HOUR EXAM 1

Naomi's Review Slides

TOPIC 1: ELECTRIC FIELDS

- $\vec{F} = q \cdot \vec{E}$
- conductors: $E = \emptyset$... charges on surface only
- volume charge density: $\rho = \frac{Q}{V}$ $\left[\frac{C}{m^3}\right]$

  surface charge density: $\sigma = \frac{Q}{A}$ $\left[\frac{C}{m^2}\right]$

  line charge density: $\lambda = \frac{Q}{L}$ $\left[\frac{C}{m}\right]$

- GAUSS' LAW $\oint \vec{E} \cdot \vec{dS} = \frac{q_{enc}}{\varepsilon_0}$

  ✲ flux $\Phi_E = \oint E \cdot dS$ ... watch out for open surfaces!

  ... choose surface so that $\oint E \cdot dS = E \cdot A$ or $\emptyset$

  - SPHERES (2 points) $E = \frac{q}{4\pi\varepsilon_0 r^2}$
  - CYLINDERS (2 lines) $E = \frac{\lambda}{2\pi\varepsilon_0 r}$
  - ∞ PLANES $E = \frac{\sigma}{2\varepsilon_0}$

  ✲ use charge ENCLOSED

  ✲ use SUPERPOSITION especially for PLANES

- E FIELD LINES
  - from ⊕ to ⊖ charges
  - density $\sim |E|$
  - don't cross
  - point in $\vec{E}$ direction
  - perpendicular to equipotential surfaces, e.g. all conductors

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TOPIC 2: ELECTRIC POTENTIAL

- $V_B - V_A = \Delta V_{A\to B} = \dfrac{W_{A\to B}}{q} = -\int_A^B \vec{E}\cdot\vec{dl}$

  $V_B - V_A > 0$ means we moved "UPHILL" going from $A \to B$

- POTENTIAL FUNCTION $V(r)$ can be defined once V set to $\emptyset$ @ some reference point (usually @ ∞)

$$V(r) = \cancelto{\emptyset}{V_{ref}} + \Delta V_{ref\to r}$$

... split up into regions of different $\vec{E}$:

$$V(r) = \cancelto{0}{V_{ref}} + \Delta V_{ref\to a} + \Delta V_{a\to b} + \Delta V_{b\to r}$$

$$= -\int_{ref}^{a} \vec{E}\cdot\vec{dl} - \int_a^b \vec{E}\cdot\vec{dl} - \int_b^r \vec{E}\cdot\vec{dl}$$

- POINT CHARGES: use superposition

$$V(r) = \sum_i \frac{kq_i}{r_i}$$

Potential energy of a group of charges
= work needed to assemble them from ∞

$$= \sum_{\substack{\text{PAIRS}\\ ij}} \frac{k q_i q_j}{r_{ij}}$$

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TOPIC 3: CAPACITANCE

HOUR EXAM 2: Spr

- $C = \frac{Q}{V}$
- $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$
- $C = C_1 + C_2$ (parallel combination) → V constant

  $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ (series combination) → Q constant
- C is property of geometry & materials only:

  $C = \frac{\varepsilon_0 A}{d}$ (parallel plates)

  $C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$ (cylindrical capacitor)

  $C = \frac{4\pi\varepsilon_0 ab}{(b-a)}$ (spherical capacitor)

HOUR EXAM 2

TOPIC 1: R NETWORKS

- $V = IR$
- $R = \frac{\rho \cdot L}{A}$
- $P = IV$ ... $= I^2R = V^2/R$ for resistors

(1a) 1-loop networks

- $R = R_1 + R_2$ (series) ... I constant

  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (parallel) ... V constant

★ when is it 2-loop
→ batteries on different loops

Usual technique:
- pack resistors to one effective R_{total}
- determine V & I across R_{total}
- unpack R_{total} once ... what's inside?
  - series → I constant
  - parallel → V constant
  - ⇨ determine the other one
- repeat last step as necessary

(1b) 2-loop networks

- KVL: around loop $\sum V_i = \emptyset$

  KCL: at node $\sum I_{in} = \sum I_{out}$

Usual technique:
- label all independent currents, with direction
- apply KCL to $(n-1)$ nodes where $n = \#$ nodes
- apply KVL to loops, until you have enough equ's
- solve for currents

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TOPIC 2: RC CIRCUITS

- Behaviour of C at $t=\emptyset$ (wire)
  & $t=\infty$ (open circuit)

- $\tau = RC$

- $Q(t) = Q_0 e^{-t/\tau}$ "discharging"
  $Q(t) = Q_\infty (1-e^{-t/\tau})$ "charging"

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TOPIC 3: LORENTZ FORCE

1a Force on moving charges

- $F = qv \times B$
- orbits: $R = \frac{mv}{qB}$, $\omega_c = \frac{qB}{m}$

★ NO WORK!

