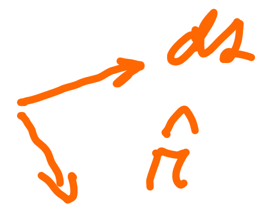
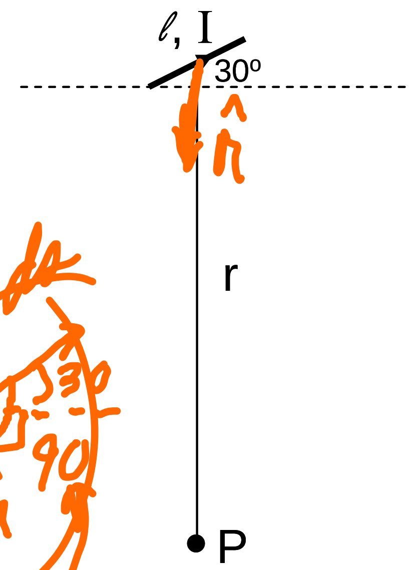
Biot-Savart Law: Allows you to find the magnetic field contribution, $d\vec{B}$, due to a tiny segment of current-carrying wire.

$$d\vec{B} = \frac{\mu_o I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

$$dB = \frac{\mu_0 I}{4\pi}\frac{ds\sin\theta}{r^2}$$



A short, straight wire segment of length carries current I and is oriented so that it makes an angle of 30º with the horizontal. Point P is a distance r below the wire segment.



15-16 F09

15. Which expression below is the best approximation for the magnetic field caused by the wire segment at point P.

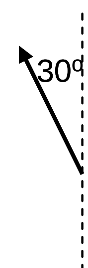
$$\frac{\mu_o I l \cos 30^\circ}{4\pi r^2} \qquad \frac{\mu_o I l \sin 30^\circ}{4\pi r^2} \qquad \frac{\mu_o I l}{4\pi r^2}$$

$$dB = \frac{\mu_o I}{4\pi} \frac{ds \sin 120}{r^2}$$

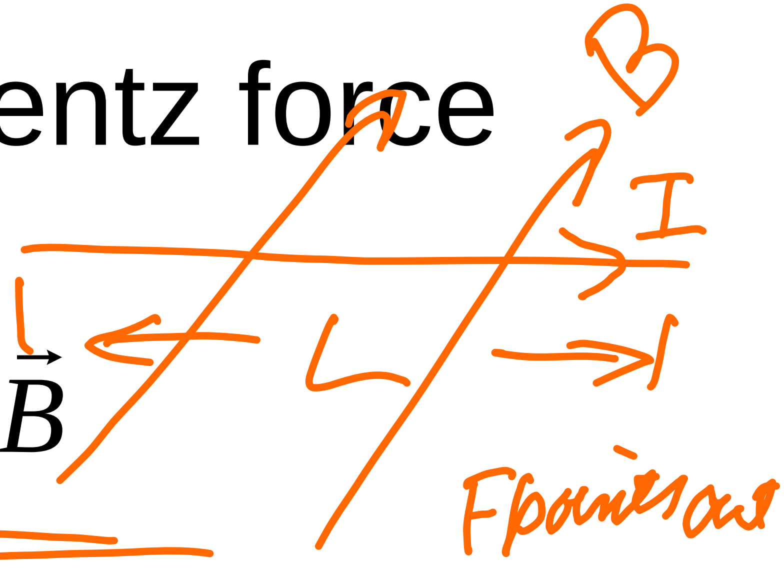
$\sin 120 = \cos 30$

16. Which direction depicted below best describes the direction of the magnetic field at point P due to the wire segment?

