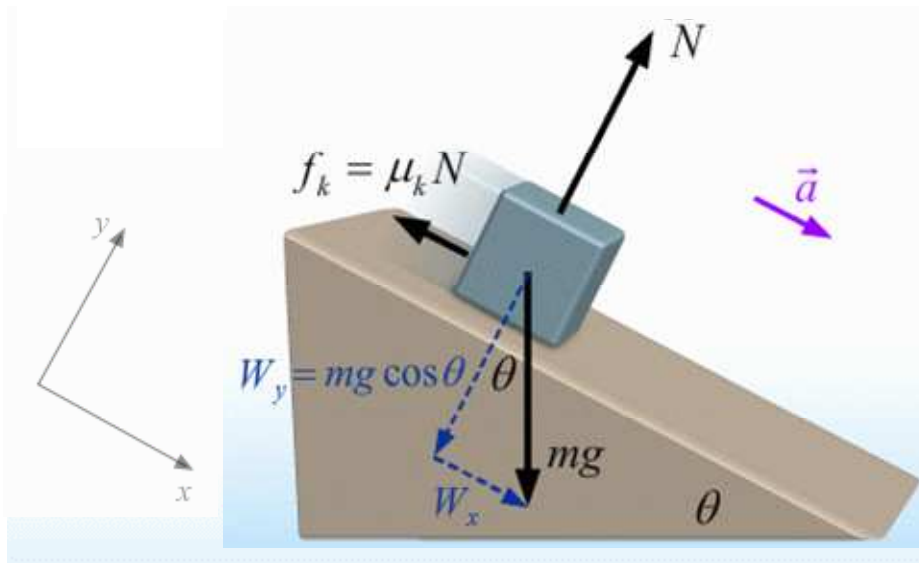

Figure 6.5
A free-body diagram for a box of mass m sliding down a ramp. The friction between the ramp and the box is specified by the coefficient of kinetic frcition, μ_K.

We now need to choose a coordinate system. As we mentioned last time, choosing one axis to be parallel to the acceleration often simplifies the calculation. Therefore, we will choose our x-axis to point down the ramp and the y-axis to be perpendicular to the ramp.

Finally we write down Newton's second law for both the x and y directions. To write down the y equation, we need to find the y-component of the weight. Since the normal force is perpendicular to the ramp and the weight is perpendicular to the horizontal, we see that the angle between these two forces must also be equal to θ, the angle the ramp makes with the horizontal. Therefore, we can use trigonometry to
$$\Sigma F_y = N - mg \cos \theta = ma_y = 0$$
determine that the y-component of the weight is just equal to the weight times the cosine of the angle the ramp makes with the horizontal. Writing down Newton's second law for the y direction, we obtain the magnitude of the normal force. This result holds for both the friction and frictionless cases.
$$N = mg \cos \theta$$
We can now obtain a value for the magnitude of the frictional force from our result for the magnitude of the normal force. The x-component of the weight is equal to the product of the weight and the sine of the angle the ramp makes with the horizontal. We can now write down Newton's second law in the x-direction in terms of this frictional
$$\Sigma F_x = mg \sin \theta - \mu_K N = ma_x$$
force and the component of the weight down the ramp. We can substitute our result for the normal force into this equation and then solve for the acceleration. We obtain the

result that the acceleration is equal to the g times a factor that is determined by the coefficient of kinetic friction and the angle the ramp makes with the horizontal.

$$a = g(\sin\theta - \mu_K \cos\theta)$$

We can remove the frictional force by setting μ_K equal to zero, to determine that the acceleration in the absence of friction is just equal to the product of g and the sine of the angle the ramp makes with the horizontal.

F) Example: Box at Rest on Ramp

We've just determined the acceleration of a box sliding down a ramp. In particular, we see that the acceleration decreases as the angle θ decreases. If we make the angle small enough, the frictional force on the box will prevent it from accelerating at all! At such an angle, what is the magnitude of the frictional force?

