# LAB \#6: Rotational Dynamics IMoments of Inertia 

A Lab Consisting of 4 Activities

Name:
Section: $\qquad$
TA: $\qquad$
Date: $\qquad$

Lab Partners: $\qquad$
$\qquad$
$\qquad$

Circle the name of the personto whose report your group printouts will be attached. Individual printouts should be attached to your own report.

## Physics Lab 211-6

## Equipment list

Motion detector
Closet door inclined plane setup:
Door
Wood barricade
Foam cushion
Large c- clamps to hold wood barricades (2)
Special rod with lattice clamps to hold the door
Table clamps (2), AL rods (2), and right angle clamps to hold special rod (2)
Green, wide- mouthed, velcroed clamp to hold the motion detector
Cart with reflector
Items per each room:
Electronic level (1)
Red- taped batons (2)
Black- taped batons (2)
Mini- basketball
Solid sphere (bocce or croquet ball)
Black plastic wheel
Two holders for the spheres when they're not in use
Objects in cart at front of room
Six sets of three hollow cylinders of equal radius but different lengths
Six sets of three hollow cylinders of different radii but equal lengths
Six sets of three solid white plastic cylinders of different radii but equal lengths
Six aluminum solid cylinders and six green plastic solid cylinders which, along with
the smallest white plastic solid cylinders, comprise six sets of three solid
cylinders of equal size but different mass

## Computer file list

MacMotion file "211-06 standards"
MacMotion file "211-06 rolling"

## Investigation 1: The Moment of Inertia

Goals:

- To study how two objects having the same mass can have dramatically different "resistances" to changes in rotational velocity (i.e., moments of inertia).
- To study how the moment of inertia of an object depends upon the object's shape, size, and construction.

Introduction: You have studied the equations describing translational motion (falling, sliding, bouncing, etc.). Now you will study in greater detail the kinematics and dynamics of rotating bodies. The equations you will use are straightforward extensions of the equations you already know for translational motion.
The new idea you will study in this laboratory is that of the moment of inertia of an object. Mathematically, the moment of inertia / relates an applied torque $\tau$ to the resulting angular acceleration $\alpha$ through the equation,

$$
\begin{equation*}
\tau=I \alpha \tag{Eq.1}
\end{equation*}
$$

The moment of inertia of an object provides a measure of how hard it is to change that object's rotational velocity. Thus, the moment of inertia is to rotational motion what the mass of an object is to translational motion. This analogy is illustrated schematically in Figure 1 below.


Figure 1. Rotational vs. translational motion

In this investigation, you will study how an object's moment of inertia affects an object's rotation. You will also study the motion of several simple objects with different moments of inertia.

## Activity 1: Spare the Rod, Spoil the Student

Introduction: In this activity you will twirl different rods or "batons" in order to (a) study their moments of inertia and (b) quench that secret longing you've always had to be a drum major or majorette.

Procedure: 1. Compare the masses of the two rods.

- Select two rods, one with black tape and the other with red tape.
- Grasp one rod in each hand and simply hold them steadily. The two rods should both have the same mass. Switch hands to make sure of this.
$\qquad$

2. Check your prediction. First, pick one of the rods (it doesn't matter which one). Now, rotate this rod about its long axis (as shown in Figure 2).

- You can do this by placing the rod between your hands and making a rolling action back and forth, as with Play-Doh ${ }^{\text {TM }}$.


Figure 2. Rotating a rod about its long axis in Activity 1
3. Now, try rotating the same rod about an axis which is perpendicular to the length of the rod and which passes through the rod's center (as shown in Figure 3).

- Twirl the rod in a propeller-like motion. Be sure to hold the rod in the center as you twirl it.
- Try rotating the rod in one sense (say, clockwise), then quickly reverse directions.


Figure 3. Rotating a rod about an axis perpendicular to the rod's long axis in Activity 1

Question: $\quad$ Was your prediction correct? About which axis was it easier to rotate the rod? If your prediction was incorrect, can you explain your result now?