⊙ ⊗

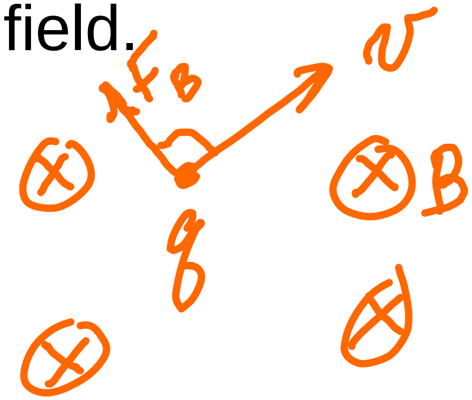


B fields and Lorentz force



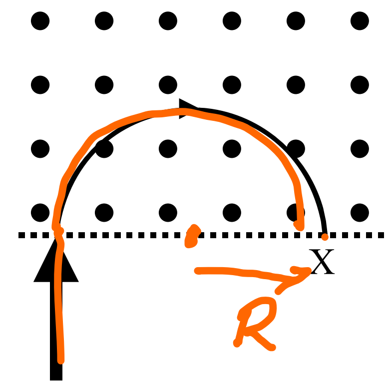
$$\vec{F} = I\vec{L} \times \vec{B} = q\vec{v} \times \vec{B}$$

Describes force on either a current-carrying wire of length L, or a charge Q moving with velocity v, in the presence of an external B field.



S09

26. A positively charged particle is injected into a region occupied by a uniform B field pointing out of the paper. The particle's initial velocity, *v*, is straight upward. It emerges from the B field region at position X, after travelling for a time Δ*t* through the field. If the particle was injected instead with velocity **2*v***, it would



(A) emerge at the same position X, but after a different amount of time Δ*t*.
(B) emerge at a different position X, but after the same amount of time Δ*t*.
(C) emerge at a different position X, and after a different amount of time Δ*t*.

Newton's 2nd Law:

$F = ma$

$qvB = m\frac{v^2}{R}$

$R = \frac{mv}{qB}$

$t = \frac{d}{v} = \frac{\pi R}{v}$

$= \frac{\pi\, mv/qB}{v}$

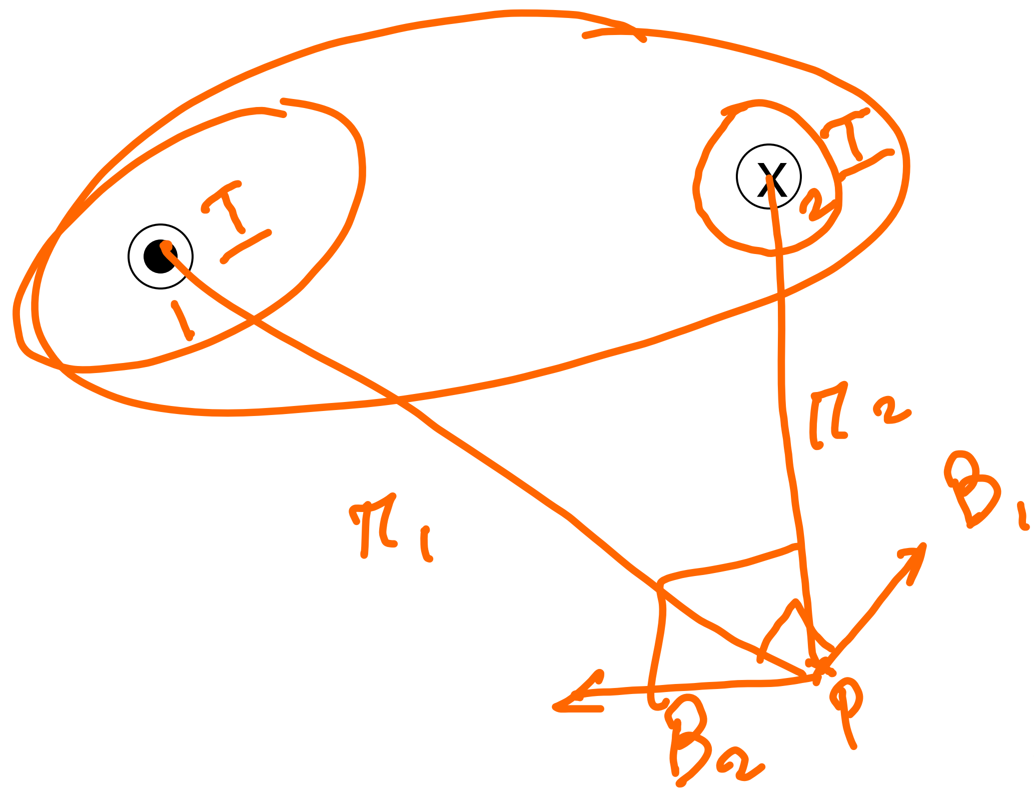
Ampere's Law

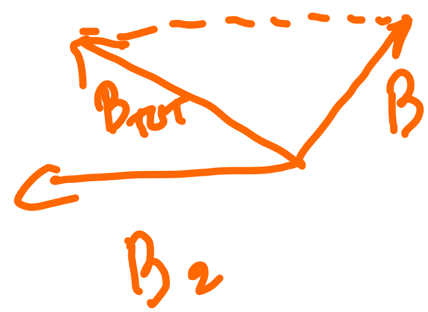
$$\oint \vec{B} \bullet d\vec{l} = \mu_o I_{enclosed}$$

