Physics 212

Lecture 15

Ampere's Law

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enclosed}$$

Music

Who is the Artist?

A) Oscar Peterson
B) Kenny Barron
C) Dave Brubeck
D) Thelonius Monk
E) Marcus Roberts

A classic for a classic day

Your Comments

"Pretty neat stuff. I like how similar this is to Gauss's law."

Easier method to calculate magnetic fields

"Need more explanation on the magnetic field directions. "

"You should probably look at getting some of the images up and running properly, its hard to deduce anything unless I try to imagine what youre asking about, which in any case i would be right all the time."

"I find this kind of confusing. Also most of the checkpoint pictures didn't work for me, so I can't go back and study from them. It makes it more confusing when I can't see the pictures to answer questions.."

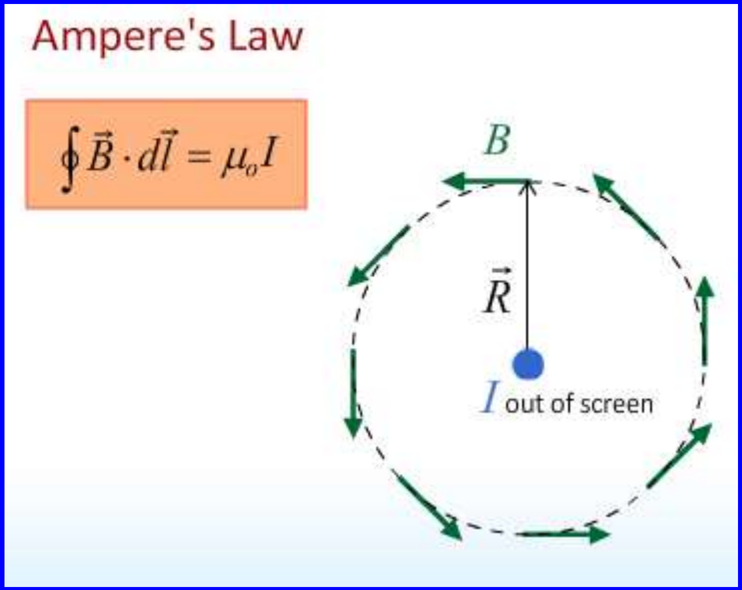
Sorry!
We'll go through the checkpoints (with pictures)

"Integrals are my greatest enemy "

Spend some time building the integrals

"Daaag man, my boy Gauss be gettin' his style cramped by dat Ampere clown."

Hour Exam II – THURSDAY Mar. 29



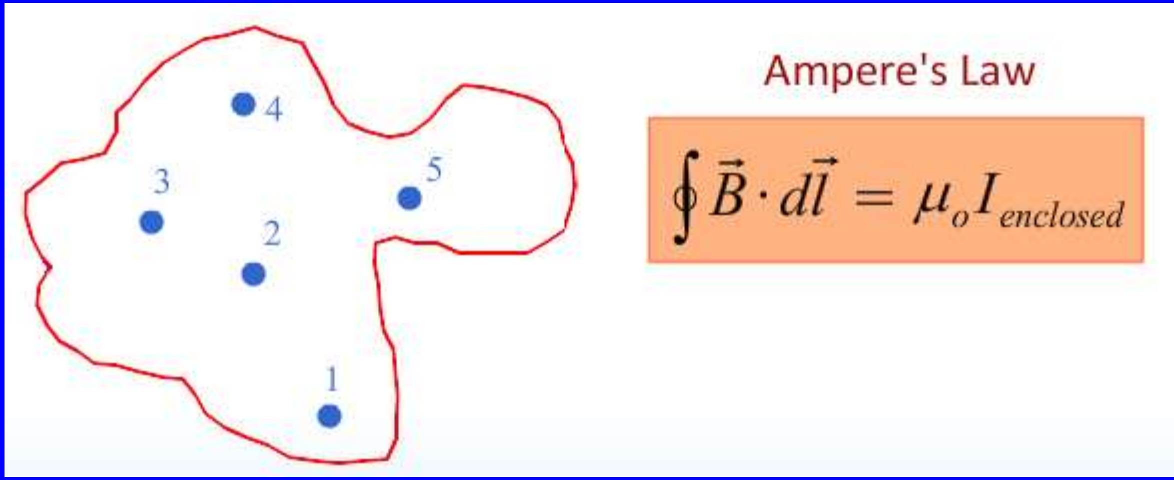
Infinite current-carrying wire

LHS: $\oint \vec{B} \bullet d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \cdot 2\pi R$

RHS: $I_{enclosed} = I$

$$B = \frac{\mu_o I}{2\pi R}$$

General Case



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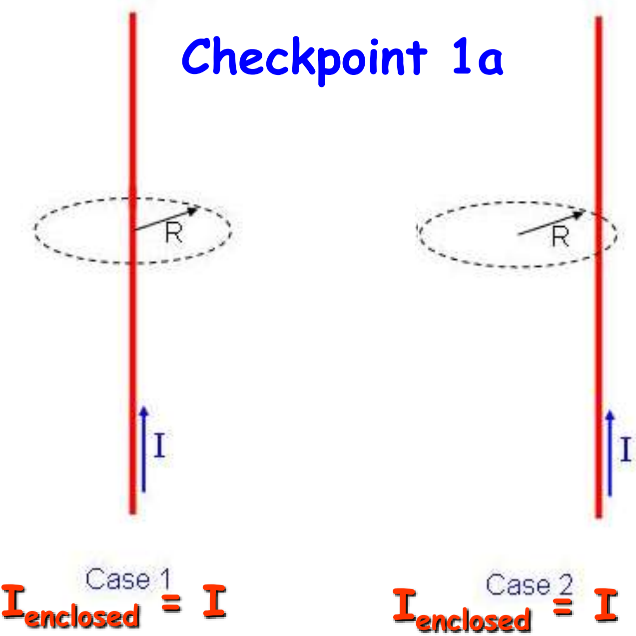
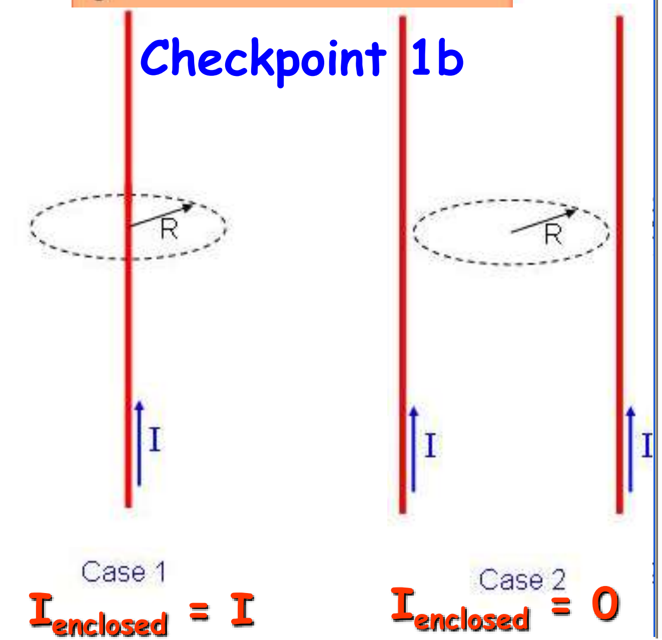
Practice on Enclosed Currents

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$



For which loop is ∫**B**·d**l** the greatest?

A. Case 1 ***B.*** Case 2 ***C.*** Same



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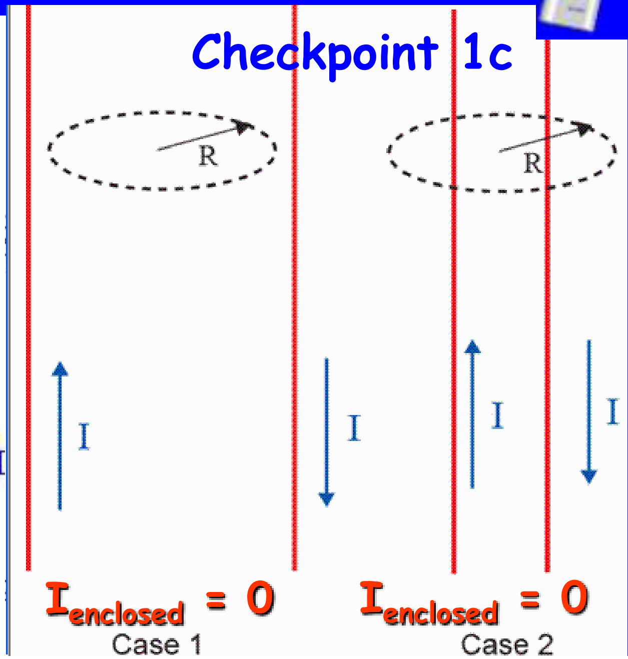

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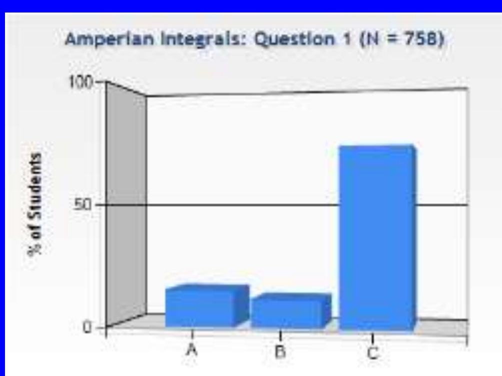

