Physics 212

Lecture 7

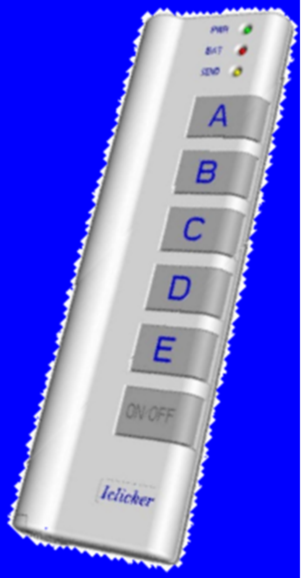
Today's Concept:

Conductors and Capacitance

Music

Who is the Artist?

A) Eric Clapton
B) Bill Frisell
C) Jimmy Page
D) Jeff Beck
E) Buddy Guy



Why?

Starting on some circuits – electric guitar

LOGISTICS

1) EXAM 1: WED Feb. 15 at 7pm

Sign Up in Gradebook for Conflict Exam at 5:15pm if desired
BY Mon. Feb. 13 at 10:00 p.m.

MATERIAL: Lectures 1 - 8

2) EXAM 1 PREPARATION?
Old exams are on-line ("Practice Exams"), also "Worked Examples"
and "Exam Prep Exercises"

Your Comments

"I'm really lost. Can you explain the examples more in lecture please?"

"I enjoyed this lecture, seems pretty straight forward."

"Soooooooooooooooooooooooooooooooooooooo confused and the homework was soooooooooooooooooooooooooooooooooooooooooooooo hard"

"How exactly does an electric field store energy in capacitors?"

"go over the conductor between the plates!"

"PLEASE!!! EXPLAIN SOME OF THE HOMEWORK IN CLASS!!!"

"I feel like all the examples were just derived from the prelecture, and I would like to go over some different examples to make sure I truly understand this stuff, and am not just repeating exactly what was previously told to me."

True, but …
We'll go through the conductor between the plates for two separate cases
We'll do a problem much like a homework problem at the end of class

"If there is a train that could hold then entire capacity of students enrolled in Physics 212, that conductor must be pretty attracted to his job."

"Ignorance is bliss"

Conductors

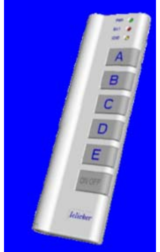
You did well on the questions on charge distributions on conductors

The Main Points

- Charges free to move
- E = 0 in a conductor (even in a cavity)
- Surface = Equipotential
- E at surface perpendicular to surface

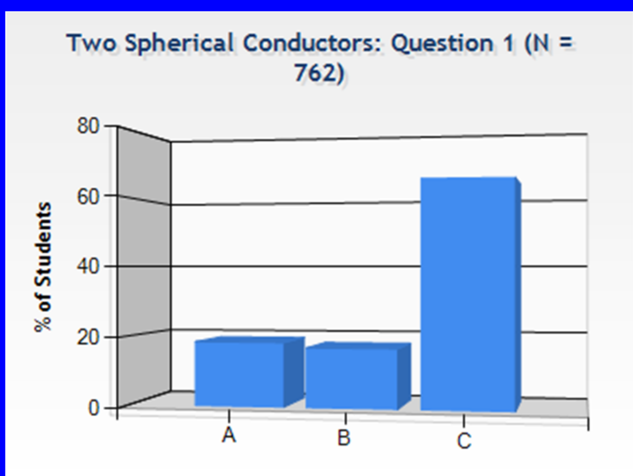
Checkpoint 1a

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B.



Compare the potential at the surface of conductor A with the potential at the surface of conductor B.

A. **$V_A > V_B$** ***B.*** **$V_A = V_B$** ***C.*** **$V_A < V_B$**

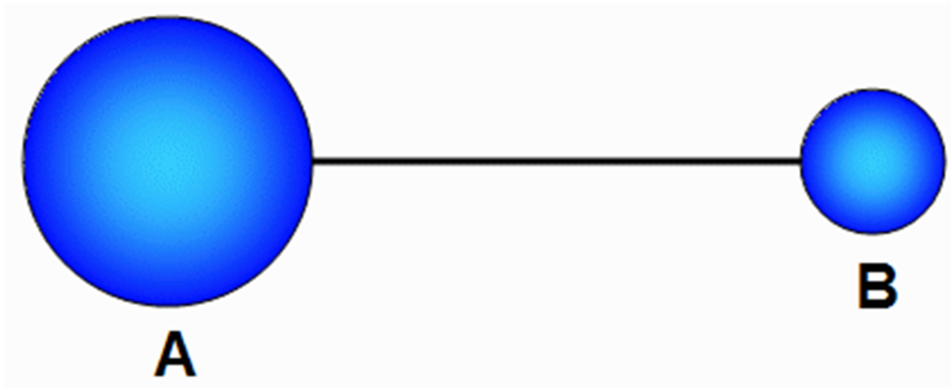


"larger area more charge"

"Conductors with the same charge are equipotential"

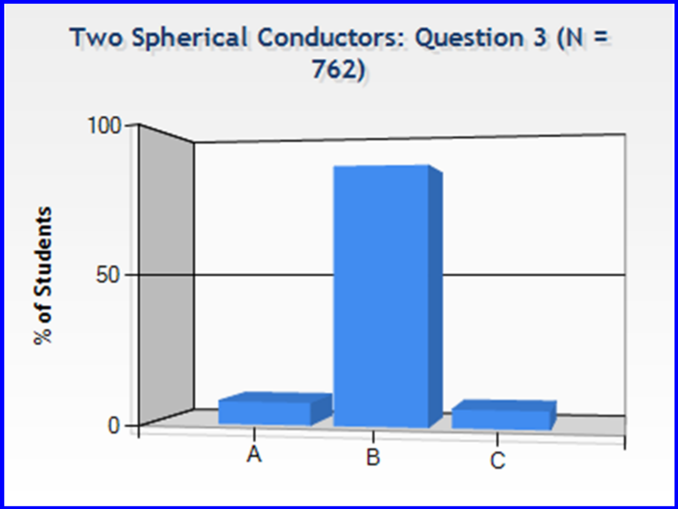
"The radius of A is 4B, and since V=kQ/r you get 4Va=Vb"

Checkpoint 1b



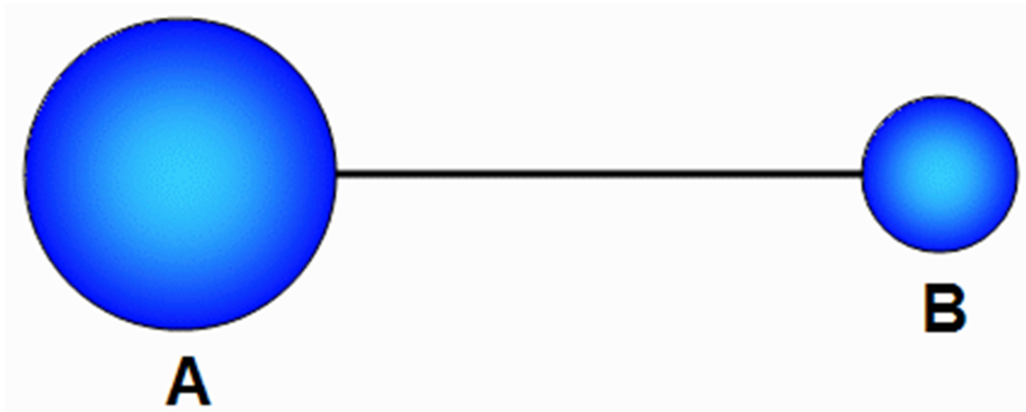
The two conductors are now connected by a wire. How do the potentials at the conductor surfaces compare now?

