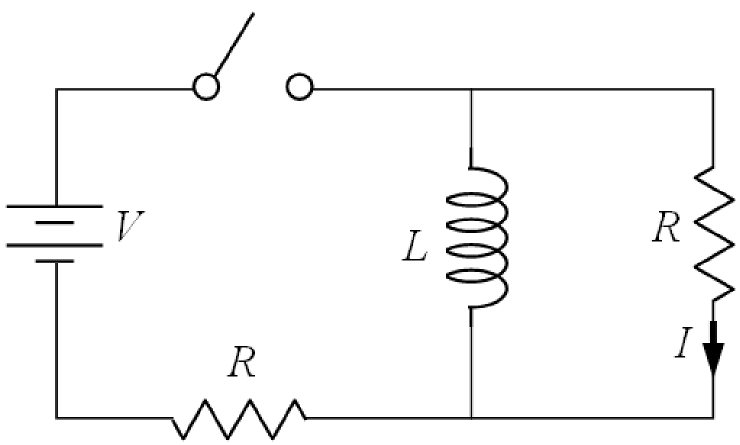
Induction and RL circuits.

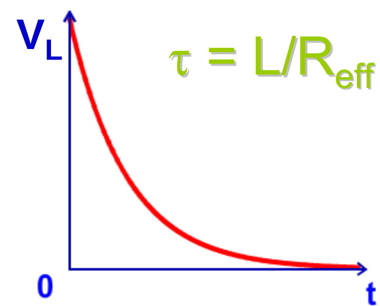
$$V_L = L\frac{dI}{dt}$$

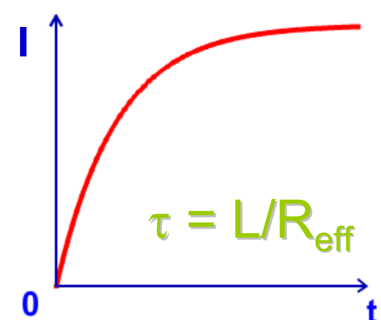
Inductors do not change their current instantaneously.
Behavior of inductors at t=0 and t=∞.

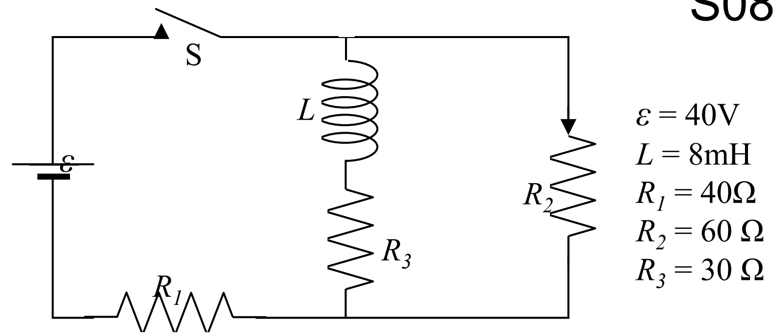
Even though I is 0 initially through inductor after switch is closed, its voltage is NOT 0.

Plot of voltage and current when switch is closed:









1. Switch S has been open for a long time. Immediately after switch S is closed, the voltage across resistor *R3* is zero.

*a. True
b. False

2. Find the magnitude of the time derivative (*dI/dt*) through the inductor immediately after switch S is closed.

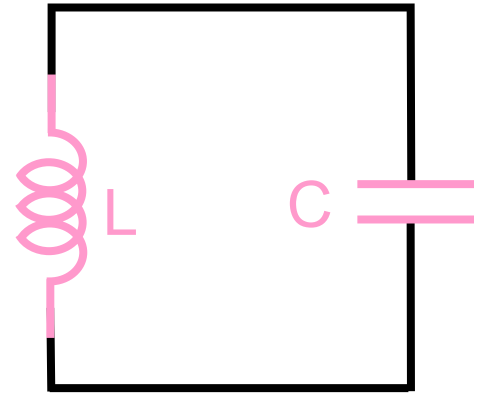
a. *dI/dt* = 0 A/s
b. *dI/dt* = 1000 A/s
c. *dI/dt* = 2000 A/s
*d. *dI/dt* = 3000 A/s
e. dI/dt = 4000 A/s

3. Now, after the circuit reaches a time-independent state, switch S is opened again. Immediately after the switch is opened, what is the direction of the current through resistor R2?

a. down (as shown by arrow)
*b. up (opposite of arrow)

LC Oscillations

Energy is not lost—it is “exchanged” back and forth
Between L and C



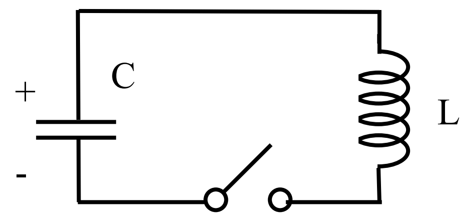
$$Energy = \frac{1}{2}LI^2 + \frac{1}{2}CV^2$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$\omega = 2\pi f$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

The switch is initially open and the capacitor initially carries a charge Q_0 = +10 μC on the top plate and $-Q_0$ on the bottom plate. At $t = 0$ the switch is closed.



C = 1 mF
L = 4 mH
Q_0 = 10 μC

1. What is the maximum current, I_{max}, in this LC circuit? (1mA = 10^{-3}A)

a. I_{max} = 1 mA *b. I_{max} = 5 mA c. I_{max} = 8 mA d. I_{max} = 10 mA e. I_{max} = 12 mA

2. At what time, t_1, is the maximum current reached for the first time? (1ms = 10^{-3}s)

a. t_1 = 6.15 ms
b. t_1 = 12.57 ms
c. t_1 = 9.36 ms
*d. t_1 = 3.14 ms
e. t_1 = 1.48 ms

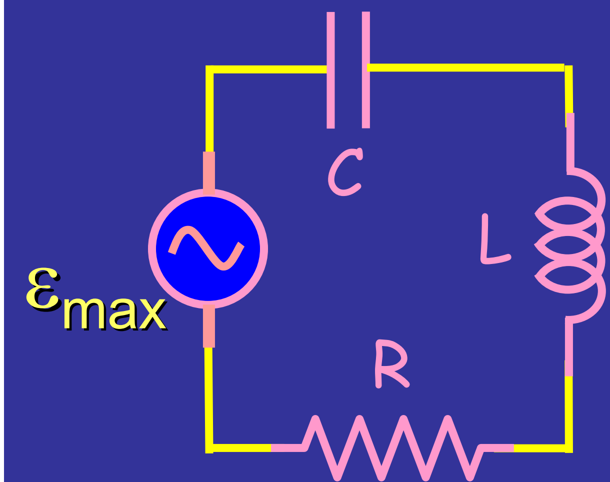
3. Calculate the rate at which the current is changing when the charge on the capacitor is 5 μC.

