# Physics 212

## Lecture 4

### Today's Concepts:

Conductors +
Using Gauss' Law

# Music

Who is the Artist?

A) Professor Longhair
B) John Cleary
C) Allen Toussaint
D) David Egan
E) Henry Butler


Theme of week: New Orleans Jazz → Mardi Gras

# Your Comments

"Everything. Week 2 and I'm desperately lost..."

"It is really hard. I want to have it all covered in lecture."

"How is there a charge induced on the inside of a conducting shell? Is there any way to think of it intuitively instead of using Gauss's Law?"

"The checkpoint questions were extremely difficult. Many explanations will be needed."

"E&M is so much more confusing than mechanics."

# Our Response

Most students are having difficulties with this topic: The Checkpoints show this clearly.

This whole way of thinking (Gauss' Law) is very unfamiliar to you: calculate a field means, first, pick a surface??

The solution? DON'T PANIC… We're confident you will master these concepts but it will take a little work.

**TODAY'S PLAN:**

- Do Checkpoints again! Try to understand the reasoning
- Do a calculation using Gauss' Law

"We'll see, won't we? Also, what do you do with overheated electrical components? Coulomb off."

# Conductors = charges free to move

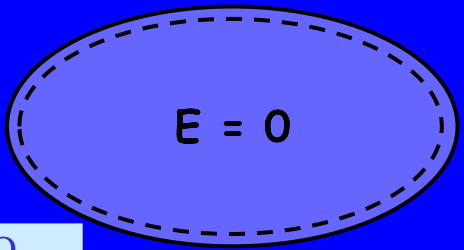
**Claim: E = 0 within any conducting material at equilibrium**

Charges in conductor move to make E field zero inside. (Induced charge distribution). If E ≠ 0, then charge feels force and moves!

**Claim: At equilibrium, excess charge on conductor only on surface**

Why?

- Apply Gauss' Law
  - Take Gaussian surface to be just inside conductor surface
  - E = 0 everywhere inside conductor → $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$
  - Gauss' Law: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$ → $Q_{enc} = 0$




SIMULATION 2

# Gauss' Law + Conductors + Induced Charges

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

ALWAYS TRUE!

If choose a Gaussian surface that is entirely in metal, then E=0 so $Q_{enclosed}$ must also be zero!

$$E = \frac{Q_{enc}}{A\varepsilon_0}$$

## How Does This Work??

Charges in conductor move to surfaces to make $Q_{enclosed} = 0$.

We say charge is induced on the surfaces of conductors

Small aside: $\varepsilon_0$ is just a constant related to $k$

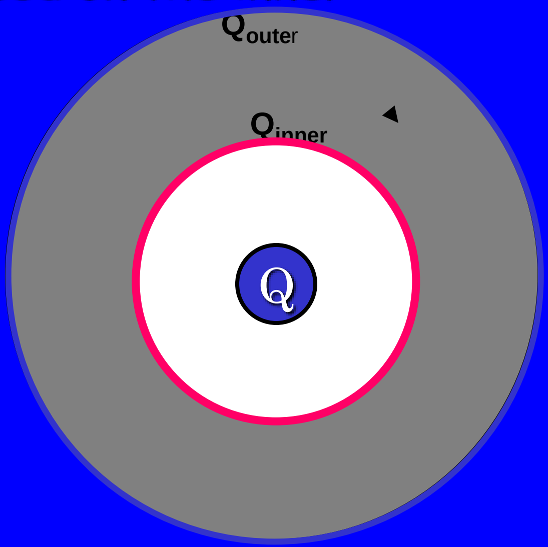
$k = 1/(4\pi\varepsilon_0)$

$k = 9\text{x}10^9$ Nm$^2$/C$^2$

$\varepsilon_0 = 8.85\text{x}10^{-12}$ C$^2$/Nm$^2$

# Charge in Cavity of Conductor

A particle with charge +Q is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?