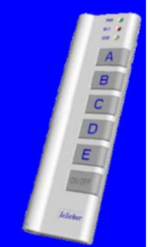
A. **$V_A > V_B$** ***B.*** **$V_A = V_B$** ***C.*** **$V_A < V_B$**



"No matter what the initial conditions are, when both spheres are making contact, their potential has to be equal since they are connected by a wire that makes them behave like a single conductor."

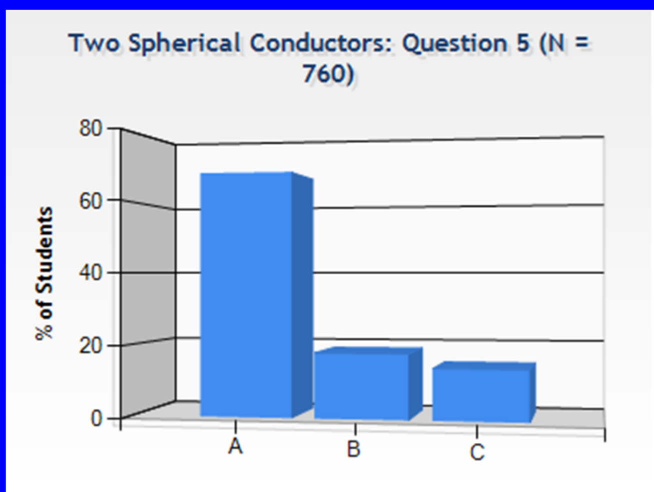
Checkpoint 1c





What happens to the charge on conductor A after it is connected to conductor B by the wire?

***A.* Q_A increases** ***B.* Q_A decreases** ***C.* Q_A doesn't change**



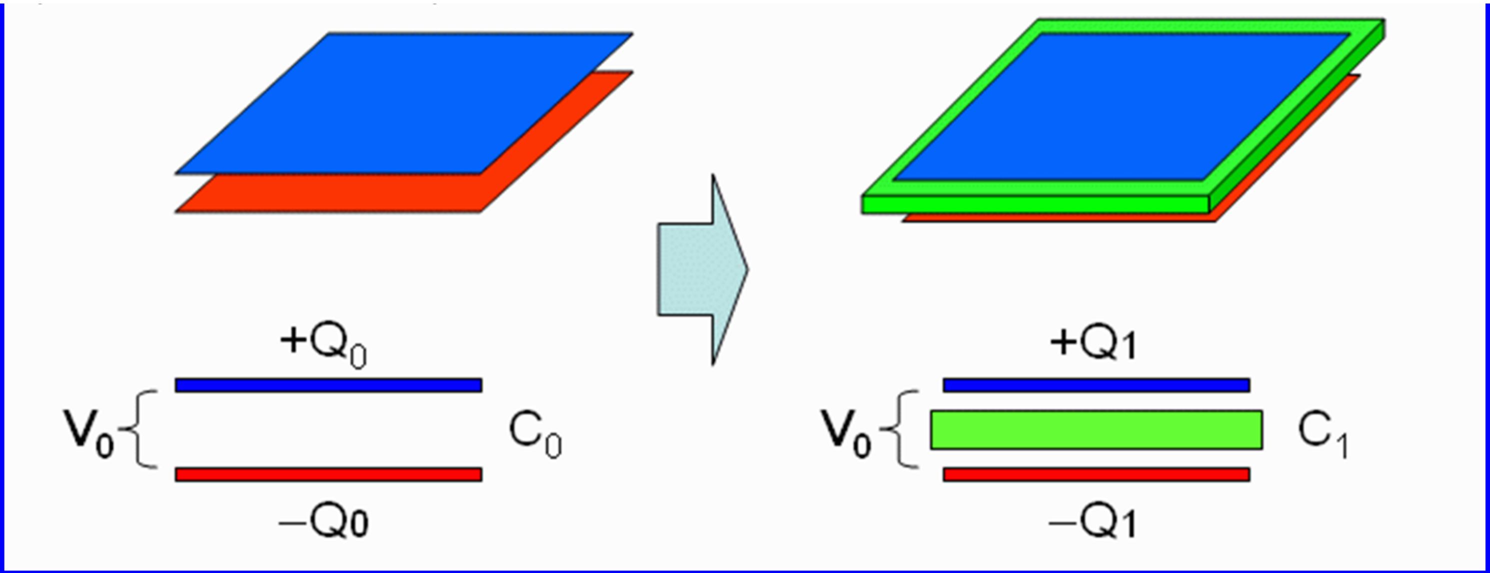
"Charge will always move to a place with lower potential, and the larger sphere has a lower potential than the smaller sphere. "

"the charge would decrease in order to compensate for the lower charge on the particle B"

"When you connect two conductors by a wire and charge moves between them as to make difference in potential of the system zero what is the charge of the wire? Or does it not matter?"

Parallel Plate Capacitor

Two parallel plates of equal area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same.



THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We'll work through the example in the Prelecture and then do the Checkpoint questions.

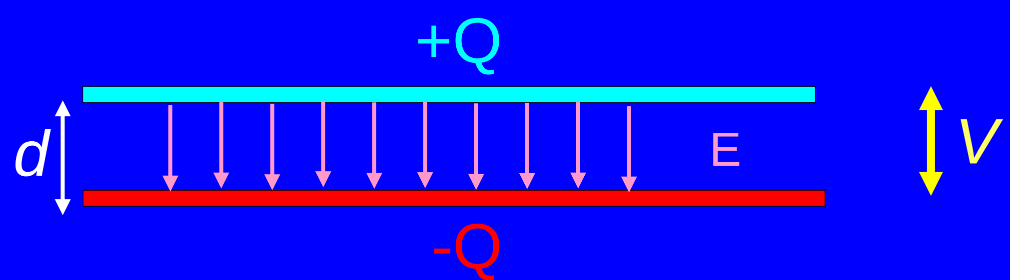
Capacitance

Capacitance is defined for any pair of spatially separated conductors

$$C \equiv \frac{Q}{V}$$

How do we understand this definition ???