*a. dI/dt = 1.3 A/s
b. dI/dt = 2.0 A/s
c. dI/dt = 5.0 A/s

Makes sense to write everything in terms of I since this is the same everywhere in a one-loop circuit:

$V_{max} = I_{max} X_C$

V 90° behind I



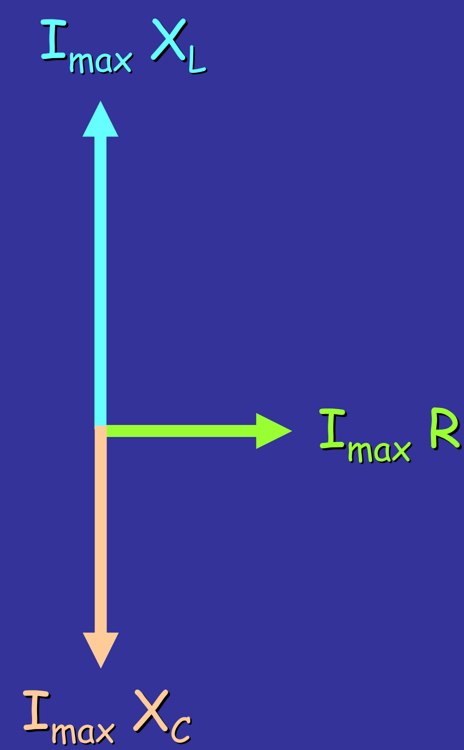
$V_{max} = I_{max} X_L$

V 90° ahead of I

$V_{max} = I_{max} R$

V in phase with I

Phasors make this simple to see



Always looks the same.
Only the lengths will change

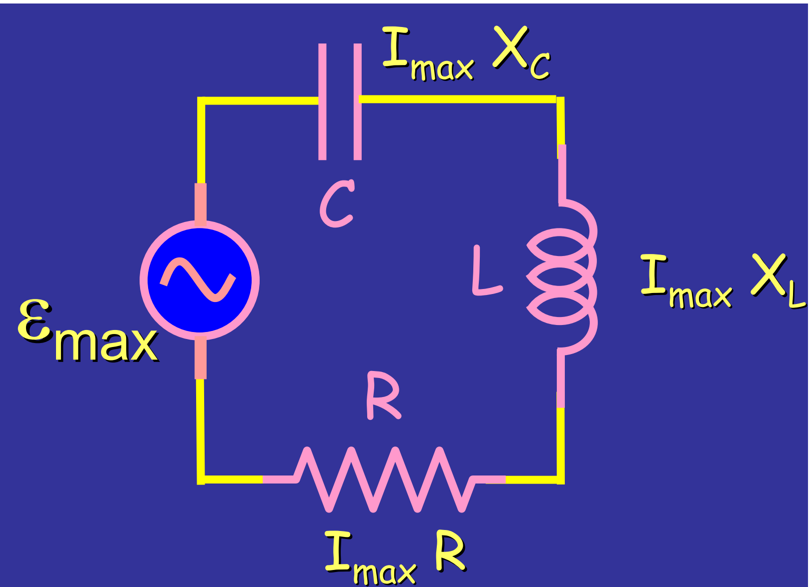
Summary:

$V_{Cmax} = I_{max} X_C$

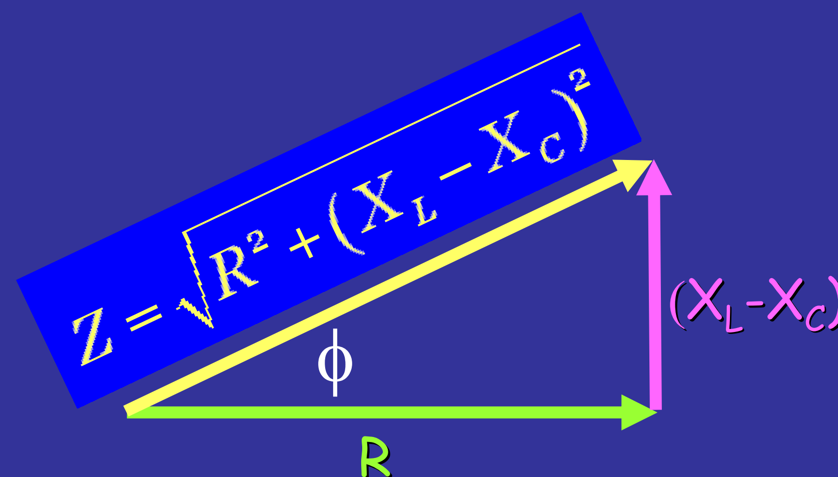
$V_{Lmax} = I_{max} X_L$

$V_{Rmax} = I_{max} R$

$\varepsilon_{max} = I_{max} Z$

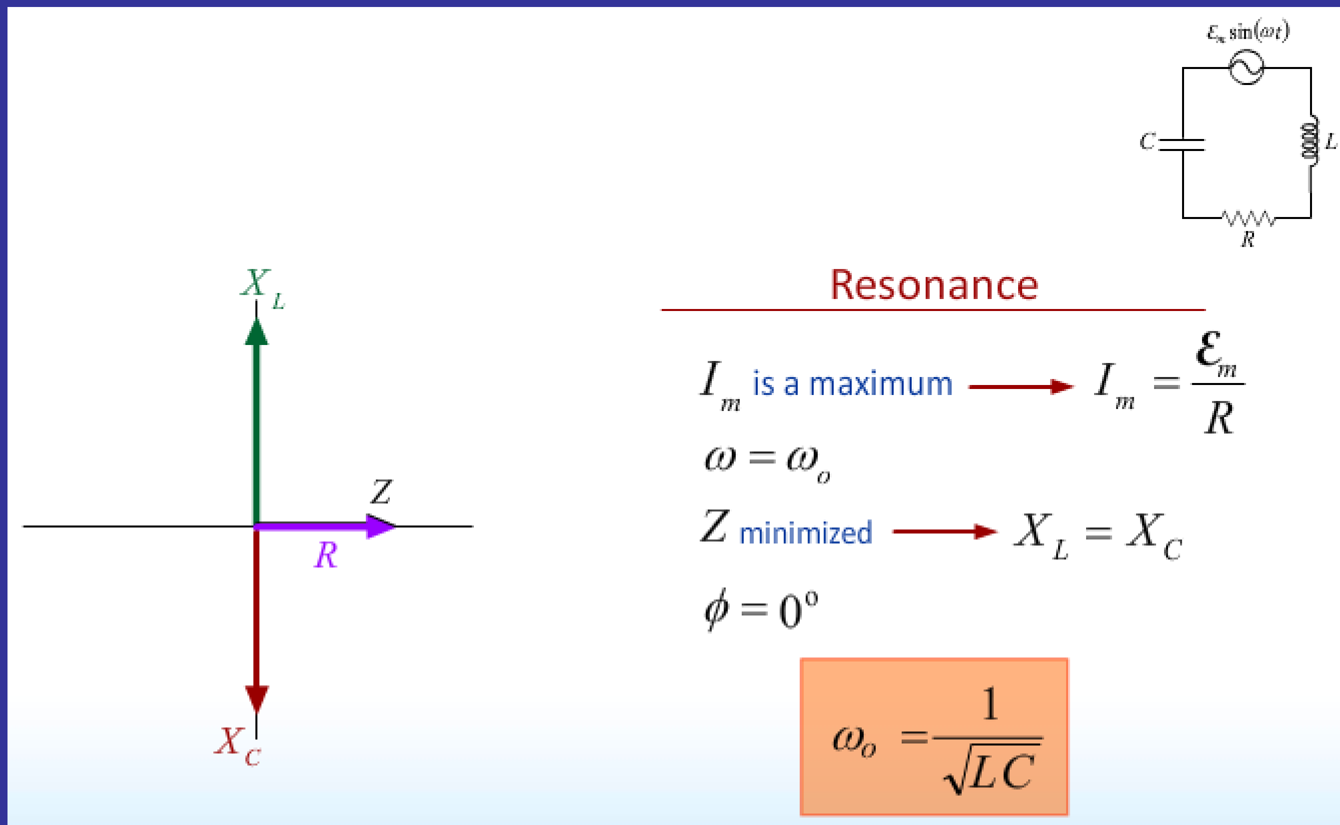


$I_{max} = \varepsilon_{max} / Z$



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

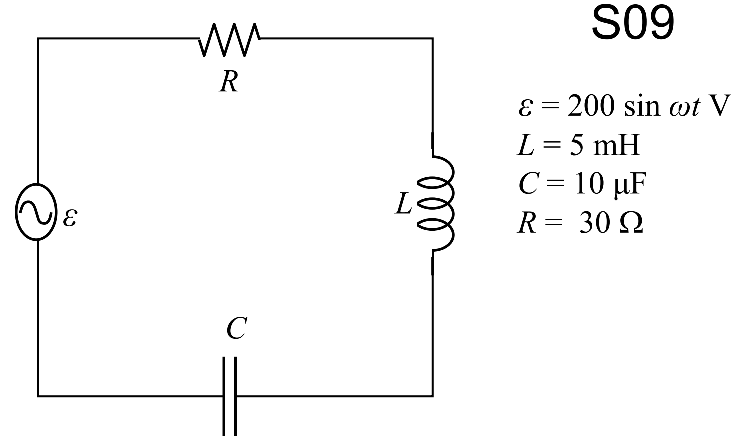


At resonance, $\omega = \omega_o = \sqrt{\frac{1}{LC}}$, Z=R, $X_L = X_C$,

current and generator voltage are in phase

6. At what frequency, f (= ω / 2π) is I_{rms} maximum?

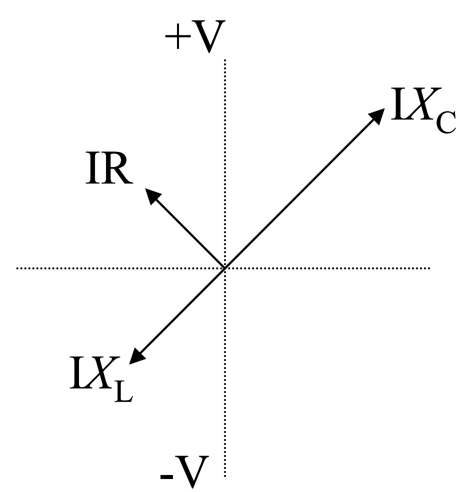
Ans: 712 Hz

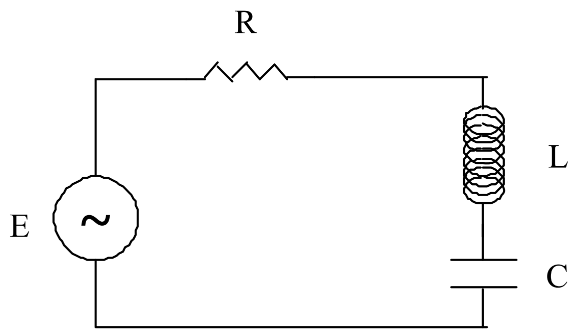


7. Fill in the blank: the <u>current</u> through the capacitor _______ the <u>voltage</u> across the resistor.

a. Leads b. Lags *c. is in phase with

9. If the phasor diagram for this circuit looks like the figure at right, is ω greater than, less than, or equal to ω_o, the resonance frequency? (Ans: less than)





