A solid sphere, a hollow sphere, and a solid cylinder have the same mass M and the same radius R. They are positioned at the top of the same ramp at the same height h = 7 m. They are released from rest and roll down the ramp without slipping.

- 1) Which object has the largest velocity when it reaches the bottom of the ramp?
 - a. The answer cannot be determined with the information provided.
 - b. The solid cylinder.
 - c. All three objects have the same velocity at the bottom of the ramp.
 - d. The solid sphere.
 - e. The hollow sphere.
- 2) What is the velocity of the solid sphere when it reaches the bottom of the ramp, V_s ?

a.
$$V_s = 9.08 \text{ m/s}$$

b.
$$V_s = 9.9 \text{ m/s}$$

c.
$$V_s = 13.1 \text{ m/s}$$

d.
$$V_s = 11.72 \text{ m/s}$$

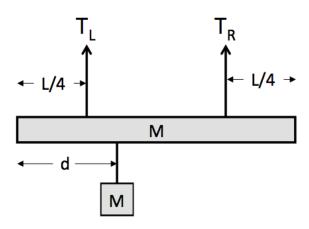
e.
$$V_s = 8.29 \text{ m/s}$$

3) Suppose the answer to the above problem is V_s . If the radius of the solid sphere was increased, but its mass and the vertical distance it rolls down the ramp was kept the same, how would its new speed at the bottom of the ramp V_{new} compare to V_s . ?

a.
$$V_{new} < V_s$$

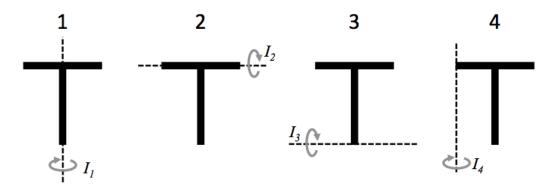
b.
$$V_{new} > V_s$$

c.
$$V_{new} = V_s$$



A beam of mass M=7 kg and length L=7.43 m is suspended by two vertical ropes as shown. The rope on the left has tension T_L and is attached a distance L/4 from the left end of the beam. The rope on the right has tension T_R and is attached a distance L/4 from the right end of the beam. A box that has the same mass M as the beam is suspended by a short rope which it attached a distance d from the left end of the beam.

- 4) If the box is hung directly under the left rope (i.e. d=L/4), how would T_L compare to T_R ?
 - a. $T_L > T_R$
 - b. $\overline{T_L} < \overline{T_R}$
 - c. $T_L = T_R$
- 5) What is the value of d for which $T_L = \frac{3}{4}Mg$?
 - a. d = 1.86 m
 - b. d = 6.19 m
 - c. d = 7.43 m
 - d. d = 4.64 m
 - e. d = 2.48 m
- 6) If the box is hung at the right end of the beam (i.e. d = L), what is the value of T_R ?
 - a. $T_R = 91.56 \text{ N}$
 - b. $T_R = 137.34 \text{ N}$
 - c. $T_R = 103.01 \text{ N}$
 - $d. T_R = 0 N$
 - e. $T_R = 68.67 \text{ N}$



A T shaped object is made by combining two identical, uniform thin rods of equal mass and length. The object can be rotated about the four different axes shown by the dashed lines in the four diagrams.

7) Rank in increasing order the moments of inertia, I_1 to I_4 for rotation about the dashed lines.

a.
$$I_2 < I_1 < I_4 < I_3$$

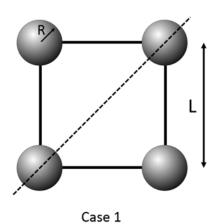
b.
$$I_1 < I_2 < I_3 < I_4$$

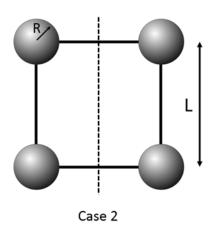
c.
$$I_1 < I_2 < I_4 < I_3$$



Choose the situation where the magnitude of the torque about the center of mass is largest.