A) inner = –Q, outer = +Q

B) inner = –Q/2 , outer = +Q/2

C) inner = 0, outer = 0

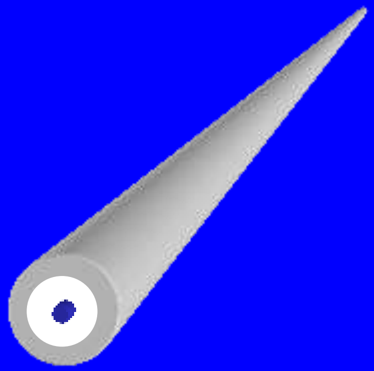
D) inner = +Q/2, outer = -Q/2

E) inner = +Q, outer = -Q

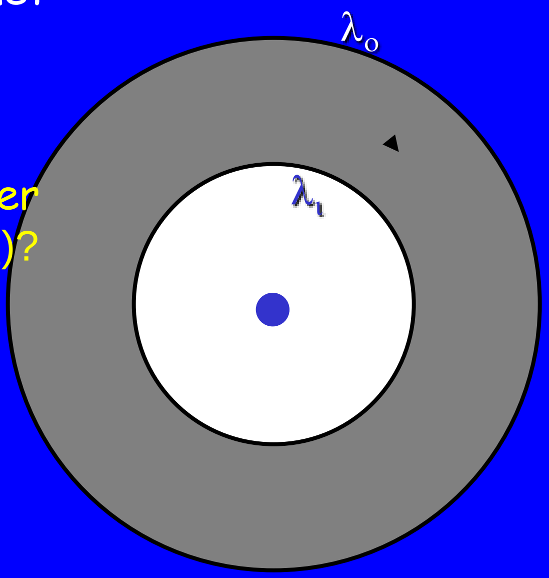
Since E=0 in conductor

- Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$ ⇒ $Q_{enc} = 0$

# Infinite Cylinders



A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of -4 C/m.




What is the linear charge density of the induced charge on the inner surface of the conducting cylinder ($\lambda_i$) and on the outer surface ($\lambda_o$)?

| | A) | B) | C) | D) | E) |
|---|---|---|---|---|---|
| $\lambda_i$: | +2.5 C/m | -4 C/m | -2.5 C/m | -2.5 C/m | 0 |
| $\lambda_o$: | -6.5 C/m | 0 | +2.5 C/m | -1.5 C/m | -4 C/m |

E = 0 in material of conducting shell → Enclosed charge = 0 → $\lambda_i$ = -2.5 C/m

$\lambda_i + \lambda_o$ = -4 C/m → $\lambda_o$ = -1.5 C/m

# Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \dfrac{Q_{enc}}{A\varepsilon_0}$

In General, integral to calculate flux is difficult…. and not useful!

To use Gauss' Law to calculate E, need to choose surface carefully!

1) Want E to be constant and equal to value at location of interest

OR

2) Want E dot A = 0 so doesn't add to integral

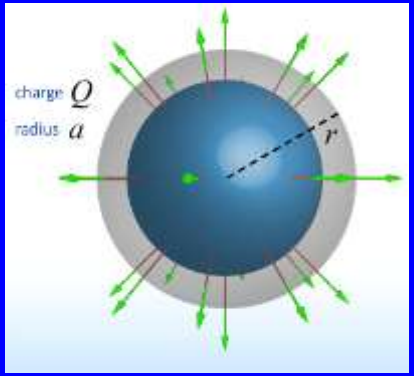
# Gauss' Law Symmetries

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \frac{Q_{enc}}{A\varepsilon_0}$