Ampere's Law is always valid. Can compute B from it IF you have lots of symmetry (cylindrical, planar). Examples of when it can be used to find B: B from long wire, B from solenoid, B from sheet of current.

If you have multiple long wires, compute B from each one, and then use superposition to add up the total contribution from all of them (remember you have vectors to add which have size and direction).

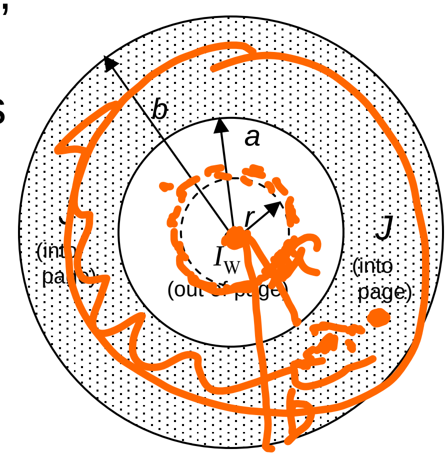
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$





$$B_1 = \frac{\mu_0 I}{2\pi r_1}$$

An infinitely long wire carries a current I_w = 3.0 A (*out* of the page), and lies along the axis of symmetry of a cylindrical shell of inner radius *a* = 2.70 cm and outer radius *b* = 7.60 cm. The shell carries a current I_2 = 1.8 A (into the page) distributed with uniform current density J.



20-22 S09

Find B at r=2 cm, r=5.4 cm, r=9 cm

$r = 2\,cm = .02\,m$

$B = \frac{\mu_0 I}{2\pi r} = 3\times10^{-5}\ T$

$r = 5.4\,cm = .054\,m$

$B_{TOT} = B_{center} - B_{shaded}$

$= \frac{\mu_0 I_{center}}{2\pi r} - \frac{\mu_0 I_{enc}}{2\pi r} = 8.2\times10^{-6}$

$I_{enc} = I\left(\frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2}\right)$

Faraday's Law

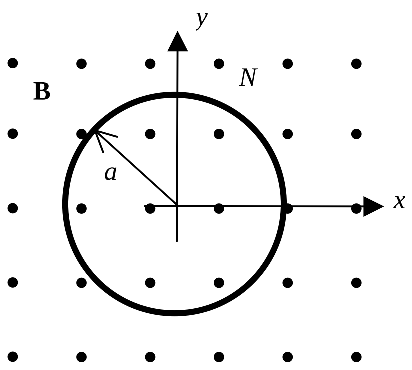
$$EMF = \oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi}{dt}$$

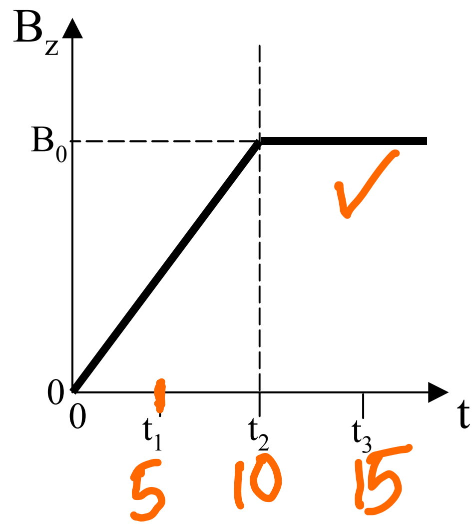
where Φ is the magnetic flux, which is computed by finding the B lines crossing an area (B times area).

When is there an induced current?

a) B changes, b) Area changes, c) orientation of area relative to B changes

Current (if it exists) in loop opposes change in flux. (put thumb of right hand in direction you need induced B to be to oppose changing flux and fingers point to direction of induced current)





In what direction is the induced current flowing at time t_1 = 5 seconds?

