Physics 212
Lecture 10
Today's Concept:
Kirchhoff's Rules
Circuits with resistors & batteries
Music

Who is the Artist?

A) Norah Jones
B) Diana Krall
C) Jane Monheit
D) Nina Simone
E) Marcia Ball

- New album with Paul McCartney (and Eric Clapton and Stevie Wonder)
- Anyone know the connection between Oscar Peterson and Diana Krall?
- Both great Canadian jazz pianists - Peterson was one of Krall’s mentors
“This stuff is loopy”

“The conventions for signs of voltage drops and gains make absolutely no sense. Why is a voltage drop considered positive and a gain negative?”

“Determining which way current flows and application of Kirchhoff's rules will need a ton of practice since this is all new to me.”

“Discuss multiple currents more. It is pretty confusing to understand.”

“I don't really understand how Kirchhoff's Rules apply to the blue wire problem. Could we please go over these problems in lecture? That would be a huge help!”

"Fluke" is not a good brand name for a multimeter. Advertisement: "If the data is good, it must have been a Fluke."
Today’s Plan:

• Summary of Kirchoff’s rules – these are the key concepts

• Example problem

• Review Checkpoints
Last Time

Resistors in series:
Current through is same.
Voltage drop across is $IR_i$

Resistors in parallel:
Voltage drop across is same.
Current through is $\frac{V}{R_i}$

Solved Circuits
Last Time
New Circuit

How Can We Solve This One?

THE ANSWER: Kirchhoff's Rules
Kirchoff’s Voltage Rule

\[ \sum \Delta V_i = 0 \]

Kirchoff’s Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

**WHY?**

The potential difference between a point and itself is zero!
Kirchoff’s Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

**WHY?**

Electric Charge is Conserved
How many potentially different currents are there in the circuit shown?

A. 3  B. 4  C. 5  D. 6  E. 7

“Two for the parallel branch and then one for the first and last resistors.”

“There are four different series connections”

“There is potentially one between every pair of resistors.”
Checkpoint 1

How many potentially different currents are there in the circuit shown?

Look at the nodes!

Top node: $I_1$ flows in, $I_2$ and $I_3$ flow out

Bottom node: $I_2$ and $I_3$ flow in, $I_1$ flows out

That's all of them!
If we are to write Kirchoff’s voltage equation for this loop in the clockwise direction starting from point a, what is the correct order of voltage gains/drops that we will encounter for resistors $R_1$, $R_2$ and $R_3$?

A. drop, drop, drop  
B. gain, gain, gain  
C. drop, gain, gain  
D. gain, drop, drop  
E. drop, drop, gain

“going with current is drop, against current is gain”

“The voltage gains when the current is flowing with the voltage”

“drops 2 times at the split then gains when merging”
If we are to write Kirchhoff’s voltage equation for this loop in the clockwise direction starting from point a, what is the correct order of voltage gains/drops that we will encounter for resistors R1, R2 and R3?

A. drop, drop, drop  
B. gain, gain, gain  
C. drop, gain, gain  
D. gain, drop, drop  
E. drop, drop, gain

With the current  \(\rightarrow\) VOLTAGE DROP

Against the current  \(\rightarrow\) VOLTAGE GAIN
In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$??

**Conceptual Analysis:**
- Circuit behavior described by Kirchhoff’s Rules:
  - KVR: $\Sigma V_{drops} = 0$
  - KCR: $\Sigma I_{in} = \Sigma I_{out}$

**Strategic Analysis**
- Write down Loop Equations (KVR)
- Write down Node Equations (KCR)
- Solve
In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$??

(1) Label and pick directions for each current
(2) Label the + and - side of each element
   This is easy for batteries
   For resistors, the “upstream” side is +

Now write down loop and node equations
In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$??

- How many equations do we need to write down in order to solve for $I_2$?
  
  (A) 1  (B) 2  (C) 3  (D) 4  (E) 5

- Why??
  - We have 3 unknowns: $I_1$, $I_2$, and $I_3$
  - We need 3 independent equations to solve for these unknowns

(3) Choose loops and directions
In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$??

- Which of the following equations is NOT correct?

(A) $I_2 = I_1 + I_3$
(B) $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
(C) $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
(D) $-V_2 - I_2R_2 + I_1R_1 + V_1 = 0$

- Why??
  - (D) is an attempt to write down KVR for the top loop
  - Start at negative terminal of $V_2$ and go clockwise
    - $V_{gain} (-V_2)$ then $V_{gain} (-I_2R_2)$ then $V_{gain} (-I_1R_1)$ then $V_{drop} (+V_1)$

(4) Write down voltage drops
(5) Write down node equation
In this circuit, assume $V_i$ and $R_i$ are known. What is $I_2$?

We have the following 4 equations:

1. $I_2 = I_1 + I_3$
2. $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
3. $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
4. $-V_2 - I_2R_2 - I_1R_1 + V_1 = 0$

Why??

- We need 3 INDEPENDENT equations
- Equations 2, 3, and 4 are NOT INDEPENDENT
  - Eqn 2 + Eqn 3 = - Eqn 4
- We must choose Equation 1 and any two of the remaining (2, 3, and 4)

Which 3 should we use?

