Physics 212
Lecture 12

Today's Concept:

Magnetic Force on moving charges

$$\vec{F} = q\vec{v} \times \vec{B}$$

Music

Who is the Artist?

A) The Meters
B) The Neville Brothers
C) Trombone Shorty
D) Michael Franti
E) Radiators

Tribute to New Orleans and Mardi Gras

Your Comments

"How can a magnetic field point in a direction? What about the wire example in the prelecture where is shown to move in a counter-clockwise/clockwise direction?"

A little at a time!

"This stuff is really neat... It is fun to actually see the calculations for magnetism. However, since this is the first time I've really seen it, it is still a bit confusing. If you could go through different examples and go over the actual concepts more, that would be great."

"Magnets. How do they work?"

"Please explain what causes magnetic fields, and why they exist. Unless we learn that later or not at all, I just don't understand."

Good questions!
But very deep!

"What are we supposed to do if we have two left hands?"

"Could you go over this right hand rule again? There are so many of them..."

RHR confused many

"I refuse to make an attempt at a bad pun."

"We most definitely need examples. Also, every time you post an awful pun, a puppy dies. Just so you know."

Key Concepts:

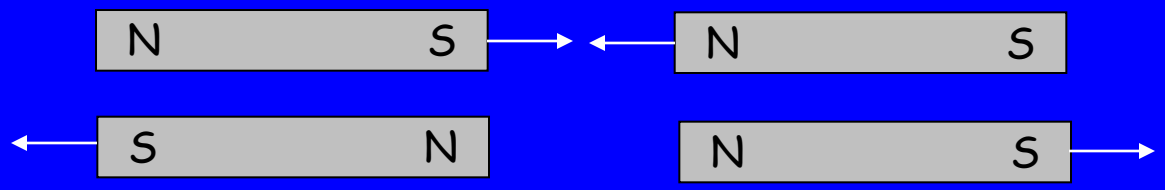
1) The force on moving charges due to a magnetic field.
2) The cross product.

Today's Plan:

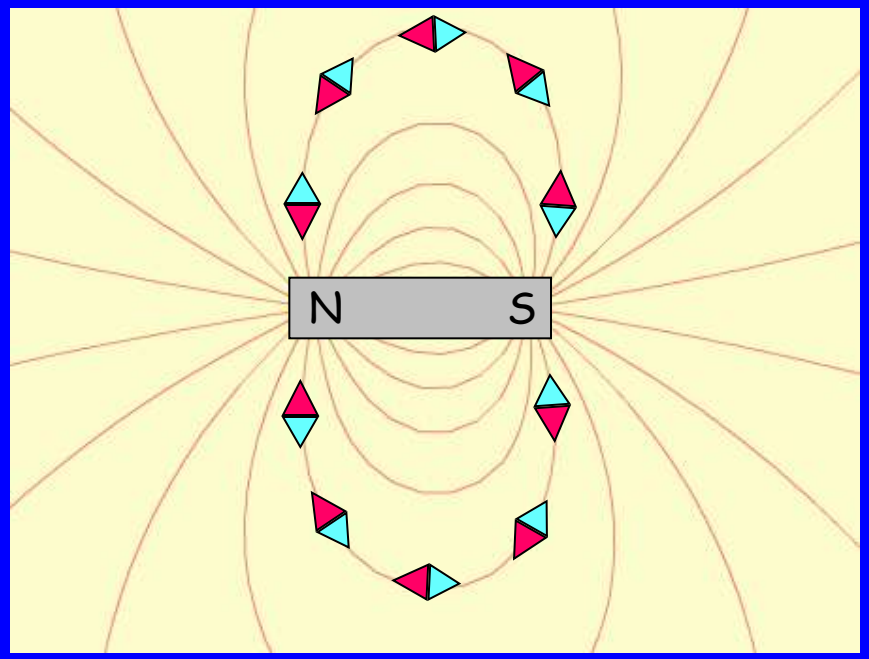
1) Review of magnetism
2) Review of cross product
3) Example problem

Magnetic Observations

- Bar Magnets



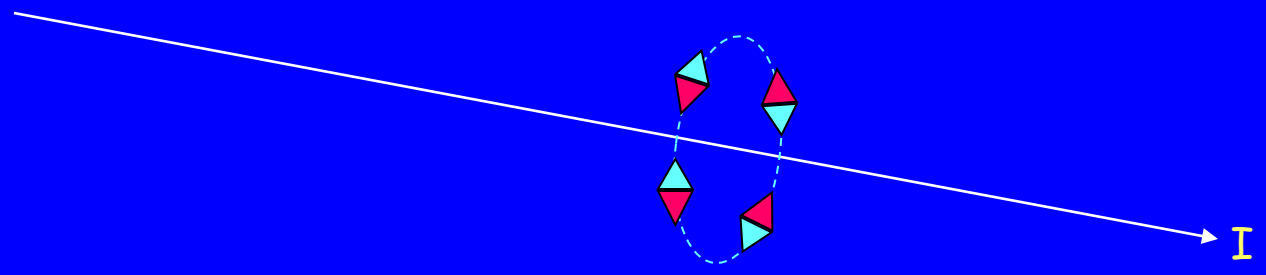
- Compass Needles



Magnetic Charge?

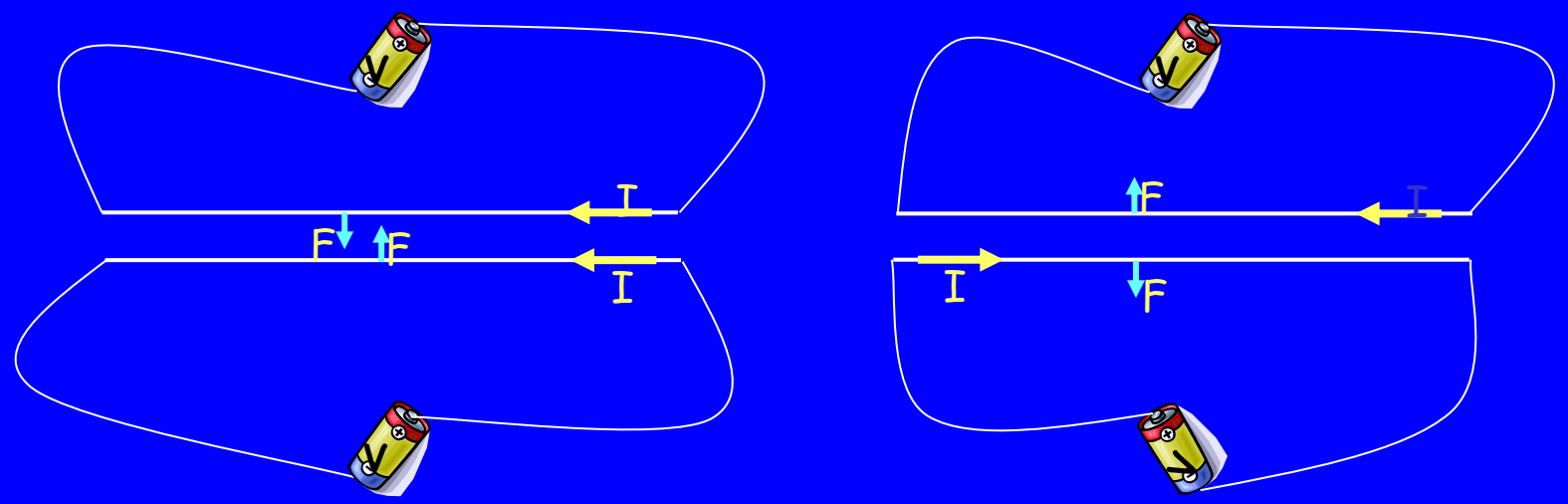
Magnetic Observations

- Compass needle deflected by electric current

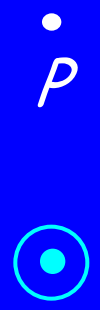


- Magnetic fields created by electric currents
  - Direction? Right thumb along I, fingers curl in direction of **B**

- Magnetic fields exert forces on electric currents (charges in motion)