Procedure: (continued)
4. Rotate both rods about their long axes (see Figure 2), in order to compare the moments of inertia of these rods about this axis.
5. Rotate both rods about an axis perpendicular to each rod (see Figure 3), in order to compare the moments of inertia of these rods about this axis.

Questions: $\quad$ Is one rod significantly harder to rotate about the long axis than the other rod? If so, which?

- Is one rod significantly harder to rotate about the perpendicular axis than the other rod? If so, which?

Questions: -What is it about the rods that gives the differences noted above? That is, can you tell what differs in the construction of the two rods?

- Why does this difference in construction affect the rotation about one axis but not the other?


## Activity 2: Sisyphus' Revenge

Introduction: The moment of inertia depends in general about which axis the object is rotated. Moments of inertia for spheres and cylinders (about the principal axes) can be written $I=\eta M R^{2}$, where $\eta$ is a constant which is dependent upon the object's mass distribution, $M$ is the object's mass and $R$ is the object's radius. This dependence on $R$ is consistent with what you should have observed in Activity 1; i.e., the farther away from the axis of rotation the mass of an object is located, the harder it is to alter the object's rotational velocity. Table 1 provides a list of moments of inertia for several standard shapes in the following instances: Spheres - for rotation about an axis through the center of the sphere; Cylinders - for rotation about the long axis of the cylinder. Notice that $\eta$ has the values $\frac{2}{5}, \frac{2}{3}, \frac{1}{2}$, and 1 , for the solid sphere, hollow sphere, solid cylinder, and hollow cylinder, respectively. In other words, the gods could have been tougher on Sisyphus! They could have made him roll a log with the same mass and radius as the stone he was given. Now, its your turn...hopefully this activity won't feel like an eternity in Hades.

| Solid Sphere | Hollow Sphere | Solid Cylinder |
| :---: | :---: | :---: |
| $\frac{2}{5} M R^{2}$ | $\frac{2}{3} M R^{2}$ | Hollow Cylinder |

Table 1. Moments of inertia for spherical and cylindrical objects

Procedure: 1. Set up the motion detector at the top of the ramp.

- Make sure that the motion detector is securely fastened in the clamp at the top of the ramp. Make sure that the motion detector "sees" the objects rolling all the way down the ramp.

2. Set up the computer.

- Start MacMotion by double-clicking its icon on the desktop.
- Open... the file Standards from the 211 Lab Files folder.

3. Determine the angle at which the table is tilted.

- Measure the angle yourself using the "digital level" your TA will provide. Record both the angle and the sine of the angle below.

$$
\theta=\square
$$

Predictions: ${ }^{\bullet}$ Consider an object rolling down an incline having an angle $\theta$ (see Figure 4).
On what parameters would you expect the acceleration, $a$, of this object to depend? Explain your reasoning.

Predictions:
(continued)


Figure 4. A rolling object of mass $M$, radius $R$, and moment of inertia I

To answer the previous prediction more quantitatively, consider an object rolling down an incline of angle $\theta$ (Figure 4). The following equations apply:
(1) The sum of the forces yielding the object's translational acceleration a along the ramp is given by

$$
\begin{equation*}
F=M g \sin \theta-F_{\text {friction }}=M a \tag{Eq.2}
\end{equation*}
$$

(2) The sum of the torques providing the objects rotational acceleration $\alpha$ about its center of mass can be written:

$$
\begin{equation*}
\sum \tau=F_{\text {friction }} R=I \alpha \tag{Eq.3}
\end{equation*}
$$

(3) Because the objects roll without slipping, one also has the following relationship between the translational and rotational accelerations

$$
\begin{equation*}
a=R \alpha \tag{Eq.4}
\end{equation*}
$$

Using Equations 2-4 above, one can derive a relationship for the acceleration of the object down the incline in terms of $\theta, \eta=I / M R^{2}$, and fundamental constants,

$$
\begin{equation*}
a=\frac{g \sin \theta}{\left(1+\frac{1}{M R^{2}}\right)}=\frac{g \sin \theta}{(1+\eta)} \tag{Eq.5}
\end{equation*}
$$