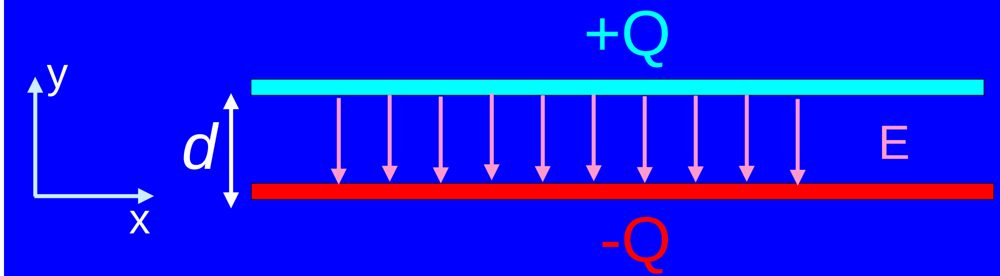
- Consider two conductors, one with excess charge = +Q and the other with excess charge = -Q



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductors
- This potential difference should be proportional to Q !!
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Example (done in Prelecture 7)

First determine E field produced by charged conductors:



$$E = \frac{\sigma}{\varepsilon_o}$$

What is σ ??

$$\sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

$$V = -\int_0^d \vec{E} \cdot d\vec{y}$$

$$V = -\int_0^d (-E dy) = E \int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

Almost everything you need for HW 1!

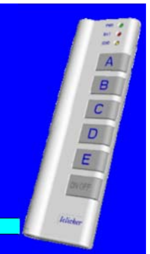
As promised, V is proportional to Q !!

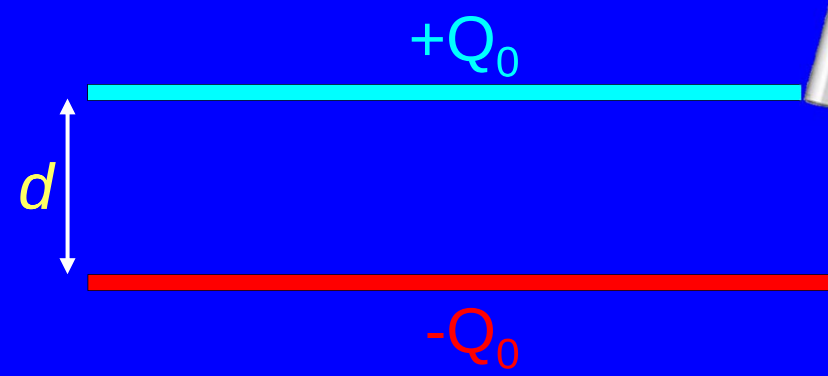
$$C \equiv \frac{Q}{V} = \frac{Q}{Qd / \varepsilon_o A}$$

$$C = \frac{\varepsilon_0 A}{d}$$

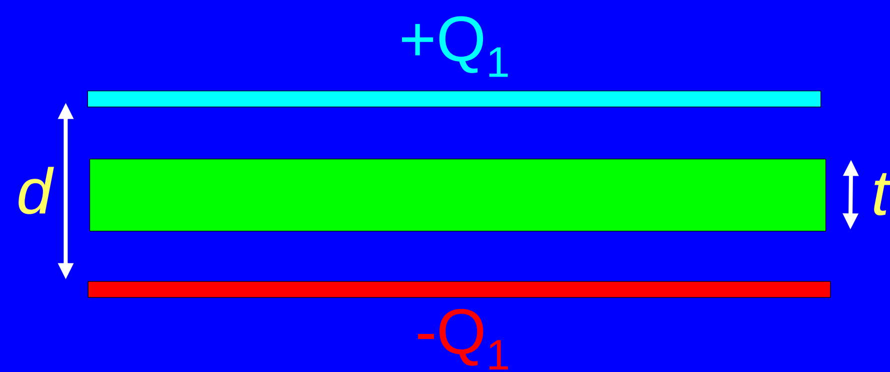
C determined by geometry !!

Question Related to Checkpoint 2





Initial charge on capacitor = Q_0



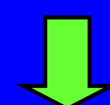
Insert uncharged conductor

Charge on capacitor now = Q_1

How is Q_1 related to Q_0 ??

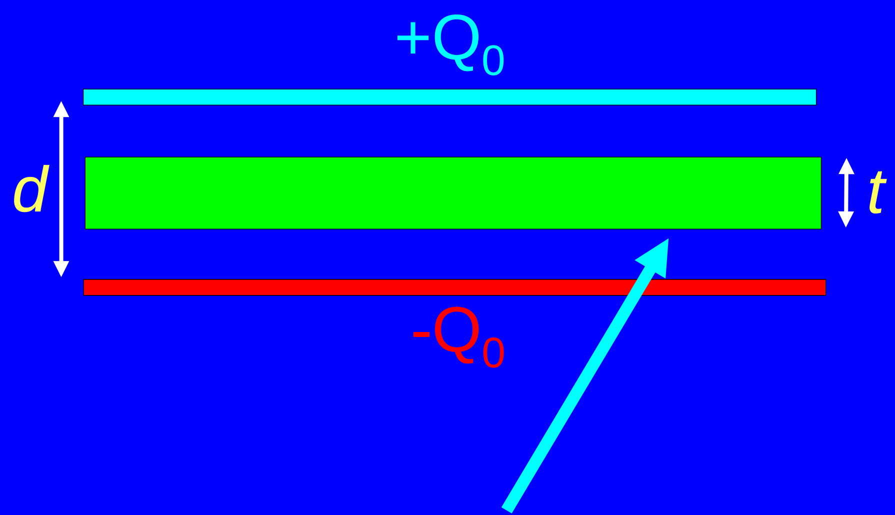
A. $Q_1 < Q_0$
B. $Q_1 = Q_0$
C. $Q_1 > Q_0$

Plates not connected to anything



CHARGE CANNOT CHANGE !!