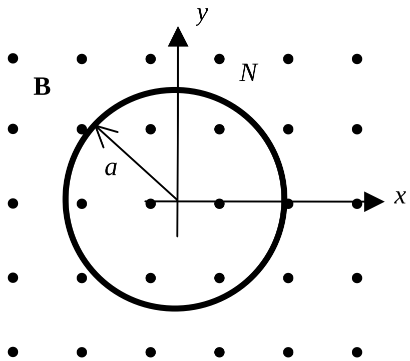
14-16 S09

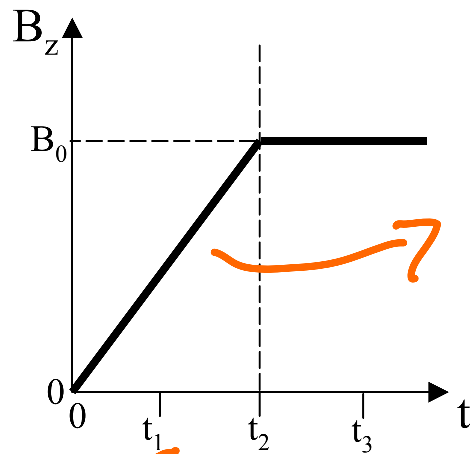
clockwise

A tightly wound circular coil with radius *a* = 3 cm and ***N* = 150 turns** lies parallel to the *x*-*y* plane. The total resistance of the coil is 5W. A spatially uniform magnetic field extends over the entire region of the coil and points in the +*z* direction (out of the page). The magnitude of the field varies with time as shown below (the maximum field B_0 = 2 T is obtained at time t_2 = 10 seconds). Neglect the effect of any B fields that might be created in the coil.

Compare I_1, the magnitude of the current induced at time t_1, to I_3, the magnitude of the current induced at time t_3 = 15 seconds

$I_1 \ge I_3 = 0$





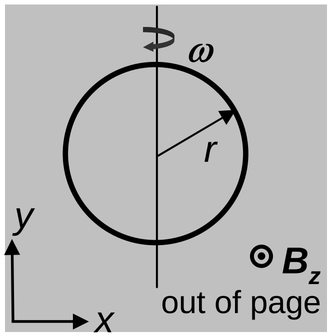
A tightly wound circular coil with radius a = 3 cm and ***N* = 150 turns** lies parallel to the *x*-*y* plane. The total resistance of the coil is 5W. A spatially uniform magnetic field extends over the entire region of the coil and points in the +*z* direction (out of the page). The magnitude of the field varies with time as shown below (the maximum field B_0 = 2 T is obtained at time t_2 = 10 seconds). Neglect the effect of any B fields that might be created in the coil.

What is I_1, the magnitude of the current induced at time t_1 = 5 seconds? (1mA=10^{-3}A)

(A) I_1 = 12 mA
(B) I_1 = 17 mA
(C) I_1 = 38 mA
(D) I_1 = 52 mA
(E) I_1 = 85 mA

A circular loop of radius r = 3 cm rotates in a region of uniform magnetic field, B_z = 8T, as shown below. at what angular velocity ω must the loop rotate in order to obtain a maximum induced EMF of $\mathcal{E}$ = 100 mV?

7 F07



r = 3 cm
B = 8 T
induced
$|E_{max}|$ = 100mV

a. ω = 0.02 rad/sec
b. ω = 2.4 rad/sec
c. ω = 3.0 rad/sec
d. ω = 4.4 rad/sec
e. ω = 13.2 rad/sec

$\phi = BA\cos\theta$

$Emf = \frac{d\phi}{dt} = BA\frac{d}{dt}\cos\theta = BA\sin\theta\frac{d\theta}{dt}$

