PHYSICS 211
LAB \#7: Rotational Dynamics II
A Lab Consisting of 4 Activities

Name:
Section: $\qquad$
TA: $\qquad$
Date: $\qquad$

Lab Partners: $\qquad$
$\qquad$
$\qquad$

Circle the name of the personto whose report your group printouts will be attached. Individual printouts should be attached to your own report.

## Physics Lab 211-7

## Equipment list

Torsion pendulum
Dual channel optical switch and cable to interface with uli (port 2)
Meter stick
Aluminum ring with two strings
Aluminum ring without strings
Two 5 g mass hangers
Two 5 g masses
Two 10 g masses
One dissect torsion pendulum per room

## Computer file list

Rotary Motion file "211-07 torqued"
Rotary Motion file "211-07 data entry 1"
Rotary Motion file "211-07 twister"

## Investigation 1: Torque, Angular Acceleration, and the Moment of Inertia

Goals:

- To study the relationship between torque, angular acceleration, and moment of inertia in a system exhibiting pure rotation.
- To investigate the energetics of an object exhibiting pure rotation.

Introduction:
In this Investigation, you will use a torsion oscillator to study the relationship between the torque on an object (in this case, the torsion pendulum), and the angular acceleration of that object. In the case of a torsion oscillator, the torque is supplied by the wire, which exerts a torque on the pendulum given by $\tau_{\text {wire }}=-\kappa \theta$, where $\kappa$ is a constant known as the torsion constant, and $\theta$ is the angle of rotation of the pendulum away from equilibrium (i.e., the position in which no torques act on the pendulum). You should recognize the similarity between this torque law, $\tau_{\text {wire }}=-\kappa \theta$, and Hooke's force law for a spring, $\mathrm{F}=-\mathrm{kx}$, where $\kappa$ is the analogue of the spring constant, $k$, and the angle $\theta$ is analogous to the linear displacement from equilibrium, $x$.


Figure 1. Torsion pendulum setup for Investigation 1

## Activity 1: Twisted Behavior

Introduction: In this activity you will study the balancing of two opposing torques on a pendulum disk, the torque due to two hanging masses and the torque exerted by a wire (see Figure 3), and then determine the torsion constant of the wire.

Procedure: 1. Double-click on the Rotary Motion icon on the Desktop to launch the application you'll use for this activity.
2. Configure the Rotary Motion graph.

- Pull down the File menu and select Open. Open the file Torqued in the Lab 7 folder. A graph like that shown in Figure 2 should appear.


Figure 2. Torqued graph for Activity 1

Procedure: (continued)
3. Prepare the experimental setup in Figure 3.

- Measure the outer radius, $r$, of the aluminum ring with the two strings attached.

$$
r=
$$

- Place the aluminum ring on the torsion pendulum, making sure to "seat" the holes in the bottom of the ring on the two "knobs" protruding from the top of the pendulum surface.


Figure 3. Torsion pendulum setup for Activity 1

Predictions: - If you hang a mass $M$ at the end of each of the two strings shown in Figure 3, will these masses exert a torque on the pendulum? If not, write " 0 " below. If so, write an expression for the total torque that is exerted by the hanging masses on the pendulum (in terms of M and other known parameters).

$$
\tau_{\text {hang }}=
$$

$\qquad$
${ }^{\bullet}$ Will the hanging masses in Fig. 3 exert a net force on the pendulum? Explain.

- Record in Table 1 the torques that you predict will be exerted on the torsion pendulum when the following masses are placed on the ends of each of the two strings (see Figure 3): 5, 10, 15, and 20 grams.
- If you hang a mass $M$ on the ends of each string as described above, how do you predict the torsion pendulum will respond in order to achieve system equilibrium? Explain your reasoning.

| Mass <br> on each <br> string $[\mathrm{kg}]$ | Expected <br> Total Torque <br> on Pendulum from <br> Hanging Masses <br> [N•m] | Measured <br> Angular Position <br> (Absolute Value) <br> $\theta$ <br> [radians] |
| :---: | :---: | :---: |
| 0 |  |  |
| 0.005 |  |  |
| 0.010 |  |  |
| 0.015 |  |  |
| 0.020 |  |  |

Table 1. Predictions and results for Activity 1

Procedure: (continued)
4. Test your predictions.