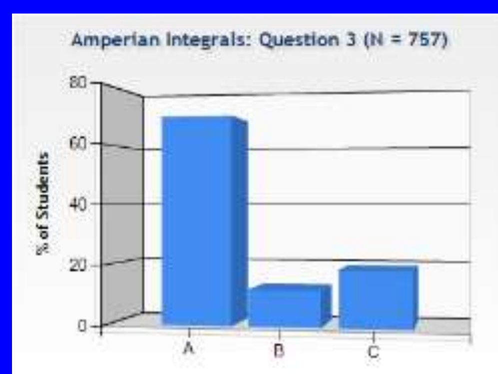

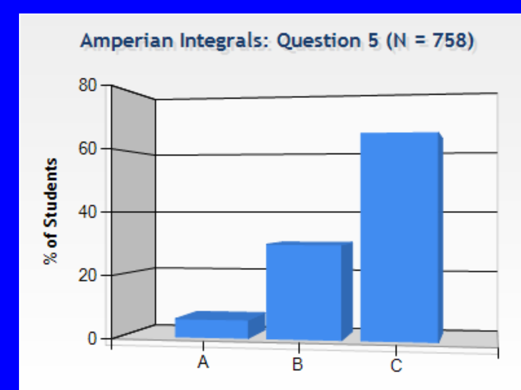

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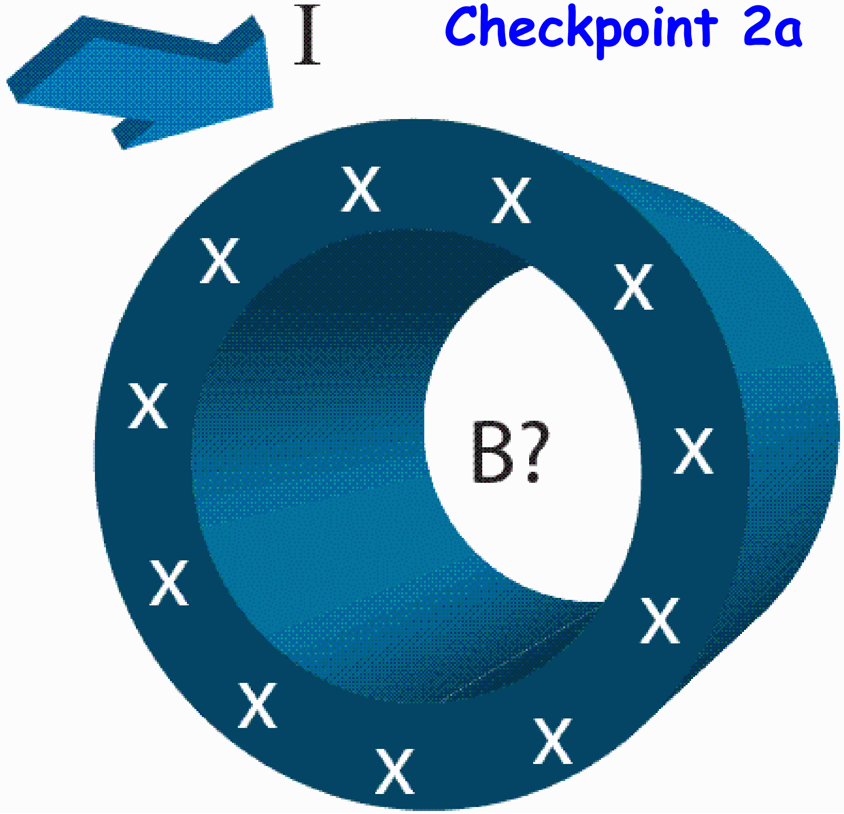
An infinitely long hollow conducting tube carries current I in the direction shown.

Checkpoint 2a



What is the direction of the magnetic field inside the tube?

A. clockwise

B. counterclockwise

C. radially inward to the center

D. radially outward from the center

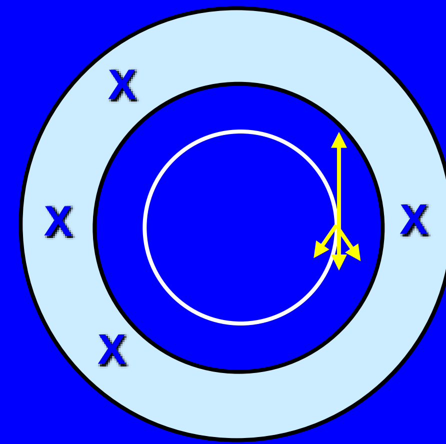
E. the magnetic field is zero

"If you point your thumb in the direction of I, your fingers curl CW."

"Force is tangent to the cylinder so according to the RHR the magnetic field must be radailly towards the middle "

"The enclosed current is zero, and if you take a non-zero closed path, then we zeo that the field MUST be zero in order for amperes law to hold true"

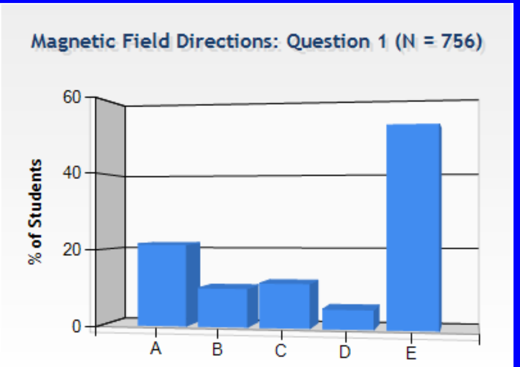
Cylindrical Symmetry



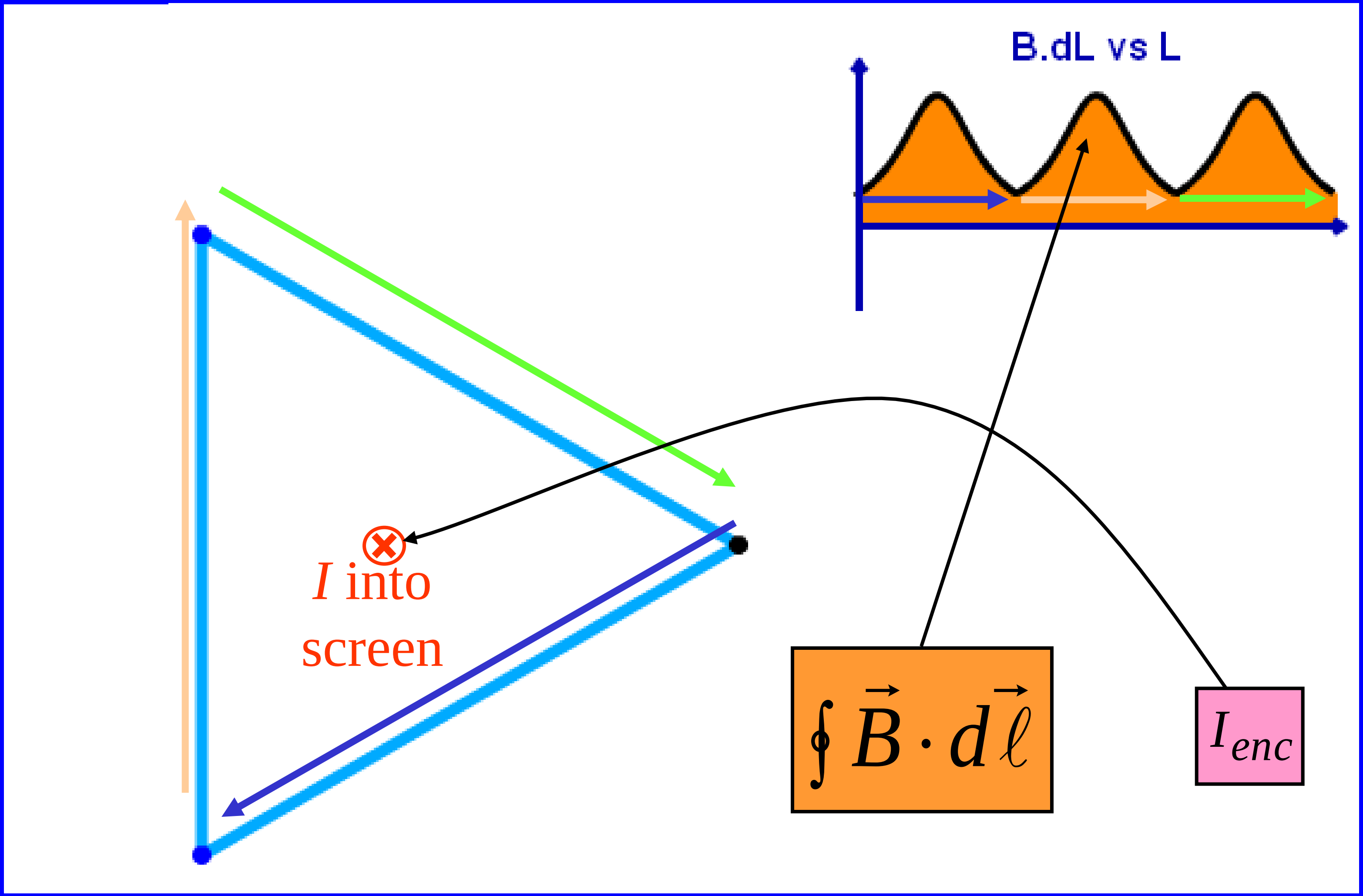
$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