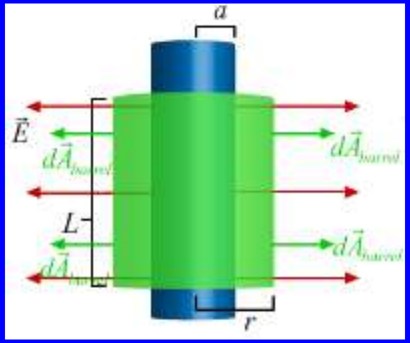
Spherical




$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0}$$

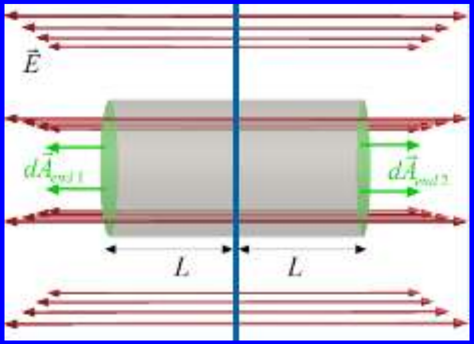
Cylindrical




$$A = 2\pi r L$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

Planar

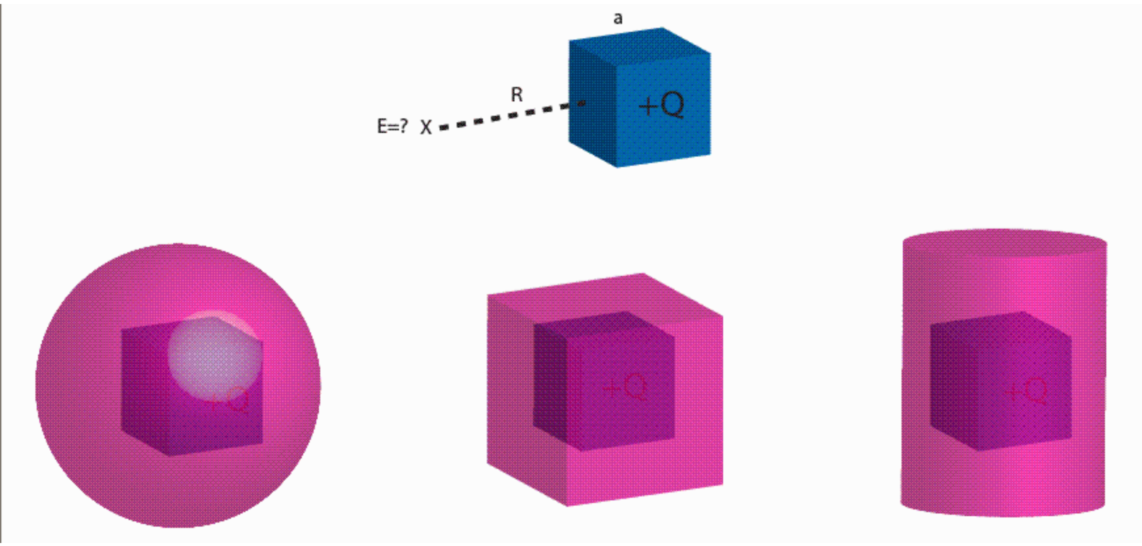



$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

# Checkpoint 1

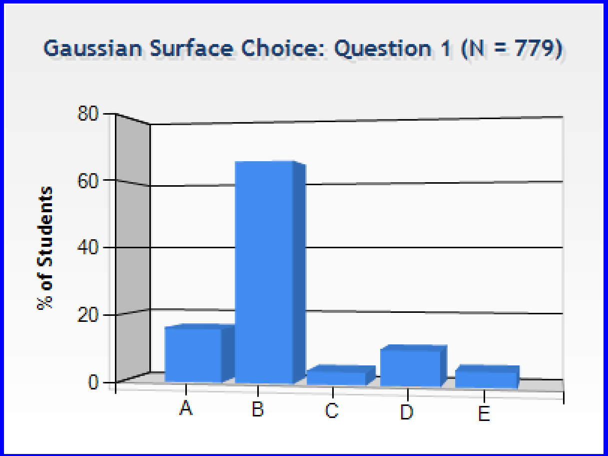
You are told to use Gauss' Law to calculate the electric field at a distance **R** away from a charged cube of dimension **a.** Which of the following Gaussian surfaces is best suited for this purpose?




(A) (B) (C)

(D) The field cannot be calculated using Gauss' Law

(E) None of the above




"A sphere catches all of the lines."

"All sides of the Gaussian surface will now be perpendicular to the electric field lines."

"The field lines are not always perpendicular to the surface and thus we cannot calculate Gauss law"

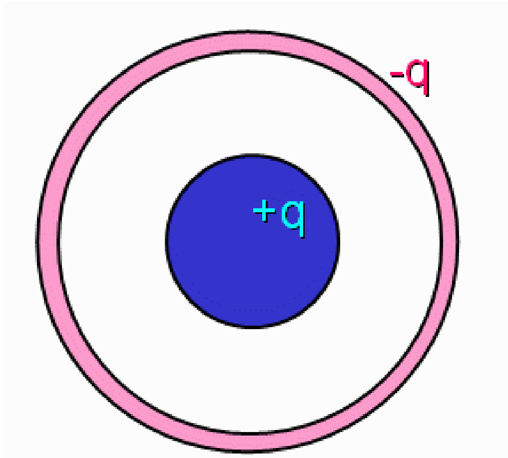
**THE CUBE HAS NO GLOBAL SYMMETRY !**

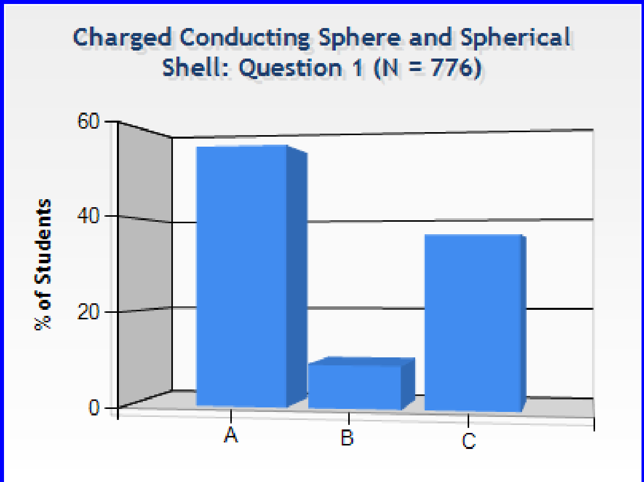
THE FIELD AT THE SURFACE OF THE CUBE IS NOT PERPENDICULAR OR PARALLEL TO THE SURFACE

| | | | |
|---|---|---|---|
| 3D | POINT | ✸ | SPHERICAL |
| 2D | LINE | ✸ | CYLINDRICAL |
| 1D | PLANE | ✸ | PLANAR |

# Checkpoint 3.1

A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.