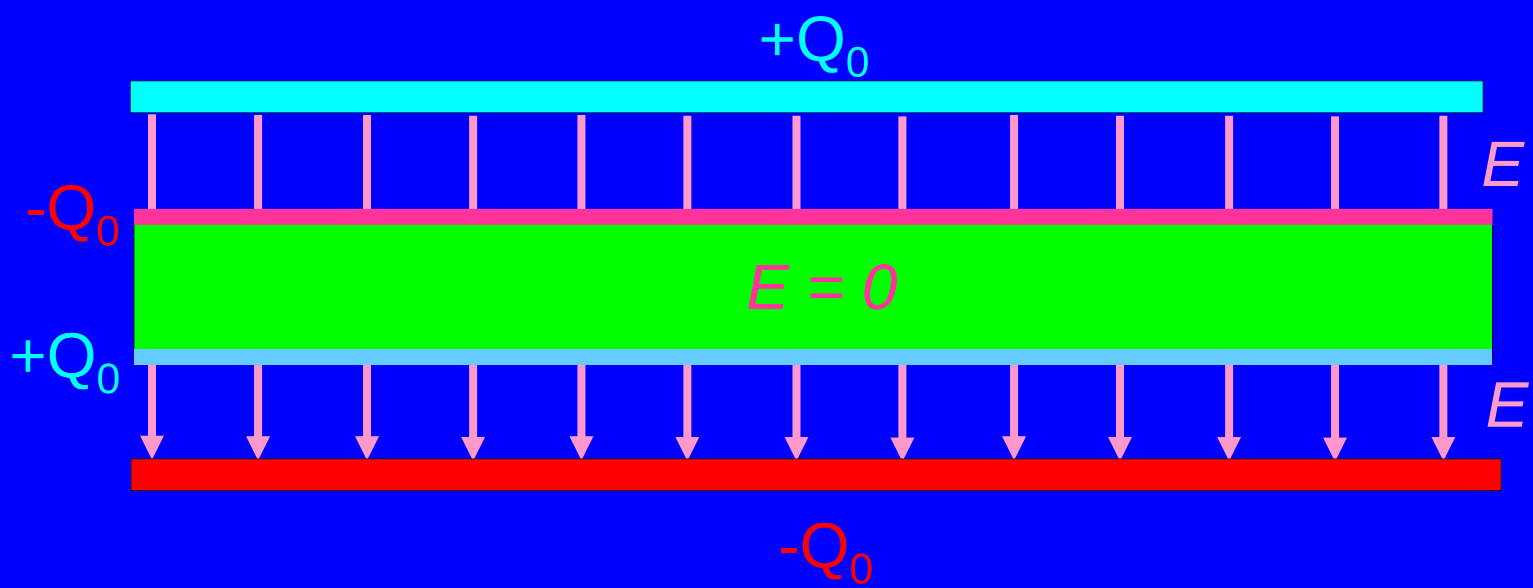
Where to Start??



What is the total charge induced on the bottom surface of the conductor?

A. $+Q_0$
B. $-Q_0$
C. 0
D. Positive but the magnitude unknown
E. Negative but the magnitude unknown

WHY ??



WHAT DO WE KNOW ???

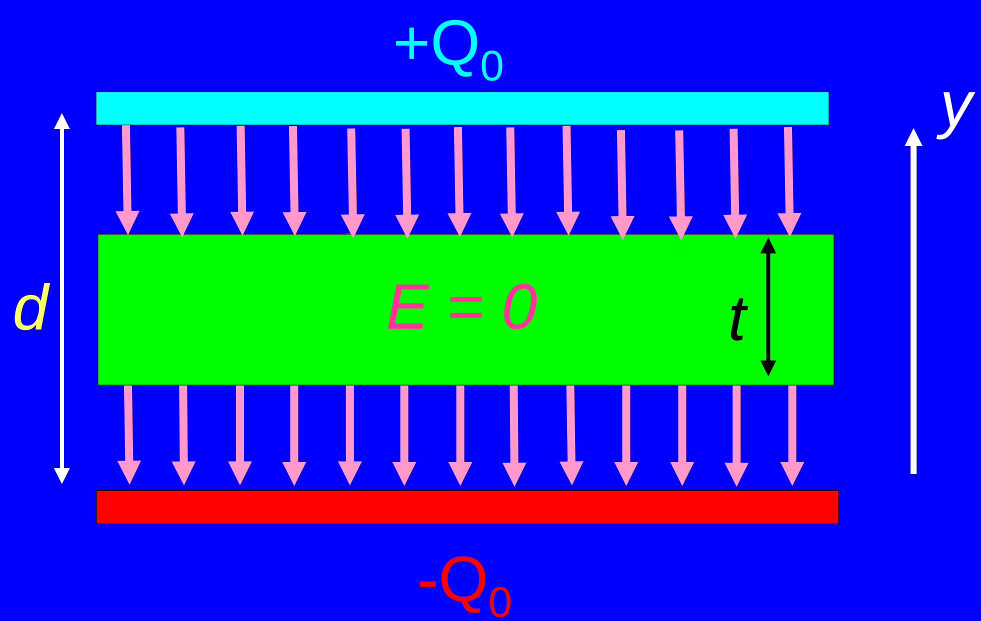
E must be = 0 in conductor !!

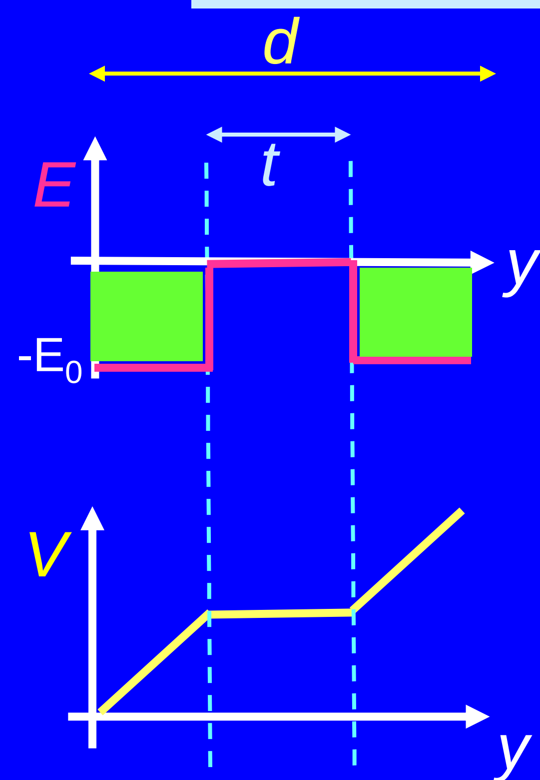
Charges inside conductor move to cancel E field from top & bottom plates

Calculate V

Now calculate V as a function of distance from the bottom conductor.