To answer this question, we start with the free-body diagram shown in Figure 6.6.
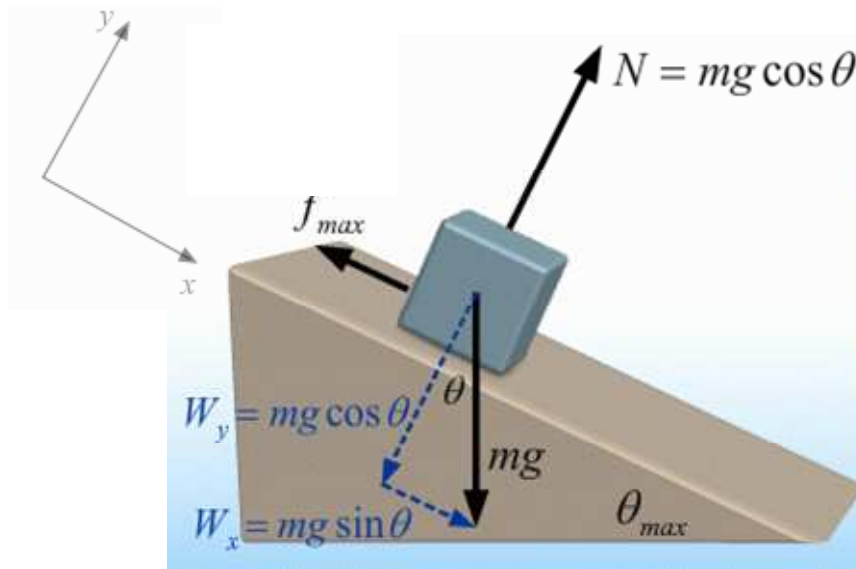

Figure 6.6
A free-body diagram for a box of mass m on a ramp whose

angle with the horizontal, θ_{max}, is the largest it can be to
prevent the box from sliding down the ramp..

We see that this diagram is essentially identical to that in Figure 6.5, as long as we just call the frictional force f. Since the box is not moving, we have a case of static friction. The magnitude of the static friction force must be determined from Newton's second law   In particular, the magnitude of the static friction force must be equal to the component of the weight down the plane.

$$\Sigma F_x = mg\sin\theta - f = 0$$

As we increase θ, both the friction force and the x-component of the weight increase. At some angle θ_{max}, though, the static friction force can increase no more and the block will begin to slide. We can determine θ_{max} by setting the static friction force equal to its

maximum value ($\mu_S N$). Substituting this value for the frictional force back into our
$$f_{max} = \mu_S N = \mu_S mg \cos\theta_{max}$$
general equation, we obtain our result for the maximum angle the ramp can make with
the horizontal to prevent the box from sliding down the ramp.
$$\mu_S mg \cos\theta_{max} = mg \sin\theta_{max}$$
$$\tan\theta_{max} = \mu_S$$

G) Example: Car Rounding a Corner

We'll close this unit by doing one more example involving friction, namely, that
of a car of mass M rounding a circular turn of radius R. If the coefficient of static friction
between the tires and the road is μ_S, how fast can the car go around the turn without
skidding off the road?

Perhaps your first question here is why in the world are we giving you the static
coefficient of friction when the car is clearly moving? The answer to this question is that
the tires are rolling: the surfaces of the tires are not sliding relative to the surface of the
road, since if they were, the car would already be skidding! During normal driving it is
the static friction between the tires and the road that makes a car speed up, slow down,
and turn corners!

We start, as always, by drawing a free body diagram for the car as shown in
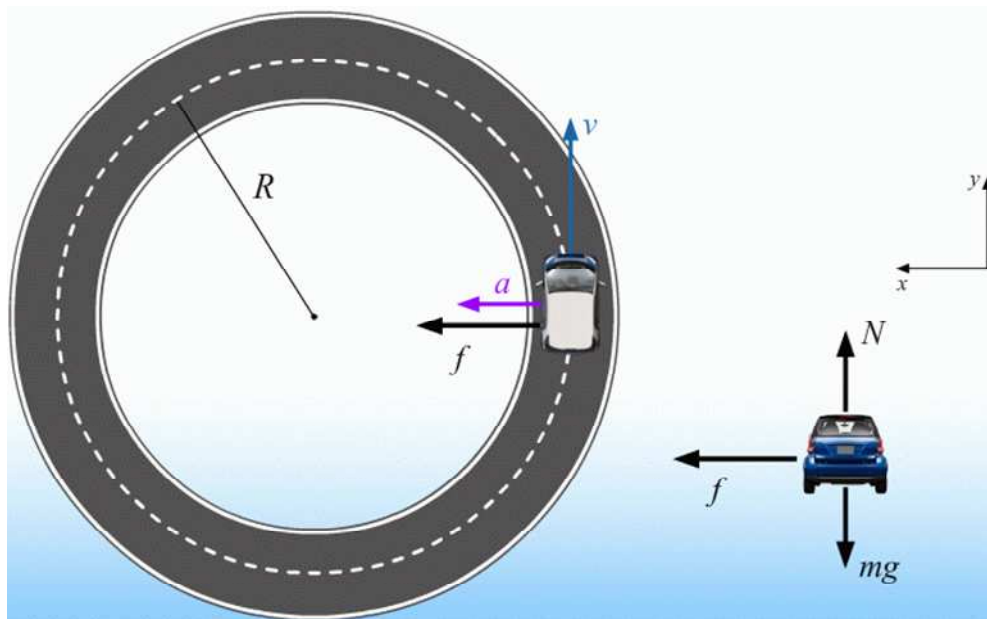Figure 6.7. From this diagram, we see it is just the frictional force f that is responsible