ω

$Emf = (BA\sin\theta)\omega$

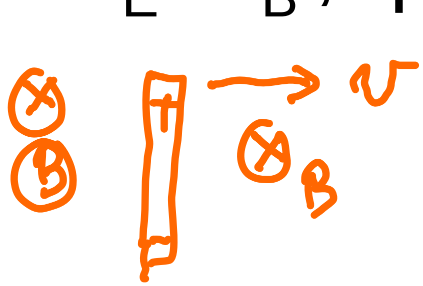
1

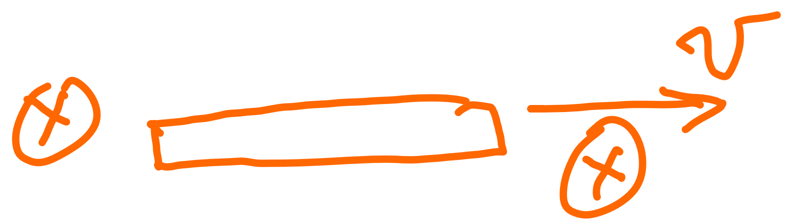
$\omega = 4.4$ rad/s

Motional EMF

- Don't need (necessarily) a complete loop to have induced voltage.
- Need a long conductor moving in "the right way" in the presence of B field.
- "right way" means that charges separate until equilibrium is reached:

$$F_E = F_B\ ,\ q\mathbf{E} = q\mathbf{v} \times \mathbf{B}$$





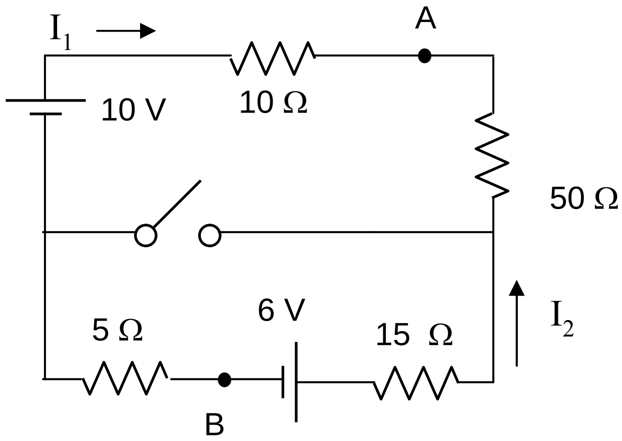
Kirchhoff's Rules

1) KVR: sum of voltages around a closed loop is 0.
2) Sum of currents entering a node = sum of currents leaving node.

Combining resistors in series and parallel.

Circuit elements in parallel have same V

F08



13. With the switch <u>open</u>, what is the current I_1 ? (A positive sign means that current flows in the direction of the arrow.)

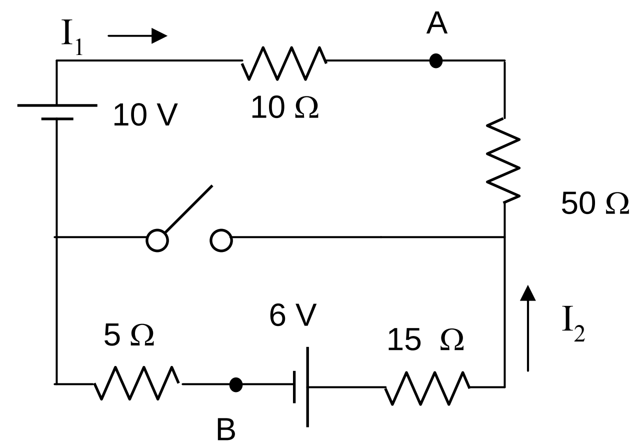
a. – 0.20 A
b. – 0.05 A
c. + 0.05 A
d. + 0.20 A
e. + 0.35 A

14. Once the switch is <u>closed</u>, what is the power dissipated in the 50 Ω resistor?

a. 0.13 W
b. 1.39 W
c. 2.47 W

15. How much current passes through the closed switch?

a. 0.05 A
b. 0.09 A
c. 0.12 A
d. 0.33 A
e. 0.47 A



16. With the switch closed, what is the voltage difference, V_A-V_B ?

a. $V_A - V_B = 10 - 10\,I_1 - 5\,I_2$
b. $V_A - V_B = 10 + 10\,I_1 + 5\,I_2$
c. $V_A - V_B = -6 + 50\,I_1 + 15\,I_2$
d. $V_A - V_B = 6 + 50\,I_1 - 15\,I_2$
e. $V_A - V_B = 6 - 50\,I_1 + 15\,I_2$

