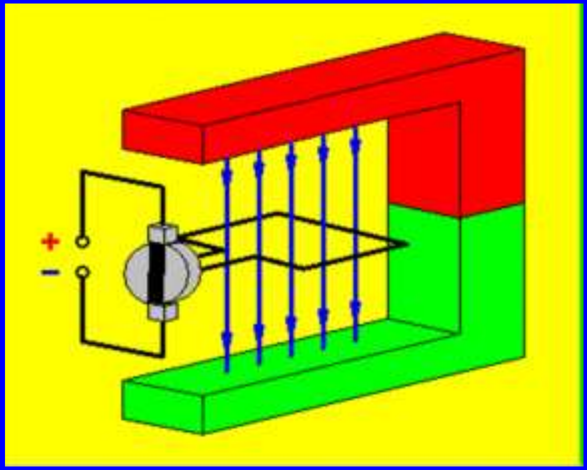
Physics 212

Lecture 13

Forces and Torques on Currents



Key Concepts:

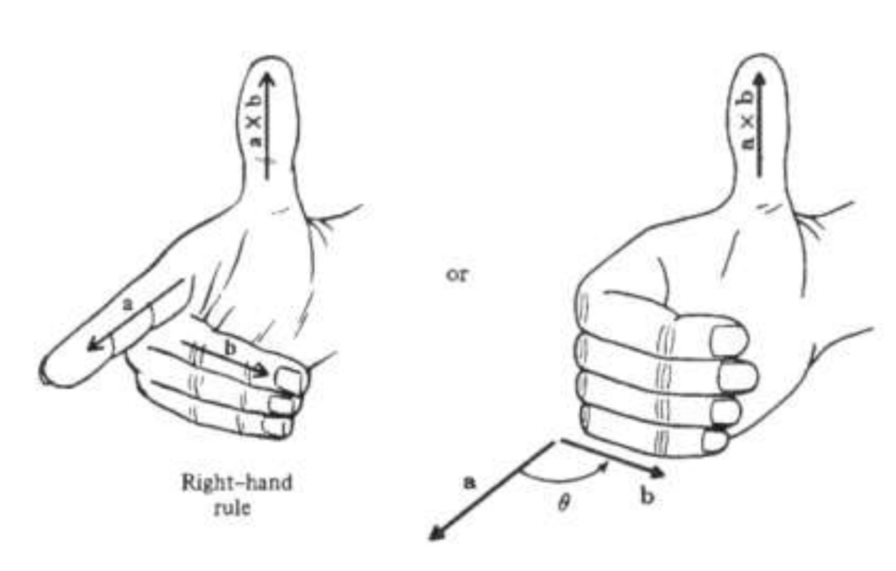
- Forces & Torques on loops of current due to a magnetic field.
- The magnetic dipole moment.

Today's Plan:

- Review of cross product
- Forces & Torques
- Magnetic dipole moment
- Example problem

Last Time: force on charge

$$\vec{F} = q\vec{v} \times \vec{B}$$



This Time: force on wire

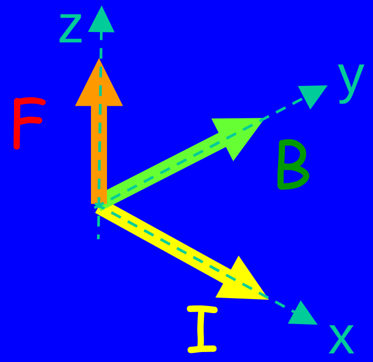
$$\vec{F} = q\sum_i \vec{v}_i \times \vec{B}$$

$$\vec{F} = qN\vec{v}_{avg} \times \vec{B}$$

$$N = nAL$$

$$I = qnAv_{avg}$$

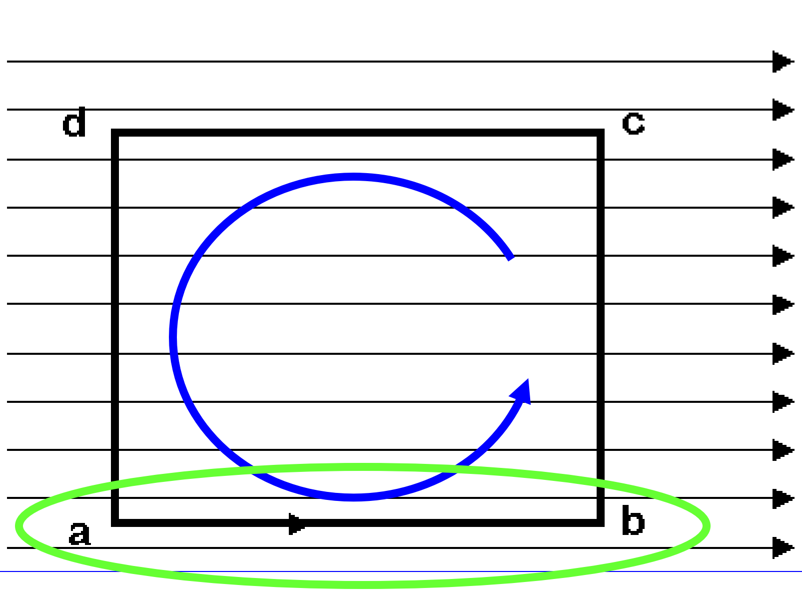
$$\Rightarrow \vec{F} = I\vec{L} \times \vec{B}$$



ACT

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



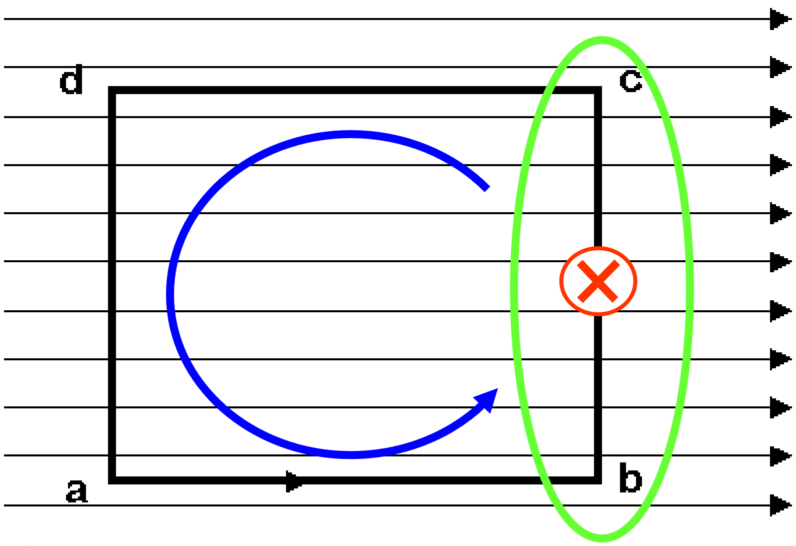
What is the force on section a-b of the loop?