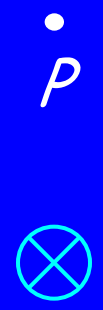


Magnetic Observations



I (out of the screen)

Case I



I (into of the screen)

Case II

The magnetic field at *P* points

A. Case I: left, Case II: right

B. Case I: left, Case II: left

C. Case I: right, Case II: left

D. Case I: right, Case II: right

WHY? Direction of $\vec{B}$: right thumb in direction of $\vec{I}$, fingers curl in the direction of $\vec{B}$

Magnetism & Moving Charges

All observations are explained by two equations:

$$\vec{F} = q\vec{v} \times \vec{B}$$ Today

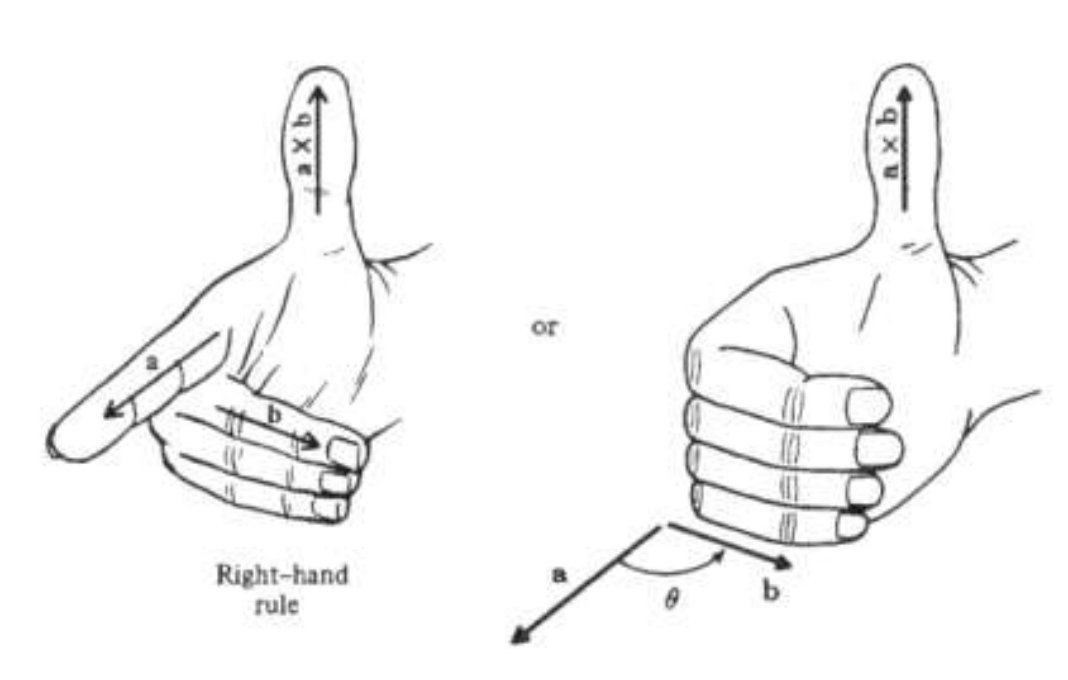
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$ Next Week

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{r}$$

Cross Product Review

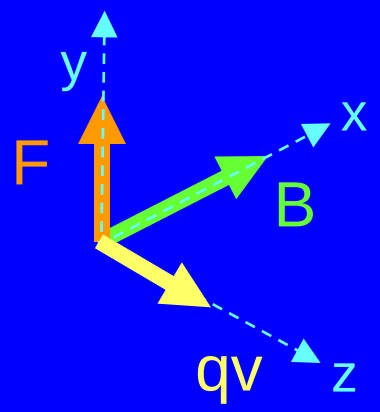
- Cross Product different from Dot Product
  - **A • B** is a scalar; **A** x **B** is a vector
  - **A • B** proportional to the component of **B** parallel to **A**
  - **A** x **B** proportional to the component of **B** perpendicular to **A**

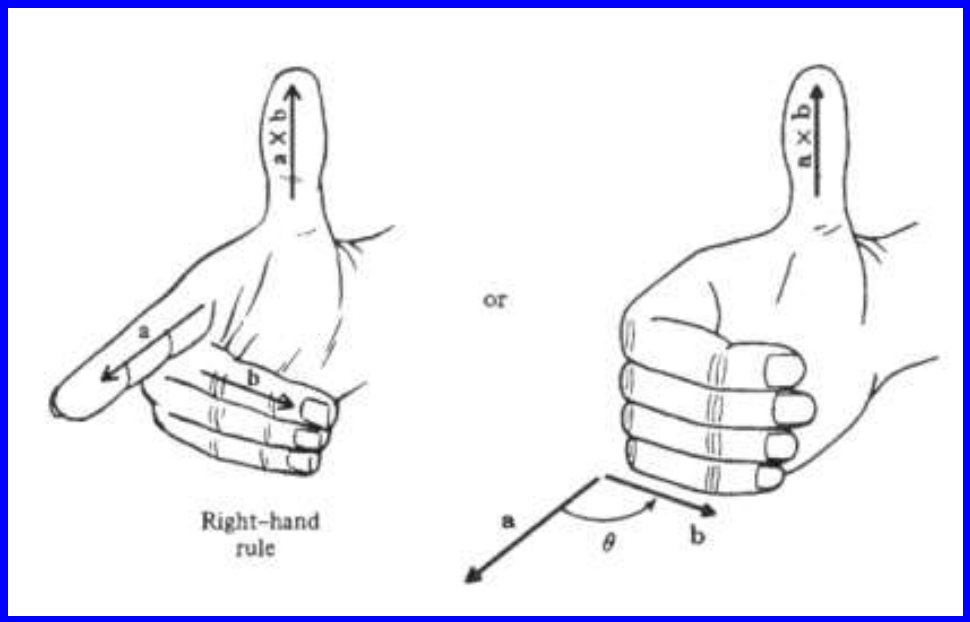
- Definition of **A** x **B**
  - **Magnitude:** $AB\sin\theta$
  - **Direction**: perpendicular to plane defined by A and B with sense given by right-hand-rule

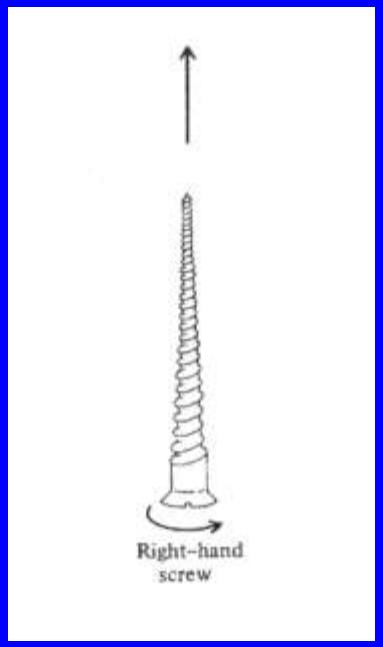


Remembering Directions: The Right Hand Rule

$$\vec{F} = q\vec{v} \times \vec{B}$$

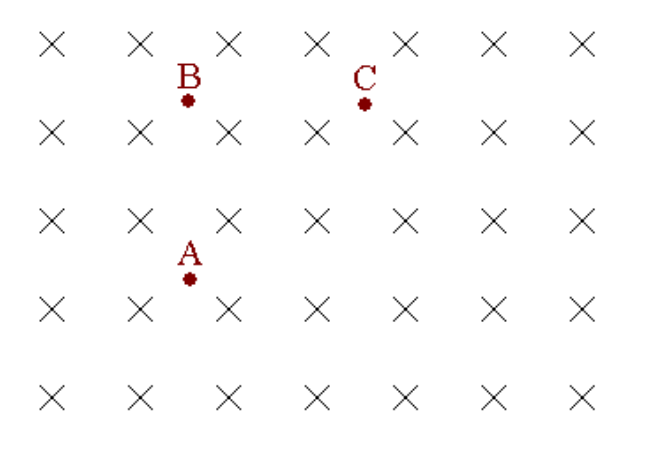






Checkpoint 1a

Three points are arranged in a uniform magnetic field. The **B** field points into the screen.