Enclosed Current = 0

Check cancellations

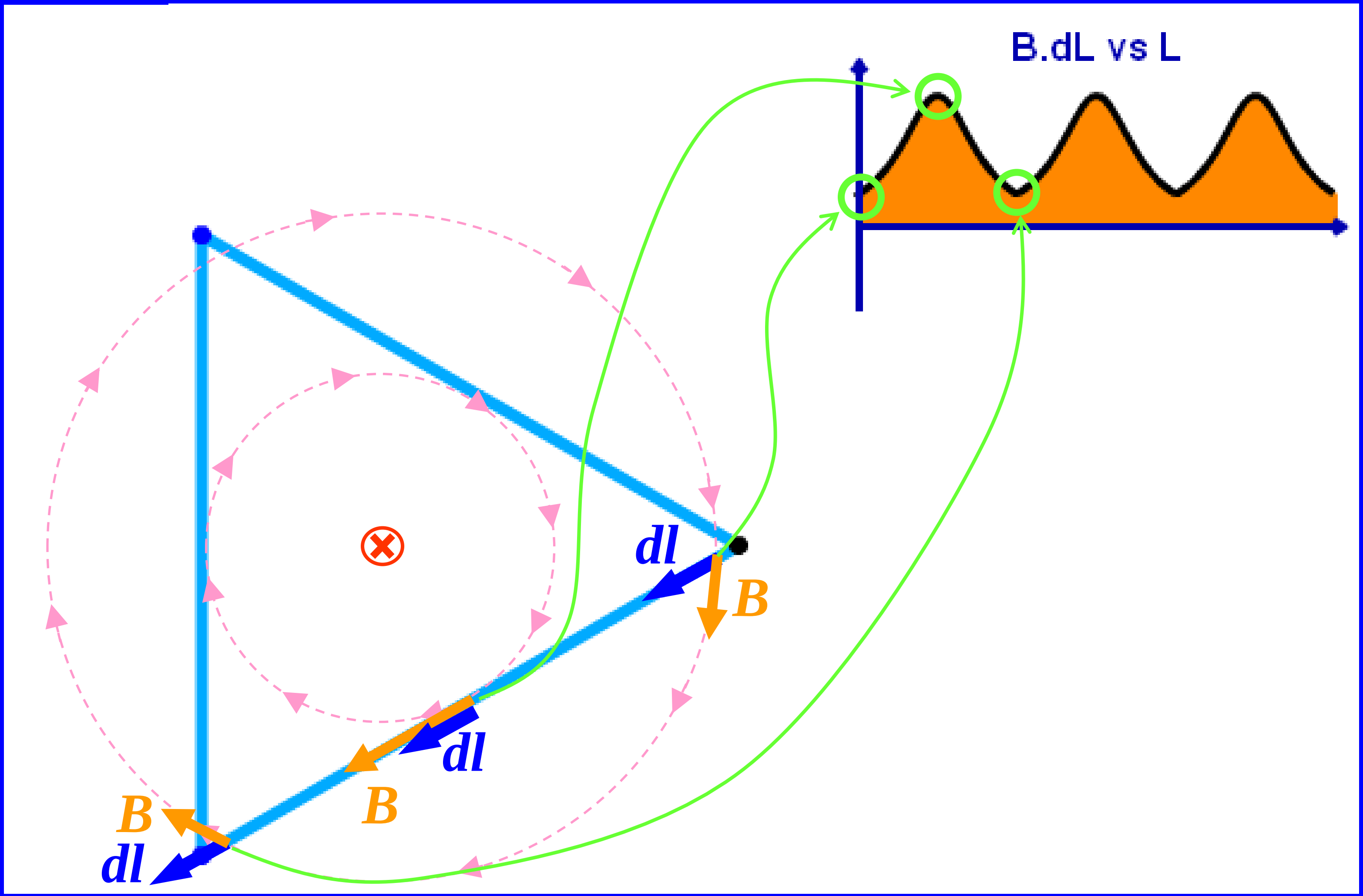


Ampere's Law
(+ integrals + magnetic field directions)



Ampere's Law

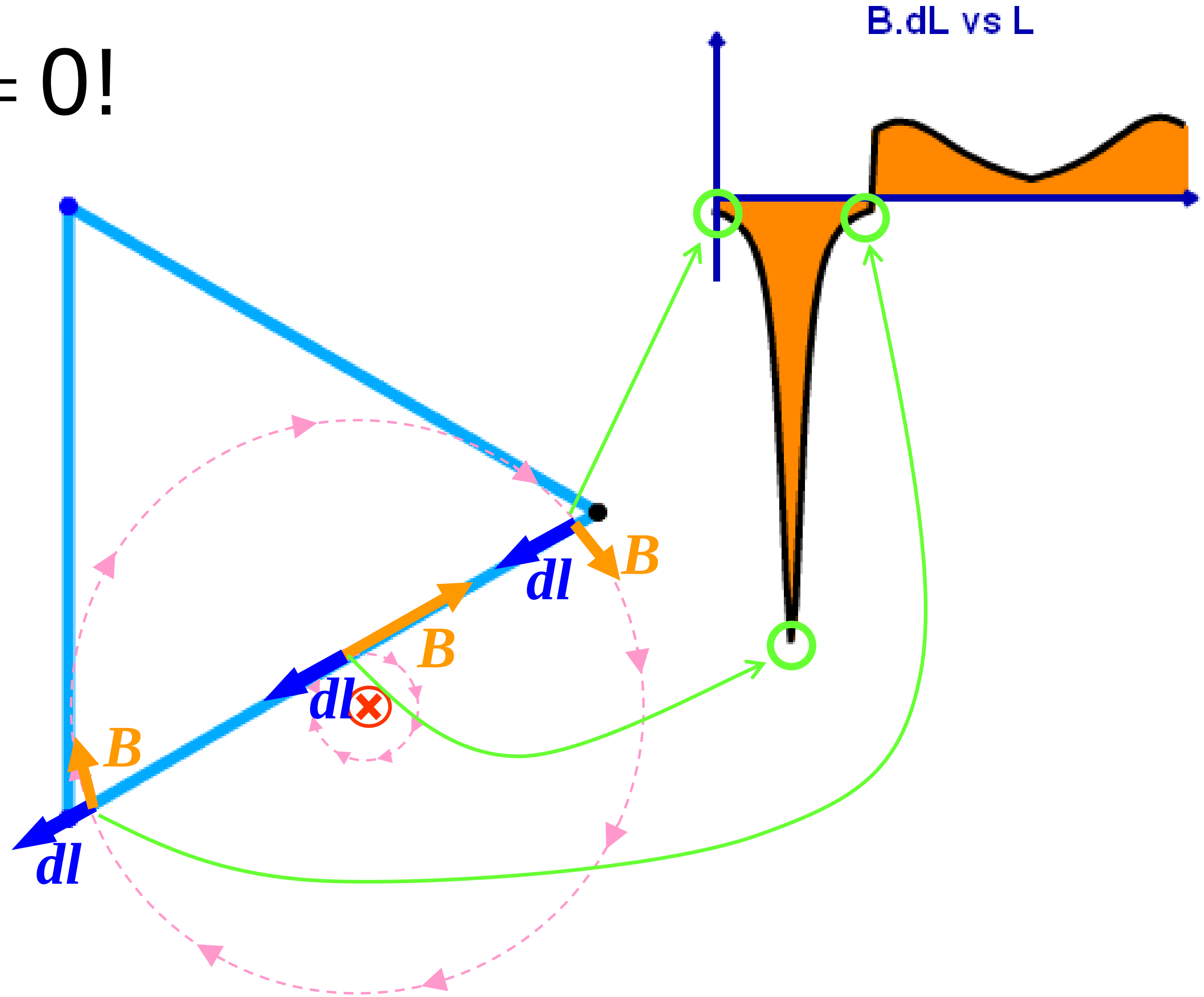
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$