- If you rolled each of the objects in Table 1 down the incline provided, as illustrated in Figure 4, what acceleration values do you predict for each of these objects? Record your predictions in Table 2.

| Object | Predicted <br> Acceleration | Measured <br> Acceleration | \% <br> Difference |
| :---: | :---: | :---: | :---: |
| Hollow |  |  |  |
| Cylinder |  |  |  |
| Solid |  |  |  |
| Cylinder |  |  |  |
| Hollow |  |  |  |
| Sphere |  |  |  |
| Solid |  |  |  |
| Sphere |  |  |  |

Table 2. Predicted and measured accelerations for Activity 2

Procedure: 4. Test your predictions by making the measurements. (continued)

- Choose one of the objects listed in Table 2. Hold it at the starting line and start graphing.
- After about a second, release the object. Make sure the motion detector can see it all the way down the ramp and that you have a nice period of constant acceleration.

5. Find the average acceleration for the object.

- Select Analyze Data A from the Analyze menu.
- Using the mouse-down-drag technique, highlight this period of constant acceleration and then select Statistics... from the Analyze menu.
- Record this mean acceleration in Trial \#1 of Table 3, under the column heading associated with the object you just measured.

6. Complete Table 3 for each of the four shapes.

- Repeat steps 4 and 5 for a total of three trials for each object, recording the data for each trial in Table 3.
- Average the results and record in Table 3.

| Trial | Hollow <br> Cylinder | Solid <br> Cylinder | Hollow <br> Sphere | Solid <br> Sphere |
| :---: | :---: | :---: | :---: | :---: |
| Trial \#1 |  |  |  |  |
| Acceleration: |  |  |  |  |
| Trial \#2 |  |  |  |  |
| Acceleration: |  |  |  |  |
| Trial \#3 |  |  |  |  |
| Acceleration: |  |  |  |  |
| Trials: |  |  |  |  |

Table 3. Measured average accelerations of standard shapes in Activity 2

Procedure: (continued)

TA Diseussion Checkbox
7. Using the average value from all the trials in Table 3, record your measured acceleration for each of the four shapes in the appropriate column of Table 2. Determine the percent difference between your measured and predicted accelerations for each shape.

Questions: The questions following concern two objects $A$ and $B$ of equal size and mass.

- Object $A$ has most of its mass placed near its center
- Object $B$ has most of its mass placed near its rim
- Which one will most quickly roll down a ramp, assuming that they both start from the same place and at rest? Circle your answer.


## Object $A \quad$ Object $B$

- If both of the objects $A$ and $B$ have the same mass, which has the greater moment of inertia? Circle your answer.

Object $A \quad$ Object $B$
${ }^{\bullet}$ Consider the black plastic wheel at your table (ask your TA if you can't find this wheel for some reason). First, make a prediction of the coefficient $\eta=$ $\mathrm{I} / \mathrm{MR}^{2}$ for this object. Record this prediction in Table 4, and explain below how you arrived at your estimate.

Next, measure the coefficient $\eta$ by rolling the object down the incline. Record your measured value in Table 4 and compare it with your prediction.

| Object | Predicted <br> $\eta$ | Measured <br> $\eta$ | \% <br> Difference |
| :---: | :---: | :---: | :---: |
| Plastic |  |  |  |
| Wheel |  |  |  |

Table 4. Predicted and measured values of $\eta$

## Activity 3: Roller Derby

Introduction: In this activity you will hold "races" between pairs of standard shapes after making predictions about the outcomes of these "races." Place your bets!


Figure 5. The Competitors

Procedure: Now that you have seen how different mass distributions affect the speed of rolling objects, you will explore what else matters in determining how fast an object rolls down a hill.