$L = 0.5$ H

$R = 100\ \Omega$

$I_{max} = 3$ A

$\omega = 300$ rad/s

$\phi = -20^{o}$

10. What is the rms power provided to the circuit?

a. $\langle P(t) \rangle$ = 330 W

*b. $\langle P(t) \rangle$ = 450 W

c. $\langle P(t) \rangle$ = 570 W

d. $\langle P(t) \rangle$ = 621 W

e. $\langle P(t) \rangle$ = 660 W

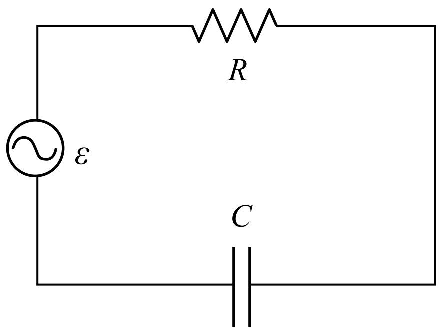
11. By changing the resistance R to an appropriate value, it is possible to make ϕ positive in the above circuit.

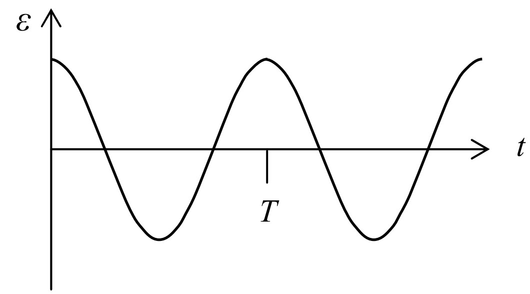
a. True

*b. False

Consider the following circuit with a resistor *R*, a capacitor *C* and an AC generator with emf *ε* whose time-dependence is given in the diagram on the right. (*R* > 0, *C* >0)

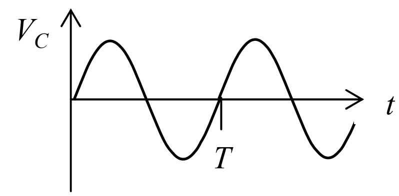
S08



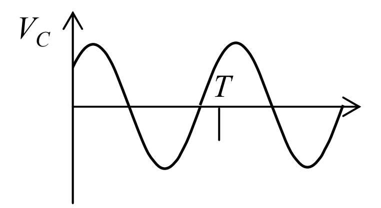


12. Which one of the following diagrams is the best description of the voltage across the capacitor as a function of time?

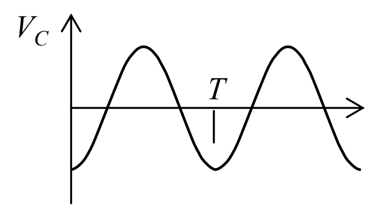
a.



b.



c.



13. If the frequency of the generator is increased, the power dissipated in the circuit increases.

*a. True   b. False

Displacement current

$$\oint \vec{B} \bullet d\vec{l} = \mu_o \left[I + I_D \right]$$

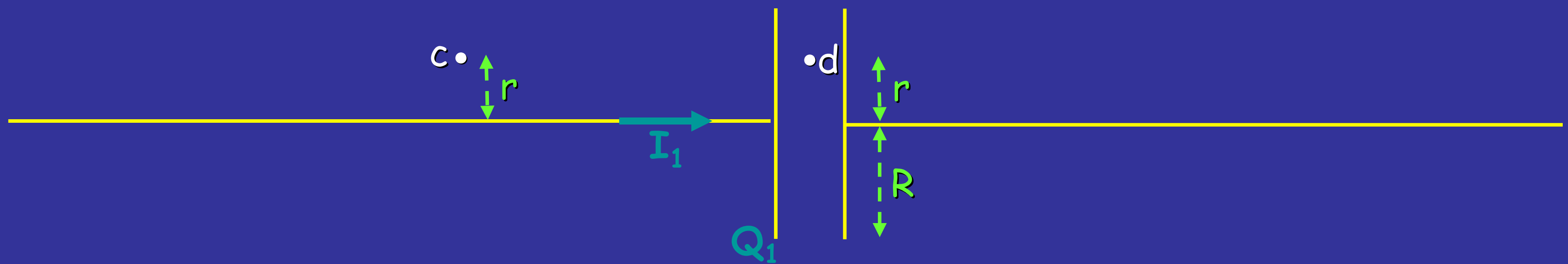
$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \varepsilon_o \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

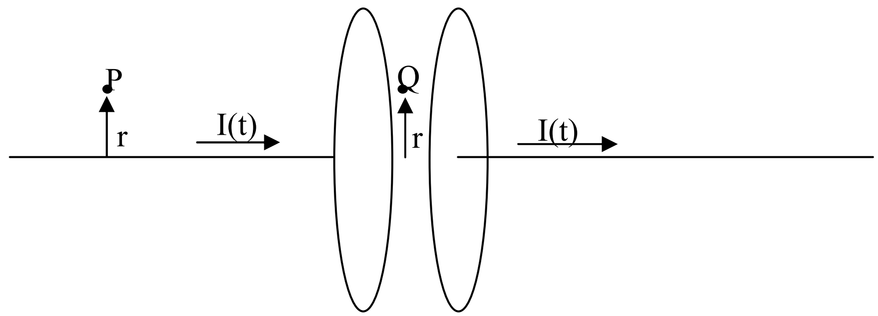
Free space

This modified Ampere's Law allows for electromagnetic waves in free space



F08

A parallel plate capacitor is charging with time. The current is flowing in the wires as shown in the figure. Point P lies a distance *r* from the wire to the left of the capacitor, and point Q lies between the capacitor plates the same distance *r* from the center of the capacitor. *r* is less than the radius of the capacitor plates. Let $\boldsymbol{B_P}$ be the magnetic field at point P and let $\boldsymbol{B_Q}$ and $\boldsymbol{E_Q}$ be the magnetic and electric fields at point Q.