Which of the following statements best describes the electric field in the region between the spheres?

***A.*** The field points radially outward

***B.*** The field points radially inward

***C.*** The field is zero

"I would expect a test charge placed in the region between the spheres to travel away from the positive inner charge and toward the outer negative charge."

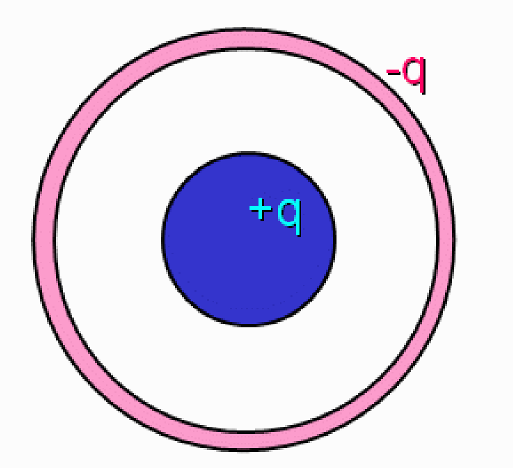
"The charge enclosed is negative. Therefore, the el

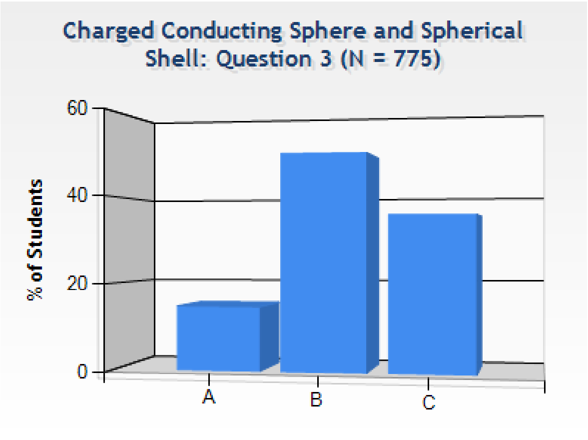
"Electric field inside a conductor should be zero."

Careful: what does inside mean?
This is always true for a solid conductor (within the material of the conductor)
Here we have a charge "inside"

# Checkpoint 3.3

A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.




Which of the following statements best describes the electric field in the region outside the red sphere?

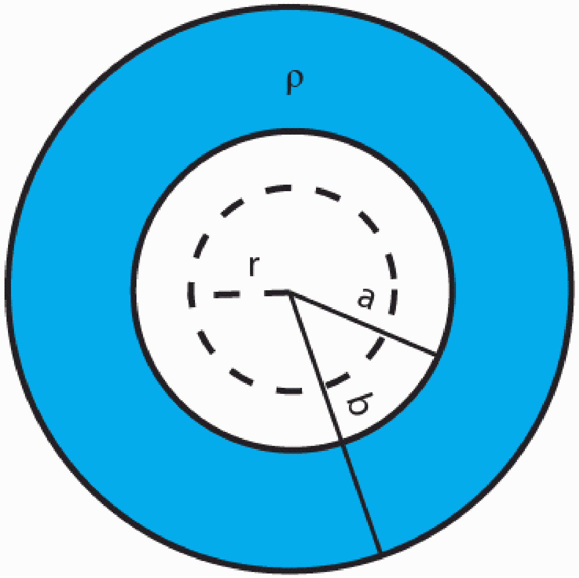
***A.*** The field points radially outward
***B.*** The field points radially inward
***C.*** The field is zero

Imagine a Gaussian sphere larger than the red sphere:

the total charge enclosed is zero!

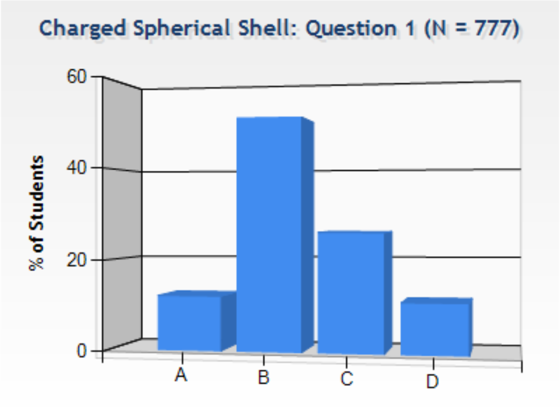
# Checkpoint 2

A charged spherical insulating shell has an inner radius *a* and outer radius *b*. The charge density of the shell is $\rho$.