In the rest of this activity, you will race different sets of objects which are similarly shaped, but which vary in certain other key attributes (such as radius, mass, length, density, color, cost, etc.). You will observe which of these attributes affects the acceleration of these objects as they roll down the ramp. In the last activity you measured this quantitatively. Now you will just compare them qualitatively (i.e., you will just race them against each other.)

In this first race, you will predict the outcome of racing (rolling) a series of solid cylinders down the ramp. These objects have the same shape and size, but are made of different materials (so each has a different mass). Note: To get a "fair" race, try holding the objects in place with a meter stick, then start the race by quickly moving the stick forward to release the objects.

1. Race A: A set of solid cylinders with different masses.

- Find and consider the set of solid cylinders in Figure 6. (Bert is made of (green) plastic, Ernie of (white) teflon, and Grover is aluminum.)


Figure 6. Solid cylinders in Race A

Predictions: ${ }^{\bullet}$ Which cylinder in Figure 6 has the largest moment of inertia? Why?
${ }^{\bullet}$ Which cylinder, if any, will win Race A? Why?
Consider this: Would the cylinder with the smallest moment of inertia roll faster? Or would it roll down more slowly because the torque due to gravity and friction would be less? Would these two effects cancel each other out?

Procedure: 2. Check your predictions by making the measurements.
(continued)

- Run Race A. Take the objects and position them at the same place at the top of the ramp. Without pushing, release the objects.
- Record these results for this race (Race A) in Table 5.

3. What if you compare a set of solid cylinders with different radii, but constant length and density?

- Race B: A set of solid cylinders of differing radii.
- Find and consider the set of cylinders in Figure 7. (They are all made of white plastic.)


Figure 7. Solid cylinders of different radii for Race $B$
$\square$

Procedure: 4. Check your prediction. Run Race B.

- Which cylinder, if any, will win Race B? Why?
- Take the objects and position them at the same place at the top of the ramp. Without pushing, release the objects.
- Record your results for this race (Race B) in Table 5.

Now, consider what would happen if you increased the radius of a hollow cylinder, while leaving the length alone. Will this produce a different outcome than in Race B? With a hollow cylinder, all the mass is out at the rim, so the change in the moment of inertia should be easier to visualize.
5. Race C: A set of hollow cylinders of differing lengths, but constant thickness, density, and radius.

- Find and examine the set of hollow cylinders depicted in Figure 8.


Figure 8. Hollow cylinders of differing lengths for Race $C$

Predictions: ${ }^{\bullet}$ In which direction does the moment of inertia increase as we view the objects in Figure 8 (to the right or left)? Why? (Hint: Think about the fundamental quantities which are different among the cylinders and whether or not they matter.)

- Which cylinder, if any, will win the race? Why? (Hint: Ask yourself if a cylinder would roll faster if you sawed it in half.)


## Procedure: <br> 6. Check your prediction. Run Race C.

(continued)

- Take the objects and position them at the same place at the top of the ramp. Without pushing, release them as discussed before.
- Record your results for Race C in Table 5.

| Race | Contestants | Race <br> Winner <br> (or tie) | Varying <br> Physical <br> Parameter(s) | Do the <br> parameters <br> matter? |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  |  |
| B |  |  |  |  |  |
| C |  |  |  |  |  |

Table 5. Daily Racing Form

Questions: ${ }^{\bullet}$ Consider the acceleration a given in Equation 5. According to this equation, on what variables does the acceleration of the rolling object depend? Which of these variables were changed in each race?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

- The acceleration of each object should depend only on the variables you listed above. Explain how your race results confirm this assertion.


## Investigation 2: Energetics of Rolling Motion

Goals:

- To study how energy is distributed when both translation and rotation occur simultaneously.
- To investigate whether total energy is conserved in this case.