A. Zero   **B.** Out of the page   **C.** Into the page

ACT

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$



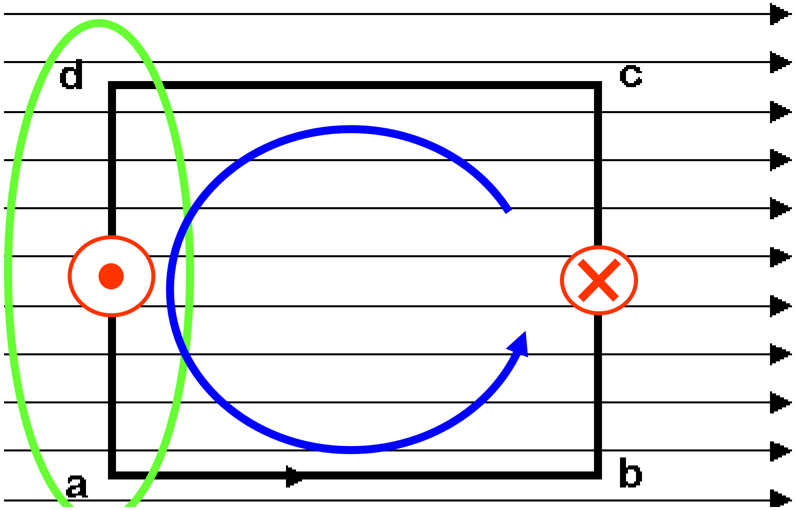
What is the force on section b-c of the loop?

A. Zero ***B.*** Out of the page ***C.*** Into the page

ACT

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.

$$\vec{F} = I\vec{L} \times \vec{B}$$

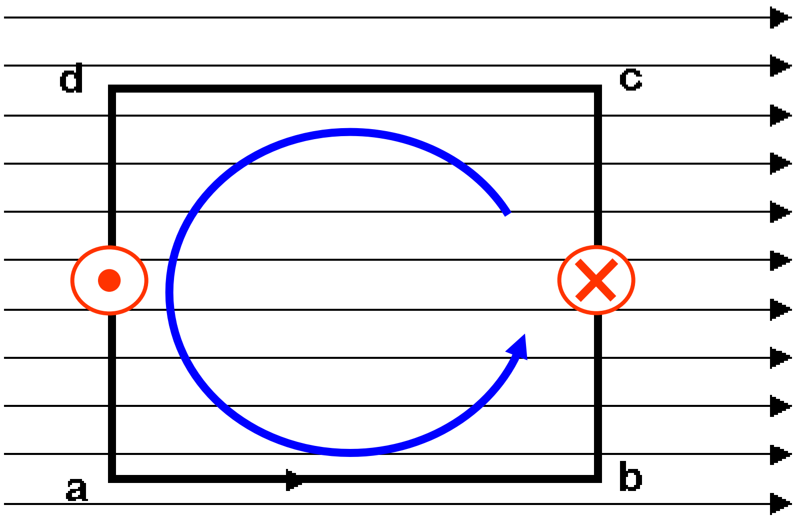


What is the force on section d-a of the loop?

A. Zero ***B.*** Out of the page ***C.*** Into the page

Checkpoint 1a

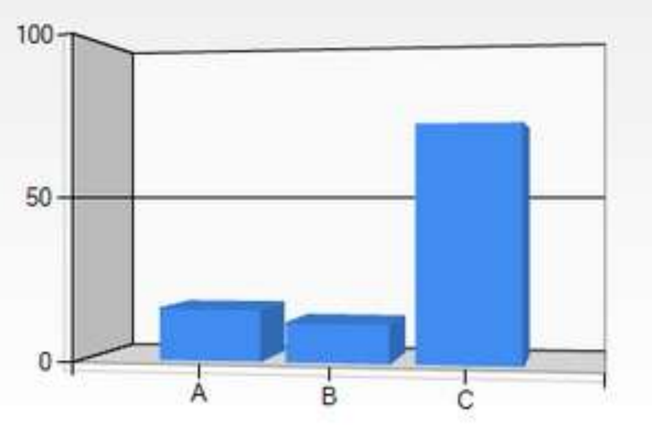
A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.



What is the direction on the net force on the loop?

A. Out of the page ***B.*** Into of the page

C. The net force on the loop is zero

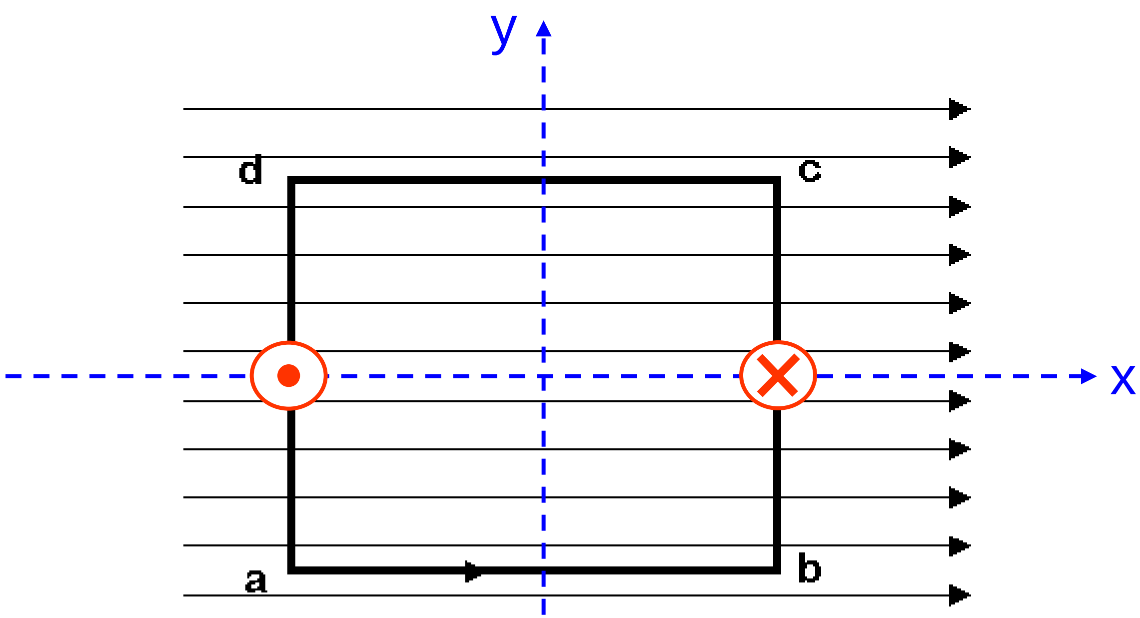


"The net force on any closed loop is zero."

Check simulations if in doubt.

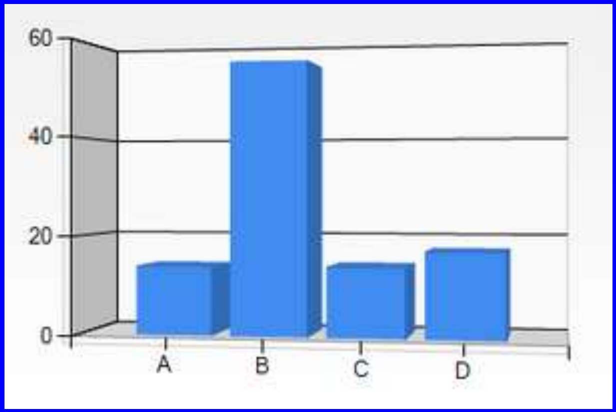
Checkpoint 1b

A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.