What is the magnitude of the E field at a distance *r* away from the center of the shell where $r < a$?

**A.** $\rho/\varepsilon_0$
**B.** zero
**C.** $\rho(b^3-a^3)/(3\varepsilon_0 r^2)$
**D.** none of the above



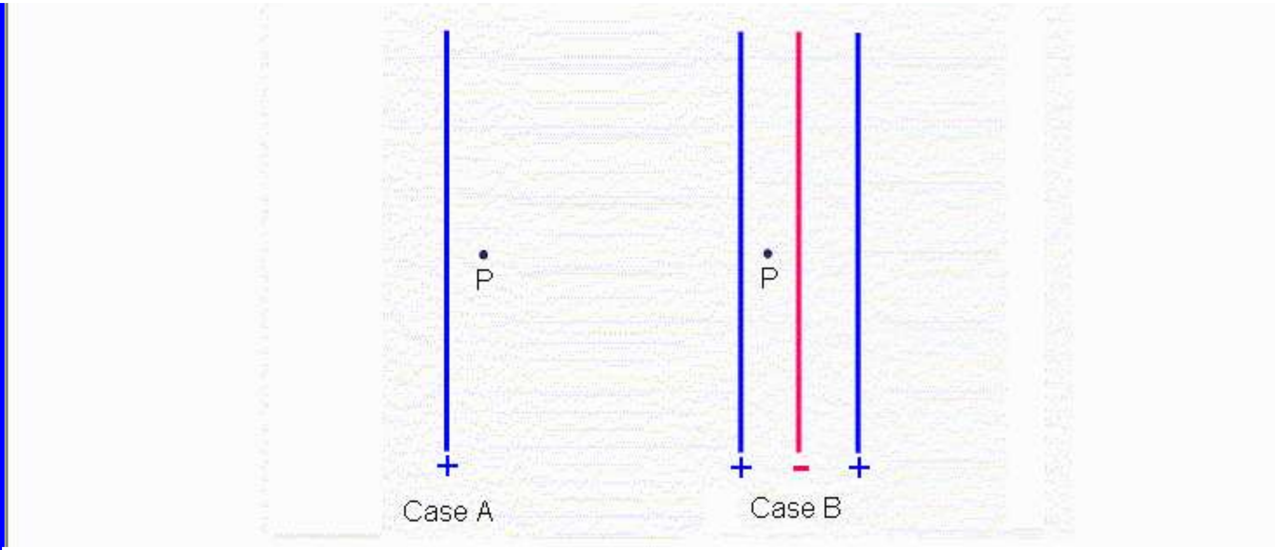

"The E-field isn't 0 and should not depend on b or a when r < a."

"There is no charge inside the Gaussian surface radius r"

"The magnitude of the electric field varies with the volume of the insulator."

# Checkpoint 4

In both cases shown below, the colored lines represent positive (blue) and negative charged planes. The magnitudes of the charge per unit area on each plane are the same.



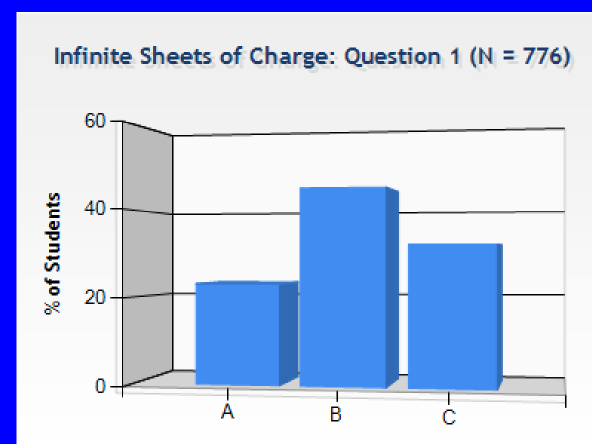

In which case is E at point P the biggest?