$$I_{enc} = 0!$$



Ampere's Law

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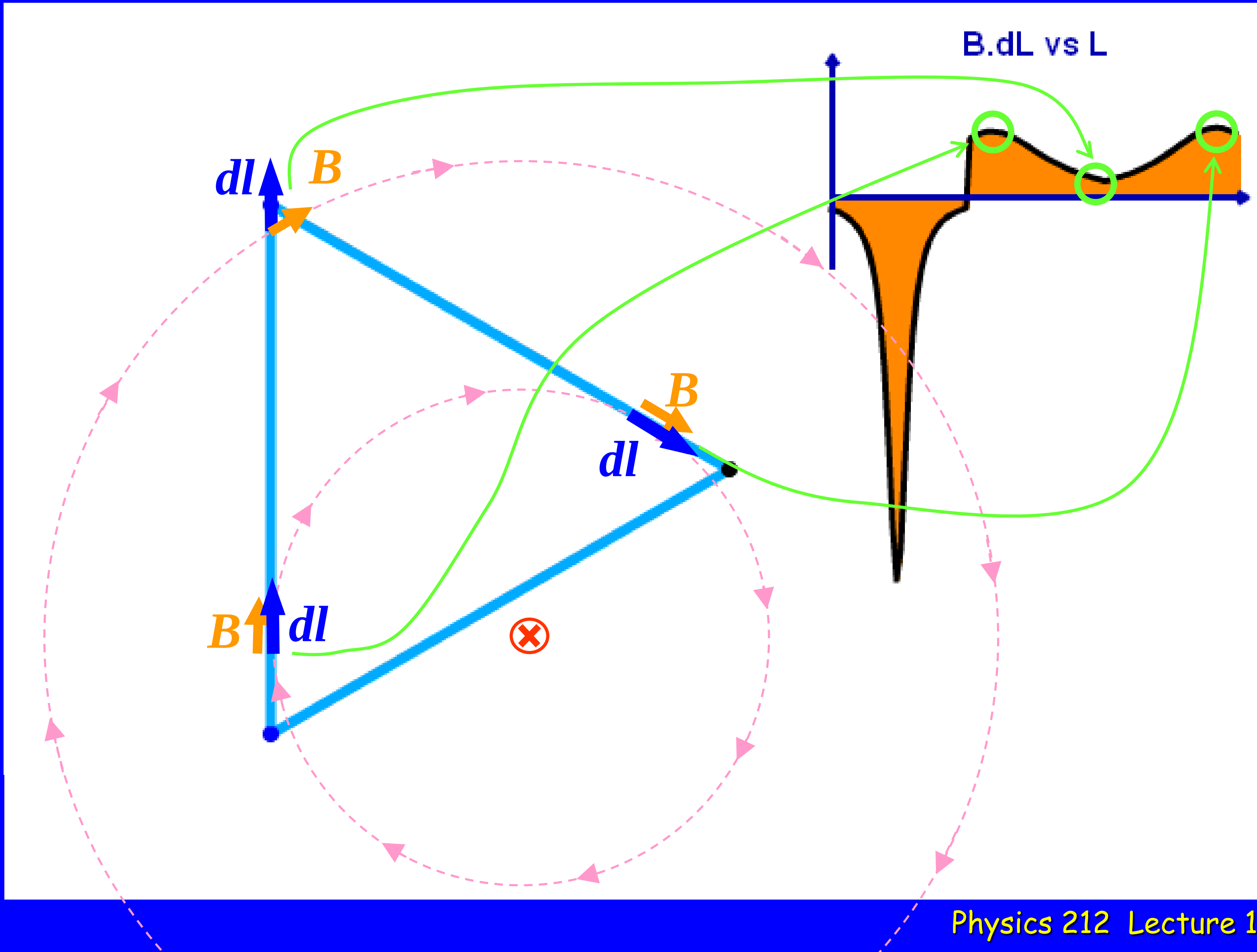
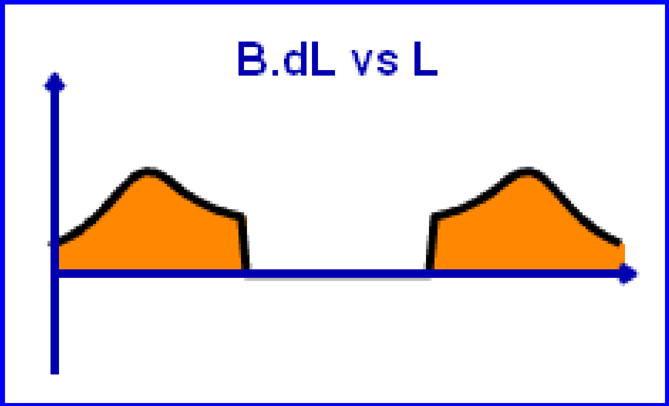
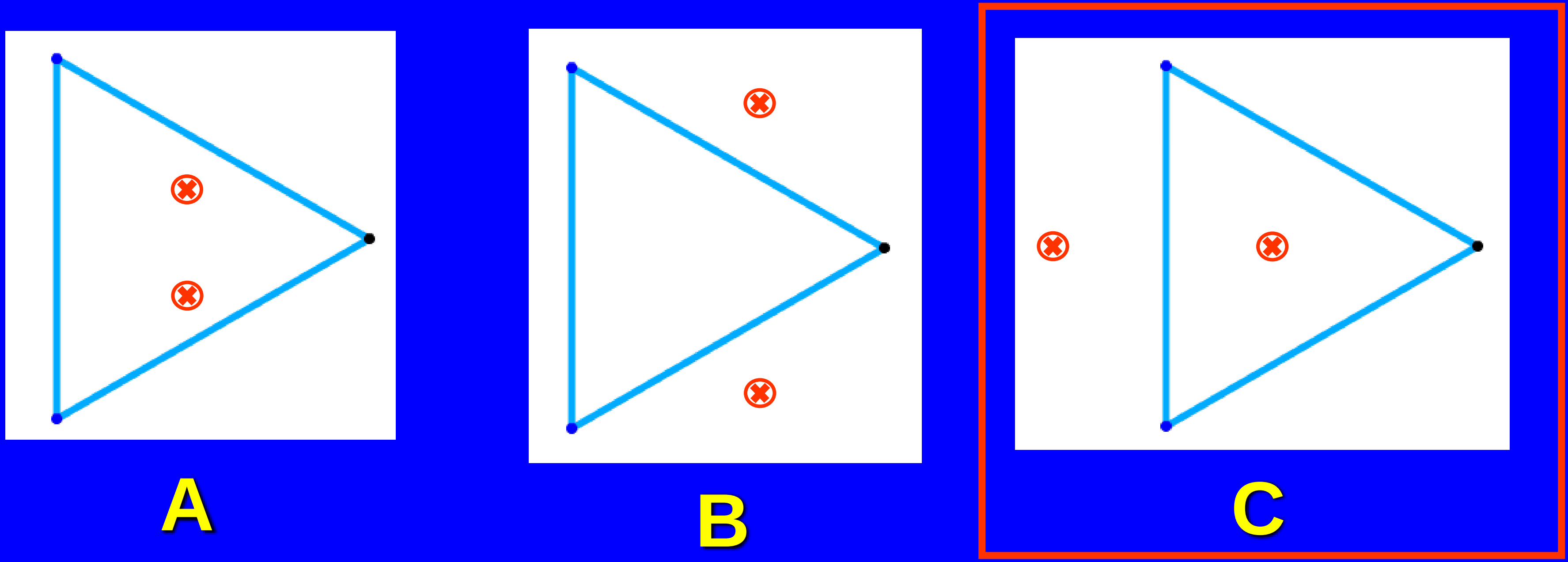
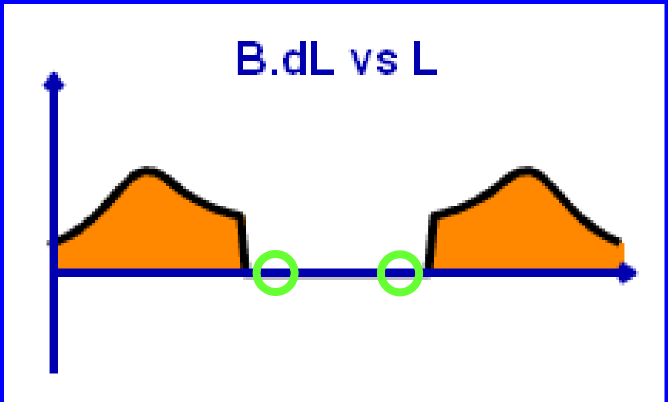
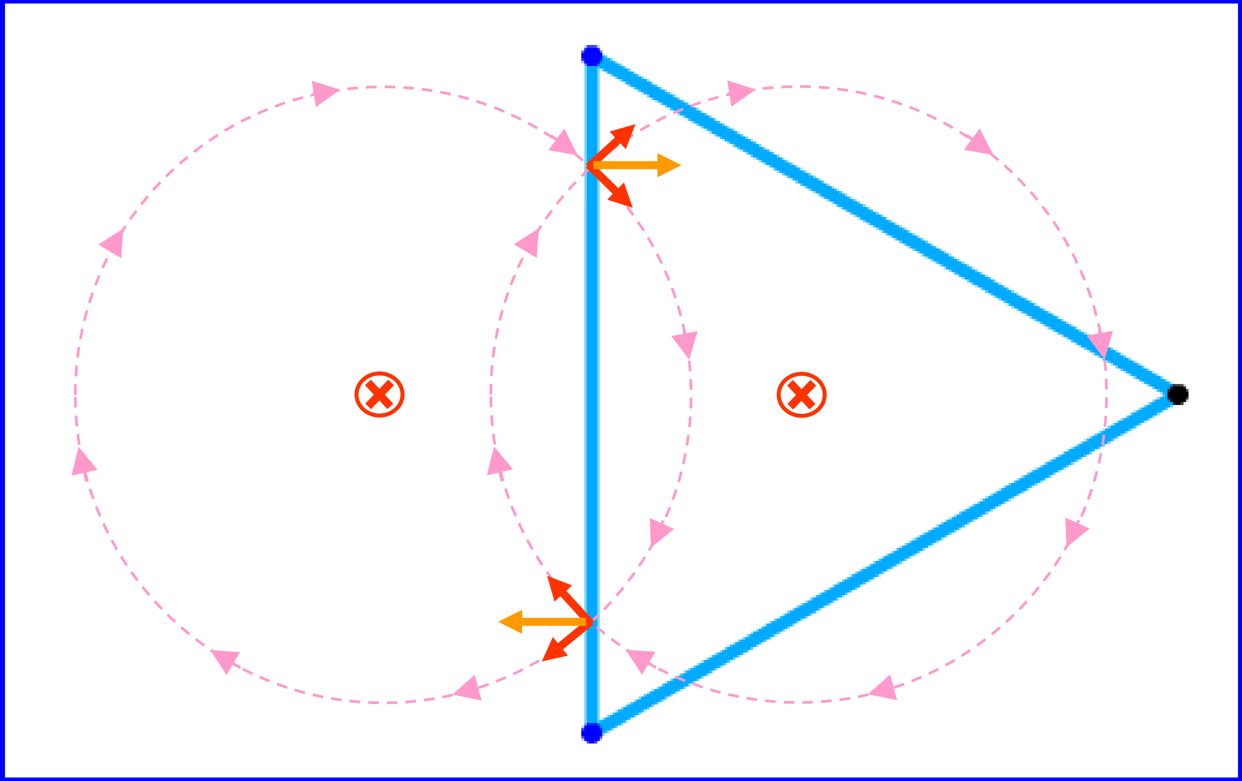


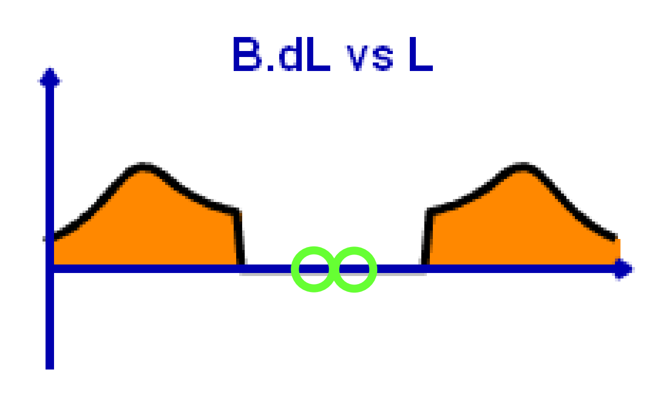
Which of the following current distributions would give rise to the B·dL distribution at the right?