A positively charged particle is located at point A and is stationary. The direction of the magnetic force on the particle is

A. right ***B.*** left ***C.*** into the screen ***D.*** out of the screen
E. zero

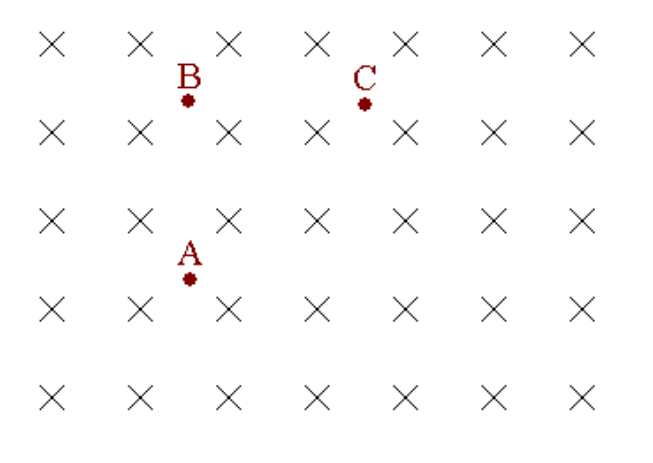
"all are into the screen"

"Right hand rule says its pointing out of the screen"

"qvXB=0 if v=0."

Checkpoint 1a

Three points are arranged in a uniform magnetic field. The **B** field points into the screen.



A positively charged particle is located at point A and is stationary. The direction of the magnetic force on the particle is

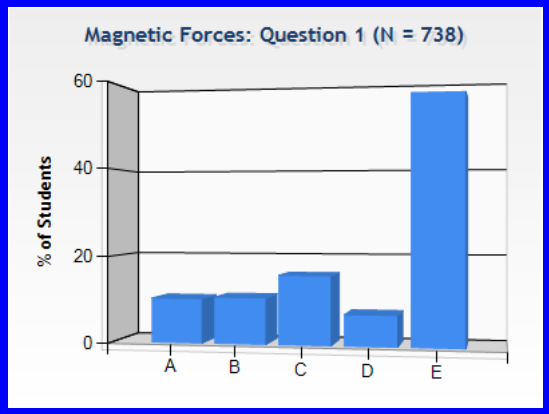
A. right ***B.*** left ***C.*** into the screen ***D.*** out of the screen

E. zero

$$\vec{F} = q\vec{v} \times \vec{B}$$

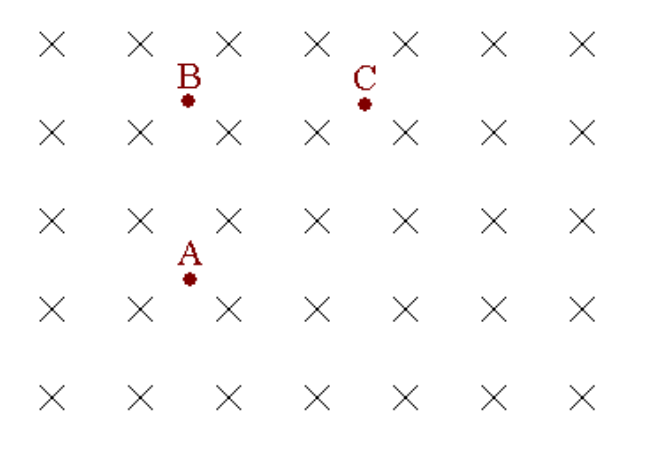
The particle's velocity is zero.

There can be no magnetic force.



Checkpoint 1b

Three points are arranged in a uniform magnetic field. The **B** field points into the screen.



The positive charge moves from A toward B. The direction of the magnetic force on the particle is

A. right ***B.*** left ***C.*** into the screen ***D.*** out of the screen
E. zero

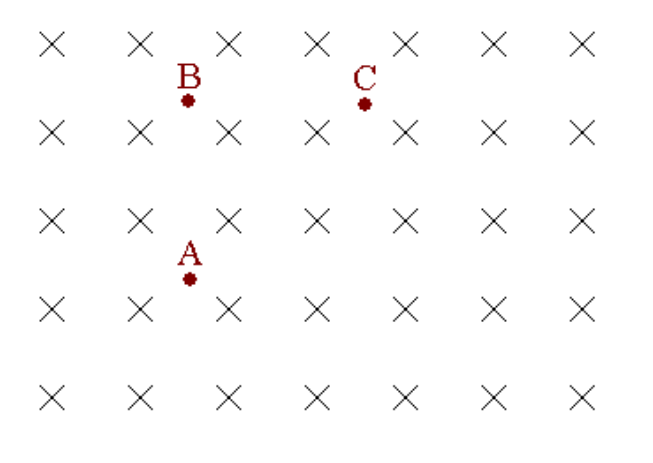
"it should be diagonally outward"

"v is up, b is into screen, right hand rule says it should be left."

"Because the magnetic force goes into the screen, so will the direction"

Checkpoint 1b

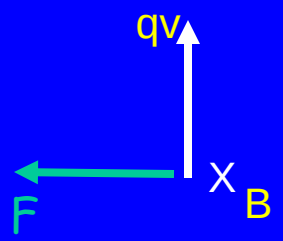
Three points are arranged in a uniform magnetic field. The **B** field points into the screen.

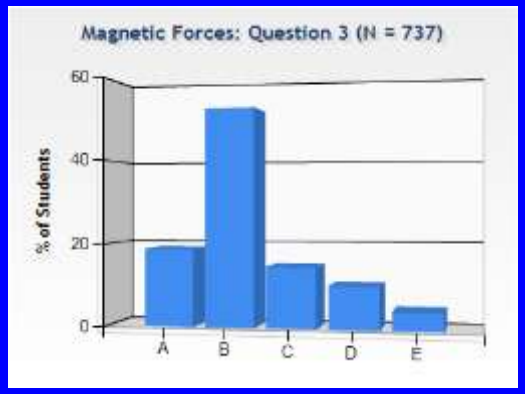


The positive charge moves from A toward B. The direction of the magnetic force on the particle is

A. right
B. left
C. into the screen
D. out of the screen
E. zero

$$\vec{F} = q\vec{v} \times \vec{B}$$





Cross Product Practice $\vec{F} = q\vec{v} \times \vec{B}$

- protons (positive charge) coming out of screen (+z direction)
- Magnetic field pointing down
- What is direction of force on POSITIVE charge?