1b Force on wires

- $dF = I d\ell \times B \Rightarrow F = IL \times B$
- F on loops: Ø in UNIFORM B

★ F between 2 wires
↑↑ attractive
↑↓ repulsive

1c Torques on loops

- $\mu = NIA$
- $U = -\vec{\mu} \cdot \vec{B}$
  $\vec{\tau} = \vec{\mu} \times \vec{B}$

★ watch for N = # turns!

★ $\vec{\mu}$ wants to align with $\vec{B}$
(i.e. $\vec{\mu} \parallel \vec{B}$ is point of minimum U)

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TOPIC 4: BIOT-SAVART & AMPERE

- $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl} \times \hat{r}}{r^2}$

- $\oint B \cdot dl = \mu_0 I_{enclosed}$

Special case formulas:

- $B = \frac{\mu_0 I}{2\pi r}$ ⇒ wires & cylinders

☆ use $I = I_{enclosed}$

- $B = \mu_0 n I$ ⇒ solenoids

  $B = \frac{\mu_0 n I}{2}$ ⇒ ∞ planes

- $B = \frac{\mu_0 N I}{2\pi r}$ ⇒ toroids

☆ watch out!

n = #turns/m

N = #turns

- $B = \frac{\mu_0 I}{2r}$ ⇒ center of circular loop

☆ $B = \emptyset$ outside ∞ solenoids & toroids

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TOPIC 5: FARADAY

- flux: $\Phi_B = \int \vec{B} \cdot d\vec{S}$

✱ Implicit N turns!

✱ In UNIFORM B

$\rightarrow \Phi_B = B \cdot A \cdot \cos\theta$

- $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$

✱ EMF: $\mathcal{E} \equiv \oint E\, dl$

= change in potential around loop

- LENZ's LAW: $\mathcal{E} \sim$ induced current OPPOSES the CHANGE in flux

✱ Induced current: $I = \mathcal{E}/R$

Types of problems:

For UNIFORM $\vec{B}$: $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}(B \cdot A \cdot \cos\theta)$

① $B(t)$ given $\longrightarrow$ $\dfrac{d\Phi_B}{dt} = \dfrac{dB}{dt} \cdot A \cdot \cos\theta$ (B changing)

② translating loops, sliding rails $\longrightarrow$ $\dfrac{d\Phi_B}{dt} = B \cdot \dfrac{dA}{dt} \cdot \cos\theta$ (A changing)

③ rotating loops $\longrightarrow$ $\dfrac{d\Phi_B}{dt} = -B \cdot A \cdot \sin\theta \cdot \dfrac{d\theta}{dt}$ (θ changing)

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TOPIC 1: INDUCTANCE & LR CIRCUITS

- $L = \dfrac{\Phi_B}{I}$ ... $\Phi_B = \int \vec{B} \cdot d\vec{A}$

✱ In UNIFORM $\vec{B}$
→ $\Phi_B = \vec{B} \cdot \vec{A}$
✱ Implicit N TURNS!

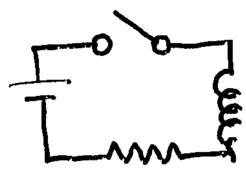
- $V = L \dfrac{dI}{dt}$

- $U = \frac{1}{2} L I^2$

- Behaviour of L at
  - $t = \phi$ ⟶ open circuit
  - $t = \infty$ ⟶ wire

✱ reverse to behaviour of capacitors!

- $\tau = \dfrac{L}{R}$



- $I(t) = I_\infty (1 - e^{-t/\tau})$ "charging"
  $I(t) = I_0 e^{-t/\tau}$ "discharging"

✱ similar to RC, except it is CURRENT which builds up or decays

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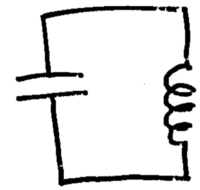
TOPIC 2: LC CIRCUITS



- V_L, V_C, I OSCILLATE @ $\omega_0 = \frac{1}{\sqrt{LC}}$

★ Remember,
$\omega = 2\pi f$ .... $T = \frac{1}{f}$

- at every quarter period, $(V_C = V_L)$ and I are ALTERNATELY maximized

- total ENERGY in circuit oscillates between
  $U_B = \frac{1}{2} L I^2$ (all here when I is max)
  $U_E = \frac{1}{2} C V^2$ (all here when V is max)

- oscillations continue indefinitely unless there is DAMPING

HOUR EXAM 3

TOPIC 3: AC CIRCUITS

- Magnitudes — 4 versions of Ohm's Law...