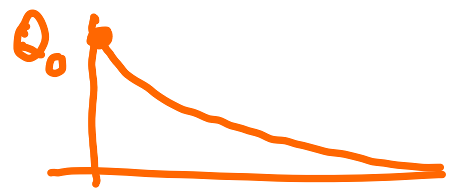
Charging and discharging capacitors

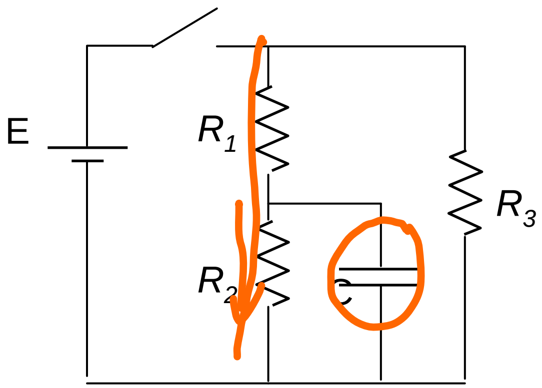
Immediately after a switch is closed and current begins to flow, an uncharged capacitor acts like a short circuit (a wire).

After a long time, a capacitor is charged and no current flows through it, thus acting like an open circuit (a cut wire).

Charging: $$Q = Q_{\infty}\left[1 - e^{-\frac{t}{\tau}}\right]$$

Discharging: $$Q = Q_0 e^{-\frac{t}{\tau}}$$



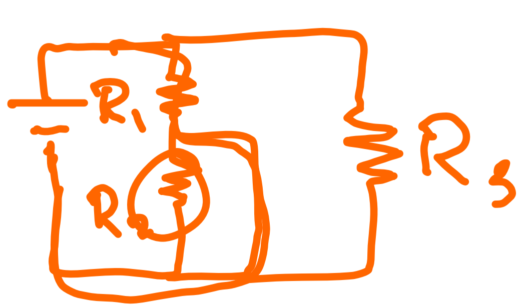


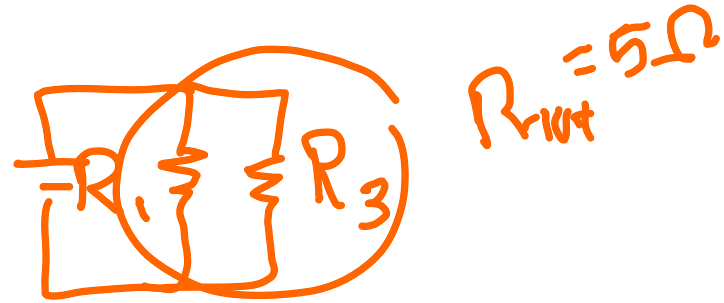
$E = 6$ V
$R_1 = R_2 = R_3 = 10\ \Omega$
$C_1 = 1\ \mu F$

9-12 F08

9. What is the current through the battery <u>immediately</u> after the switch is closed?

a. $I = 0.6$ A
b. $I = 0.9$ A
c. $I = 1.2$ A



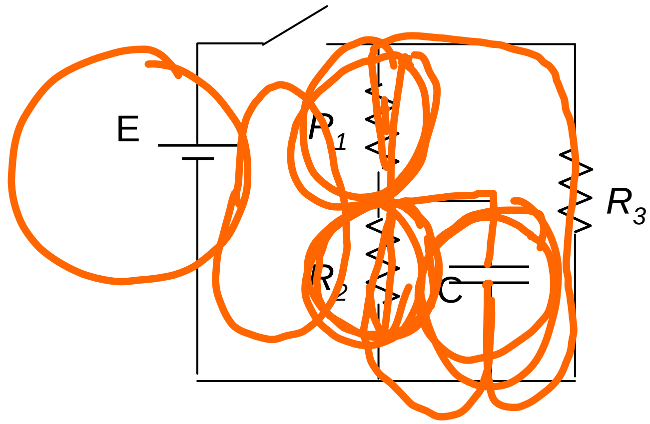


10. After the switch is closed, the current through resistor R_1,

a. Increases with time
b. Decreases with time
c. Remains constant with time

11. Find the charge on the capacitor a long time after the switch is closed.

a. $Q = 2.0\ \mu C$
b. $Q = 3.0\ \mu C$
c. $Q = 4.0\ \mu C$
d. $Q = 5.0\ \mu C$
e. $Q = 6.0\ \mu C$