A) A B) B C) the same

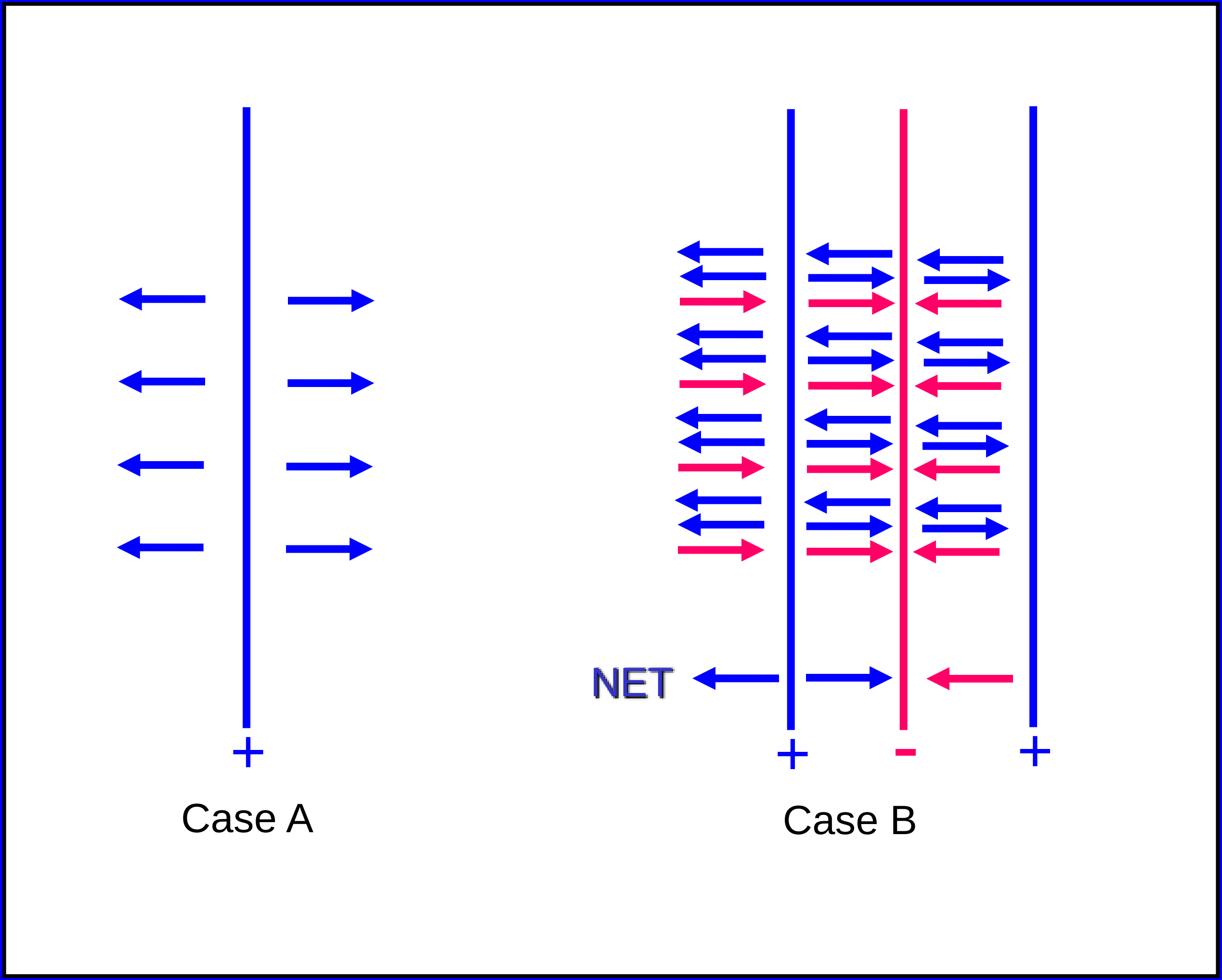
"In case B the positive and negative planes around P will create an electric field of 0, so the only other contribution is the positive plane on the right, which is farther away than the plane in case A."

"Because there are multiple charges around P in case B, the field is larger there."

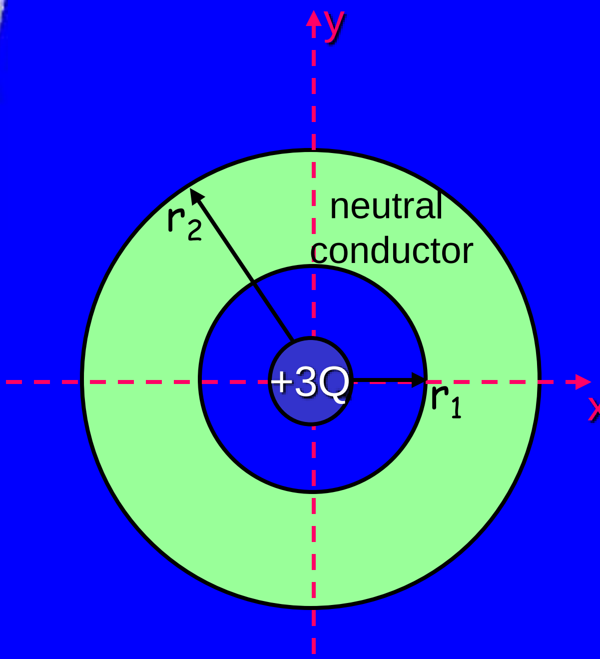
"distance does not matter for charged planes. So one negative and one positive plane will cancel out in Case B."