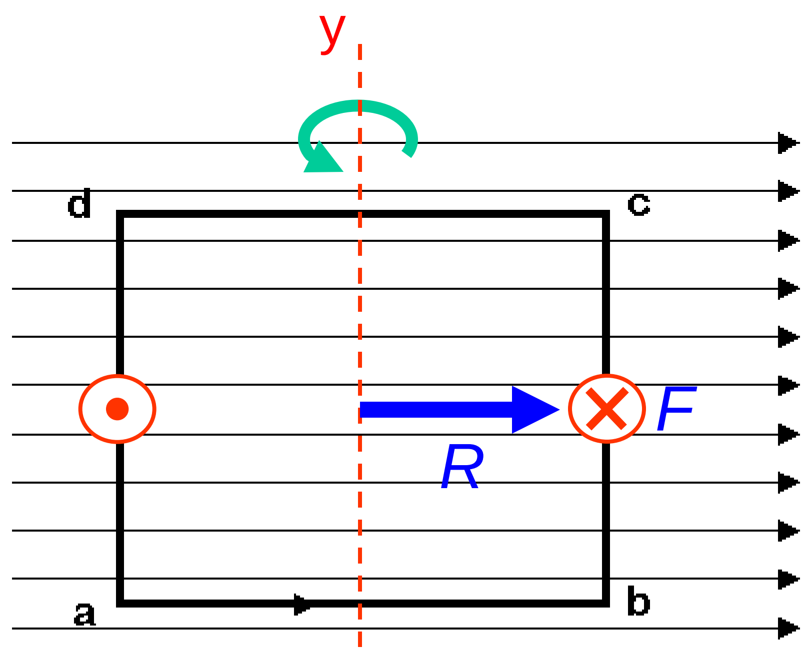
In which direction will the loop rotate (assume the z axis is out of the page)?

A. Around the *x* axis
B. Around the *y* axis
C. Around the *z* axis
D. It will not rotate



Checkpoint 1c

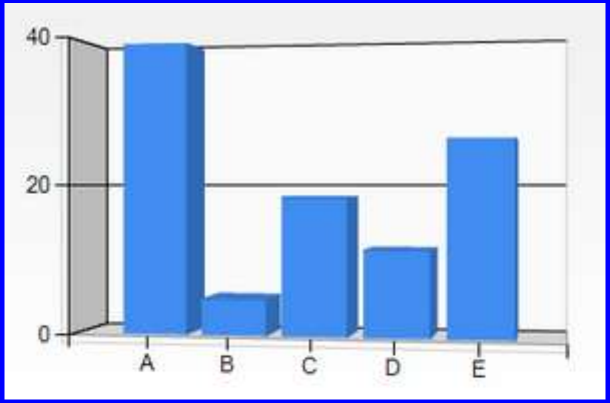
A square loop of wire is carrying current in the counterclockwise direction. There is a horizontal uniform magnetic field pointing to the right.



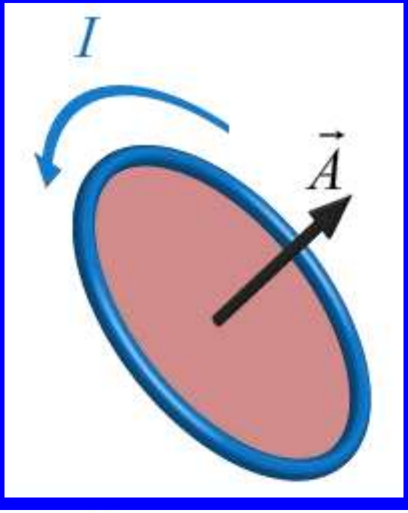
$$\vec{\tau} = \vec{R} \times \vec{F}$$

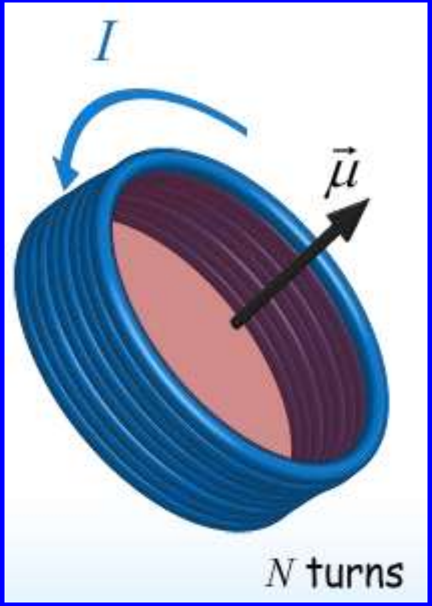
What is the direction of the net torque on the loop?

A. Up ***B.*** Down
C. Out of the page ***D.*** Into the page
E. The net torque is zero



Magnetic Dipole Moment





Area vector

Magnitude = Area

Direction uses R.H.R.

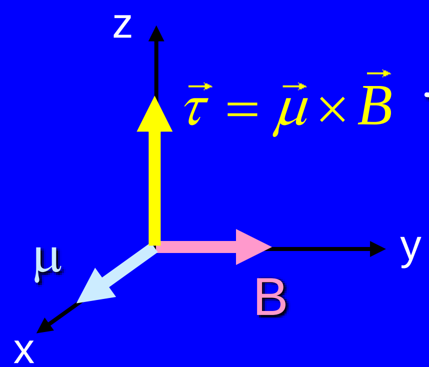
Magnetic Dipole moment

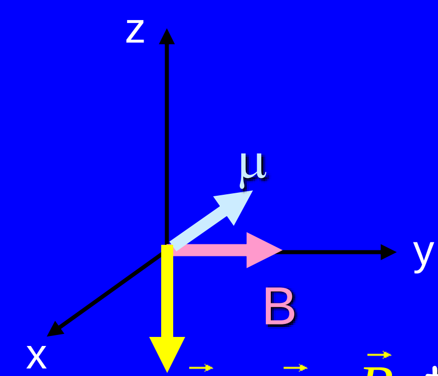
$$\vec{\mu} \equiv NI\vec{A}$$

μ Makes Torque Easy!

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

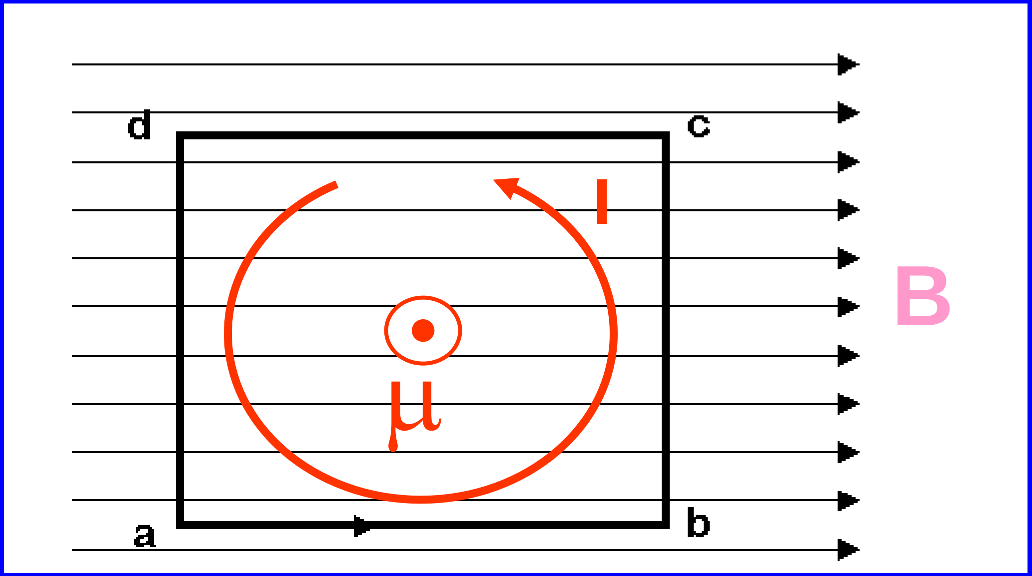
The torque always wants to line μ up with **B** !