- Drape the two strings over the pulleys as shown in Figure 3.
- Starting with no hanging masses initially on the strings, and with the pendulum completely motionless, click Start. After about 10 seconds, add one 5 gram hanger to each of the strings. Stabilize the pendulum after adding the mass so that it is motionless (i.e., does not oscillate).
- Repeat the above step for each of the masses listed in Table 1.

5. Select Analyze Data A. For each of the hanging masses in Table 1, measure the corresponding stable angular positions of the pendulum. Record the absolute value of these measured angles in Table 1.

Question: $\quad$ As you added the hanging masses, what did the behavior of the torsion pendulum indicate about the torques acting on the pendulum? Did you predict this?
$\qquad$
$\qquad$

Predictions: ${ }^{\bullet}$ Based on your previous results, is there a relationship between the torque exerted by the hanging masses, and the torque exerted by the wire? If so, write an expression for this relationship.
${ }^{\bullet}$ Can you describe a procedure by which the torsion constant, $\kappa$, of the wire can be determined from your previous data?

Procedure: (continued)
6. Plot the results of your previous experiment.

- Pull down the File menu and select Open. Open the file Data Entry 1 in the Lab 7 folder.
- Select Data A Table under the Windows menu. You should see a new window labeled "Data A" similar to that shown in Figure 4. Enter numerical data from Table 1 directly into the Data A table in the spaces shown.
- When all your data has been entered, click once on the graph window to bring it to the front so that you can view your points.


Figure 4. Table for entering the torque and angular position for Activity 1

Question: $\quad$ Do your results confirm the functional relationship for the torque exerted by the wire on the pendulum $\tau_{\text {wire }}$ shown in Figure 1. Why or why not?

Procedure: (continued)
7. Determine the torsion constant, $\kappa$, of the wire by "fitting" your data.

- Select Fit... under the Analyze menu. A window like that in Figure 5 will appear.


Figure 5. Window for defining the fitting function for Activity 1

- Make sure that Linear is selected, then click Maintain Fit. You should now see a straight line on top of your graph, which is the computer's "best fit" through all the data points.

8. Read the computer's "best fit" estimate of the torsion constant of the wire by selecting Fit Results... under the Analyze menu. Record the fit result below (Note - your value should have at least three significant figures!):

$$
\kappa=\ldots[\mathrm{N}-\mathrm{m} / \mathrm{rad}]
$$

9. Make a record of this fit to your data.

- Set Graph Title... to TORSION Constant and append your group's names.
- Print one copy of this graph.
- Record your measured value for $\kappa$ on the plot.


## Activity 2: Look Who's Torque-ing

Introduction: In this activity you will test the relationship between the torque exerted by the wire and the angular acceleration of the pendulum, $\tau=l \alpha$. We don't have a "Torque Probe" to independently monitor the torque exerted by the wire on the torsion pendulum. But in the last activity you should have carefully measured the dependence of this torque on the angular position, $\theta$. Therefore, by recording $\theta$ as a function of time, the computer can calculate the torque (exerted by the wire) versus time.


Figure 6. (a) Torsion pendulum and (b) large aluminum ring used in Activity 2


| Quantity <br> $\left[\mathrm{kg}-\mathrm{m}^{2}\right]$ | Predicted <br> Value | Measured <br> Value | \% <br> Difference |
| :---: | :---: | :---: | :---: |
| $\mathrm{I}_{0}$ |  |  |  |
| $\mathrm{I}_{\text {ring }}$ |  |  |  |
| $\mathrm{I}_{\text {total }}$ |  |  |  |

Table 2: Results and predictions for Activity 2

Procedure: 1. Test your predictions. Prepare to graph the torque, $\tau$, angular acceleration, $\alpha$, and the ratio torque/angular acceleration, $\tau / \alpha$.

- Pull down the File menu and select Open. Open the file Twister in the Lab 7 folder. A graph like that shown in Figure 8 should appear.


Figure 8. Twister graph format for Activity 2
2. Define the torque graph.