$$V(y) = -\int_0^y \vec{E} \cdot d\vec{y}$$





What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

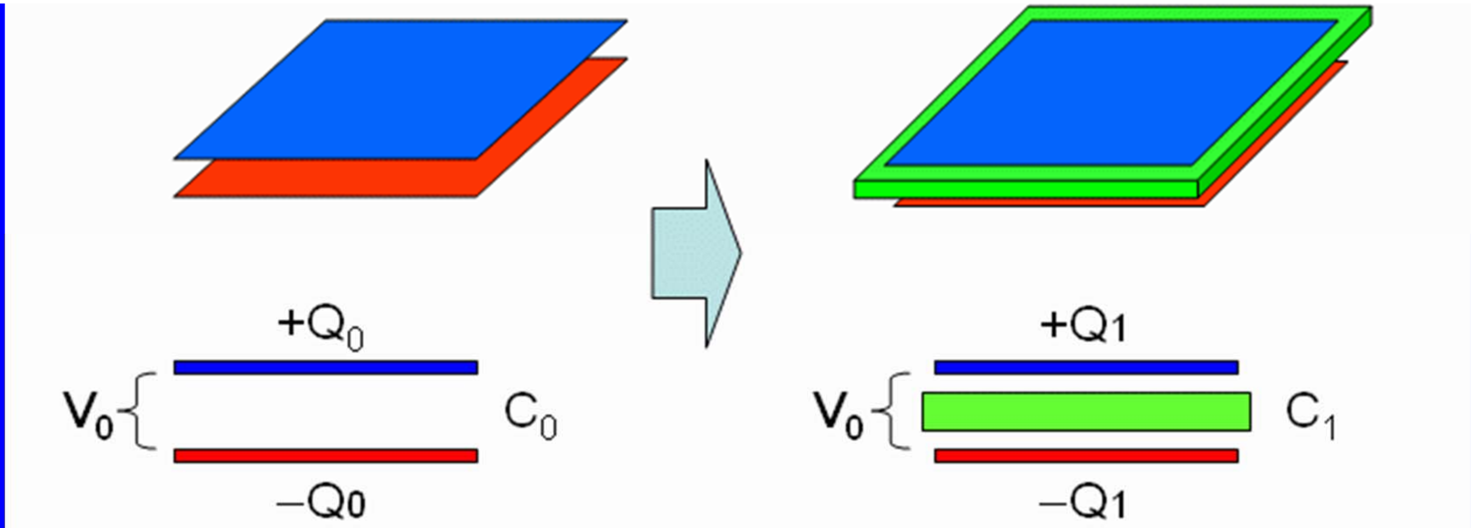
B) $\Delta V = E_0(d - t)$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

Back to Checkpoint 2a

Two parallel plates of equal area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same.

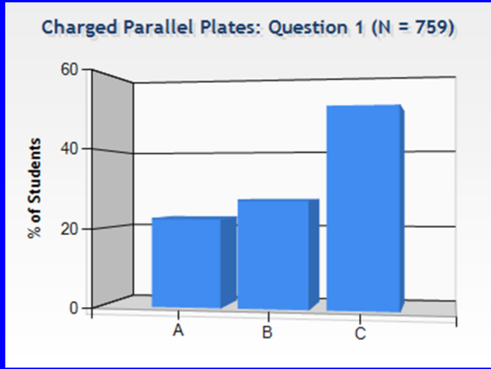


A) $Q_1 < Q_o$ **B) $Q_1 = Q_o$** **C) $Q_1 > Q_o$**

"The air space in between Q0 is greater than Q1 so Q0 must be greater to achieve the same potential difference."

"The potential difference is just the difference in charge between the plates. Adding a conductor in the center doesn't change that"

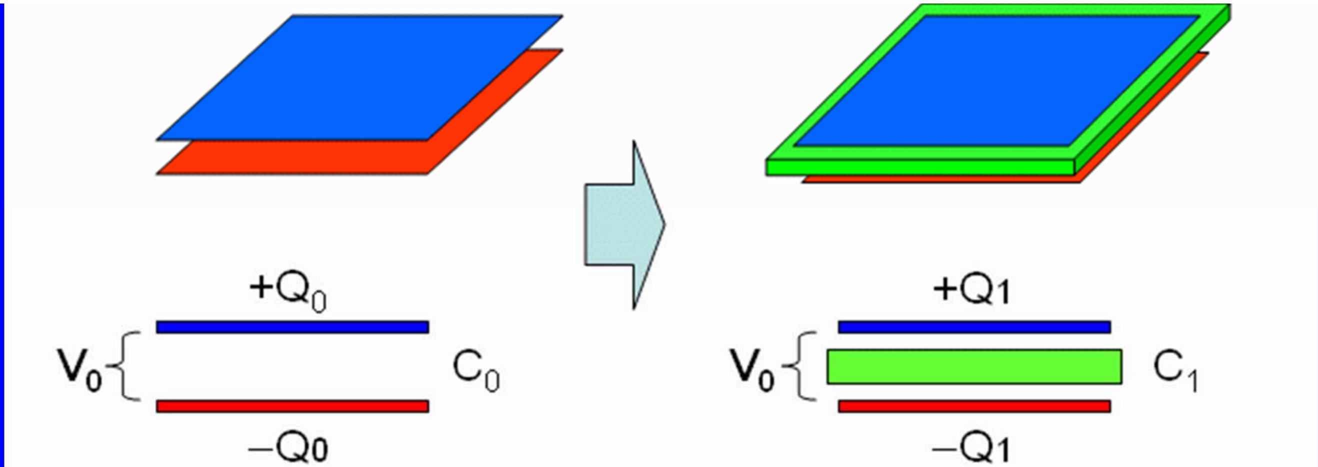
"Q1 needs a greater charge to have the same potential since part of its electric field is zero."



How do you get the same V_0 in 'less space'?

Checkpoint 2b

Two parallel plates of equal area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same. ***What happens to C_1 relative to C_0?***



A) $C_1 > C_o$ B) $C_1 = C_o$ C) $C_1 < C_o$

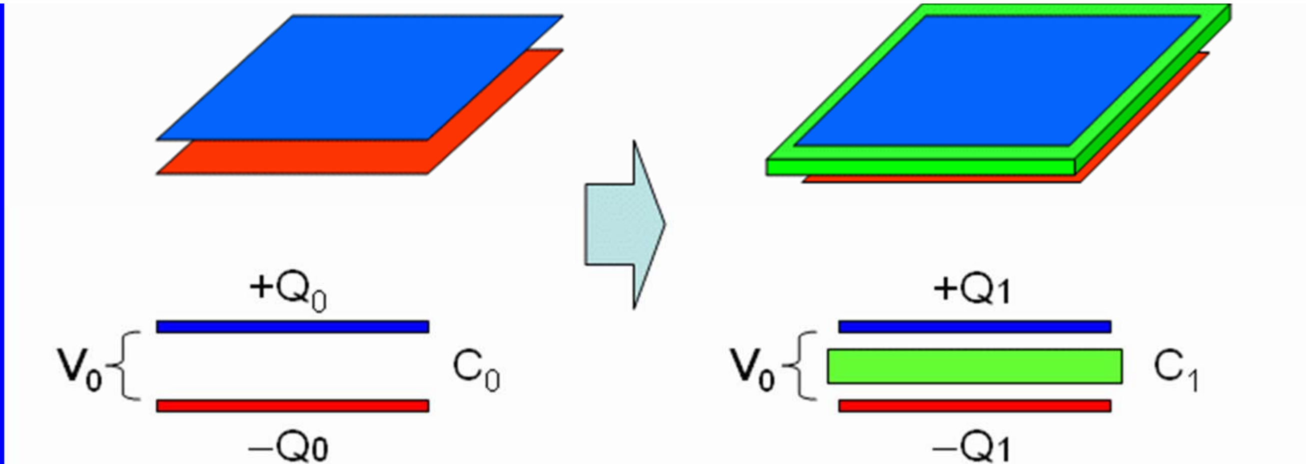
“Capacitance is directly proportional to the charge, so if in case 1, the charge is greater than in case 0, that means the Capacitance is greater.”