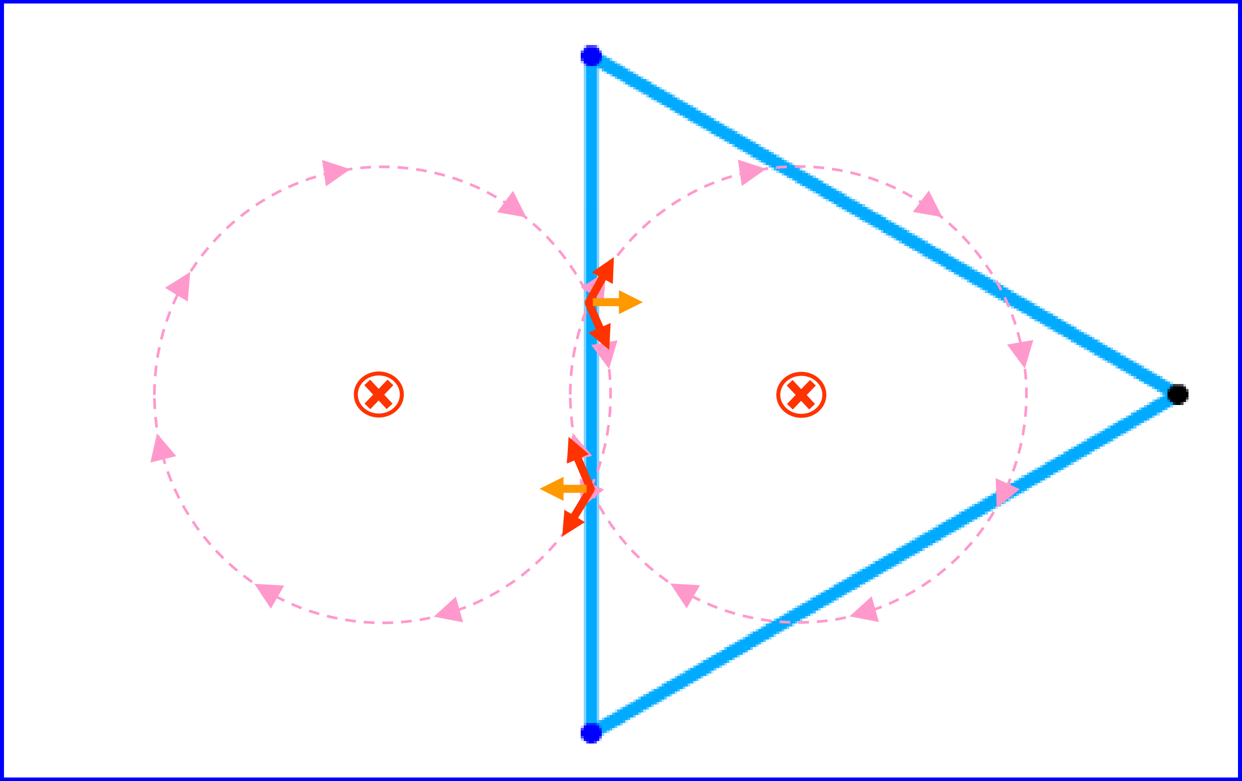


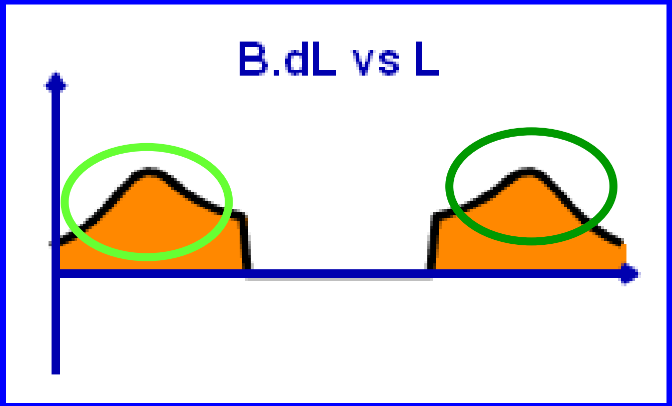


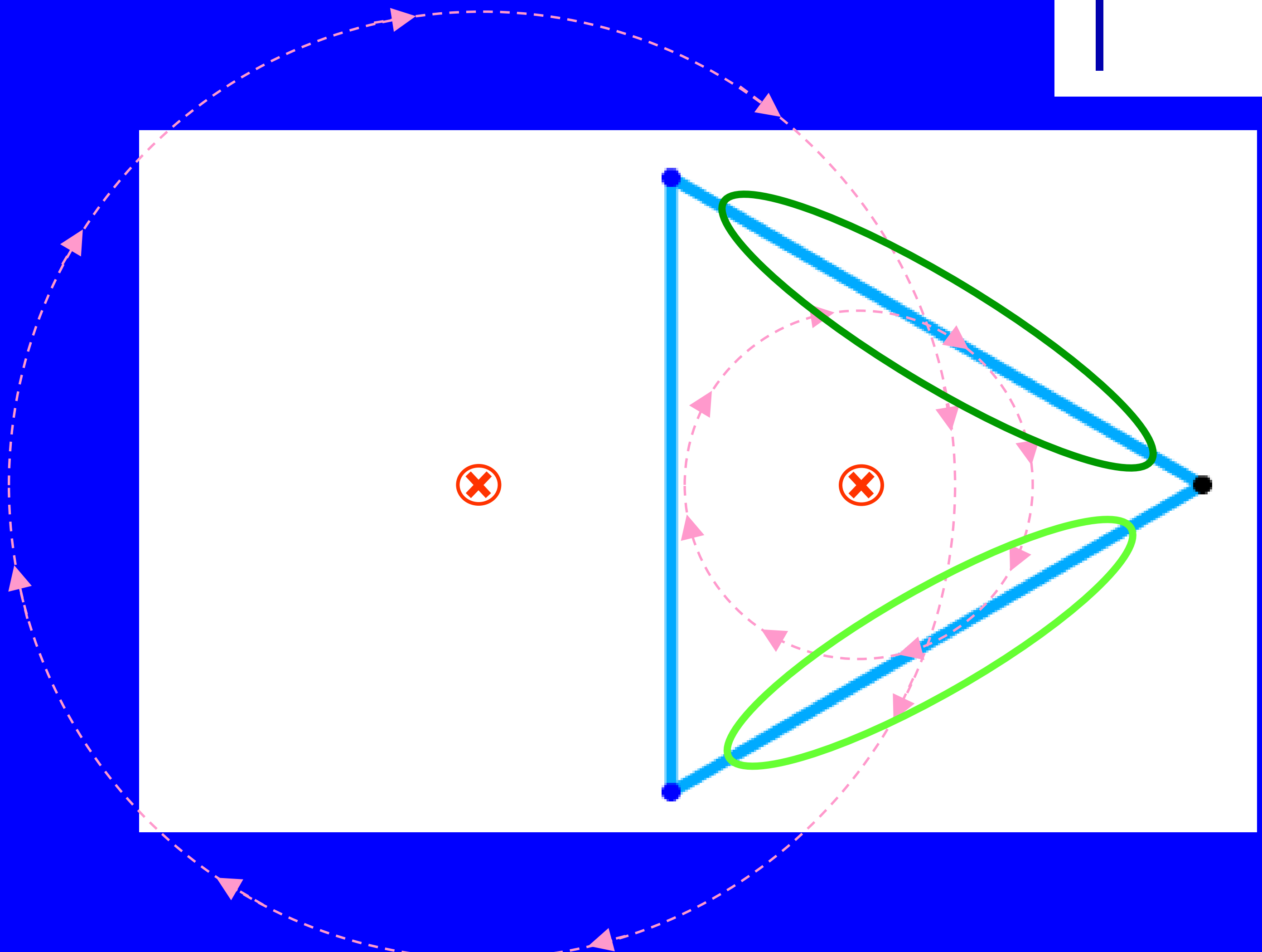

B.dL vs L



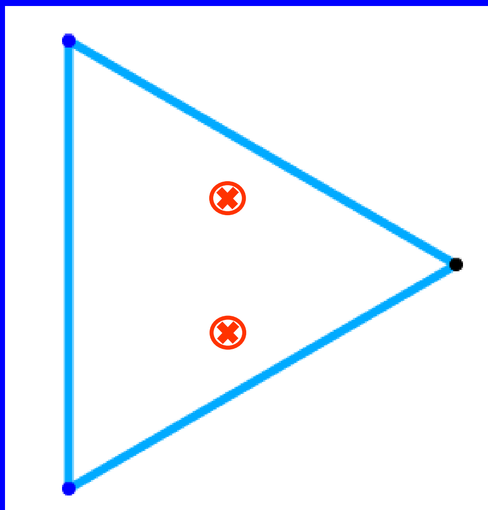

B.dL vs L



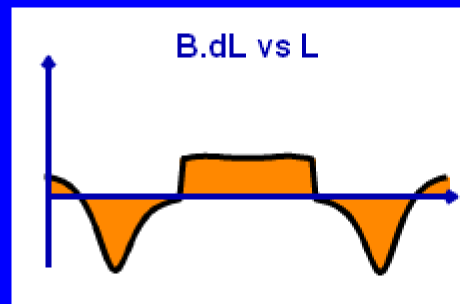

B.dL vs L

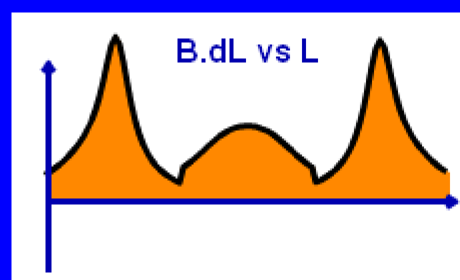


Match the other two:

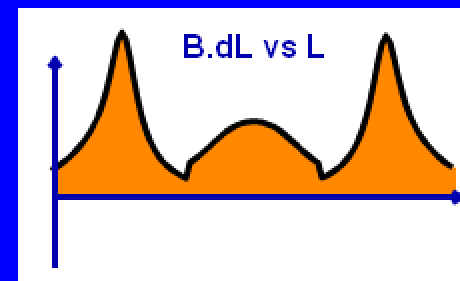


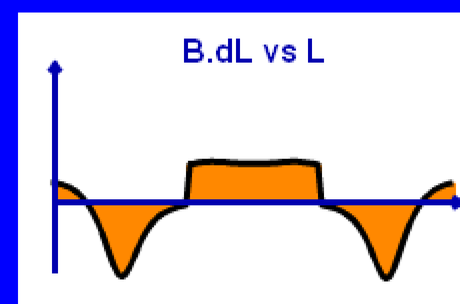
A

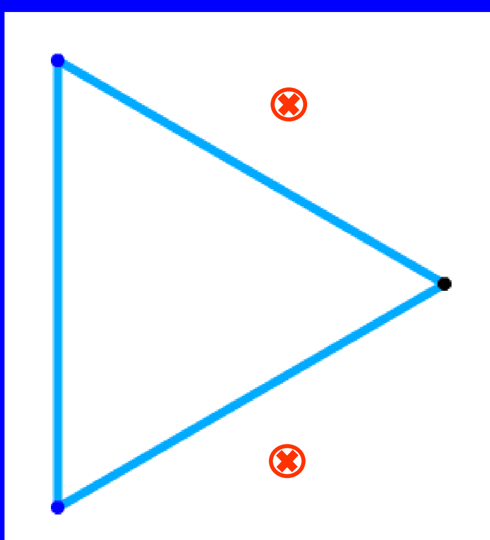




B







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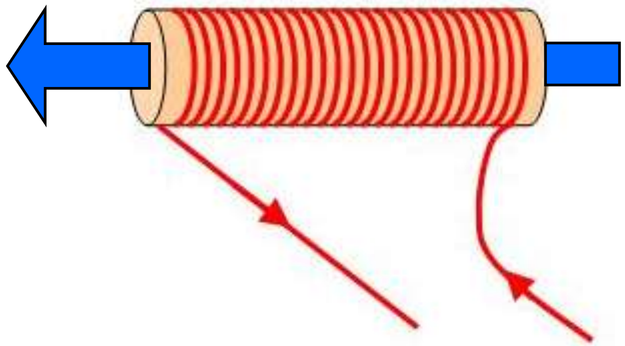
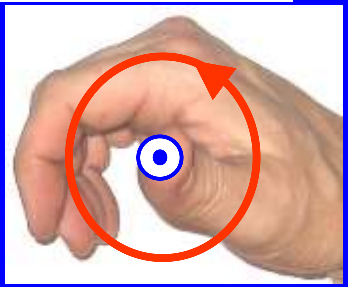
A current carrying wire is wrapped around a cardboard tube as shown below.