22. While the capacitor is charging, which of the following statements is true?

a. $\boldsymbol{B_P} \neq 0$, $\boldsymbol{B_Q} = 0$, $\boldsymbol{E_Q} = 0$
b. $\boldsymbol{B_P} = 0$, $\boldsymbol{B_Q} \neq 0$, $\boldsymbol{E_Q} = 0$
c. $\boldsymbol{B_P} = 0$, $\boldsymbol{B_Q} \neq 0$, $\boldsymbol{E_Q} \neq 0$
d. $\boldsymbol{B_P} \neq 0$, $\boldsymbol{B_Q} = 0$, $\boldsymbol{E_Q} \neq 0$
*e. $\boldsymbol{B_P} \neq 0$, $\boldsymbol{B_Q} \neq 0$, $\boldsymbol{E_Q} \neq 0$

23. While the capacitor is charging, which of the following statements is true?

a. $|\boldsymbol{B_P}| < |\boldsymbol{B_Q}|$ b. $|\boldsymbol{B_P}| = |\boldsymbol{B_Q}|$ *c. $|\boldsymbol{B_P}| > |\boldsymbol{B_Q}|$

24. After a long time the capacitor is fully charged and dQ/dt is zero everywhere. Which statement is now true:

a. $\boldsymbol{B_Q} = 0$, $\boldsymbol{E_Q} = 0$ b. $\boldsymbol{B_Q} \neq 0$, $\boldsymbol{E_Q} = 0$ *c. $\boldsymbol{B_Q} = 0$, $\boldsymbol{E_Q} \neq 0$

Total Energy Density

$$u = \varepsilon_o E^2$$

Average Energy Density

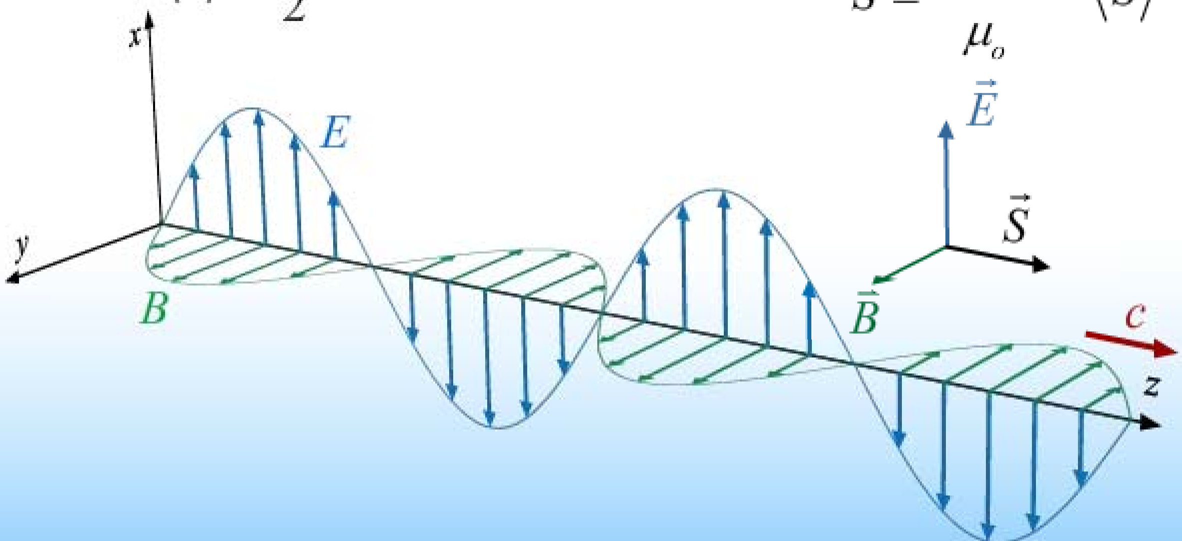
$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\,2}$$

Intensity

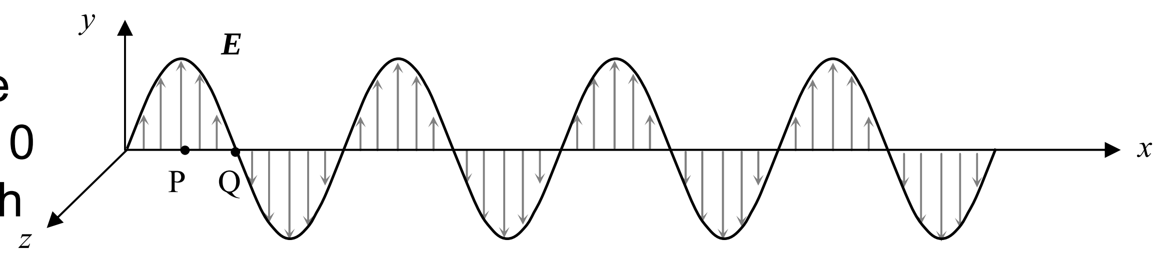
$$I = \frac{1}{2} c\,\varepsilon_o E_o^{\,2} = c\,\langle u \rangle$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \qquad \langle S \rangle = I$$



A plane electromagnetic wave propagates in vacuum. The figure represents a snapshot at time $t = 0$ of the electric field associated with this wave, which is described by $\mathbf{E} = E_0 \sin(\omega t - kx)$, with $E_0 = 1000$ V/m.



S09

The frequency is $f = 300$ kHz. Points P and Q are along the x-axis.