“Capacitance is equal to charge over voltage, both of which are the same.”

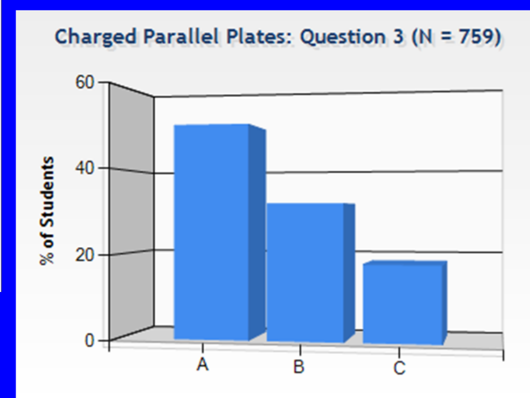
“C=(1/2)*((Q^2)/U). Thus, if Q decreases, then C will decrease.”

Checkpoint 2b

Two parallel plates of equal area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same. ***What happens to C_1 relative to C_0?***



A) $C_1 > C_o$ B) $C_1 = C_o$ C) $C_1 < C_o$



We can determine C from either case
same V (Checkpoint)
same Q (Prelecture)
C depends only on geometry !!

$E = Q/\varepsilon_0 A$

Same V:

$V_0 = E_0 d$
$V_0 = E_1(d - t)$

$C_0 = Q_0/E_0 d$
$C_1 = Q_1/(E_1(d - t))$

$C_0 = \varepsilon_0 A/d$
$C_1 = \varepsilon_0 A/(d - t)$

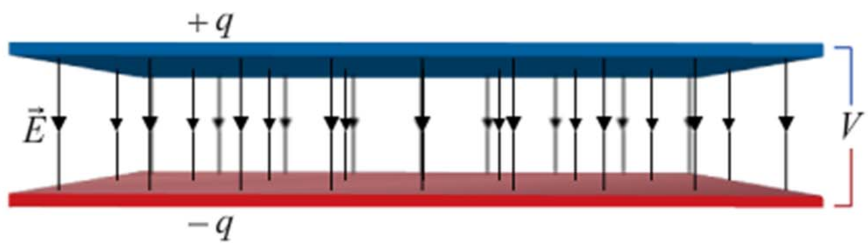
Energy in Capacitors

Energy Stored in Capacitors

$$U = \frac{1}{2} QV \quad \text{or} \quad U = \frac{1}{2}\frac{Q^2}{C} \quad \text{or} \quad U = \frac{1}{2} CV^2$$

Energy Density

$$u = \frac{1}{2} \varepsilon_o E^2$$

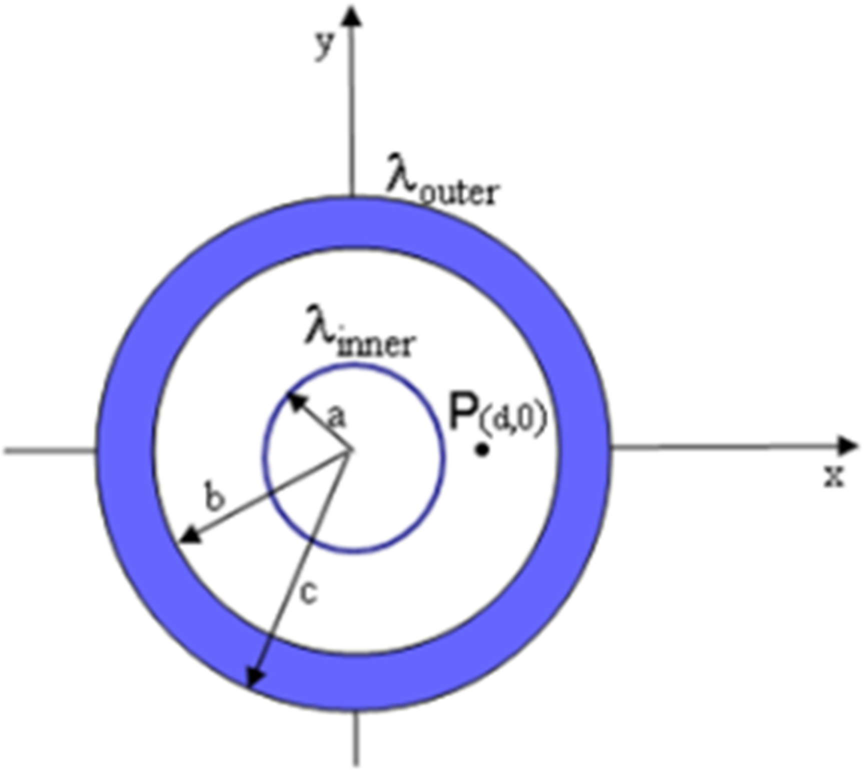


BANG

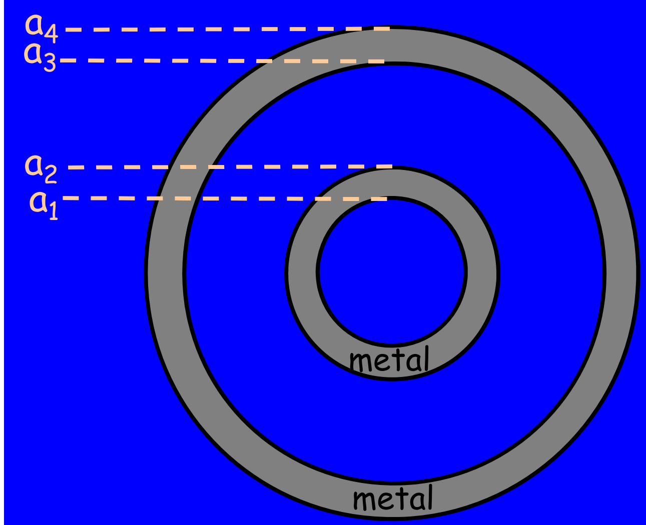
Homework for Lec. 7&8

Concentric Cylindrical Conducting Shells

An infiinitely long solid conducting cylindrical shell of radius a = 3.6 cm and negligible thickness is positioned with its symmetry axis along the z-axis as shown. The shell is charged, having a linear charge density λ_{inner} = -0.55 μC/m. Concentric with the shell is another cylindrical conducting shell of inner radius b = 14.7 cm, and outer radius c = 19.7 cm. This conducting shell has a linear charge density λ_{outer} = 0.55μC/m.