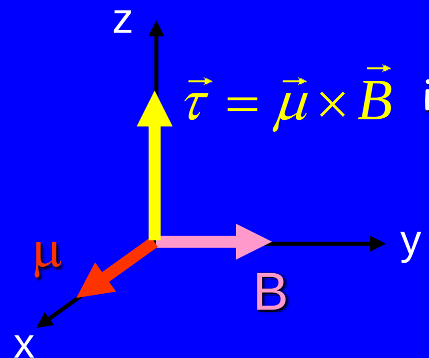


Practice with μ and τ

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



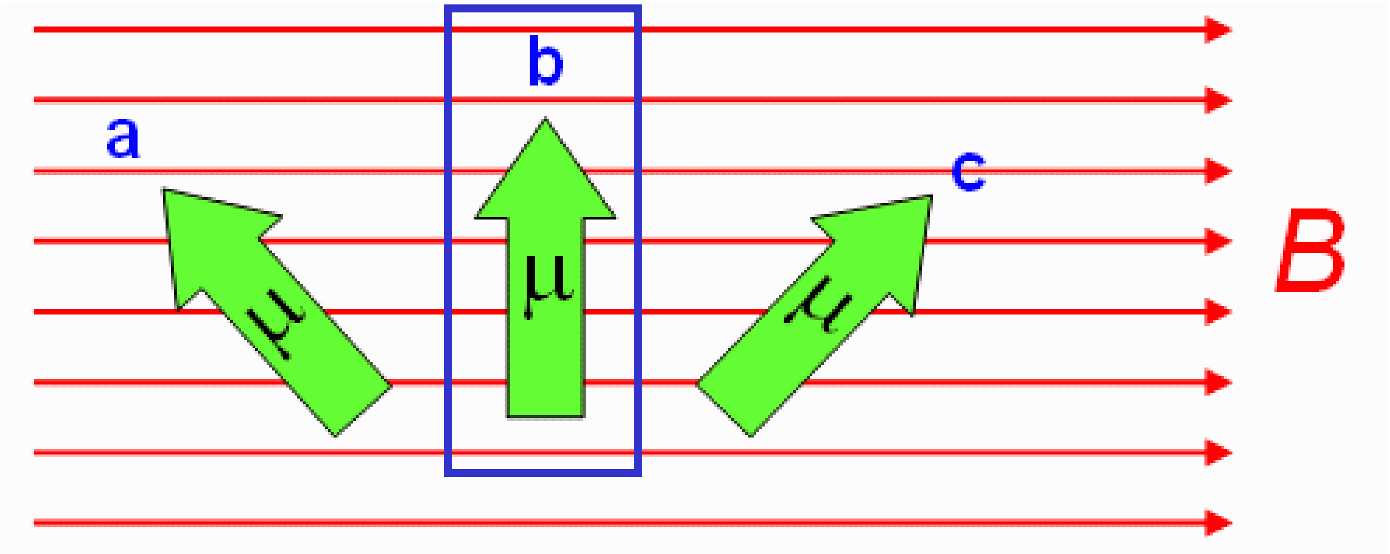
In this case μ is out of the page (using right hand rule)

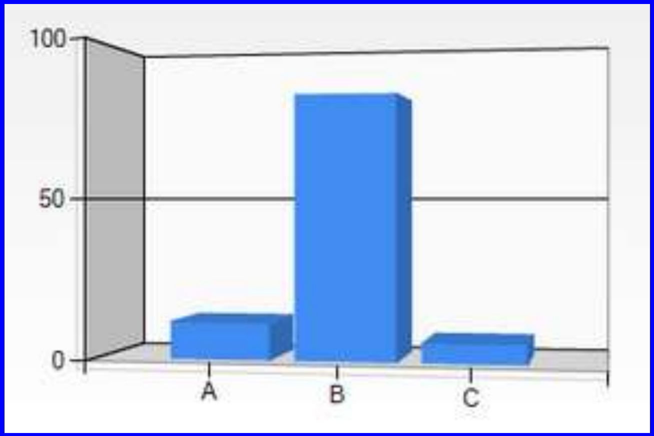


$\vec{\tau} = \vec{\mu} \times \vec{B}$ is up (turns μ toward B)

Checkpoint 2a

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. Which orientation results in the largest magnetic torque on the dipole?





$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Biggest when $\vec{\mu} \perp \vec{B}$

Magnetic Field can do Work on Current

From Physics 211: $W = \int \tau d\theta$

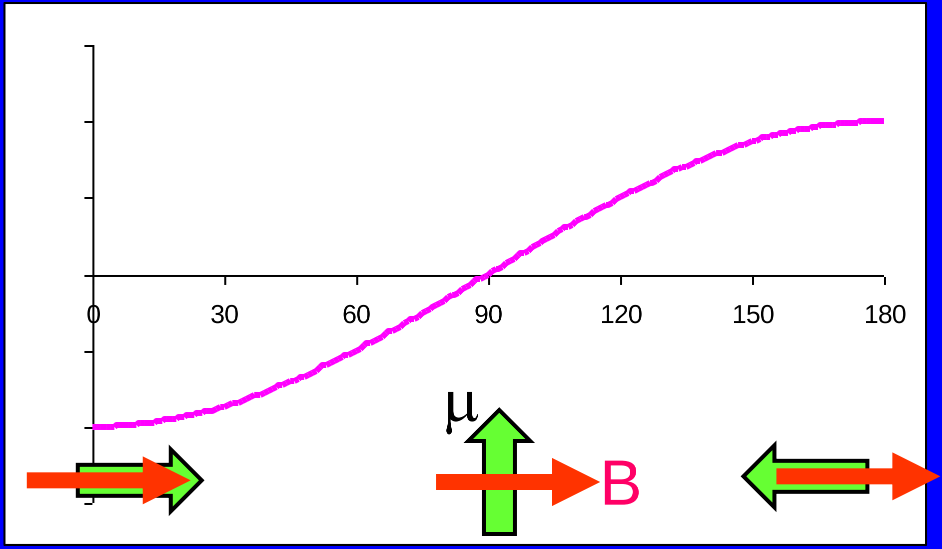
From Physics 212: $\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin(\theta)$

$$W = \int \mu B \sin(\theta) d\theta = \mu B \cos(\theta) = \vec{\mu} \cdot \vec{B}$$