Introduction: It should make intuitive sense to you by now that an object has energy by virtue of the fact that it is moving through space, sliding across a track, or falling. We have identified this type of energy as translational kinetic energy. Similarly, the mass elements that make up a rotating object are also moving through space; this results in an additional kinetic energy contribution which we call rotational kinetic energy. For example, a moving merry-go-round has rotational energy associated with the rotational motion of all the mass elements, especially the faster-moving ones near the rim.

## Activity 4: We're Rollin' Now

Introduction: In this activity you will compare the motions of a cart, a ball (solid sphere), and a hollow disk in order to examine how the shape of an object influences the relative contributions of rotational and translational kinetic energy.

Procedure: 1. Measure the masses of the cart, solid sphere, and hollow cylinder.
$\mathrm{m}_{\text {cart }}=$ $\qquad$ [kg]
$\mathrm{m}_{\text {sphere }}=$ $\qquad$ [kg]
$\mathrm{m}_{\text {cylinder }}=$ $\qquad$ [kg]
2. Record again the value you determined for $\sin \theta$ in Activity 1 , where $\theta$ is the inclination angle of the experimental setup you measured previously in Activity 1 (see Figure 9).

$$
\sin \theta=
$$

$\qquad$
motion detector


Figure 9. Experimental setup for Activity 5

Predictions: - If you held a race down the incline between the cart, the solid sphere, and the hollow cylinder, in what order would these objects finish the race?

- Assuming that the origin is located at the motion detector as shown in Figure 10, and assuming the masses of all the objects are the same, sketch in Figure 11 your predictions for the gravitational potential energy, P.E._grav, the rotational kinetic energy, K.E._rot, the translational kinetic energy, K.E._trans, and the total energy, T.E., for each of the following objects traveling down an incline of angle $\theta$ : (a) [solid line] a cart that slides without friction (ignore any contribution due to the small wheels); (b) [dashed line] a solid sphere that rolls without slipping; and (c) [dotted line] a hollow cylinder that rolls without slipping. Be sure to emphasize any differences or similarities you expect to observe.


Figure 10. Experimental parameters for an object sliding or rolling down an incline


Figure 11. Predictions for potential, rotational kinetic, translational kinetic, and total energies

How would you determine the velocity of a rolling object after it has rolled a distance $\Delta d=d_{2}-d_{1}=1$ meter down the incline from a stationary initial position?
The following facts apply:
(1) Energy is conserved, $\mathrm{E}_{\text {init }}=\mathrm{E}_{\text {final }}$.

$$
\begin{gather*}
\mathrm{E}_{\text {init }} \\
-\mathrm{mgd}_{1} \sin \theta \tag{Eq.6}
\end{gather*}=\quad-\mathrm{E}_{\text {final }}
$$

where $I=\eta m R^{2}$ is the moment of inertia of the object, $\eta$ is the constant prefactor determined by the object's shape ( $\eta=0.4$ and 1 for a solid sphere and hollow cylinder, respectively), v is the translational velocity of the object, $\omega$ is the angular velocity of the object, and $d_{1}$ and $d_{2}$ are the initial and final distances of the object from the origin.
(2) The rolling objects roll without slipping,

$$
\begin{equation*}
\omega=\mathrm{V} / \mathrm{R} \tag{Eq.7}
\end{equation*}
$$

Combining these two relationships and solving for the velocity v gives a relationship that depends only on the known parameters $\mathrm{m}, \mathrm{g}, \eta, \theta$, and $\Delta \mathrm{d}(=$ $\mathrm{d}_{2}-\mathrm{d}_{1}$ ) (make sure you know how to derive this!),

$$
\begin{equation*}
v=\sqrt{\frac{2 g\left(d_{2}-d_{1}\right) \sin \theta}{1+\eta}}=\sqrt{\frac{2 g \Delta d \sin \theta}{1+\eta}} \tag{Eq.8}
\end{equation*}
$$

- If a cart, a solid sphere, and a hollow cylinder are released at the top of the incline illustrated in Figure 9, what do you predict will be the velocity of each of the objects when they have traveled a distance of $\Delta \mathrm{d}=1$ meter? Record your predictions in Table 6.