Calculation



A capacitor is constructed from two conducting *cylindrical* shells of radii a_1, a_2, a_3, and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this device ?

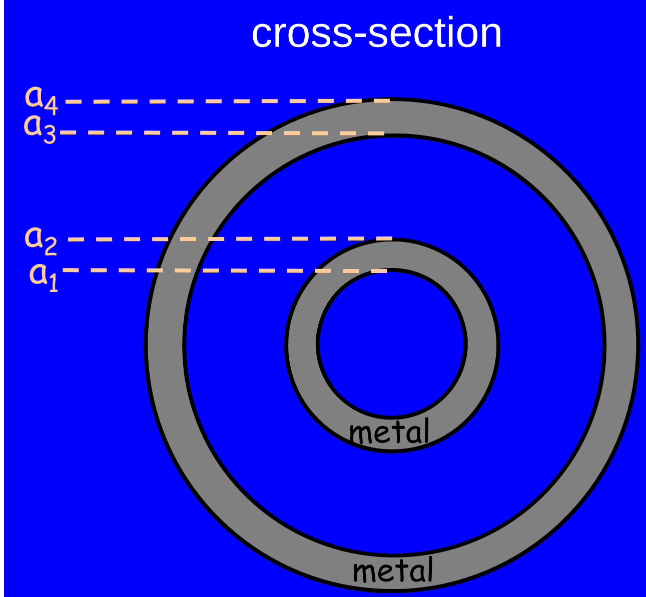
- Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

But what is **Q** and what is **V**? They are not given??

- Important Point: C is a property of the object!! (concentric cylinders here)
  - Assume some Q (i.e., $+Q$ on one conductor and $-Q$ on the other)
  - These charges create E field in region between conductors
  - This E field determines a potential difference V between the conductors
  - V should be proportional to Q; the ratio Q/V is the capacitance.

Calculation



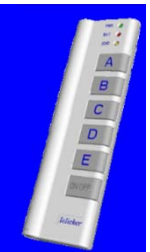
A capacitor is constructed from two conducting *cylindrical* shells of radii a_1, a_2, a_3, and a_4 and length L (L >> a_i).

What is the capacitance C of this capacitor ?

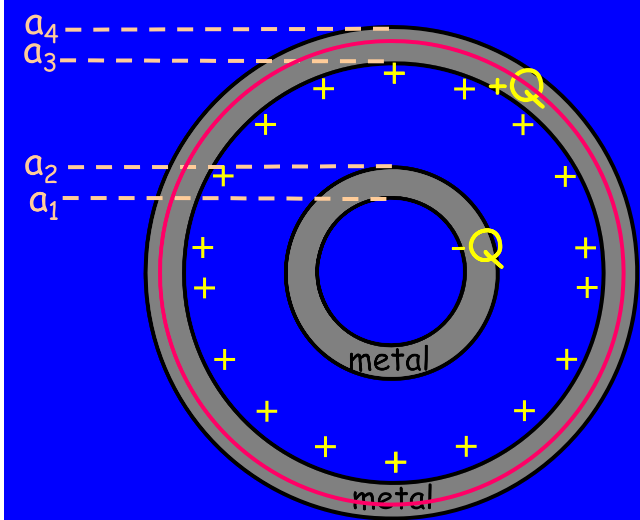
$$C \equiv \frac{Q}{V}$$

- Strategic Analysis:
  - Put +Q on outer shell and –Q on inner shell
  - Cylindrical symmetry: Use Gauss' Law to calculate E everywhere
  - Integrate E to get V
  - Take ratio Q/V: should get expression only using geometric parameters (a_i, L)

Calculation



cross-section



A capacitor is constructed from two conducting *cylindrical* shells of radii a_1, a_2, a_3, and a_4 and length L (L >> a_i).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is +Q on outer conductor located?

(A) at $r=a_4$ (B) at $r=a_3$ (C) both surfaces (D) throughout shell

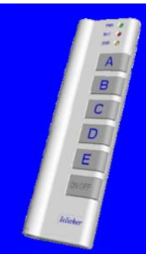
Why?

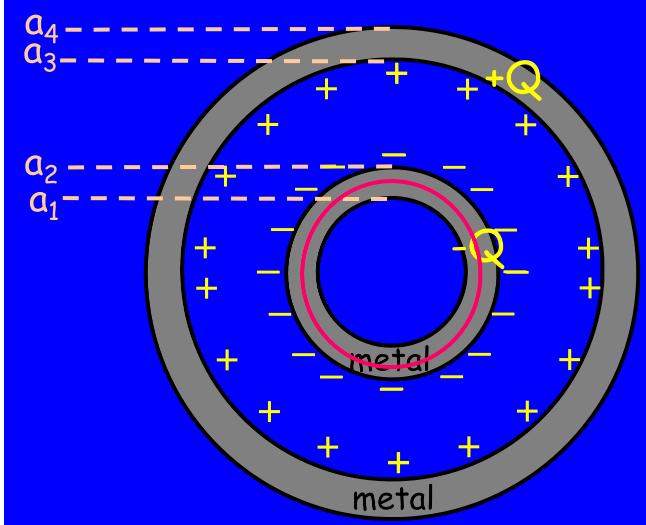
Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$

We know that E = 0 in conductor (between a_3 and a_4) ⇒ $Q_{enclosed} = 0$

$Q_{enclosed} = 0$ ⇒ +Q must be on inside surface (a_3), so that $Q_{enclosed} = +Q - Q = 0$

Calculation





A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L (L >> a_i).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is -Q on inner conductor located?

(A) at $r=a_2$ (B) at $r=a_1$ (C) both surfaces (D) throughout shell

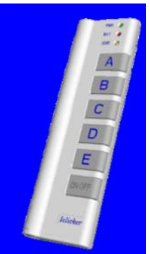
Why?