$$\Delta U = -W$$

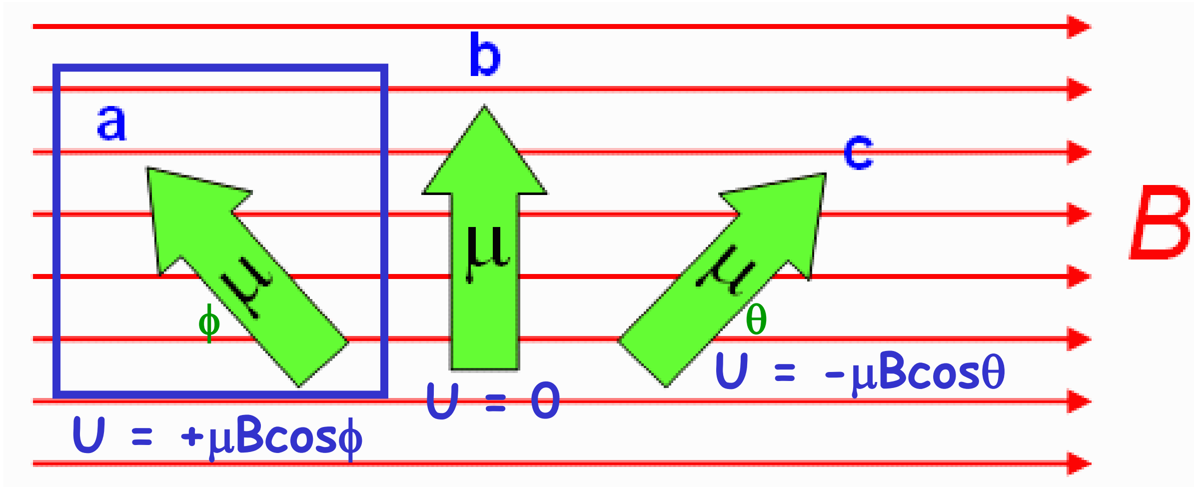
Define U = 0 at position of maximum torque

$$U \equiv -\vec{\mu} \cdot \vec{B}$$

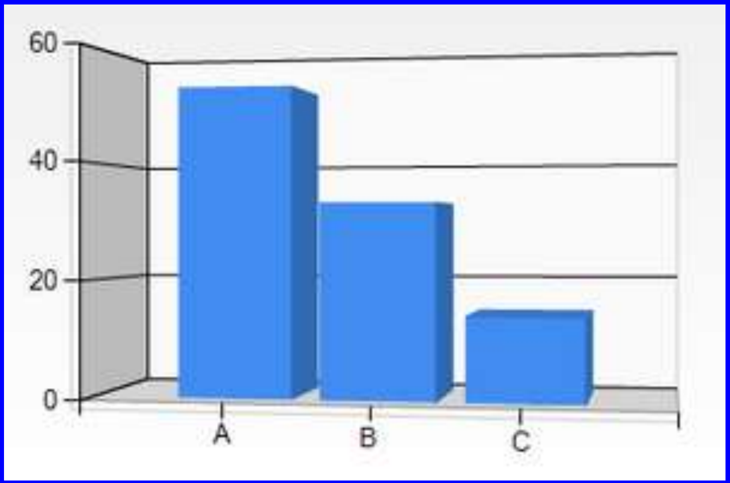


Checkpoint 2b

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. Which orientation has the most potential energy?

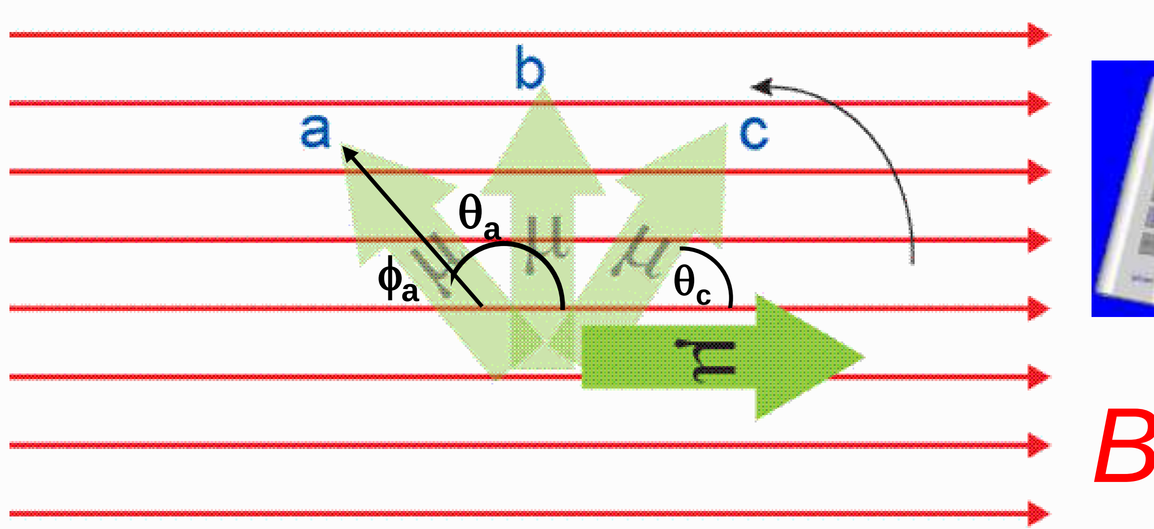


$$U = -\vec{\mu} \cdot \vec{B}$$



ACT

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. We want to rotate the dipole in the CCW direction.



First, consider rotating to position c. What are the signs of the work done by you and the work done by the field?

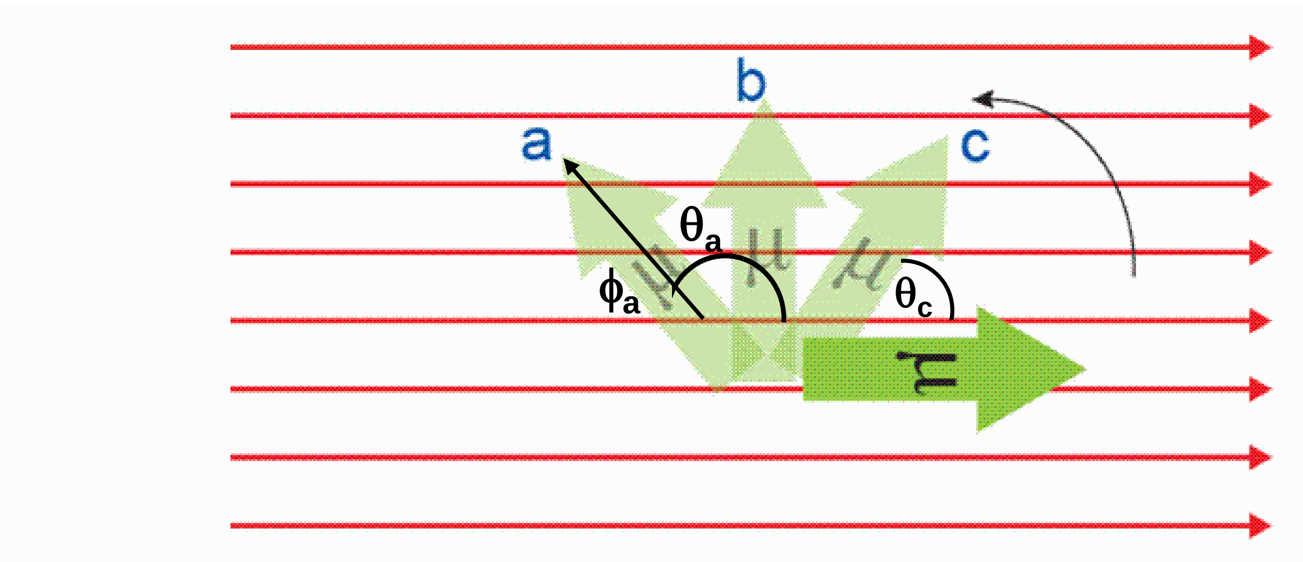
$$W_{field} = -\Delta U$$

A) $W_{you} > 0$, $W_{field} > 0$
B) $W_{you} > 0$, $W_{field} < 0$
C) $W_{you} < 0$, $W_{field} > 0$
D) $W_{you} < 0$, $W_{field} < 0$