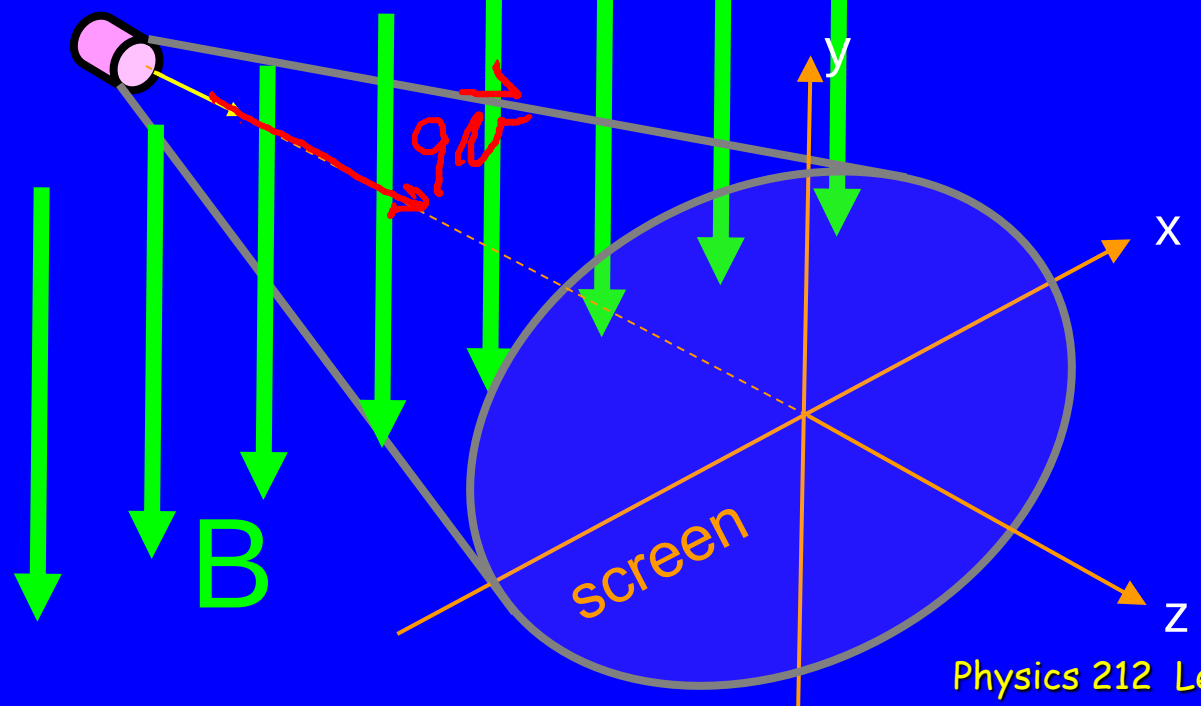
A) Left
- x

B) Right
+ x

C) UP
+ y

D) Down
- y

E) Zero



Motion of Charge q in Uniform B Field

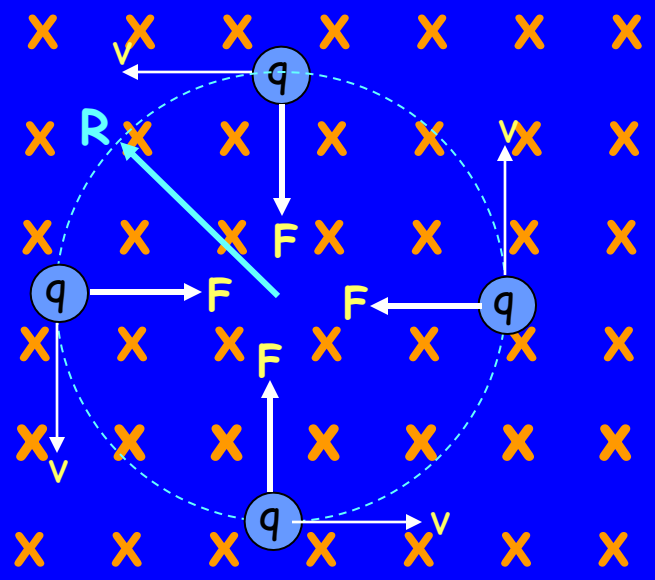
- Force is perpendicular to v
  - Speed does not change
  - Uniform Circular Motion

- Solve for R:

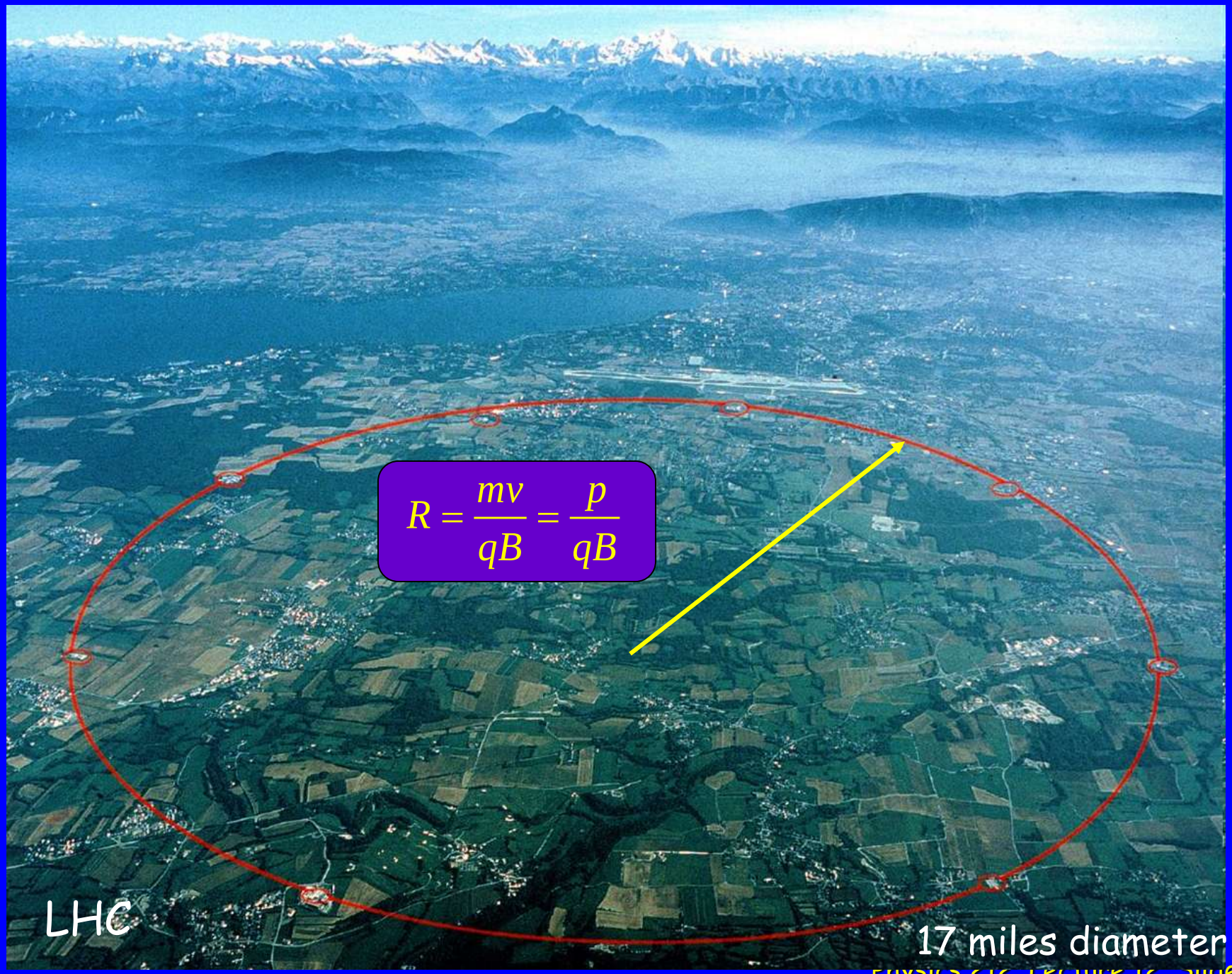
$$\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB$$

$$a = \frac{v^2}{R}$$

$$qvB = m\frac{v^2}{R} \Rightarrow \boxed{R = \frac{mv}{qB}}$$



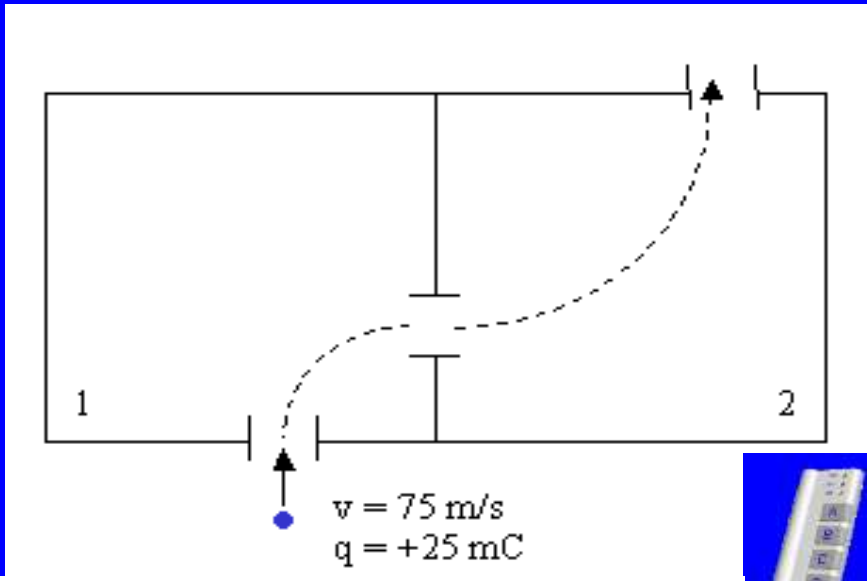
Uniform B into page



$$R = \frac{mv}{qB} = \frac{p}{qB}$$

Checkpoint 2a

The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.