It is also interesting to consider the fraction of total kinetic energy associated with rotational motion, K.E.rot/(K.E.trans + K.E.rot), for different objects rolling down an incline. Using the relationships K.E.trans $=1 / 2 \mathrm{mv}^{2}$, K.E.rot $=1 / 2 \mid \omega^{2}=$ $1 / 2 \eta m R^{2} \omega^{2}=1 / 2 \eta m v^{2}$, and Equation 7 , one finds

$$
\begin{equation*}
\frac{\text { K.E. }_{\text {rot }}}{\text { K.E.total }}=\frac{\text { K.E.rot }}{\text { K.E.rot }}+\text { K.E.trans } \quad=\frac{1 / 2 \eta m R^{2} \omega^{2}}{1 / 2 \eta m R^{2} \omega^{2}+1 / 2 m v^{2}}=\frac{\eta}{\eta+1} \tag{Eq.9}
\end{equation*}
$$

- For each of the three objects described above, record in Table 6 the value $\eta /(\eta+1)$, which is associated with the ratio of the rotational kinetic energy to the total kinetic energy, K.E.rot/(K.E.trans + K.E.rot).

| Object | Predicted <br> velocity <br> after $\Delta \mathrm{d}=1 \mathrm{~m}$ | Measured <br> velocity <br> after $\Delta \mathrm{d}=1 \mathrm{~m}$ | \% <br> Difference | $\eta /(\eta+1)$ | Measured <br> K.E. rot $/$ K.E. <br> total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| cart |  |  |  |  |  |
| solid <br> sphere |  |  |  |  |  |
| hollow <br> cylinder |  |  |  |  |  |

Table 6. Predictions and results for Activity 4

## Procedure:

 (continued)3. Prepare to test your predictions. First, configure the MacMotion graph.

- Open the file Rolling in the Lab 6 folder. A graph like that shown in Figure 12 should appear.


Figure 12. Experimental setup for Activity 4
4. Modify the total energy formula for the cart.

- Select Modify... under the Data menu, then select Total Energy.
- Replace the "0" in the "formula" space with the expected relationship for the cart's total energy,
"P.E._grav" + "K.E._rot" + "K.E._trans"

Click on OK when you're satisfied with your answer.

Procedure:
(continued)

TA Discussion Checkbox

Procedure: (continued)
11. Repeat steps $5-10$ above for the case of a solid sphere rolling down the incline.

- When you Modify the formulas for this experiment, make sure you use the correct mass.
- Make sure you Modify the rotational kinetic energy formula to include the relationship appropriate for this situation (write the formula in terms of the velocity, "vel"). Write this rotational kinetic energy equation below.

$$
\text { K.E.rot }=
$$

- Analyze your data as described in step 10, and record your results in Table 6.

Questions: ${ }^{\bullet}$ What did you notice about the velocity of the sphere after rolling $\Delta \mathrm{d}=1$ meter compared to that of the cart after rolling the same distance? Did you predict this?

- Based on your observations of the cart and the sphere, will an object with a larger value of K.E.rot K.E.total have a larger or smaller acceleration down the incline than an object with a smaller value of K.E.rot K.E.total? Explain.

12. Repeat steps 5-10 above for the case of a hollow cylinder rolling down the incline.

- When you Modify the formulas for this experiment, make sure you use the correct mass.
- Make sure you Modify the rotational kinetic energy formula to include the relationship appropriate for this situation. Write this rotational kinetic energy equation below.

$$
\text { K.E.rot }=
$$

$\qquad$

- Analyze your data as described in step 10, and record your results in Table 6.

Questions: $\quad$ Provide an explanation for the order in which a cart, a solid sphere, and a hollow cylinder would finish a race down an incline in terms of the different rotational and translational kinetic energies of the objects.

- Is energy conserved in all cases? Did you expect this? Why or why not?
$\qquad$
$\qquad$