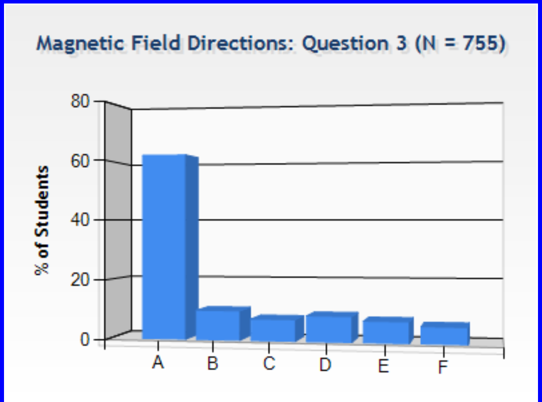
Checkpoint 2b





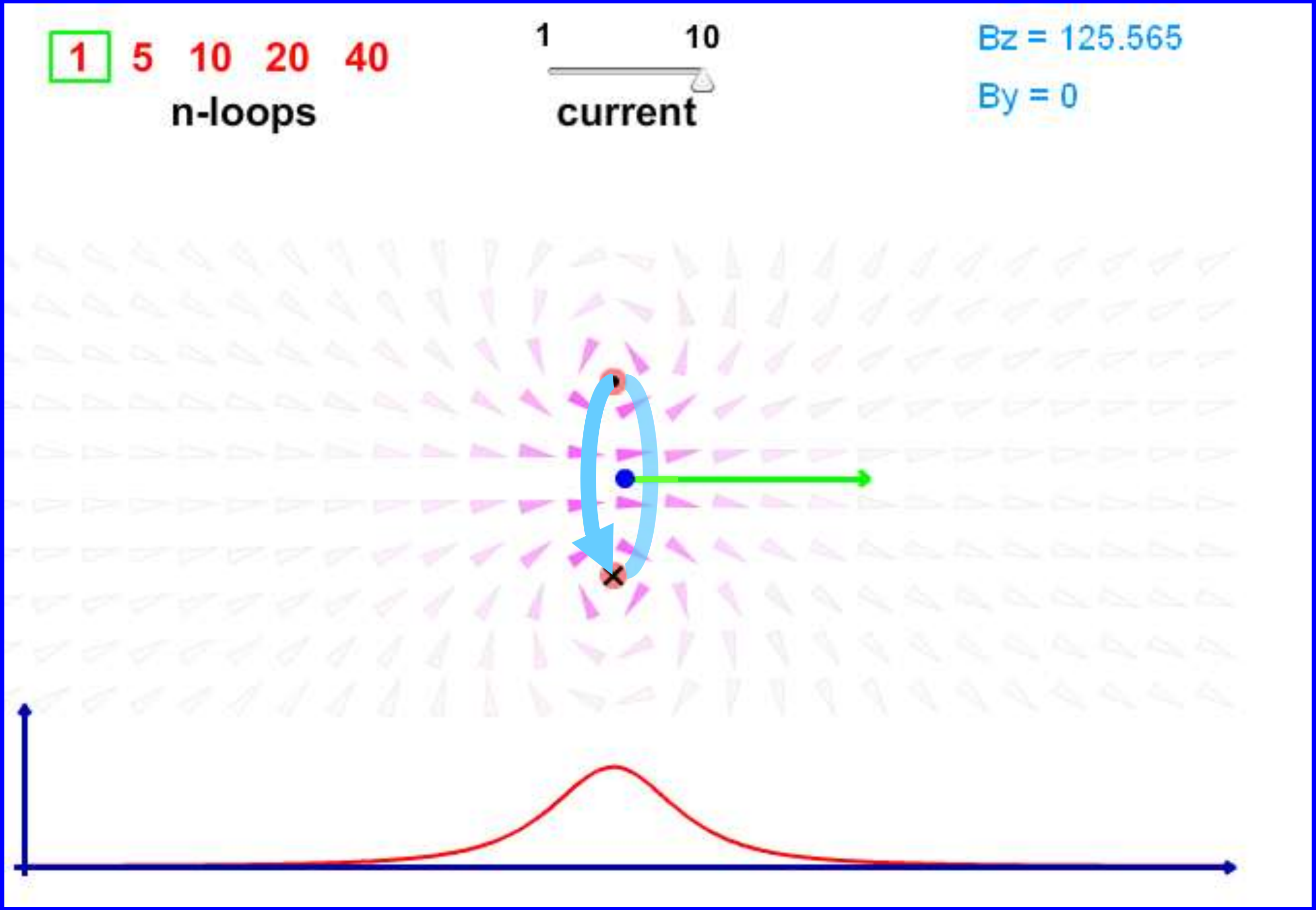
In which direction does the magnetic field point inside the tube?

A. Left **B.** Right **C.** Up **D.** Down **E.** Out of screen



Use the right hand rule and curl your fingers along the direction of the current.

Simulation



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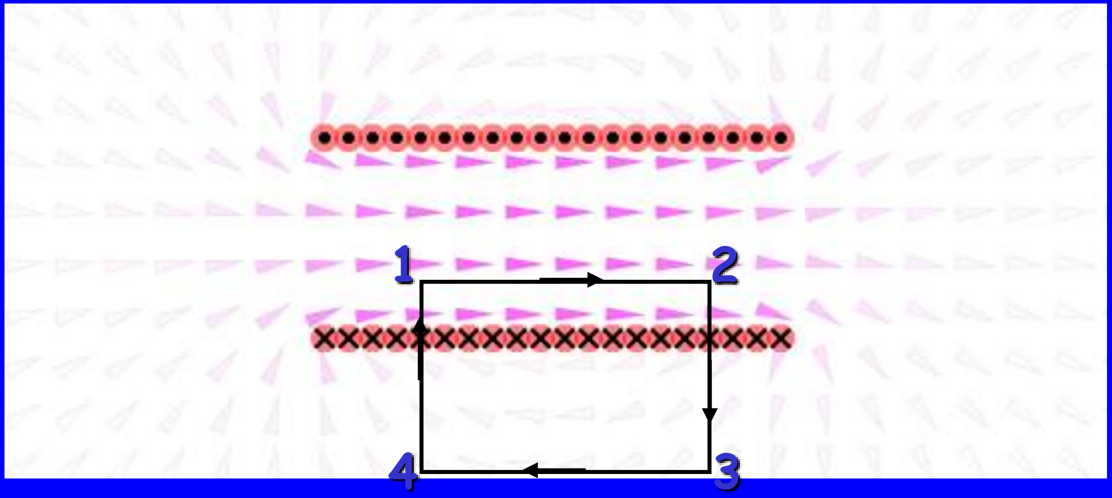
Solenoid

Several loops packed tightly together form a uniform magnetic field inside, and nearly zero magnetic field outside.



From this simulation, we can assume a constant field inside the solenoid and zero field outside the solenoid, and apply Ampere's law to find the magnitude of the constant field inside the solenoid !!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$\int_1^2 \vec{B} \bullet d\vec{\ell} + \int_2^3 \vec{B} \bullet d\vec{\ell} + \int_3^4 \vec{B} \bullet d\vec{\ell} + \int_4^1 \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$BL + 0 + 0 + 0 = \mu_o I_{enc}$$