- $\Delta U > 0$, so $W_{field} < 0$. W_{you} must be opposite W_{field}
- Also, torque and displacement in opposite directions → $W_{field} < 0$

ACT

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. Consider rotating the dipole to each of the three final orientations shown.



Do the signs depend on which position (a, b, or c) the dipole is rotated to?

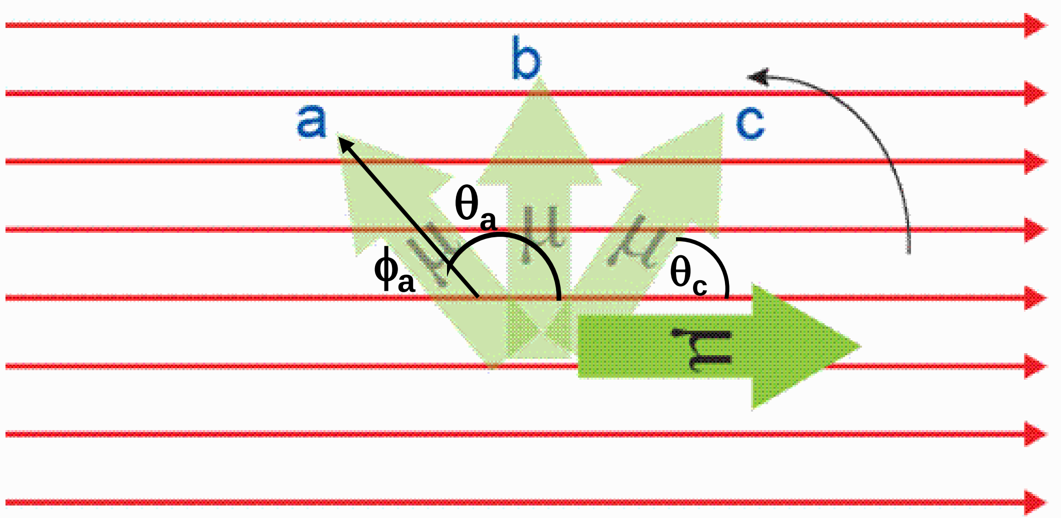
A) Yes
B) No

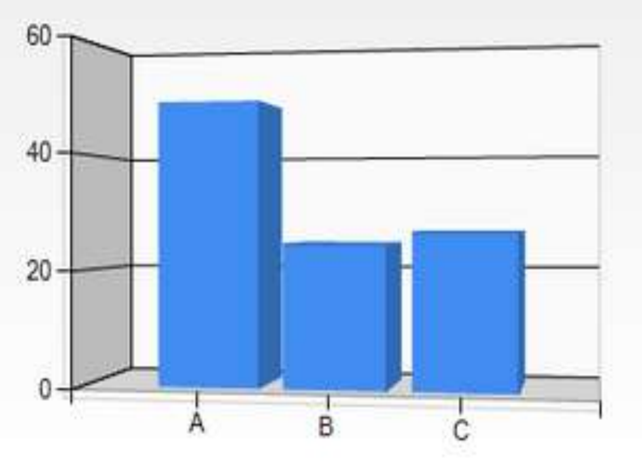
$$U = -\vec{\mu} \cdot \vec{B}$$

The lowest potential energy state is with dipole parallel to B. The potential energy will be higher at any of a, b, or c.

Checkpoint 2c

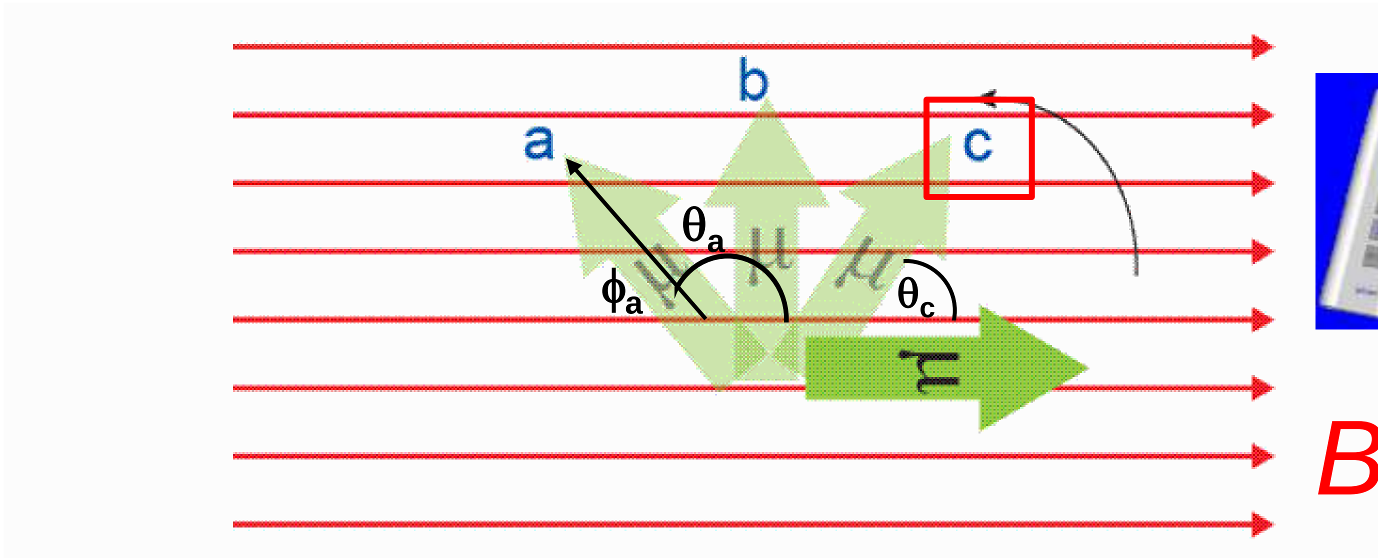
Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. In order to rotate a horizontal magnetic dipole to the three positions shown, which one requires the most work done by the magnetic field?