Figure 6.7
The free-body diagram for a car of mass m moving at constant speed v in a circle of radius R..

for the centripetal acceleration of the car. Next, we choose the x-axis to point toward the
center of the circle to align it with the direction of the acceleration. We choose the y-axis
to be vertically up.

Writing Newton's second law for the y-direction, we see that the magnitude of the

$$\Sigma F_y = N - mg = ma_y = 0$$

normal force is just equal to the weight of the car. Writing Newton's second law for the x-direction, we see that the frictional force must be equal to the mass of the car times the

$$\Sigma F_x = f = ma_x$$

centripetal acceleration. We know the centripetal acceleration is equal to the square of the speed divided by the radius of the turn. Therefore as the car's speed increases, the

$$a_x = \frac{v^2}{R}$$

frictional force must also increase. There is a limit though, as to how much this frictional force can increase. This limit is determined by the coefficient of static friction, μ_S. This maximum frictional force then produces the maximum centripetal acceleration which determines the maximum velocity. Therefore, we obtain the maximum speed that the car

$$f_{max} = \mu_S N = \mu_S mg$$

$$a_{max} = \frac{v_{max}^2}{R}$$

$$v_{max} = \sqrt{a_{max} R} = \sqrt{\frac{f_{max}}{m} R} = \sqrt{\mu_S gR}$$

can make the turn without skidding is proportional to the square root of the product of the turning radius and the coefficient of static friction. This result makes sense: the car's maximum speed should increase if either the friction increases or the turning radius increases.

Main Points

- **Friction Forces**

  Frictional forces refer to the component of the contact force between two srufaces that is parallel to the surfaces

- **Kinetic Friction**

  Kinetic friction exists between surfaces in relative motion

  The direction of kinetic friction always opposes the relative motion

  The magnitude of kinetic friction is proportional to the normal force

  Kinetic Friction
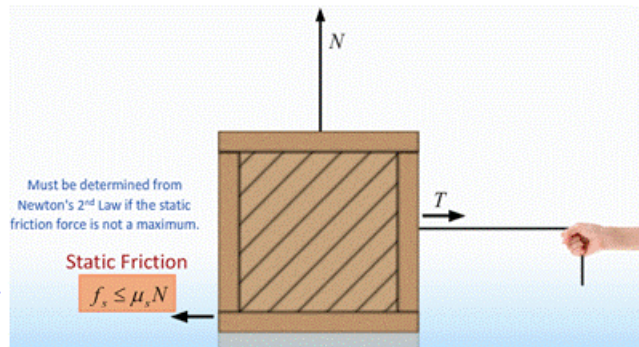
  $$f_k = \mu_k N$$

  N

  v
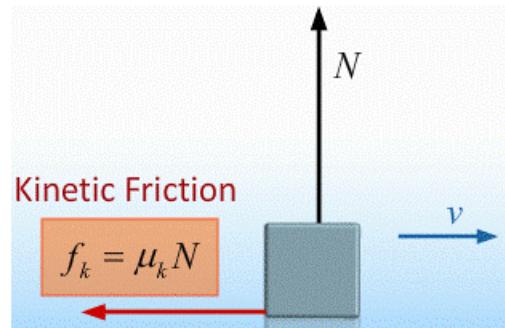
- **Static Friction**

  Static friction exists between surfaces NOT in relative motion

  The direction of static friction always opposes the relative motion that would exist in the absence of friction.

  The magnitude of static friction must be determined from Newton's second law. Its maximum value is proportional to the normal force.

  The static force is what it has to be to do what it does!

  N

  Must be determined from Newton's 2nd Law if the static friction force is not a maximum.

  Static Friction

  $$f_s \le \mu_s N$$

  T