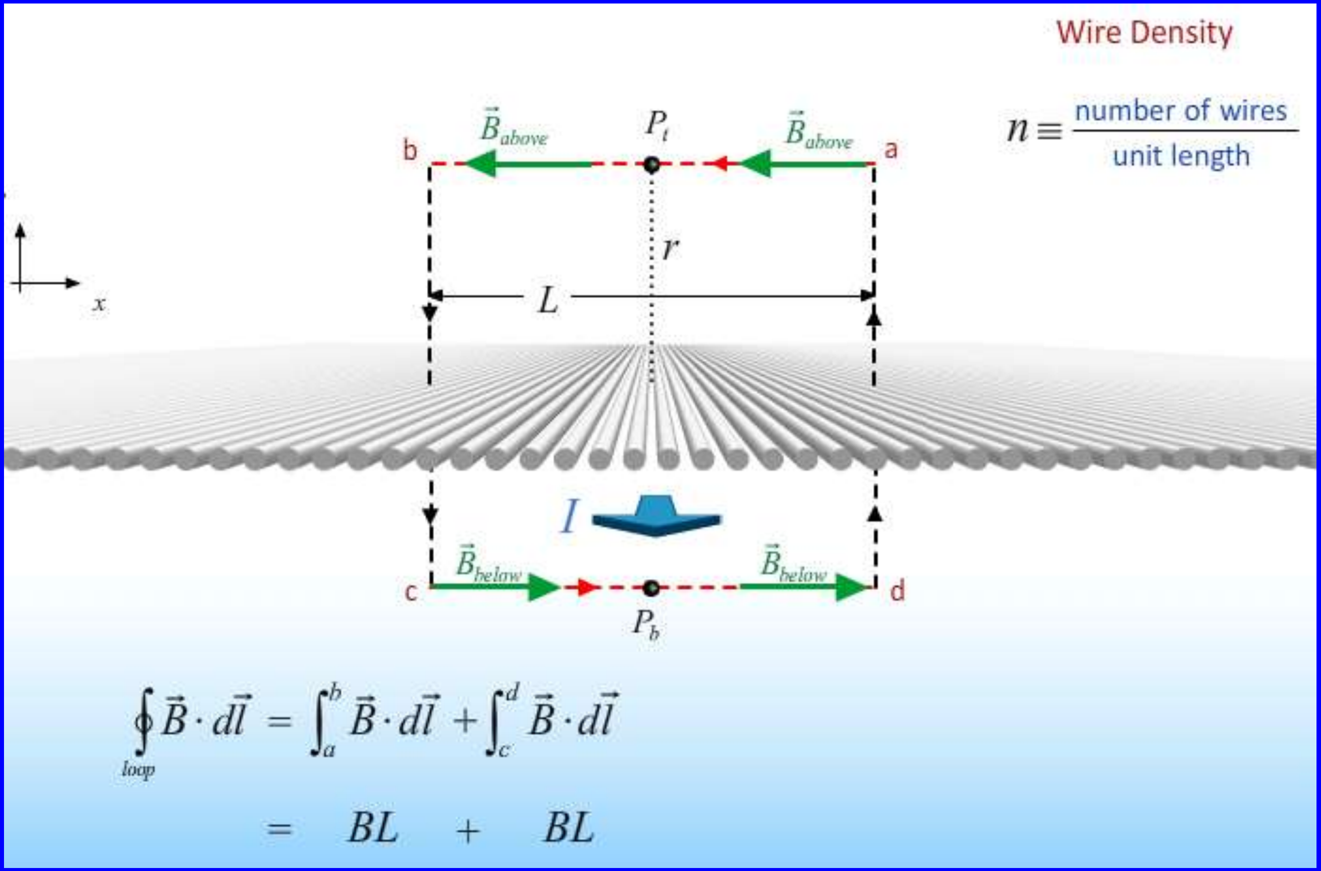
$$BL = \mu_o nLI$$

$$B = \mu_o nI$$

:28

n = # turns/length

Similar to the Current Sheet



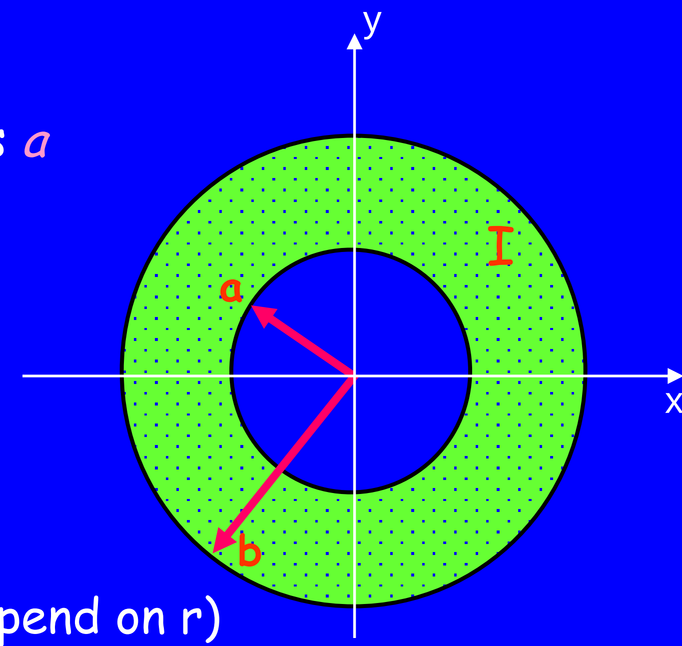
- Total integral around the loop

$$\oint_{loop} \vec{B} \bullet d\vec{\ell} = 2BL = \mu_0 I_{enclosed}$$

$$\therefore B = \frac{\mu_0 N I}{2L} = \frac{\mu_0 n I}{2}$$

Example Problem

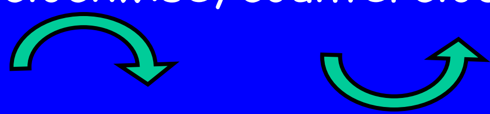
An infinitely long cylindrical shell with inner radius *a* and outer radius *b* carries a uniformly distributed current I ***out of the screen.***



Sketch |B| as a function of *r*.

- Conceptual Analysis
  - Complete cylindrical symmetry (can only depend on r)
    ⇒ can use Ampere's law to calculate B
  - B field can only be clockwise, counterclockwise or zero!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$



$$B \oint d\ell = \mu_o I_{enc}$$

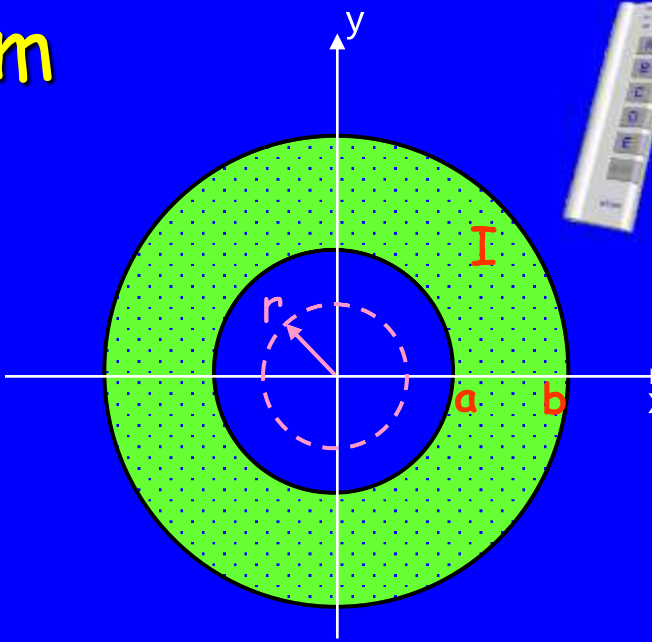
For circular path concentric w/ shell

- Strategic Analysis

  Calculate B for the three regions separately:

  1) $r < a$
  2) $a < r < b$
  3) $r > b$

Example Problem

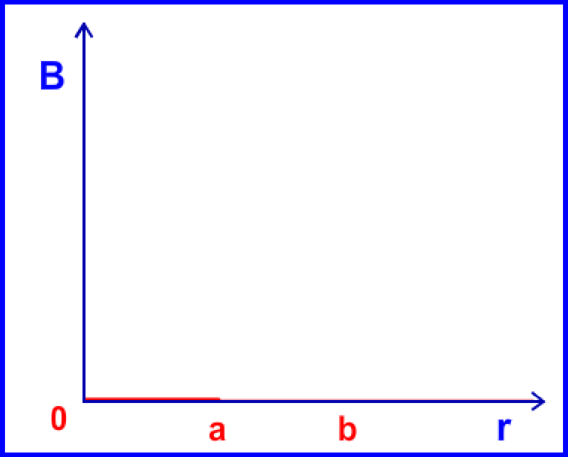


What does |B| look like for r < a ?

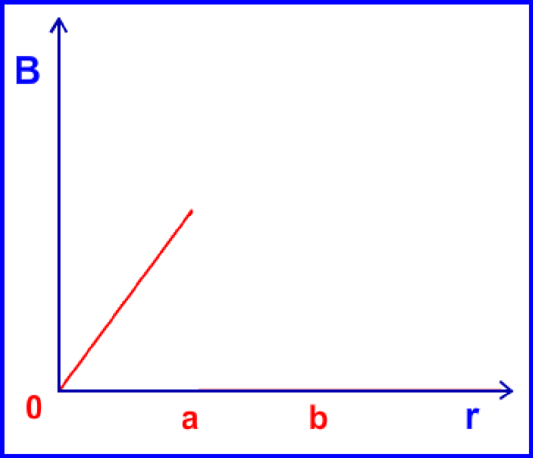
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

0

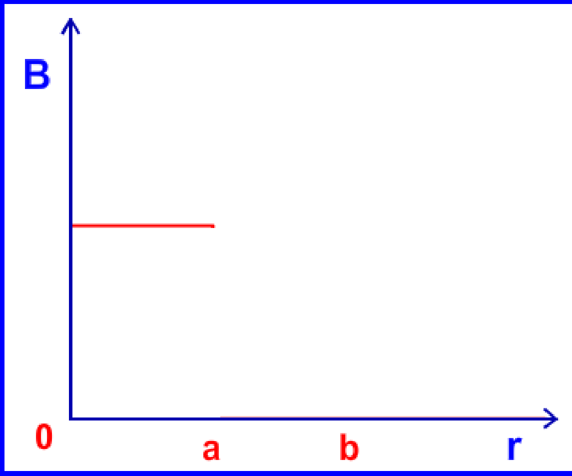
so $\vec{B} = 0$



(A)



(B)



(C)

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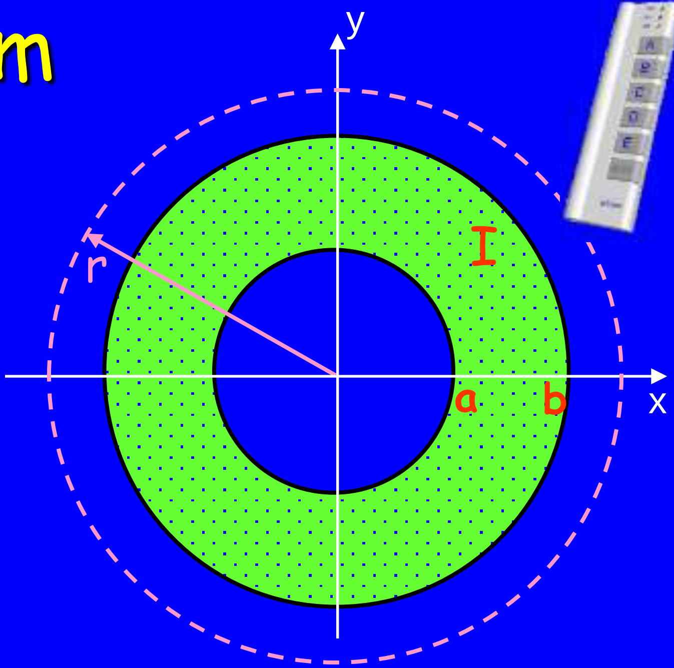
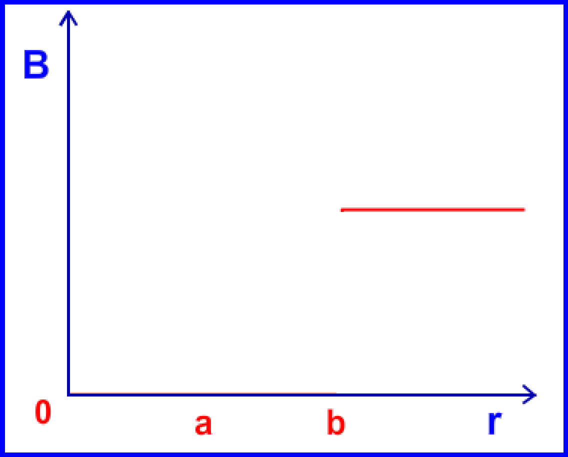
Example Problem



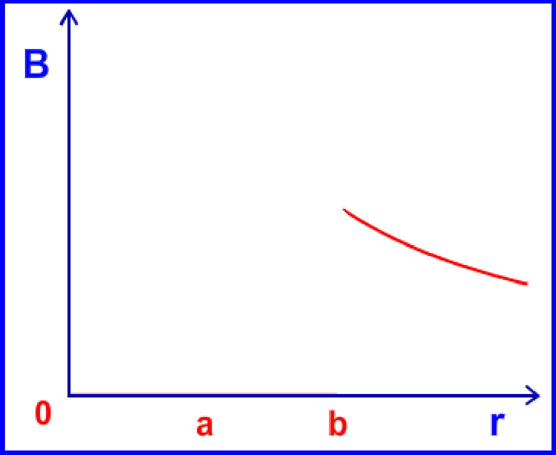
What does |B| look like for r > b ?