- a. Situation C
- b. Situation A
- c. Situation B





Four solid spheres with mass M and radius R are connected together with massless rods to form a square of length L. In Case 1, the object is rotated about an axis along the diagonal of the square. In Case 2, the object is rotated about an axis that bisects the sides of the square.

9) Compute the ratio of the moments of inertia, I_1/I_2 for the two cases.

a.
$$I_1/I_2 = \sqrt{2}$$

b. $I_1/I_2 = 1$
c. $I_1/I_2 = 1/2$

b.
$$I_1/I_2 = 1$$

c.
$$I_1/I_2 = 1/2$$

d.
$$I_1/I_2 = 1/2$$

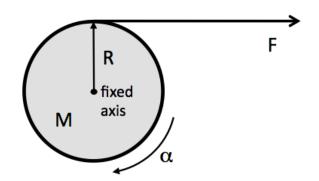
e. $I_1/I_2 = 1/\sqrt{2}$
e. $I_1/I_2 = 2$

e.
$$I_1/I_2 = 2$$

10) If the solid spheres were replaced with hollow spheres, how would the ratio I_1/I_2 change?

- a. The ratio would increase.
- b. The ratio would decrease.
- c. The ratio would not change.

A solid cylinder of radius R = 0.45 m starts from rest and can rotate without friction about an axis through its center of mass as shown. A string is wrapped around the circumference of the cylinder and is pulled with a constant force F=3.6 N. The rotation axis is fixed so that the center of mass of the cylinder does not move as the cylinder rotates. The magnitude of the angular acceleration of the cylinder is $\alpha = 11 \text{ rad/s}^2$.



- 11) Which of the following statements is true?
 - a. There is a non-zero net torque on the cylinder, but the net force on the cylinder is zero.
 - b. There is both a non-zero net torque and a non-zero net force on the cylinder.
 - c. There is a non-zero net force on the cylinder, but the net torque on the cylinder is zero.
- 12) What is the mass of the cylinder M?

a.
$$M = 2.91 \text{ kg}$$

b.
$$M = 1.45 \text{ kg}$$

c.
$$M = 0.73 \text{ kg}$$

d.
$$M = 0.29 \text{ kg}$$

e.
$$M = 0.15 \text{ kg}$$

13) What is the angular displacement θ_t of the cylinder after the force has been pulling for time t = 8 seconds?

a.
$$\theta_t = 88 \text{ rad}$$

b.
$$\theta_t = 1408 \text{ rad}$$

c.
$$\theta_t = 704 \text{ rad}$$

d.
$$\theta_t = 44 \text{ rad}$$

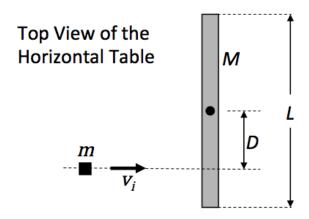
e.
$$\theta_t = 352 \text{ rad}$$

14) Suppose the kinetic energy of the cylinder after 8 seconds of pulling is K_t . How does the kinetic energy after pulling for twice as much time, K_{2t} , compare to K_t ?

a.
$$K_{2t} < 2K_t$$

b.
$$K_{2t} > 2K_t$$

c.
$$K_{2t} = 2K_t$$



A block (which you should treat like a point particle) has mass m = 1.785 kg and slides with initial speed v_i in the +x direction. It collides with a rod of length L = 1.1 m and mass M = 35.7 kg, which is initially at rest and oriented perpendicular to the path of the block. The block hits the rod a distance D = 0.275 from the center of the rod. After the collision, the block is at rest and the rod spins with angular velocity of $\omega_f = 25$ rad/s. Everything is on top of a horizontal frictionless table, and the rod has a fixed frictionless pivot through its center that allows it to rotate freely but keeps its center from moving.