Checkpoint 2c

Three different orientations of a magnetic dipole moment in a constant magnetic field are shown below. In order to rotate a horizontal magnetic dipole to the three positions shown, which one requires the most work done by the magnetic field?



$$W_{by\_field} = -\Delta U = U_i - U_f$$

$$U = -\vec{\mu} \cdot \vec{B}$$

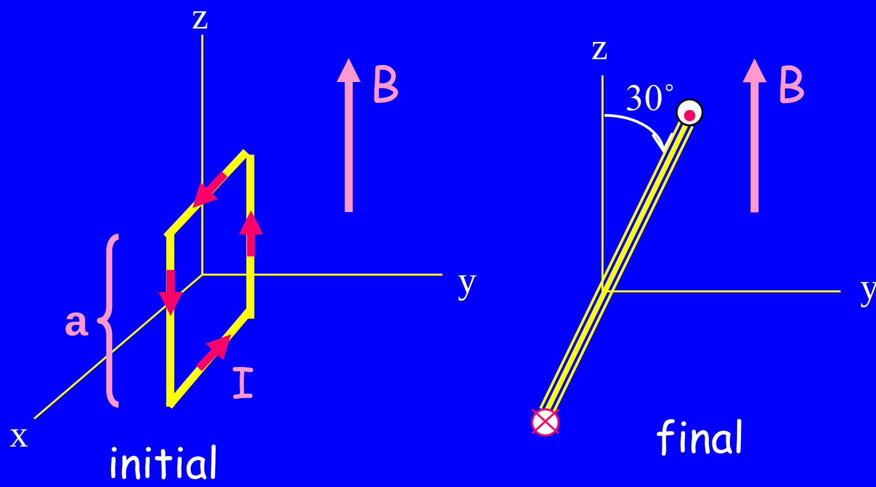
(c): $W_{by\_field} = -\mu B - (-\mu B \cos\theta_c) = -\mu B(1 - \cos\theta_c)$

(b): $W_{by\_field} = -\mu B - 0 = -\mu B$

(a): $W_{by\_field} = -\mu B - (-\mu B \cos\theta_a) = -\mu B(1 + \cos\phi_a)$

Calculation

A square loop of side a lies in the x-z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the +z axis. Assume a, I, and B are known.



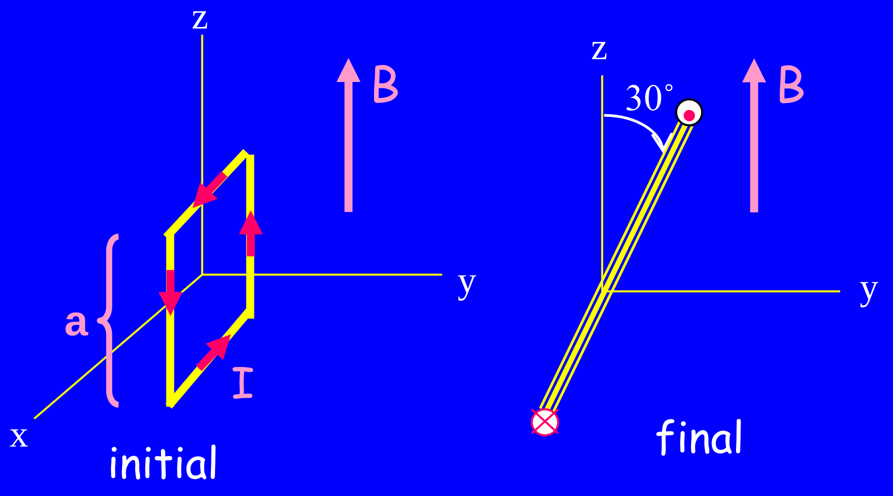
How much does the potential energy of the system change as the coil moves from its initial position to its final position.

- Conceptual Analysis
  - A current loop may experience a torque in a constant magnetic field
    - $\tau = \mu \times B$
  - We can associate a potential energy with the orientation of loop
    - $U = -\mu \cdot B$

- Strategic Analysis
  - Find μ
  - Calculate the change in potential energy from initial to final

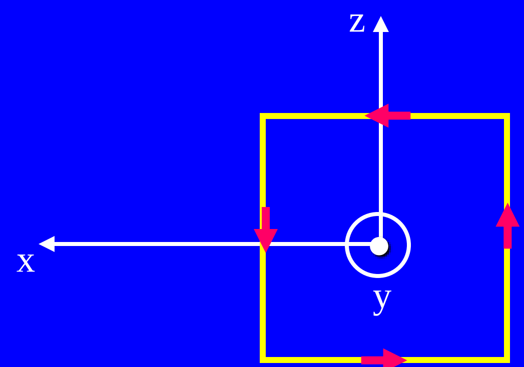
Calculation

A square loop of side a lies in the x-z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the +z axis. Assume a, I, and B are known.

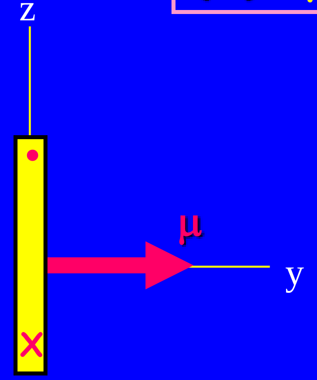


• What is the direction of the magnetic moment of this current loop in its initial position?

(A) +x (B) -x (C) +y (D) -y



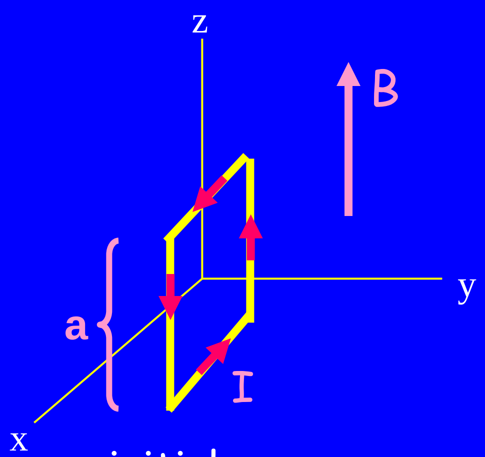
$\vec{\mu} = I\vec{A}$



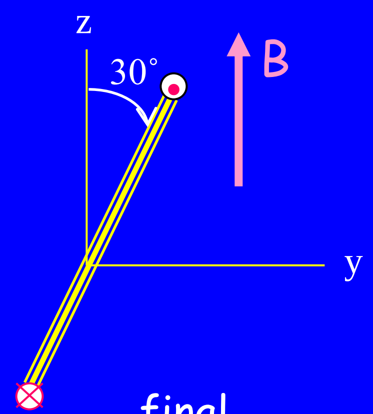
Right Hand Rule

Calculation

A square loop of side a lies in the x-z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the +z axis. Assume a, I, and B are known.



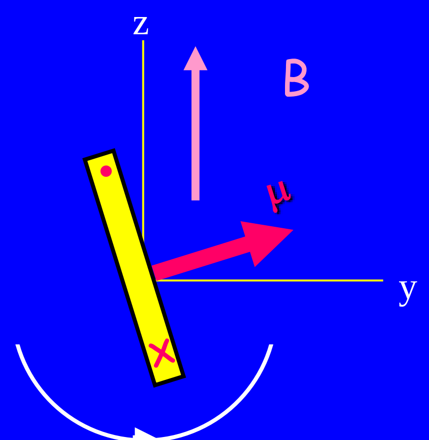
initial



final

• What is the direction of the torque on this current loop in the initial position?