- Prepare to plot the torque exerted by the wire on the pendulum. Under the Data menu, select Modify... and then select Torque. Replace the " 0 " in the formula space with the formula for the torque exerted by the wire:

$$
-(\kappa) \text { *"angle" }
$$

where $\kappa$ is the actual value for the torsion constant of the wire which you measured in Activity 1. Click OK.

Procedure: 3. Define the "Torque/Angular Acceleration" graph.

TA Discussion Checkbox

Questions: - What quantity is given by the ratio $\tau / \alpha$ ? If your measured and predicted values for this ratio differed significantly, provide possible explanations.
-Did you find that the angular acceleration vs. time plot "follows" the torque vs. time plot in the sense that both torque and angular acceleration increase and decrease at the same times (have maximum and minimum values at the same times, etc.)? Is this what you predicted?

Procedure: (continued)
7. Perform the measurement with the aluminum ring added to the pendulum.

- Place the aluminum ring onto the pendulum as shown in Figure 9. Make sure that you "seat" the two holes in the ring on the two protruding knobs on the top of the pendulum.
- Repeat steps 4 and 5 above in this new configuration.
- Analyze your new results, and record in Table 2 your measured value for the moment of inertia, $I_{\text {total }}$. Also, determine the percent difference between your measurement and prediction for $\mathrm{I}_{\text {total }}$.


Figure 9. Torsion pendulum setup for Activity 2
8. Using your measured values for the moments of inertia of both the bare pendulum, $I_{0}$, and the bare pendulum + aluminum ring, $I_{\text {total }}$, deduce the value for the moment of inertia of the ring, $I_{\text {ring }}$, and record this measured value in Table 2. Compute the percent difference between your measurements and predictions for $I_{\text {ring }}$.
9. Make a record of your measurements.

- Set Graph Title... to TwISTER and add your group's names.
- Print one copy of this graph. Note points of significant disagreement between your results and predictions.

10. DO NOT ERASE your results from this activity as you will need them later!

Question: ${ }^{\bullet}$ If you had a small wheel with an unknown moment of inertia, $l_{\text {unk }}$, describe an extension of your previous procedure with which you could determine $l_{\text {unk. }}$.

## Activity 3: Harnessing the Energy of a Twister

Procedure: 1. Temporarily remove Data B from the screen by selecting Hide Data B under the Data menu for each of the three graphs on the screen.


Figure 10. Predictions for Activity 3
${ }^{\bullet}$ What relationships are there, if any, between the total energy and the maximum kinetic and potential energies? If relationships exist, write mathematical expressions describing these relationships if possible.

- Use an estimate of the maximum potential energy from your data to predict the total energy associated with the torsion oscillator system during your last experiment (pendulum + aluminum ring). Record your numerical prediction below and in Table 3. HINT - You will need to change the top graph ("Torque") to display the angular position, $\theta$, of your recorded data.

Total Energy $=$ [J]

| Quantity | Predicted <br> Value | Measured <br> Value | \% <br> Difference |
| :---: | :---: | :---: | :---: |
| Total Energy <br> $[\mathrm{J}]$ |  |  |  |

Table 3: Results and predictions for Activity 3

Procedure: (continued)
2. Test your predictions. Click on the top graph to select it. Now, Modify the total energy formula for the torsion pendulum for Data A (the pendulum with the aluminum ring added).

- Select Modify... under the Data menu, then select Total Energy.
- Replace the " 0 " in the "formula" space with the expected relationship for the cart's total energy.
"P.E." + "K.E._rot"

3. Click on the middle graph ("A.Accel.") to select it. Next, Modify the kinetic energy formula for the torsion oscillator.

- Under the Data menu, first select Modify..., then select K.E._rot.
- Replace the " 0 " in the "formula" space with the relationship for the torsion oscillator's kinetic energy, K.E. $=1 / 2 I_{\text {total }} \omega^{2}$ :

$$
\text { K.E. }=0.5^{*}\left(I_{\text {total }}\right) \text { * "A. Vel"^2 }
$$

where $I_{\text {total }}$ is the value for the moment of inertia you measured earlier.
4. Click on the lower graph ("Torque/A.Accel") to select it. Next, Modify the potential energy formula.