17. Which one of the following options correctly relates B_P, the magnitude of the magnetic field at point P at time $t = 0$, to B_Q, the magnitude of the magnetic field at point Q at time $t = 0$?

a. $B_P = B_Q = 0$ *b. $B_P > B_Q = 0$ c. $B_P = B_Q > 0$

18. In what direction is the wave propagating?

*a. +x b. –x c. +y

19. If the amplitude of the magnetic field associated with this electromagnetic wave is doubled, the intensity I of this electromagnetic wave changes by a factor of:

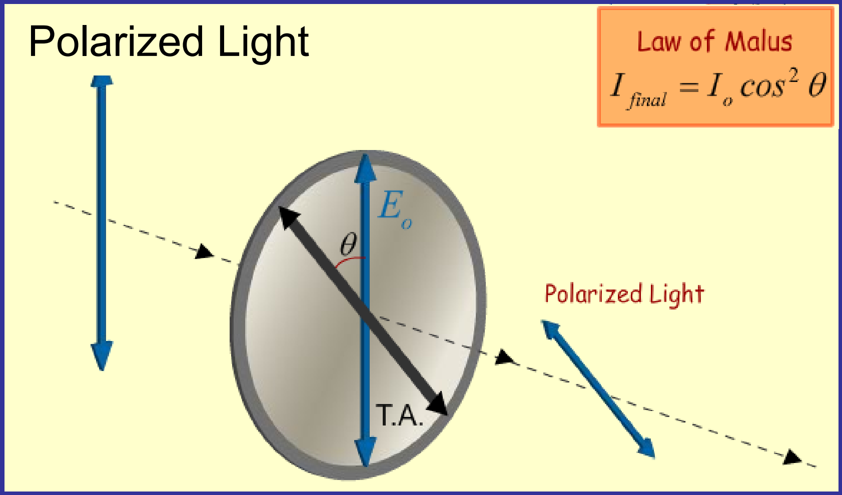
a. 2 b. ½ *c. 4

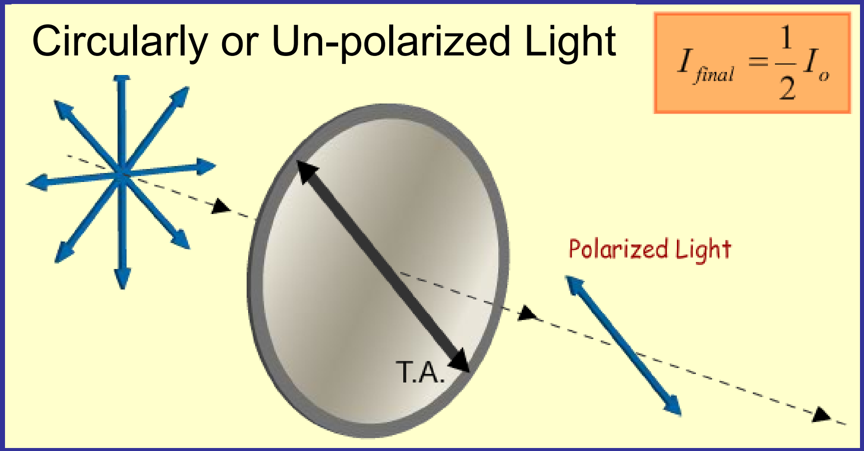
20. The wavelength λ associated with this electromagnetic wave is:

a. $\lambda = 0.1$ m b. $\lambda = 1.0$ m c. $\lambda = 100$ m *d. $\lambda = 1000$ m e. $\lambda = 10000$ m

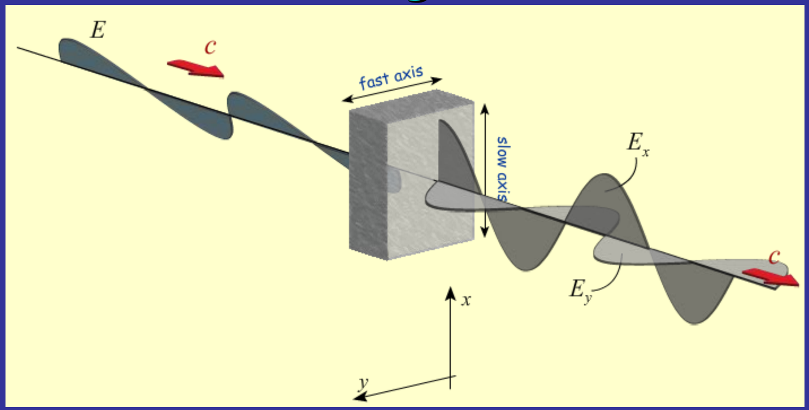
Executive Summary:

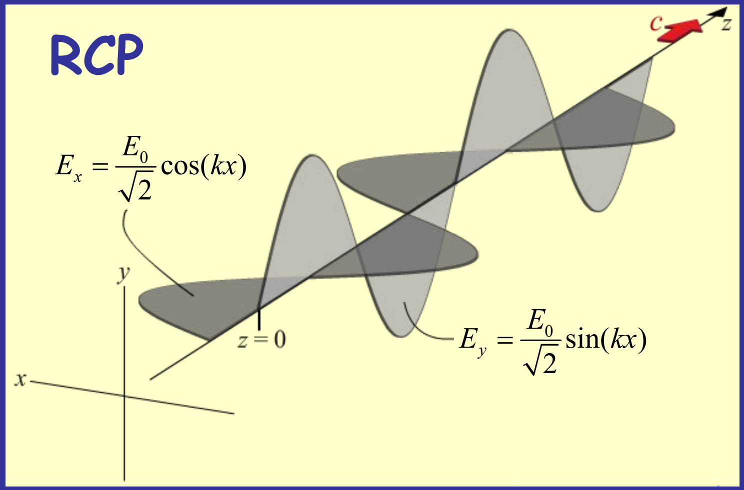
Polarizers & QW Plates:



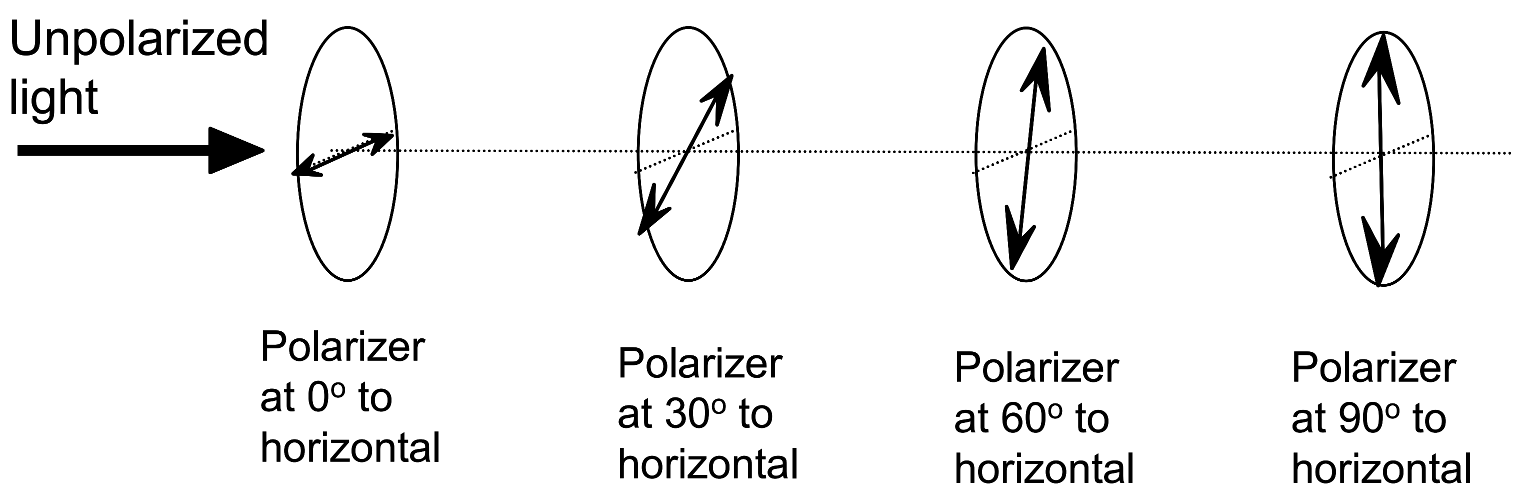


Birefringence





Consider the following arrangement of 4 polarizers. The first has a horizontal transmission axis, the second is oriented at 30^o with respect to the horizontal, the third at 60^o to the horizontal, and the fourth at 90^o to the horizontal. The light incident on the first polarizer from the left is unpolarized.



1. The ratio of the final intensity of the transmitted light to the intensity of the incident light is

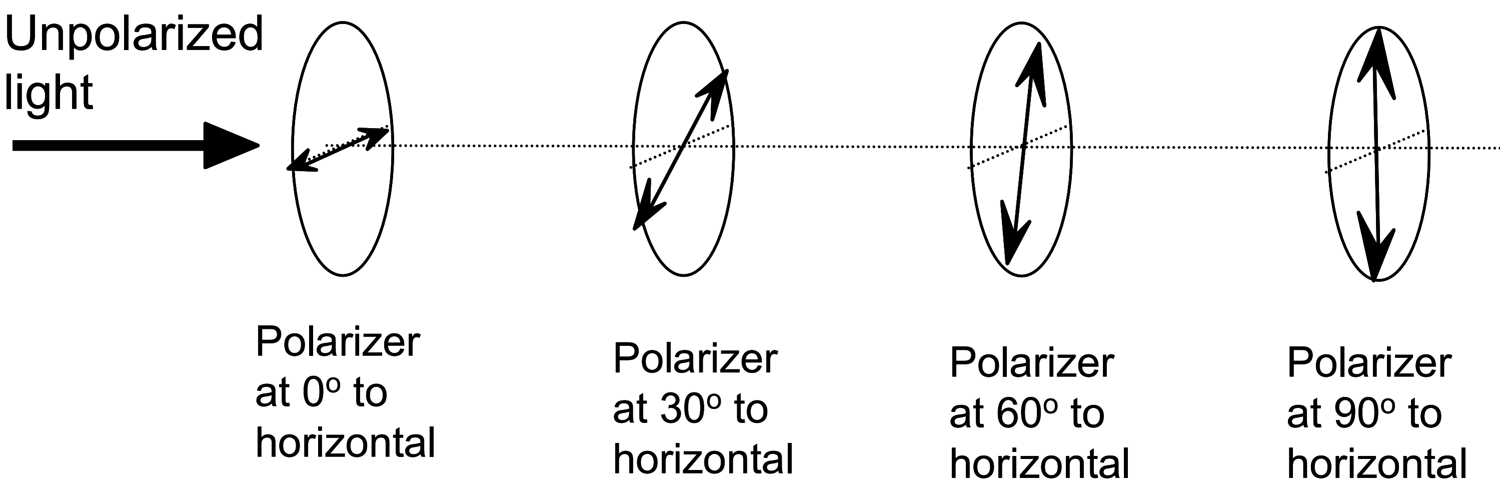
a. 0.65 b. 0.32 *c. 0.21

2. If the third and fourth polarizers are interchanged, the final intensity of the transmitted light will

a. increase b. remain the same *c. decrease

F09 1-3

Consider the following arrangement of 4 polarizers. The first has a horizontal transmission axis, the second is oriented at 30^o with respect to the horizontal, the third at 60^o to the horizontal, and the fourth at 90^o to the horizontal. The light incident on the first polarizer from the left is unpolarized.



3. Let's start with the original situation at the top of the page. If the first polarizer is replaced with a quarter wave plate, the final intensity of the transmitted light will be:

a. Larger by a factor of 2

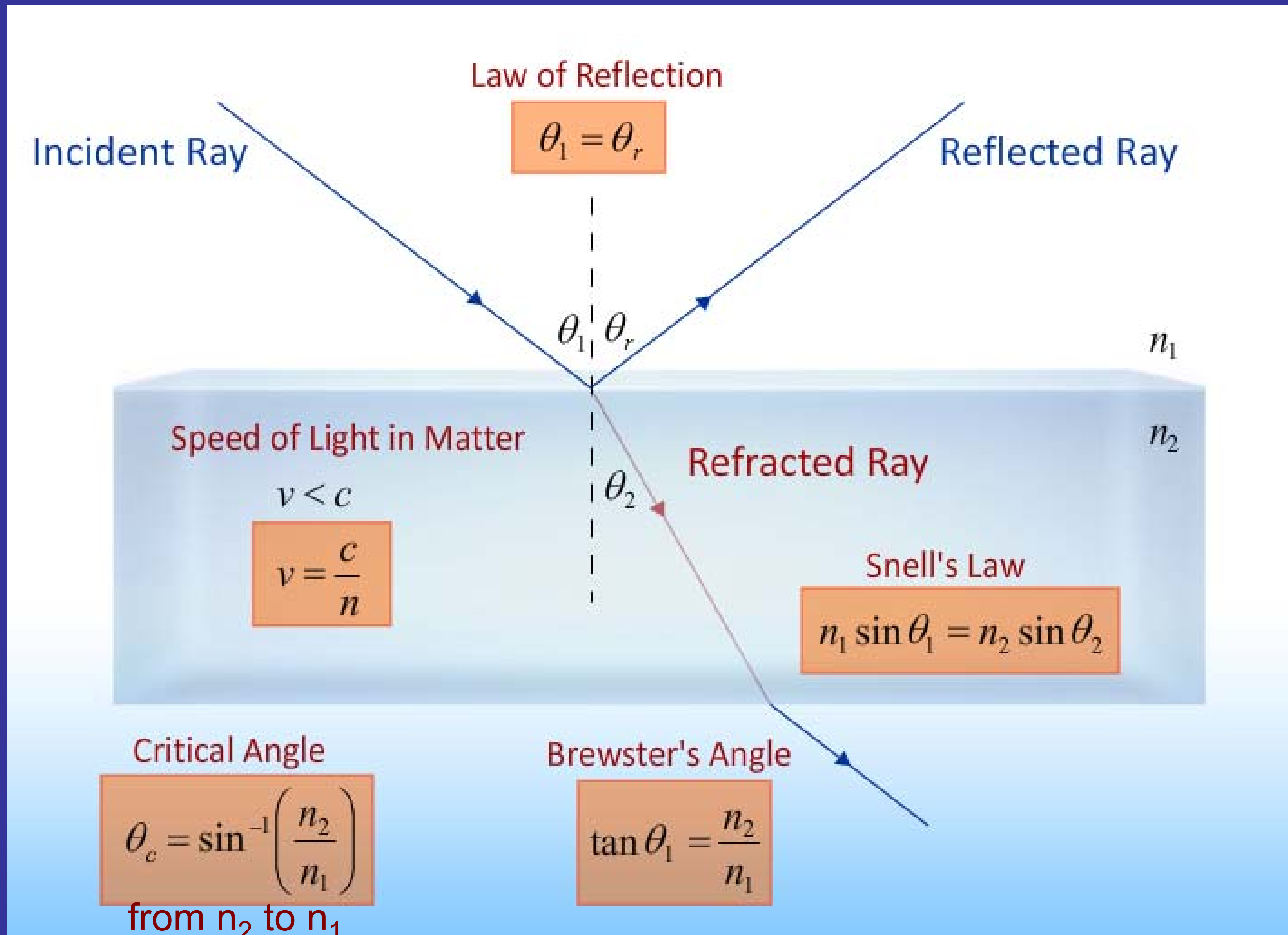
b. Smaller by a factor of 1/2

*c. Larger by a factor of $1/\cos^2 30^o$

d. Smaller by a factor of $\cos^2 30^o$

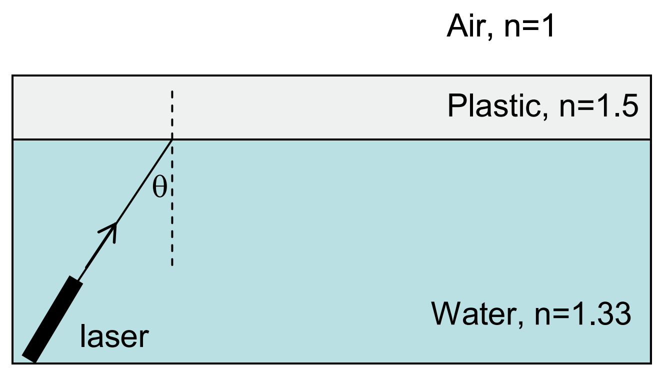
e. The same as in the original situation

Reflection & Refraction



F09 6

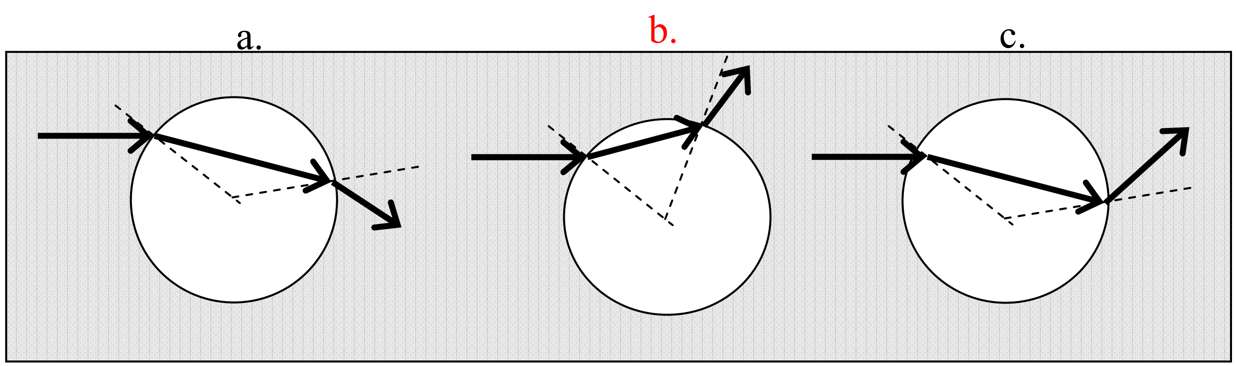
A laser sends a beam of light from water toward a plastic slab at the surface of the water. Above the plastic slab there is air.



6. What is the maximum value of the angle, θ, that the laser beam can make with the vertical and still have the beam of light emerge into the air above the plastic?

a. 41.81° *b. 48.75° c. 60.07°

15. Consider an **<u>air bubble in water</u>**. A laser beam is directed at the bubble from the left as shown, slightly above the central axis (see figures). Which of the following diagrams best indicates the trajectory of the light? (Note, the dotted lines cross in the center of each bubble, and therefore indicate the normal to the surface.)



F07