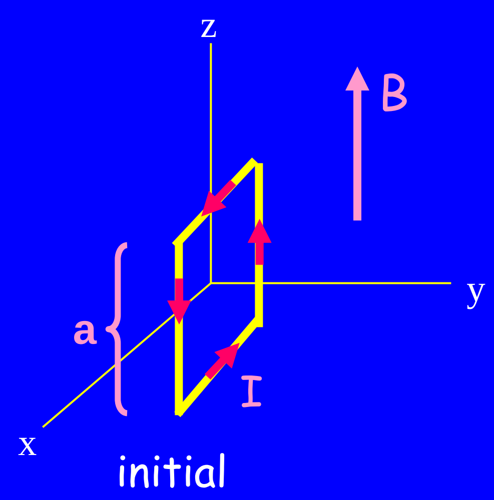
(A) +x (B) -x (C) +y (D) -y

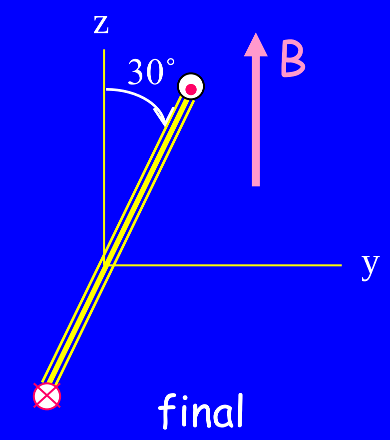


Calculation

A square loop of side a lies in the x-z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the +z axis. Assume a, I, and B are known.

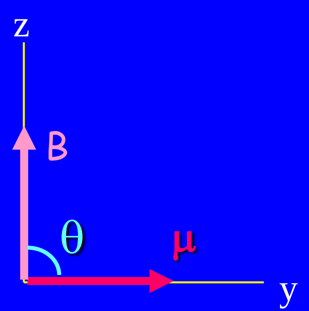
$$U = -\vec{\mu} \cdot \vec{B}$$





- What is the potential energy of the initial state?

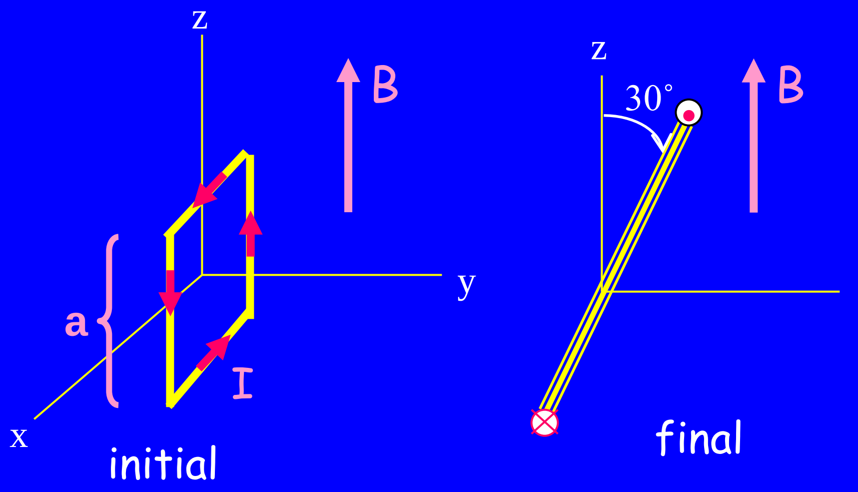
(A) $U_{initial} < 0$ (B) $U_{initial} = 0$ (C) $U_{initial} > 0$



$\theta = 90^0$ ⇒ $\vec{\mu} \bullet \vec{B} = 0$

Calculation

A square loop of side a lies in the x-z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the +z axis. Assume a, I, and B are known.



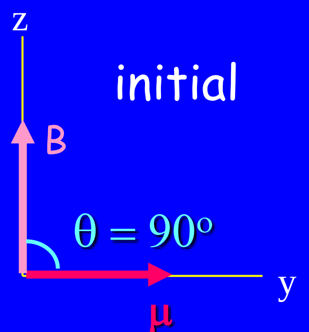
$$U = -\vec{\mu} \cdot \vec{B}$$

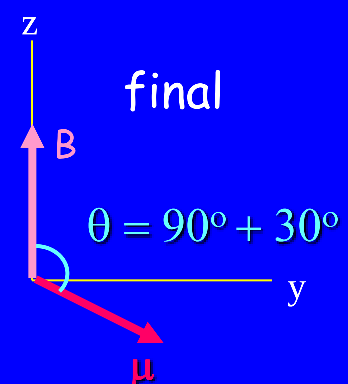
- What is the sign of the potential energy in the final state?

(A) $U_{final} < 0$ (B) $U_{final} = 0$ (C) $U_{final} > 0$

Check: μ moves away from B

Energy must increase !





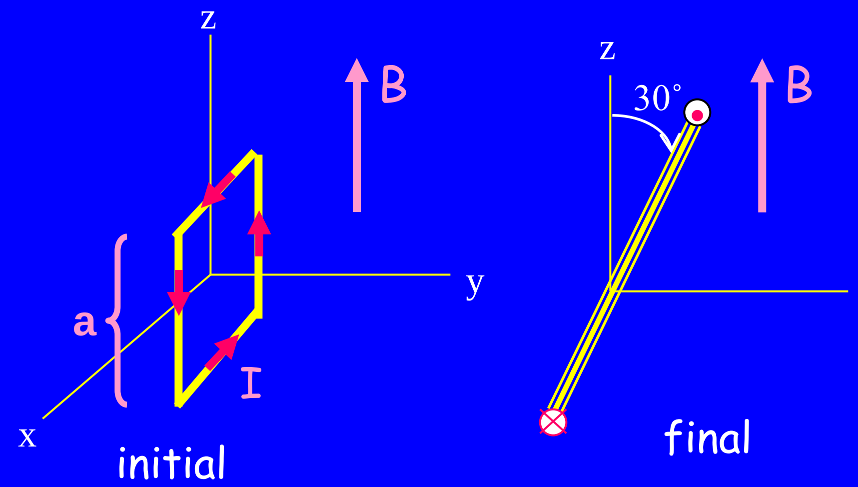
$\theta = 120^0$

$\vec{\mu} \bullet \vec{B} < 0$

$U = -\vec{\mu} \cdot \vec{B} > 0$

Calculation

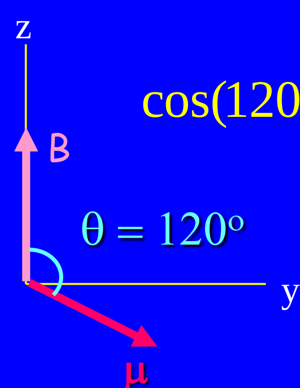
A square loop of side a lies in the x-z plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points along the +z axis. Assume a, I, and B are known.



$$U = -\vec{\mu} \cdot \vec{B}$$

- What is the potential energy of the final state?

(A) $U = Ia^2 B$ (B) $U = \frac{\sqrt{3}}{2} Ia^2 B$ (C) $U = \frac{1}{2} Ia^2 B$



$$\cos(120^o) = -\frac{1}{2}$$

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos(120^o) = \frac{1}{2} \mu B$$

$$\mu = Ia^2$$

$$U = \frac{1}{2} Ia^2 B$$