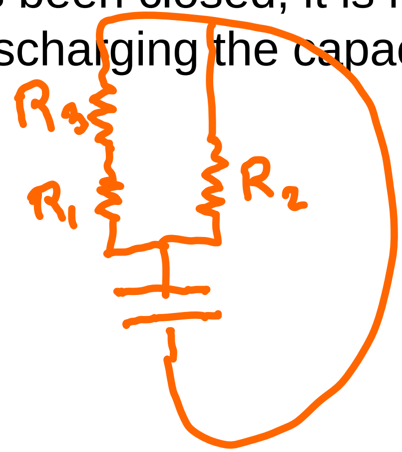
$E = 6$ V
$R_1 = R_2 = R_3 = 10\ \Omega$
$C_1 = 1\ \mu F$

9-12 F08

$V_{c\infty} = 3$ Volts $C = \frac{Q}{V}$

12. A long time after the switch has been closed, it is re-opened. What is the time constant for discharging the capacitor?

a. 2.4 μs
b. 3.3 μs
c. 6.7 μs
d. 10.0 μs
e. 30.0 μs



$\tau = R C$
R_{eq}

Torque on current loops

Magnetic moment:

$$\vec{\mu} = NIA \text{ (direction by right hand rule)}$$

Torque of loop in presence of B field:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Magnetic moment and B want to be aligned if loop released and allowed to rotate on its own

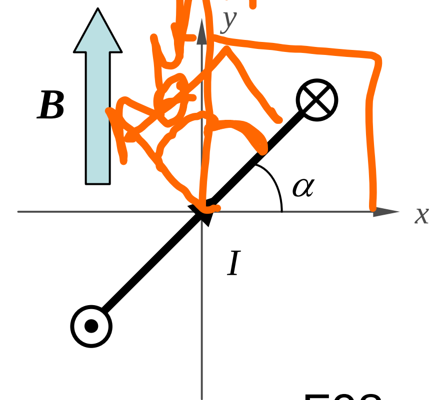
Torque on current loops

Potential energy:

$$U = -\vec{\mu} \bullet \vec{B}$$

If released, loop wants to rotate in direction so as to reduce its potential energy. Potential energy is lowest when μ and B are aligned.

The figure at right depicts a square wire coil with 4 loops. The length of each side of the square is *L*. The coil is situated in a region of constant magnetic field ***B*** = 0.2 T pointing in the +*y* direction. A current *I* = 20 amps flows in the coil in the direction shown (the black arrowhead indicates the current direction on the side of the square nearest you.) The square coil makes an angle of a with the *xz*-plane and the coil has a magnetic dipole moment with magnitude 25 A m^2



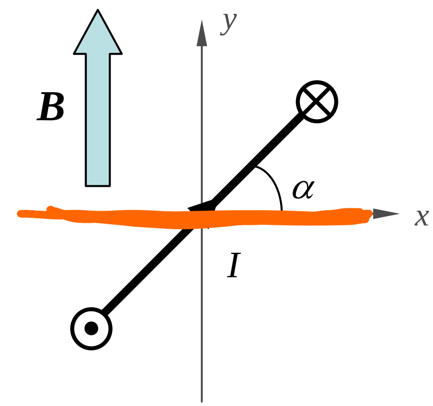
F08

1. What is the length of a side of the square loop?

2. What is the magnitude of the torque on loop with α=30?

3. What is direction of torque on the loop when α=30?

The figure at right depicts a square wire coil with 4 loops. The length of each side of the square is *L.* The coil is situated in a region of constant magnetic field ***B*** = 0.2 T pointing in the +*y* direction. A current *I* = 20 amps flows in the coil in the direction shown (the black arrowhead indicates the current direction on the side of the square nearest you.) The square coil makes an angle of a with the *xz*-plane and the coil has a magnetic dipole moment with magnitude 25 A m^2



4. At which angle, α, does the coil have the lowest potential energy?

$\alpha = 0$, μ & B lined up, lowest U

5. What is the magnitude of the change in potential energy of the coil when it is rotated from α=30 to α=0?

$U = \mu \cdot B = \mu B(\cos 0 - \cos 30)$

.866