What is the direction of the magnetic field in chamber 1?

A. up ***B.*** down ***C.*** into the page ***D.*** out of the page

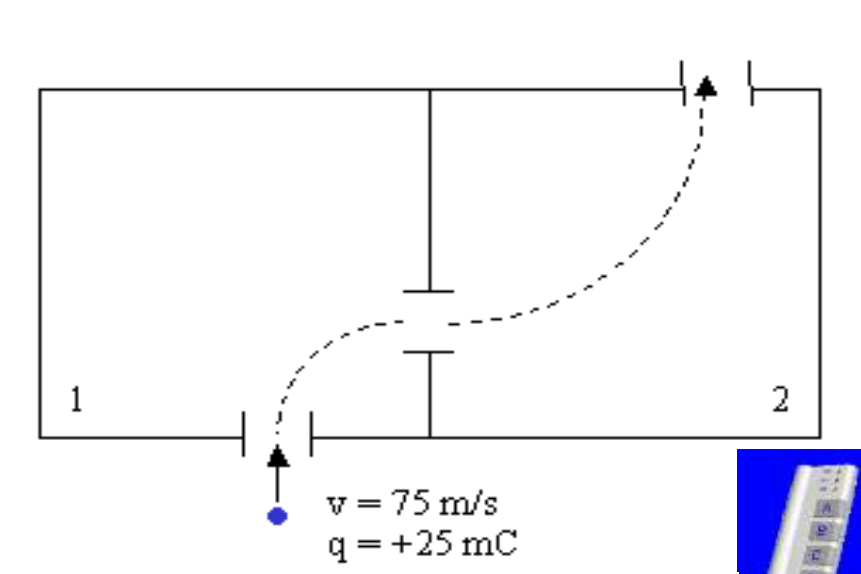
"Since the direction of the magnetic field has to be perpendicular to the charged particle's direction, it will point up.."

"Having a magnetic field going into the screen indicates counterclockwise motion."

"Thumb points to the left because particle is deflected to the left and fingers point up because that is velocity vector. Fingers curl out of the screen."

Checkpoint 2a

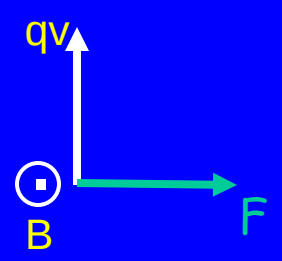
The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.

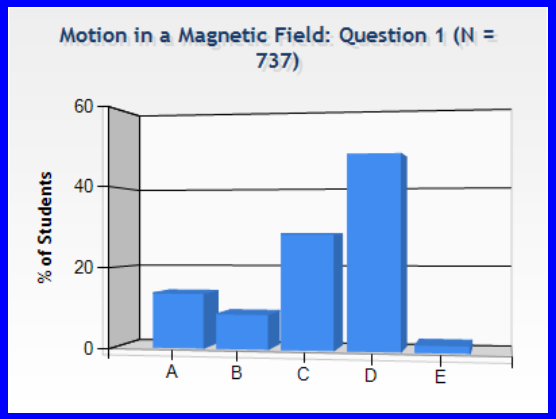


What is the direction of the magnetic field in chamber 1?

A. up ***B.*** down ***C.*** into the page ***D.*** out of the page

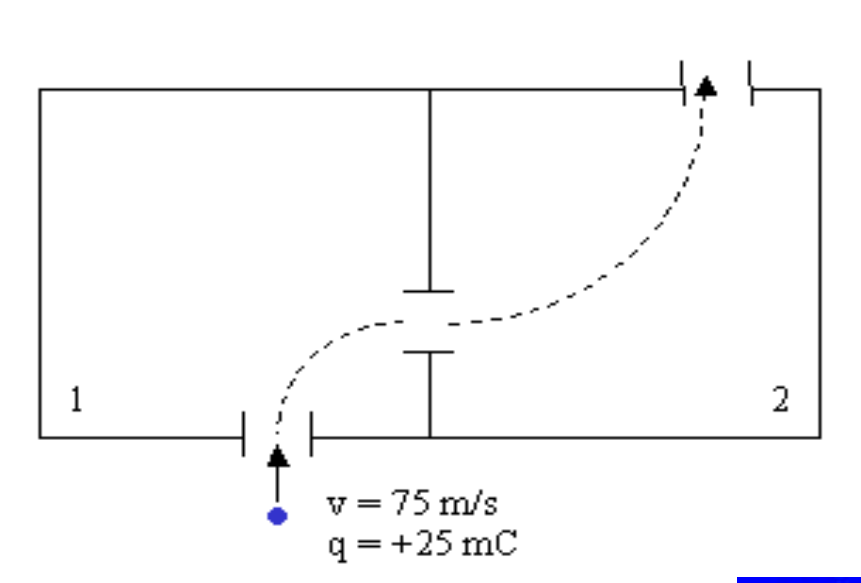
$$\vec{F} = q\vec{v} \times \vec{B}$$





Checkpoint 2b

The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.



Compare the magnitude of the magnetic field in chamber 1 to the magnitude of the magnetic field in chamber 2

A. $|B_1| > |B_2|$ ***B.*** $|B_1| = |B_2|$ ***C.*** $|B_1| < |B_2|$

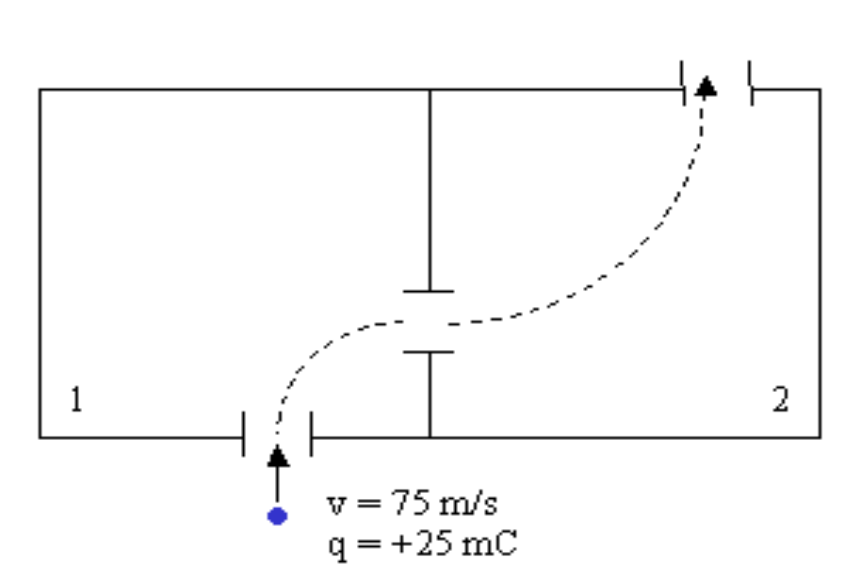
"because the curve is much sharper inside box 1"

"It equals out."

"Because the particle changed direction more slowly in the second chamber."

Checkpoint 2b

The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.