- Under the Data menu, first select Modify..., then select P.E..
- Replace the " 0 " in the "formula" space with the relationship for the torsion oscillator's potential energy, P.E. $=1 / 2^{\kappa} \theta^{2}$ :

$$
\text { P.E. }=0.5 \text { * }(\kappa) \text { * "Angle"^2 }
$$

where $\kappa$ is the value of the torsion constant you measured earlier.
5. Make sure that you rescale the $y$-axis of the total energy graph so that the finite value of the energy is clearly visible (try a range from 0 to 0.01 J ).
6. Make a record of your measurements:

- Set Graph Title... to Twister Energy and add your group's names.
- Print one copy of this graph for your group.

Questions: ${ }^{\bullet}$ Did the potential and kinetic energies of the torsion pendulum behave the way you predicted?

- Was energy conserved? What are the possible sources of energy loss in this system, if any?
- What was the observed relationship between the total energy and the maximum potential and kinetic energies?


## Activity 4: An Important Period in Your Life

Introduction: One of the obvious features of your results in the previous activities which we have ignored thus far is the oscillatory motion of the pendulum. You haven't been introduced yet to periodic motion in Lecture, but in this activity you'll identify one or two significant results related to periodic motion which you'll learn soon in lecture and study more carefully in Laboratory 8.
Procedure: 1. Recall Data B (pendulum only) for use in this activity by selecting Show Data B under the Data menu, for each of the three graphs on the screen. Data A (pendulum + aluminum ring) should already be present (red curves).

| Predictions: | Notice that both the torque and angular acceleration in the last activity exhibit <br> periodic behavior, i.e., the magnitudes of these quantities repeat themselves in <br> equal intervals of time; this time interval is known as the oscillatory period, T. |
| :--- | :--- |
| © Do you expect the oscillatory period of the torsion pendulum to depend on <br> the moment of inertia, I, of the pendulum? Explain your answer. <br>  <br> - If you answered "yes" in the prediction above, do you expect the period to <br> increase or decrease with increasing moment of inertia? Why? |  |

## 2. Test your predictions.

- Change one of your graphs to either Torque or Angular Acceleration. Rescale this graph if necessary to see the oscillations clearly.
- By measuring the time-interval between consecutive peaks in the torque or angular acceleration graphs, find the oscillatory periods of the torsion pendulum:
(a) $\mathbf{T}_{\mathbf{o}}$ - before the aluminum ring was added (Analyze Data B)
(b) $\mathbf{T}_{\mathbf{1}}$ - after the aluminum ring was added, (Analyze Data A).

$$
\mathrm{T}_{0}=\ldots[\mathrm{sec}] \quad \mathrm{T}_{1}=\square[\mathrm{sec}]
$$

3. Record below and in the appropriate column of Table 4 your measured value of the ratio $T_{1} / T_{0}$.

Measured $T_{1} / T_{0}=$ $\qquad$
4. Record below your measured value for the ratio of the moment of inertia with the aluminum ring, $I_{\text {total }}$, to the moment of inertia without the ring, $I_{0}$ (from Table 2): $I_{\text {total }} / I_{0}$.

Measured $I_{\text {total }} I_{0}=$ $\qquad$

Procedure: 5. For each "trial" value of the exponent $\varepsilon$ in Table 4, compute the quantity (continued) $\left(I_{\text {total }} / I_{0}\right)^{\varepsilon}$ and determine the percent difference between this quantity and your measured ratio $T_{1} / T_{0}$. Put a check mark $(\sqrt{ })$ in the row that gives the best comparison.

| Trial <br> exponent <br> $\varepsilon$ | $\left(I_{\text {total }} / I_{o}\right)^{\varepsilon}$ | Measured <br> $\mathrm{T}_{1} / \mathrm{T}_{\mathrm{o}}$ | $\%$ <br> Difference | Best <br> Comparison |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| $1 / 2$ |  |  |  |  |
| -1 |  |  |  |  |
| $-1 / 2$ |  |  |  |  |

Table 4: Results and predictions for Activity 4
Questions: ${ }^{\bullet}$ Were your predictions for the correlation, or absence of a correlation, between the period, T , and the moment of inertia of the torsion pendulum, I, correct? If you found a correlation, write a mathematical relationship describing the proportionality between T and I .

- Can you think of any other parameters on which the period $T$ should depend? Justify your answer(s).