# Superposition:



# Calculation




Point charge +3Q at center of neutral conducting shell of inner radius $r_1$ and outer radius $r_2$.

a) What is E everywhere?

First question: Do we have enough symmetry to use Gauss' Law to determine E?

Yes.. Spherical Symmetry (what does this mean???)

A ☐ Magnitude of E is fcn of r

B ☐ Magnitude of E is fcn of $(r-r_1)$

C ☐ Magnitude of E is fcn of $(r-r_2)$

D ☐ None of the above

A ☐ Direction of E is along $\hat{x}$

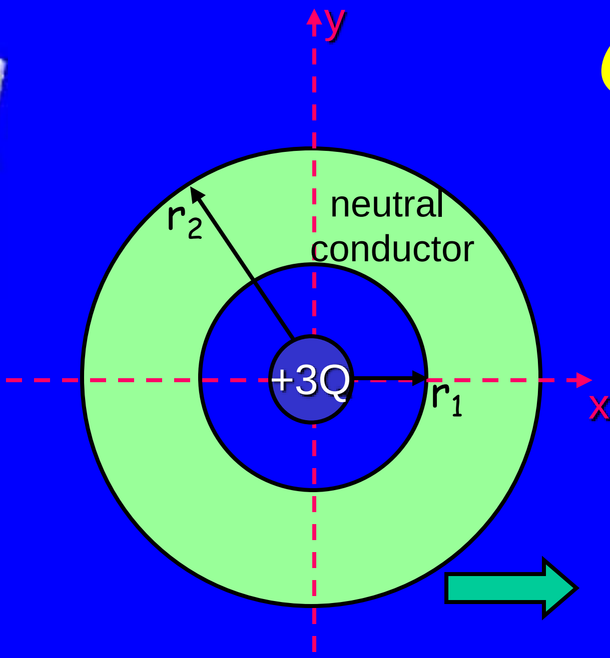
B ☐ Direction of E is along $\hat{y}$

C ☐ Direction of E is along $\hat{r}$

D ☐ None of the above

SPHERICAL SYMMETRY IS GENERATED BY A POINT !!

# Calculation




Point charge +3Q at center of neutral conducting shell of inner radius $r_1$ and outer radius $r_2$.

a) What is E everywhere?

We know:

magnitude of E is fcn of r

direction of E is along $\hat{r}$

We can use Gauss' Law to determine E

Use Gaussian surface = sphere centered on origin

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

**$r < r_1$**

$$\oint \vec{E} \cdot d\vec{A} = E\,4\pi r^2$$

$$Q_{enc} = +3Q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$$

**$r_1 < r < r_2$**

A $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$

B $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r_1^2}$

C $E = 0$

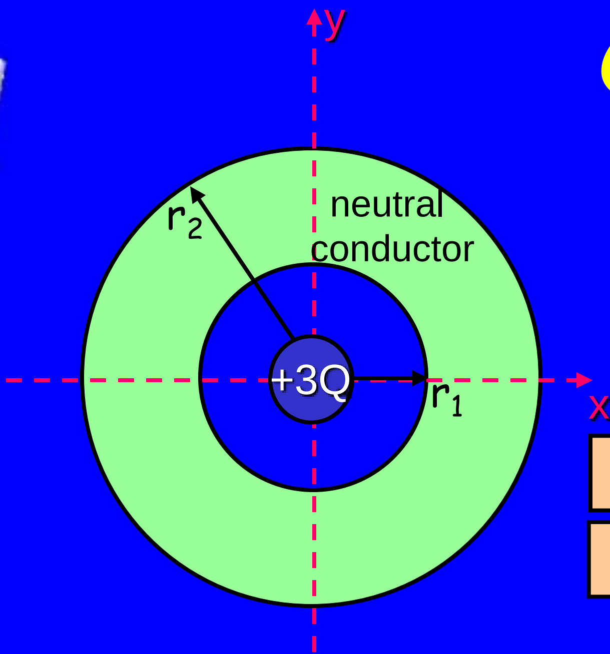
**$r > r_2$**

A $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$

B $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{(r - r_2)^2}$

C $E = 0$

# Calculation




Point charge +3Q at center of neutral conducting shell of inner radius $r_1$ and outer radius $r_2$.

a) What is E everywhere?