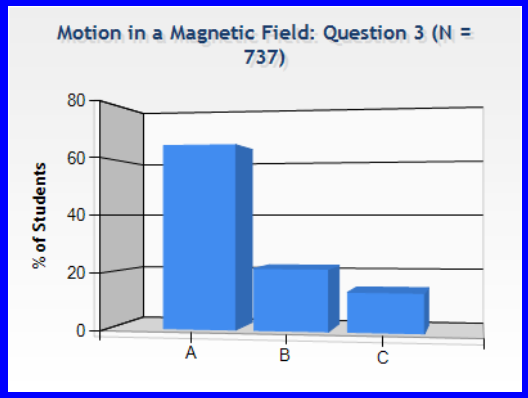


Compare the magnitude of the magnetic field in chamber 1 to the magnitude of the magnetic field in chamber 2

A. $|B_1| > |B_2|$ ***B.*** $|B_1| = |B_2|$ ***C.*** $|B_1| < |B_2|$

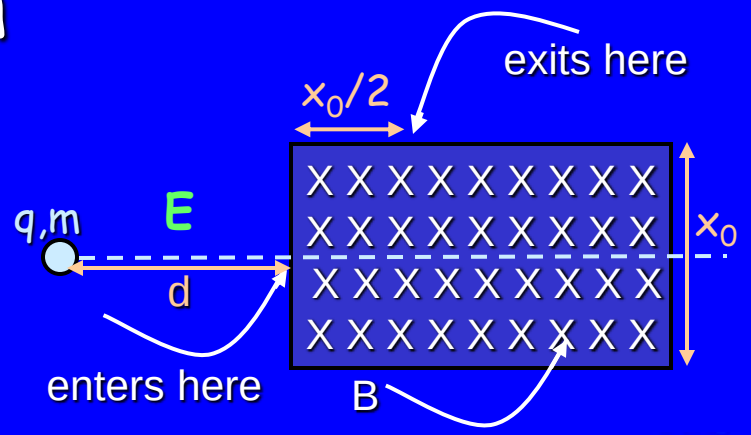
Observation: $R_2 > R_1$

$$R = \frac{mv}{qB} \Rightarrow |B_1| > |B_2|$$



Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



What is B?

- Conceptual Analysis
  - What do we need to know to solve this problem?

electric potential

(A) Lorentz Force Law $(\vec{F} = q\vec{v} \times \vec{B} + q\vec{E})$ (B) E field definition (C) V definition

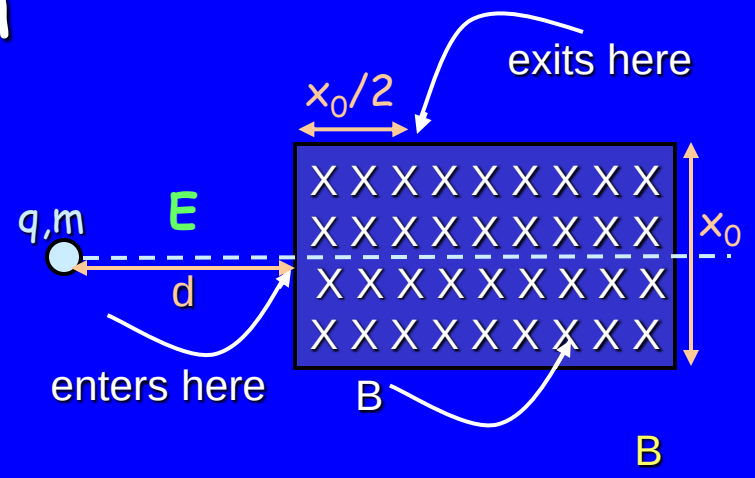
(D) Conservation of Energy/Newton's Laws (E) All of the above

- Absolutely ! We need to use the definitions of V and E and either conservation of energy or Newton's Laws to understand the motion of the particle before it enters the B field.
- We need to use the Lorentz Force Law (and Newton's Laws) to determine what happens in the magnetic field.

Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.

What is B?

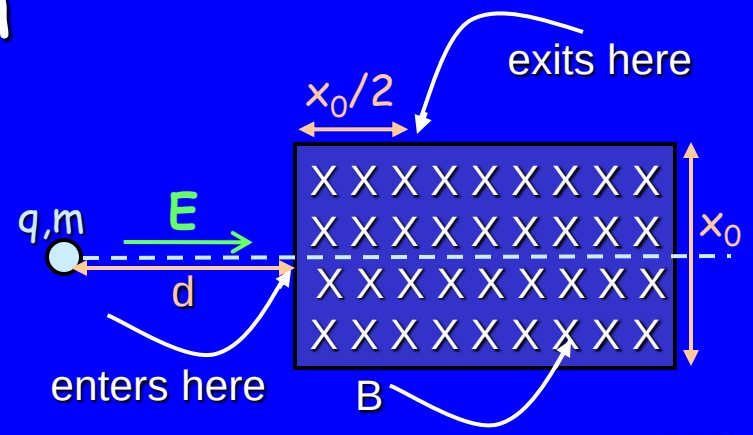


- Strategic Analysis
  - Calculate v, the velocity of the particle as it enters the magnetic field
  - Use Lorentz Force equation to determine the path in the field as a function of B
  - Apply the entrance-exit information to determine B

Let's Do It !!

Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



What is B?

- What is v_0, the speed of the particle as it enters the magnetic field ?

$$v_o = \sqrt{\frac{2E}{m}}$$ (A)

$$v_o = \sqrt{\frac{2qEd}{m}}$$ (B)

$$v_o = \sqrt{2ad}$$ (C)

$$v_o = \sqrt{\frac{2qE}{md}}$$ (D)

$$v_o = \sqrt{\frac{qEd}{m}}$$ (E)

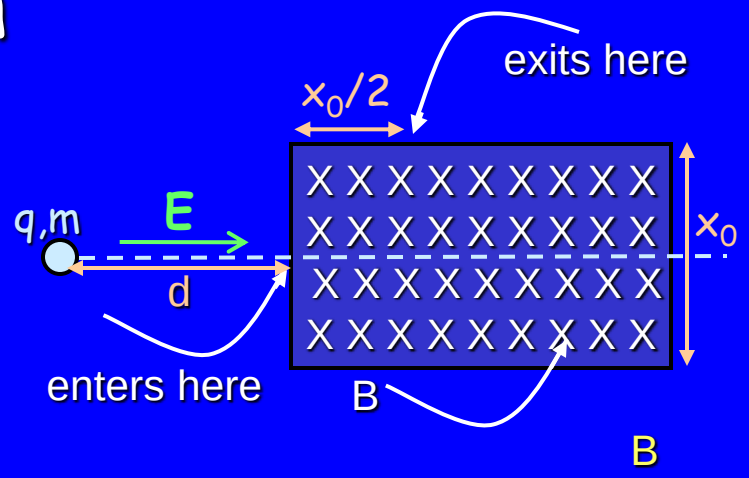
- Why??
  - Conservation of Energy
    - Initial: Energy = U = qV = qEd
    - Final: Energy = KE = $\frac{1}{2} mv_0^2$

    → $\frac{1}{2} m v_o^2 = qEd$ → $v_o = \sqrt{\frac{2qEd}{m}}$
  - Newton's Laws
    - a = F/m = qE/m
    - $v_0^2 = 2ad$

    → $v_o^2 = 2\frac{qE}{m} d$ → $v_o = \sqrt{\frac{2qEd}{m}}$

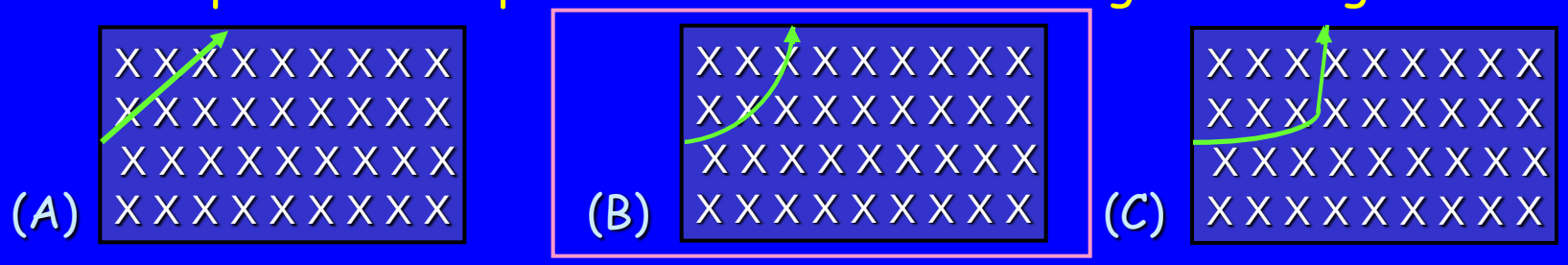
Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



What is B? $v_o = \sqrt{\frac{2qEd}{m}}$

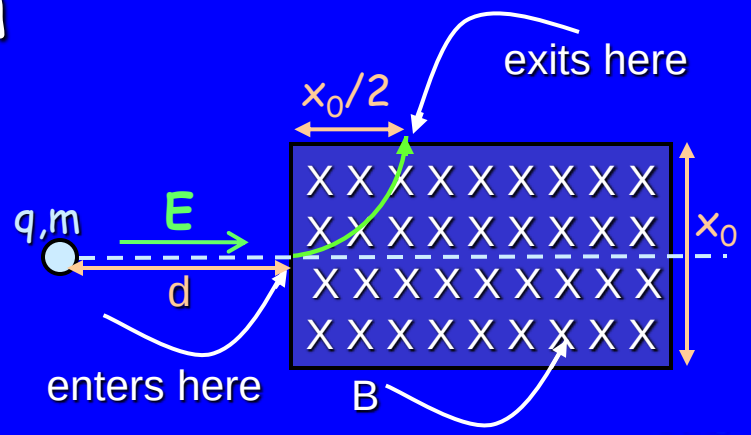
- What is the path of the particle as it moves through the magnetic field?