A) Any 3 will do
B) 1, 2, and 4
C) 2, 3, and 4
In this circuit, assume $V_i$ and $R_i$ are known.

What is $I_2$??

- We have 3 equations and 3 unknowns.
  
  \[ I_2 = I_1 + I_3 \]
  
  \[ V_1 + I_1 R_1 - I_3 R_3 + V_3 = 0 \]
  
  \[ V_2 - I_2 R_2 - I_1 R_1 + V_1 = 0 \]

(6) Solve the equations

- The solution will get very messy!
  
  Simplify: assume $V_2 = V_3 = V$
  
  \[ V_1 = 2V \]
  
  \[ R_1 = R_3 = R \]
  
  \[ R_2 = 2R \]
Calculation: Simplify

In this circuit, assume $V$ and $R$ are known. What is $I_2$? 

- We have 3 equations and 3 unknowns.
  - $I_2 = I_1 + I_3$
  - $-2V + I_1R - I_3R + V = 0$ (outside)
  - $-V - I_2(2R) - I_1R + 2V = 0$ (top)

- With this simplification, you can verify:
  - $I_2 = \frac{1}{5} V/R$
  - $I_1 = \frac{3}{5} V/R$
  - $I_3 = \frac{-2}{5} V/R$
Follow-Up

- We know:
  \[ I_2 = \frac{1}{5} \frac{V}{R} \]
  \[ I_1 = \frac{3}{5} \frac{V}{R} \]
  \[ I_3 = -\frac{2}{5} \frac{V}{R} \]

- Suppose we short \( R_3 \): What happens to \( V_{ab} \) (voltage across \( R_2 \)?)
  (A) \( V_{ab} \) remains the same
  (B) \( V_{ab} \) changes sign
  (C) \( V_{ab} \) increases
  (D) \( V_{ab} \) goes to zero

Bottom Loop Equation:
\[ V_{ab} + V - V = 0 \]
\[ V_{ab} = 0 \]
Is there a current flowing between A and B?

A) Yes  
B) No

A & B have the same potential  
No current flows between A & B

Current flows from battery and splits at A  
Some current flows down  
Some current flows right
Checkpoint 3a

Consider the circuit shown below. Note that this question is *not* identical to the similar looking one you answered in the prelecture.

Which of the following best describes the current flowing in the blue wire connecting points $a$ and $b$?

A. Positive current flows from $a$ to $b$  
B. Positive current flows from $b$ to $a$  
C. No current flows between $a$ and $b$

“Energy flows toward least resistance, which is $1R$.”

“because $b$ has higher voltage than $a$”

“The top half is the same as the bottom half.”
Consider the circuit shown below. Note that this question is not identical to the similar looking one you answered in the prelecture.

Which of the following best describes the current flowing in the blue wire connecting points a and b?

A. Positive current flows from a to b
B. Positive current flows from b to a
C. No current flows between a and b

\[ I_1R - I_2(2R) = 0 \quad \Rightarrow \quad I_2 = \frac{1}{2} I_1 \]

\[ I_4R - I_3(2R) = 0 \quad \Rightarrow \quad I_4 = 2I_3 \]

\[ a: I_1 = I + I_3 \]
\[ b: I + I_2 = I_4 \]

\[ I_1 - I_3 + \frac{1}{2} I_1 = 2I_3 \]

\[ I_1 = 2I_3 \]

\[ I = +I_3 \]
Prelecture

What is the same? Current flowing in and out of the battery

Checkpoint

What is different? Current flowing from a to b
The diagram shows a circuit with two resistors, R and 2R, in parallel. The voltage source V is applied across the circuit, and the currents and voltages are indicated as follows:

- Currents:
  - \( \frac{2}{3}I \) through R
  - \( \frac{1}{3}I \) through 2R

- Voltages:
  - \( \frac{V}{2} \) across the circuit

The text on the slide appears to be a question about current division in parallel circuits, possibly asking for the calculation of the total current or the current through a specific resistor.
Consider the circuit shown below.

**Checkpoint 3b**

In which case is the current flowing in the blue wire connecting points a and b the largest?

A. Case A  
B. Case B  
C. They are both the same

“Case A has a lower Req”

“The resistors have a value of 4R in Case B, so the current will be more apt to bypass this section of the circuit”

“The total resistance in the path taken is the same for both”
Consider the circuit shown below.

**Checkpoint 3b**

In which case is the current flowing in the blue wire connecting points a and b the largest?

A. Case A  
B. Case B  
C. They are both the same

Current will flow from left to right in both cases

In both cases, \( V_{ac} = V/2 \)

\[
I_{2R} = 2I_{4R}
\]

\[
I_A = I_R - I_{2R} = I_R - 2I_{4R}
\]

\[
I_B = I_R - I_{4R}
\]
Model for Real Battery: Internal Resistance

Usually can’t supply too much current to the load without voltage “sagging”
Kirchhoff’s Laws

(1) **Label all currents**
   Choose any direction

(2) **Label +/- for all elements**
   Current goes + ⇒ - (for resistors)
   Battery signs *fixed!*

(3) **Choose loops and directions**
   Must start on wire, not element.

(4) **Write down voltage drops**
   First sign you hit is sign to use.

(5) **Write down node equations**
   \[ I_{in} = I_{out} \]

(6) **Solve set of equations**