$V_R^{max} = I^{max} \cdot R$

$V_L^{max} = I^{max} \cdot X_L$

$V_C^{max} = I^{max} \cdot X_C$

$\mathcal{E}^{max} = I^{max} \cdot Z$

* can equally well relate RMS values as max values $V^{RMS} = V^{max}/\sqrt{2}$

* X_L, X_C, Z are FREQUENCY-dependent

- Phases

V_R IN PHASE with I

V_L LEADS I by 90°

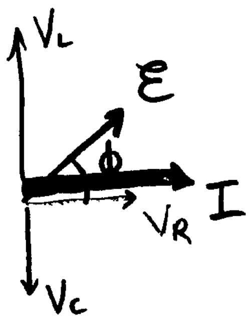
V_C LAGS I by 90°

$\mathcal{E}$ related to I by $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

* ELI ICE

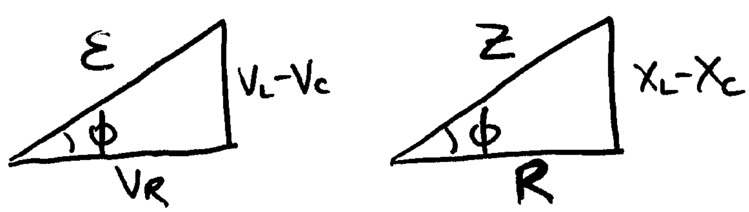
* SIGN of ϕ determined by $X_L - X_C$ → who's winning?

- Phasor Diagram



* add voltages like VECTORS on this diagram

- Useful Triangles



* obtain the formulas you need!

$R = Z\cos\phi$

$\tan\phi = (X_L - X_C)/R$

$Z^2 = R^2 + (X_L - X_C)^2$

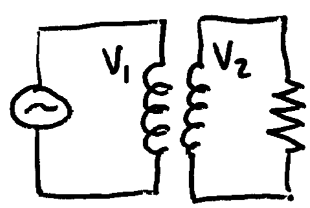
etc...

- Resonance
  - $Z = \frac{R}{\cos\phi} = \sqrt{R^2 + (X_L - X_C)^2}$ MINIMIZED when $\begin{cases} X_L = X_C \\ \phi = 0° \end{cases}$ * $Z_{min} = R$
  - $\langle P \rangle = \mathcal{E}_{rms}\, i_{rms} \cos\phi$ * $\cos\phi$ = "power factor"

    ... MAXIMIZED at resonance

    * Note: $\langle P \rangle = V_{R,rms}\, i_{rms}$

    average power is entirely due to loss in resistor
  - can bring circuit into resonance by adjusting $\omega \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ * i.e. balance X_L & X_C
  - Q factor describes sharpness of $\langle P \rangle$ vs. ω
- Transformers
  - $V_2 = \frac{N_2}{N_1} \cdot V_1$



* V_2 behaves like new AC generator placed across load except voltage has been stepped UP or DOWN

  - $\langle P \rangle = I_{1\,rms} V_{1\,rms}$

    $= I_{2\,rms} V_{2\,rms}$

HOUR EXAM 3

TOPIC 4: EM WAVES

- LINEARLY polarized plane wave

$$\vec{E} = E_0 \hat{\imath} \cos(kz - \omega t)$$

amplitude of E

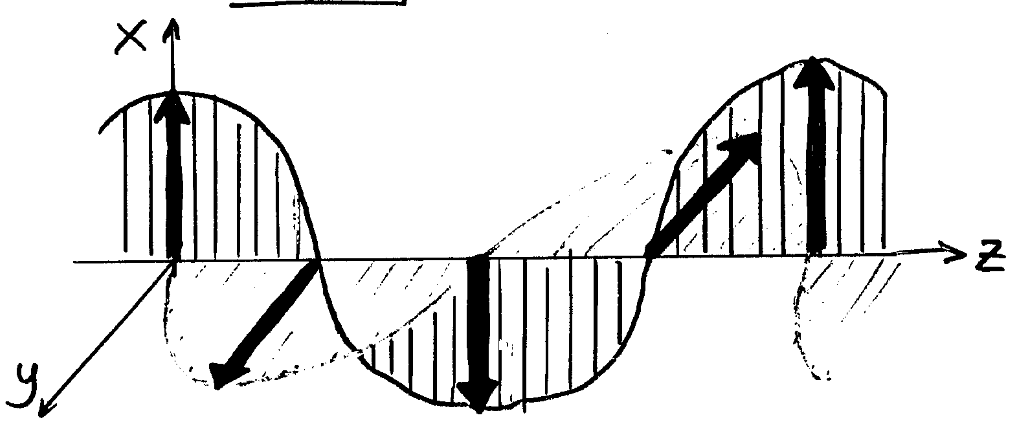
direction of $\vec{E}$ = axis of polariz^{n}

direction of wave ($\hat{s}$) = $+\hat{z}$ here

- CIRCULARLY polarized plane wave

$$\vec{E} = E_0 \left[\hat{\imath} \cos(kz - \omega t) + \hat{\jmath} \sin(kz - \omega t) \right]$$