- Why??
  - Path is circle !
    - Force is perpendicular to the velocity
    - Force produces centripetal acceleration
    - Particle moves with uniform circular motion

Calculation

A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



What is B? $v_o = \sqrt{\frac{2qEd}{m}}$

- What is the radius of path of particle?

$R = x_o$ (A)

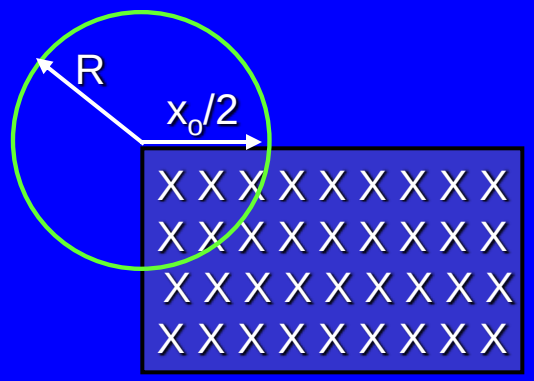
$R = 2x_o$ (B)

$R = \frac{1}{2}x_o$ (C)

$R = \frac{mv_o}{qB}$ (D)

$R = \frac{v_o^2}{a}$ (E)

- Why??



Calculation

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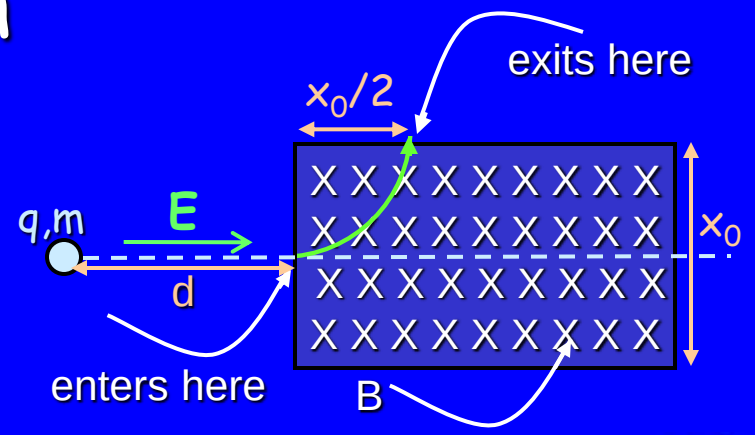


What is B?

$$v_o = \sqrt{\frac{2qEd}{m}}$$

$$R = \frac{1}{2}x_0$$

(A) $B = \frac{2}{x_o}\sqrt{\frac{2mEd}{q}}$

(B) $B = \frac{E}{v}$

(C) $B = E\sqrt{\frac{m}{2qEd}}$

(D) $B = \frac{1}{x_o}\sqrt{\frac{2mEd}{q}}$

(E) $B = \frac{mv_o}{qx_o}$

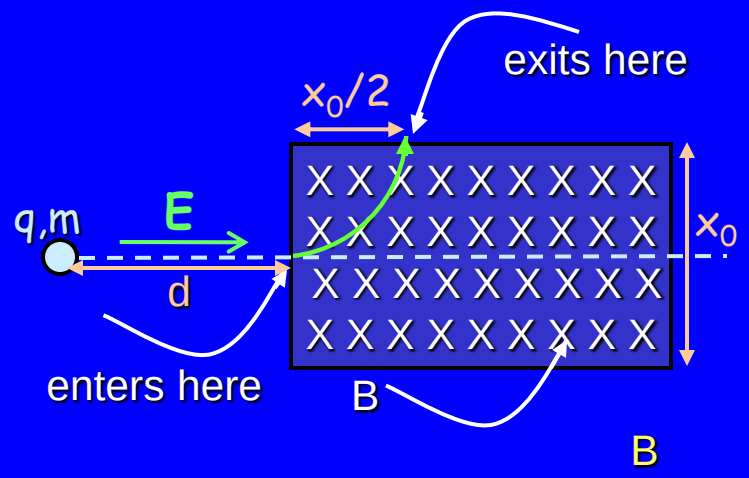
- Why??

$$\vec{F} = m\vec{a} \Rightarrow qv_oB = m\frac{v_o^2}{R} \Rightarrow B = \frac{m}{q}\frac{v_o}{R} \Rightarrow B = \frac{m}{q}\frac{2}{x_o}\sqrt{\frac{2qEd}{m}}$$

$$\Rightarrow B = \frac{2}{x_o}\sqrt{\frac{2mEd}{q}}$$

Follow-Up

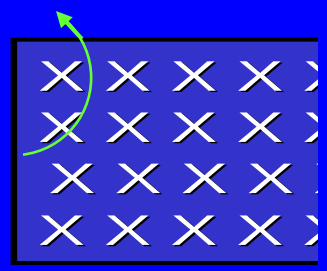
A particle of charge q and mass m is accelerated from rest by an electric field E through a distance d and enters and exits a region containing a constant magnetic field B at the points shown. Assume q,m,E,d, and x_0 are known.



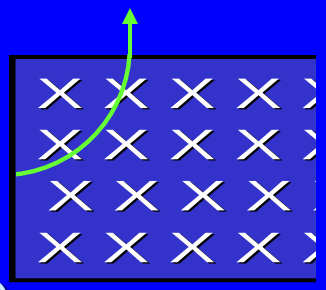
What is B?

$$B = \frac{2}{x_0}\sqrt{\frac{2mEd}{q}}$$

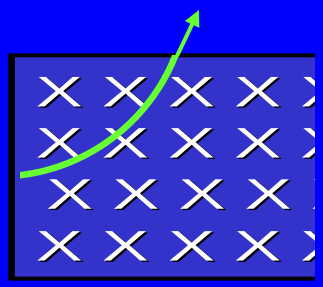
- Suppose the charge of the particle is doubled (Q = 2q),while keeping the mass constant. How does the path of the particle change?



(A)



(B)



(C)

No slam dunk.. As Expected !
Several things going on here

1. q changes ➡ v changes
2. q & v change ➡ F changes
3. v & F change ➡ R changes

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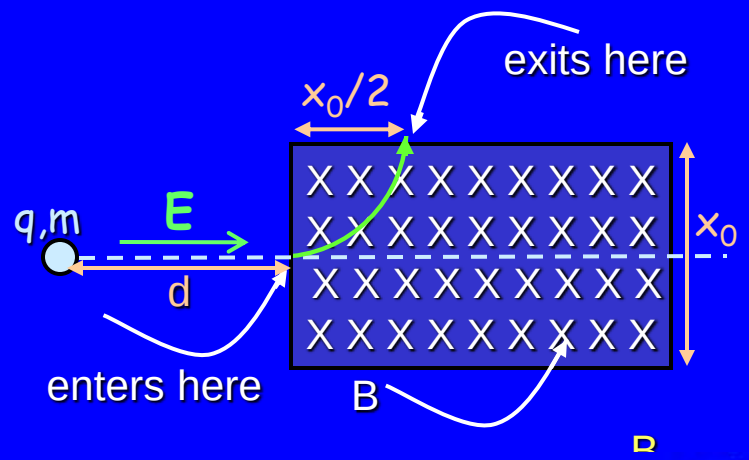

What is B?

$$B = \frac{2}{x_o}\sqrt{\frac{2mEd}{q}}$$

- Suppose the charge of the particle is doubled (Q = 2q),while keeping mass constant. How does the path of the particle change?