We know:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$

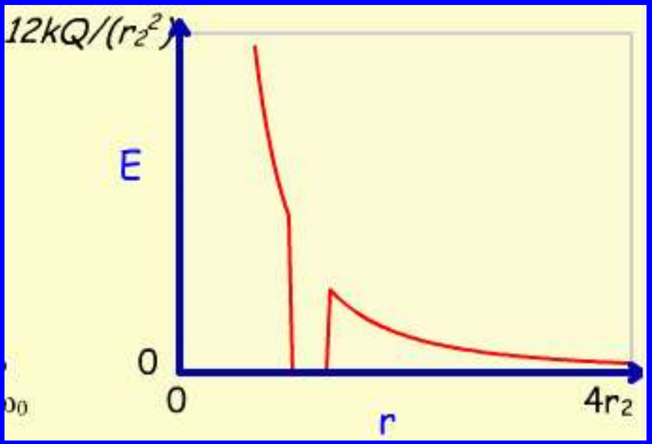
$r < r_1$

$r > r_2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$$

$r_1 < r < r_2$

$$E = 0$$

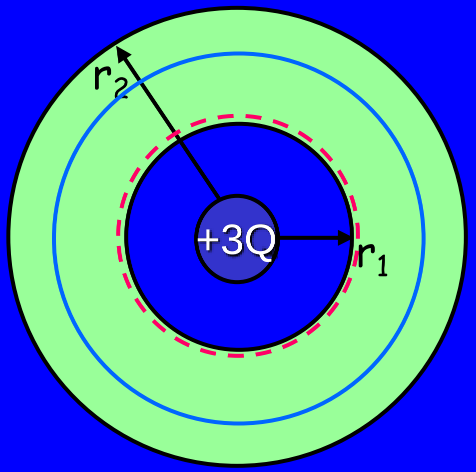



b) What is charge distribution at $r_1$?

A ☐ $\sigma < 0$

B ☐ $\sigma = 0$

C ☐ $\sigma > 0$



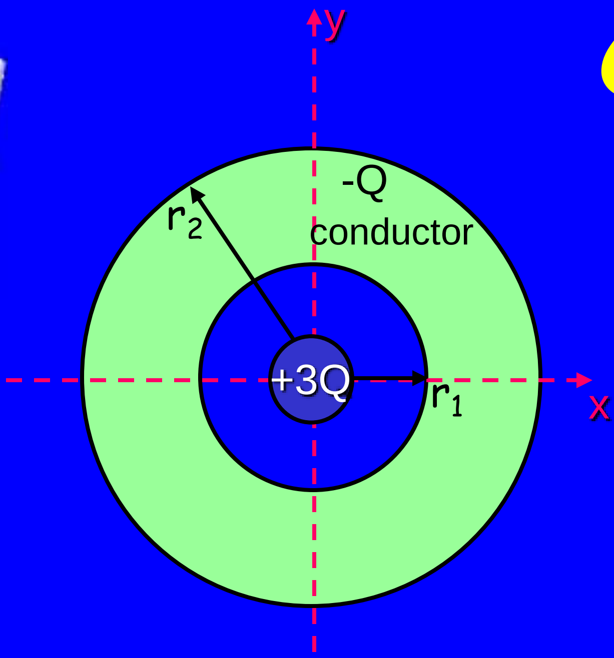

Gauss' Law:

$$E = 0 \Rightarrow Q_{enc} = 0 \Rightarrow \sigma_1 = \frac{-3Q}{4\pi r_1^2}$$

Similarly:

$$\sigma_2 = \frac{+3Q}{4\pi r_2^2}$$

# Calculation

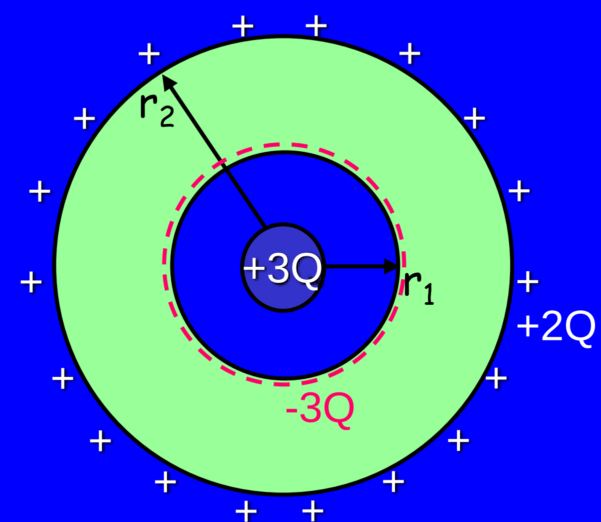



Suppose give conductor a charge of -Q

a) What is E everywhere?

b) What are charge distributions at $r_1$ and $r_2$?

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_o}$$




**$r < r_1$**

A ☐ $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

B ☐ $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

C ☐ $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

**$r > r_2$**

A ☐ $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

B ☐ $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

C ☐ $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

**$r_1 < r < r_2$**

$$E = 0$$