⇨ SUM of two linearly polarized waves, with perpendicular polarization axes, and OUT OF PHASE by 90°



* now magnitude of $\vec{E}$ is CONSTANT

... only its direction changes

- speed of wave in vacuum: $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

- B-FIELD always accompanies E:
  - in phase
  - $\hat{E} \times \hat{B} = \hat{S}$
  - $B_0 = E_0/c \Longrightarrow u_E = u_B$

- (Average) INTENSITY: $I = \frac{\langle E^2 \rangle}{Z_0} = \begin{cases} \frac{E_0^2}{2Z_0} & \text{(linear pol}^{n}\text{)} \\ \frac{E_0^2}{Z_0} & \text{(circular pol}^{n}\text{)} \end{cases}$ ✫ remember units: W/m^2

  ... also $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ ... $\langle \vec{S} \rangle = I$

- LINEAR POLARIZERS
  - turn anything into linearly polarized light
  - linearly pol. light: $I_1 = I_0 \cos^2\theta$
    circularly pol. light: $I_1 = I_0/2$

- QUARTER WAVE PLATES
  - turn linearly pol. light $\longrightarrow$ circularly pol.
  - turn circularly pol. light $\longrightarrow$ linearly pol.
  - never reduce intensity

- ANTENNAS: Transmit and receive EM waves with $\vec{E}$ direction along antenna

HOUR EXAM 3

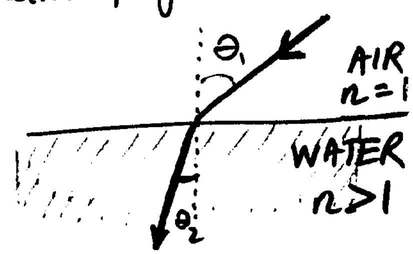
TOPIC 5: REFLECTION & REFRACTION

- Reflection: $\theta_i = \theta_r$

  Refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

★ angles measured with respect to NORMAL

★ when going to MORE dense medium, light bends "IN"



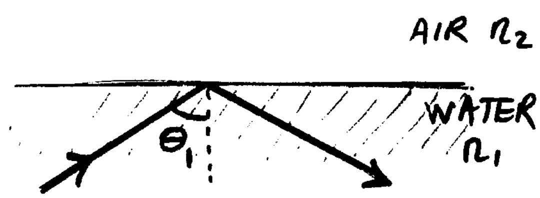
- waves in MEDIA:

  $\omega' = \omega$

  $\lambda' = \lambda/n$

  $v' = c/n$

- TOTAL INTERNAL REFLECTION:



When $\sin\theta_2 > 1$,
there IS NO refracted ray!

This occurs when

$$\sin\theta_1 = \frac{n_2 \sin\theta_2}{n_1} > \frac{n_2}{n_1}$$

... i.e. for angles θ_1 greater than the CRITICAL ANGLE

★ usual case: going from medium n to air/vacuum

$$\Longrightarrow \sin\theta_c = \frac{1}{n}$$

FINAL EXAM ONLY

Topic 1: Mirrors & Lenses

- $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

  Sign conventions: ("forward" = direction of light)

  $s > 0 \Rightarrow$ object BEFORE lens/mirror

  $s' > 0 \Rightarrow$ image AFTER lens/mirror

  $f > 0 \Rightarrow$ lens/mirror CONVERGING

✲ REAL image → physical rays DO intersect

... VIRTUAL image → only extensions of rays intersect

- magnification: $M = -s'/s$
  - $|M| = h'/h$
  - $M > 0 \Rightarrow$ image & object have SAME orientation

- focal lengths from geometry:
  - spherical mirrors: $f = R/2$
  - lenses: $\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

- principal rays

	PART 1: BEFORE mirror/lens	PART 2: AFTER mirror/lens
①	parallel to optical axis	through focal point
②	through center of mirror/lens	continue straight (lens) bounce back @ equal angle (mirror)
③	through focal point	parallel to optical axis

✲ which focal point to use? → depends on converging/diverging

✲ ALL part 1 rays MUST pass through OBJECT