How does v, the new velocity at the entrance, compare to the original velocity v_0?

(A) $v = \frac{v_o}{2}$ (B) $v = \frac{v_o}{\sqrt{2}}$ (C) $v = v_o$ (D) $v = \sqrt{2}v_o$ (E) $v = 2v_o$

- Why??

$$\frac{1}{2}mv^2 = QEd = 2qEd = 2\frac{1}{2}mv_o^2 \Rightarrow v^2 = 2v_o^2 \Rightarrow v = \sqrt{2}v_o$$

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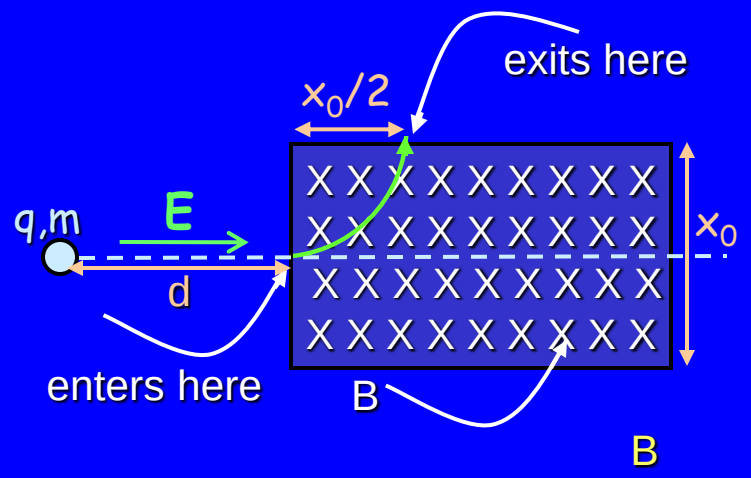

What is B?

$$B = \frac{2}{x_o}\sqrt{\frac{2mEd}{q}}$$

- Suppose the charge of the particle is doubled (Q = 2q),while keeping the mass constant. How does the path of the particle change?

$$v = \sqrt{2}v_o$$

How does F, the magnitude of the new force at the entrance, compare to F_0, the magnitude of the original force?

(A) $F = \frac{F_o}{\sqrt{2}}$ (B) $F = F_o$ (C) $F = \sqrt{2}F_o$ (D) $F = 2F_o$ (E) $F = 2\sqrt{2}F_o$

- Why??

$$F = QvB = 2q \cdot \sqrt{2}v_o \cdot B \quad \Rightarrow \quad F = 2\sqrt{2}F_o$$

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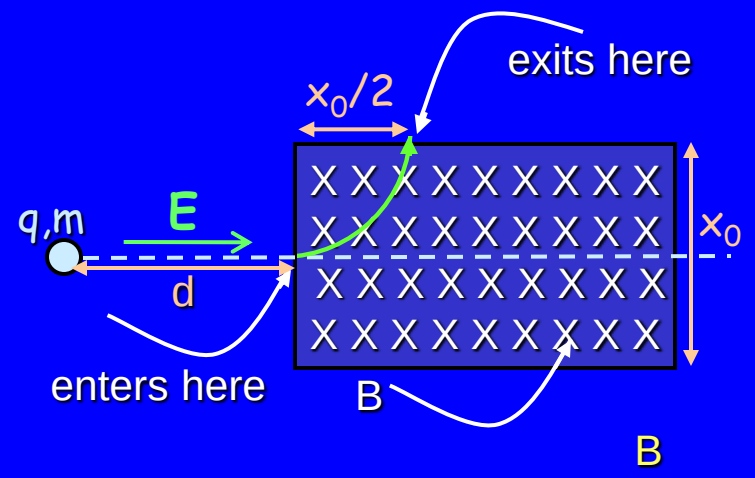

What is B?

$$B = \frac{2}{x_o}\sqrt{\frac{2mEd}{q}}$$

- Suppose the charge of the particle is doubled (Q = 2q),while keeping the mass constant. How does the path of the particle change?

$$v = \sqrt{2}v_o \qquad F = 2\sqrt{2}F_o$$

How does R, the radius of curvature of the path, compare to R_0, the radius of curvature of the original path?

(A) $R = \frac{R_o}{2}$ (B) $R = \frac{R_o}{\sqrt{2}}$ (C) $R = R_o$ (D) $R = \sqrt{2}R_o$ (E) $R = 2R_o$

- Why??

$$F = m\frac{v^2}{R} \Rightarrow R = m\frac{v^2}{F} \Rightarrow R = m\frac{2v_o^2}{2\sqrt{2}F_o} = m\frac{v_o^2}{\sqrt{2}F_o} = \frac{R_o}{\sqrt{2}}$$

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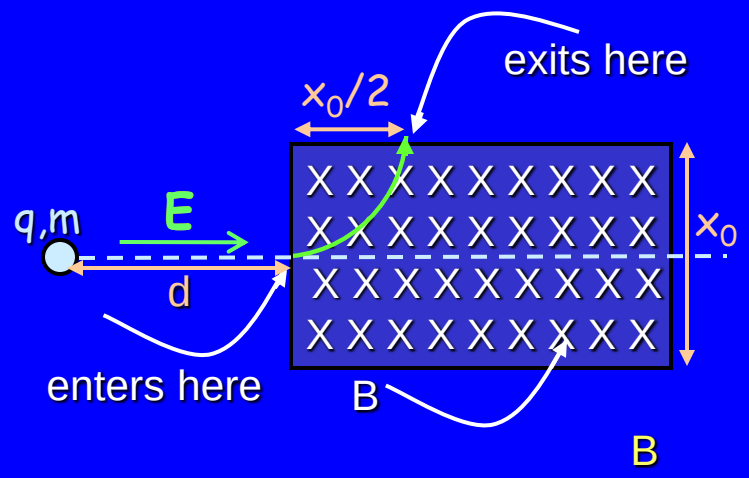

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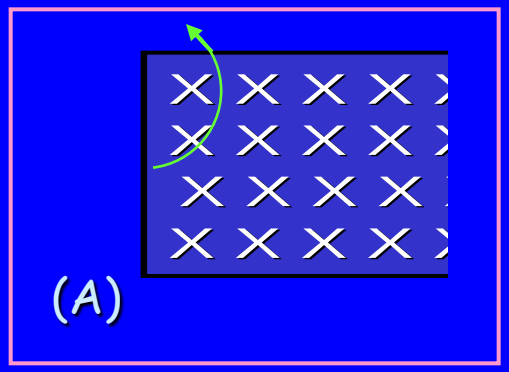

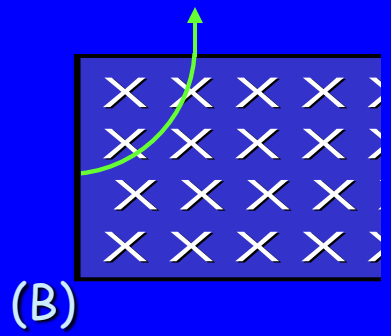

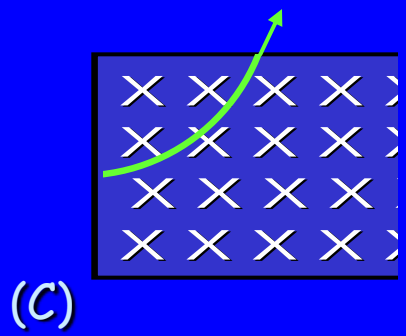

$$R = \frac{R_o}{\sqrt{2}}$$

A Check: (Exercise for Student)
Given our result for B (above), can you show:

$$R = \frac{1}{B}\sqrt{\frac{2mEd}{Q}} \Rightarrow R = \frac{R_o}{\sqrt{2}}$$