- 15) Which of the following statements best describes the collision?
 - a. The angular momentum about the pivot is conserved, but the linear momentum is not conserved.
 - b. The angular momentum about the pivot and the linear momentum are both conserved.
 - c. Neither the angular momentum about the pivot nor the linear momentum are conserved.
- 16) What was the initial speed of the block?

a.
$$v_i = 35.5 \text{ m/s}$$

b.
$$v_i = 1100 \text{ m/s}$$

c.
$$v_i = 733.33 \text{ m/s}$$

d.
$$v_i = 71 \text{ m/s}$$

e.
$$v_i = 183.33 \text{ m/s}$$

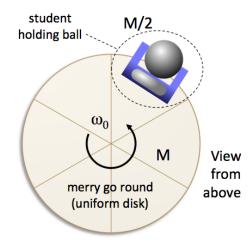
17) Suppose the experiment is repeated using new block that has the same mass and initial speed as in the original situation, but is made of a different material so that it bounces back and ends up moving in the -x direction after the collision. How would the final angular velocity of the rod in this new case, ω_{new} , compare to the final angular velocity of the rod in the original case ω_f ?

a.
$$\omega_{new} < \omega_f$$

b.
$$\omega_{new} > \omega_f$$

c.
$$\omega_{new} = \omega_f$$

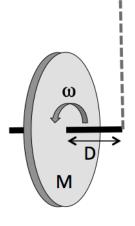
A student holding a heavy ball stands facing outward on the outer edge of a merry go round. As viewed from above, the merry go round rotates counter-clockwise in the horizontal plane around a frictionless vertical axis through its center, and the magnitude of its initial angular velocity is ω_0 .



- 18) Suppose the student throws the ball straight outward, directly away from the center of the merry go round. What happens to the magnitude of the angular velocity of the merry go round?
 - a. It increases.
 - b. It decreases.
 - c. It does not change.
- 19) Instead of throwing the ball outward, suppose she throws the ball to her right. What happens to the magnitude of the angular velocity of the merry go round?
 - a. It decreases.
 - b. It increases.
 - c. It does not change.
- 20) Suppose instead that the student holds on to the ball and walks from the outer edge of the merry go round to its center. After she reaches the center, what is the magnitude of the new angular velocity of the system ω_{new} ? Assume the merry go round is a uniform solid disk of mass M, and treat the combination of the student and the ball as a point particle that has mass M/2.
 - a. $\omega_{new}~=~3\omega_0/2$
 - b. $\omega_{new} = 3\omega_0$
 - c. $\omega_{new} = \omega_0$
 - d. $\omega_{new}~=~4\omega_0/3$
 - e. $\omega_{new} = 2\omega_0$

A gyroscope made from a solid disk of mass M and radius R hangs from a vertical rope attached to the ceiling. The disk spins around a horizontal axle through its center in the direction shown by the arrow, and the rope is attached to one end of this axle at a distance D = 1.39 m from the center of mass of the disk.

The moment of inertia of the disk is $I = 2.14 \text{ kg-m}^2$. The angular momentum of the spinning disk is $L = 64.9 \text{ kg-m}^2/\text{s}$. The time it takes the gyroscope to make one complete revolution in the horizontal plane (its precession period) is 11.8 seconds).



21) What is the magnitude of the angular velocity of the spinning disk?

a.
$$\omega = 45.5 \text{ rad/s}$$

b.
$$\omega = 30.4 \text{ rad/s}$$

c.
$$\omega = 72.9 \text{ rad/s}$$

d.
$$\omega = 91.1 \text{ rad/s}$$

e.
$$\omega = 36.4 \text{ rad/s}$$

22) What is the mass of the gyroscope?

a.
$$M = 1.27 \text{ kg}$$

b.
$$M = 0.81 \text{ kg}$$

c.
$$M = 56.16 \text{ kg}$$

d.
$$M = 3.52 \text{ kg}$$

e.
$$M = 2.53 \text{ kg}$$

23) Suppose the same gyroscope is moved to the surface of a new planet where the acceleration of gravity on the surface is bigger than it is on Earth. How does the precession period change?

- a. It stays the same.
- b. It decreases.
- c. It increases.