Gauss' law: $$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

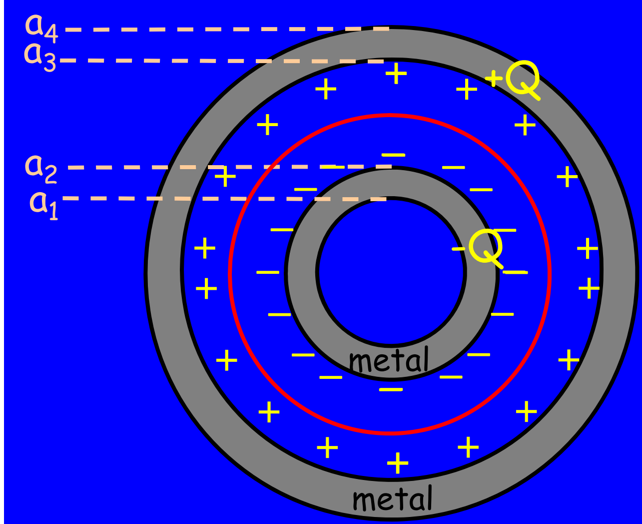
We know that E = 0 in conductor (between a_1 and a_2) ⇒ $$Q_{enclosed} = 0$$

$$Q_{enclosed} = 0$$ ⇒ +Q must be on outer surface (a_2), so that $Q_{enclosed}$ = 0

Calculation



cross-section



A capacitor is constructed from two conducting *cylindrical* shells of radii a_1, a_2, a_3, and a_4 and length L ($L \gg a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is E(r)?

(A) 0

(B) $\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$

(C) $\frac{1}{2\pi\varepsilon_o}\frac{Q}{Lr}$

(D) $\frac{1}{2\pi\varepsilon_o}\frac{2Q}{Lr}$

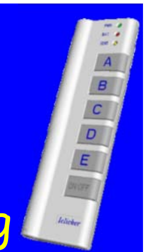
(E) $\frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2}$

Why?

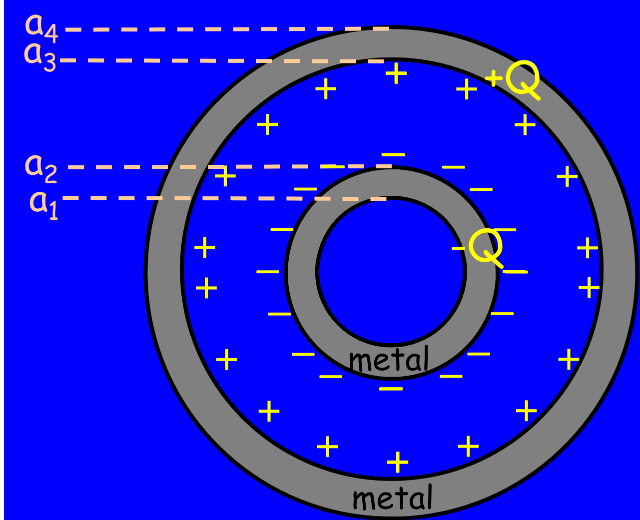
Gauss' law: $\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ ⇒ $E\cdot 2\pi rL = \frac{Q}{\varepsilon_o}$ ⇒ $E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$

Direction: Radially In

Calculation



cross-section



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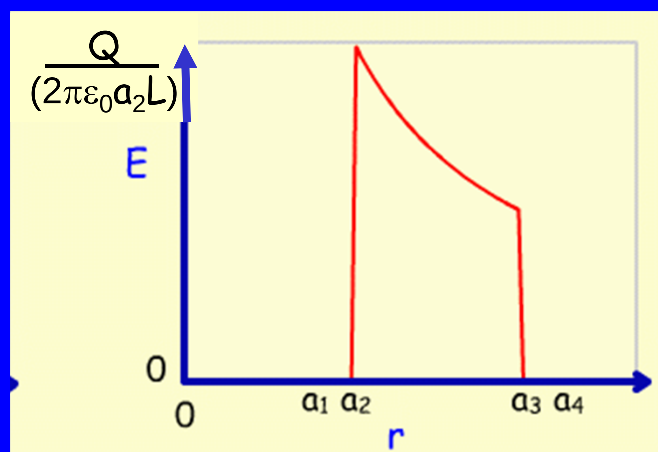
What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: $$E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

$r < a_2$: $E(r) = 0$

since $Q_{enclosed} = 0$

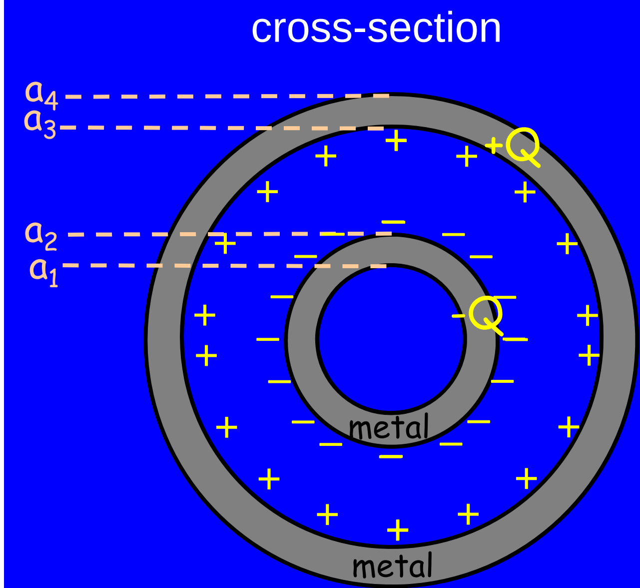


- What is V?
  - The potential difference between the conductors

What is the sign of $V = V_{outer} - V_{inner}$?

(A) $V_{outer} - V_{inner} < 0$ (B) $V_{outer} - V_{inner} = 0$ (C) $V_{outer} - V_{inner} > 0$

Calculation



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What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$:

$$E = \frac{1}{2\pi\varepsilon_o}\frac{Q}{rL}$$

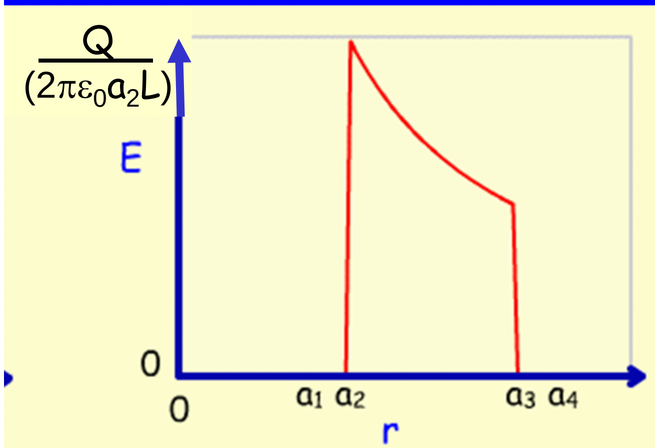
What is $V \equiv V_{outer} - V_{inner}$?

(A) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_1}{a_4}$

(B) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_4}{a_1}$

(C) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_3}{a_2}$

(D) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_2}{a_3}$



$$V = -\int_{a_2}^{a_3}\frac{-Q}{2\pi\varepsilon_o L}\frac{dr}{r} \Rightarrow V = \frac{Q}{2\pi\varepsilon_o L}\int_{a_2}^{a_3}\frac{dr}{r} \Rightarrow V = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{a_3}{a_2}$$

V proportional to Q, as promised

$$\Rightarrow C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(a_3/a_2)}$$