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

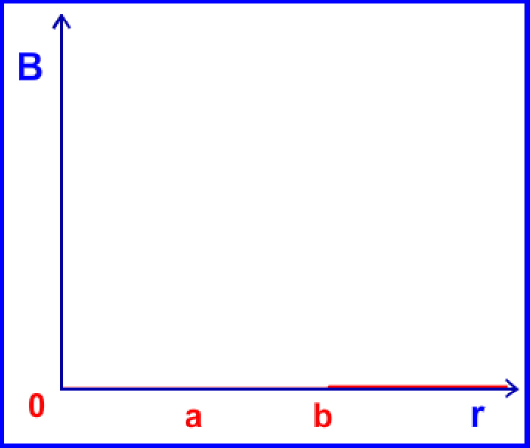
I



(A)



(B)



(C)

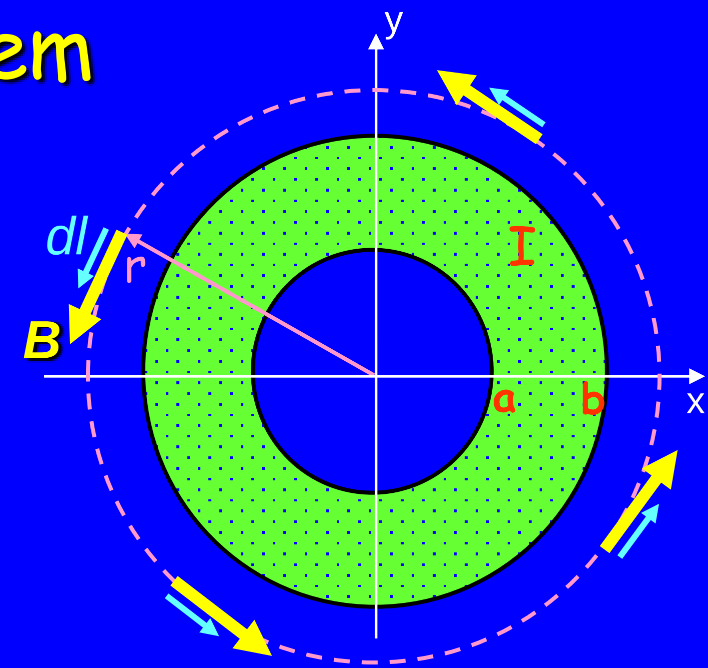
Example Problem

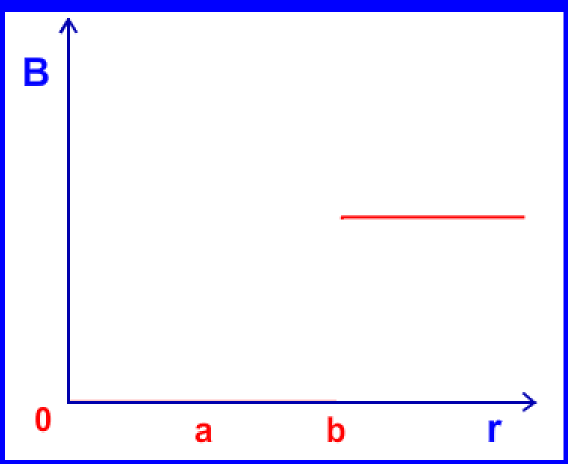
What does |B| look like for r > b ?

LHS: $\oint \vec{B} \bullet d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \cdot 2\pi r$

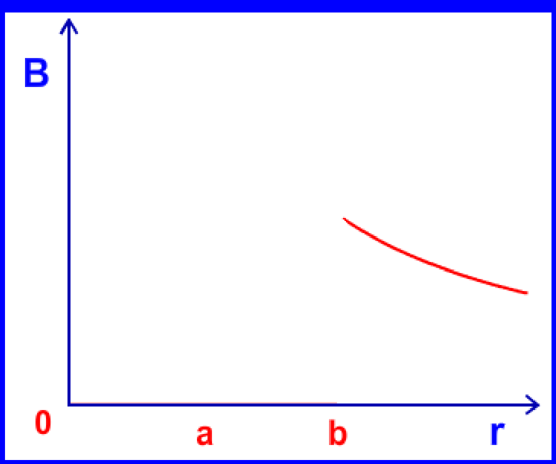
RHS: $I_{enclosed} = I$

$$B = \frac{\mu_o I}{2\pi r}$$

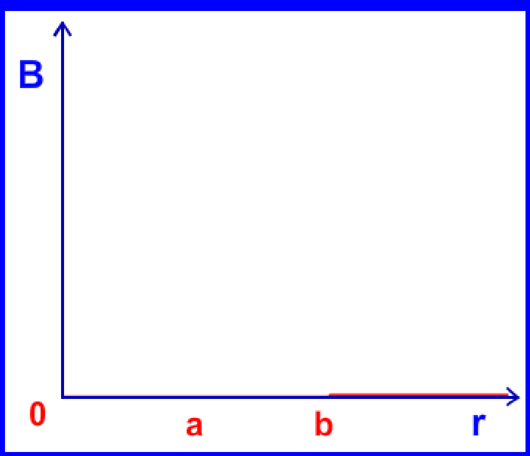




(A)

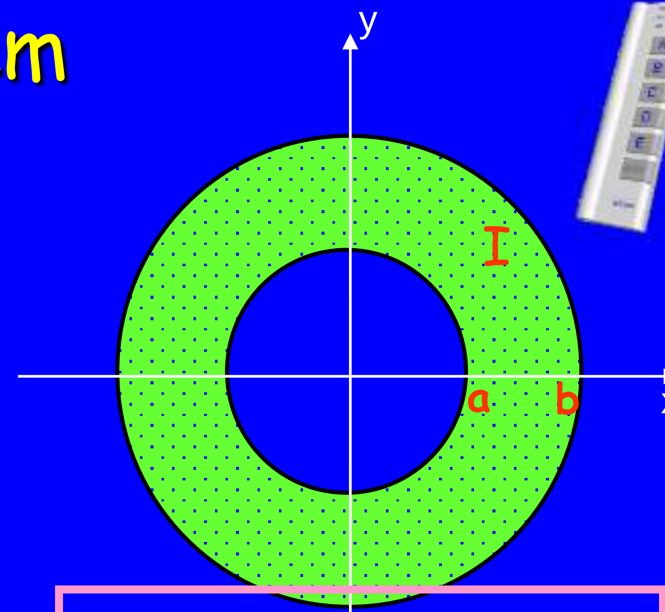


(B)



(C)

Example Problem



What is the current density j (Amp/m^2) in the conductor?

(A) $j = \dfrac{I}{\pi b^2}$

(B) $j = \dfrac{I}{\pi b^2 + \pi a^2}$

(C) $j = \dfrac{I}{\pi b^2 - \pi a^2}$

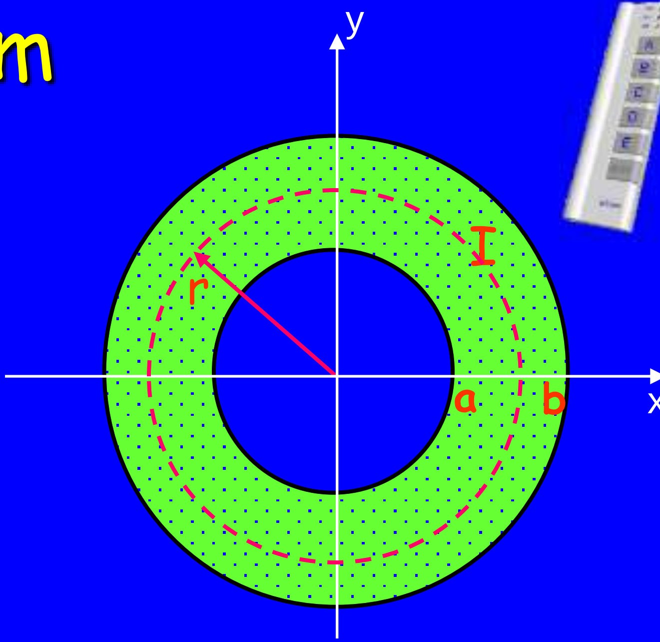
$j = I / area$

$area = \pi b^2 - \pi a^2$

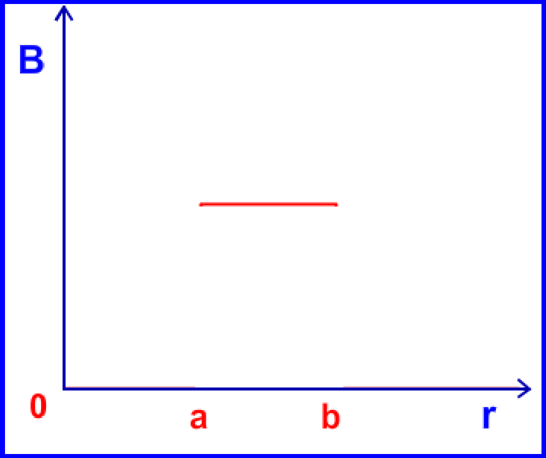
$$j = \frac{I}{\pi b^2 - \pi a^2}$$

:40

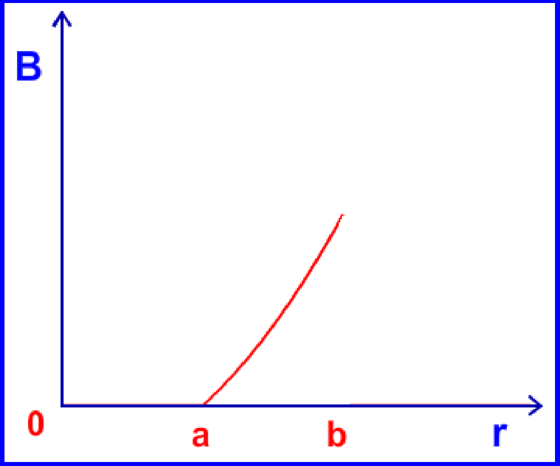
Example Problem



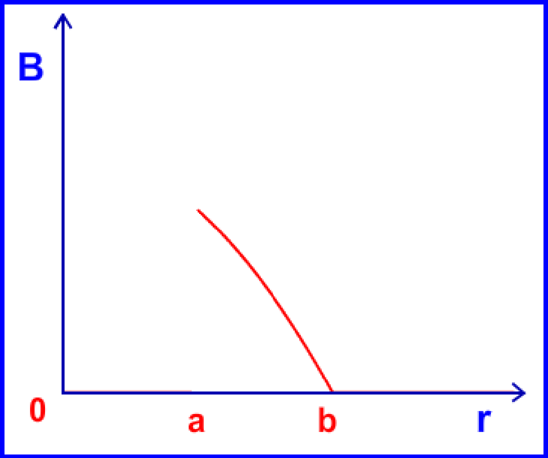
What does |B| look like for a < r < b ?



(A)



(B)

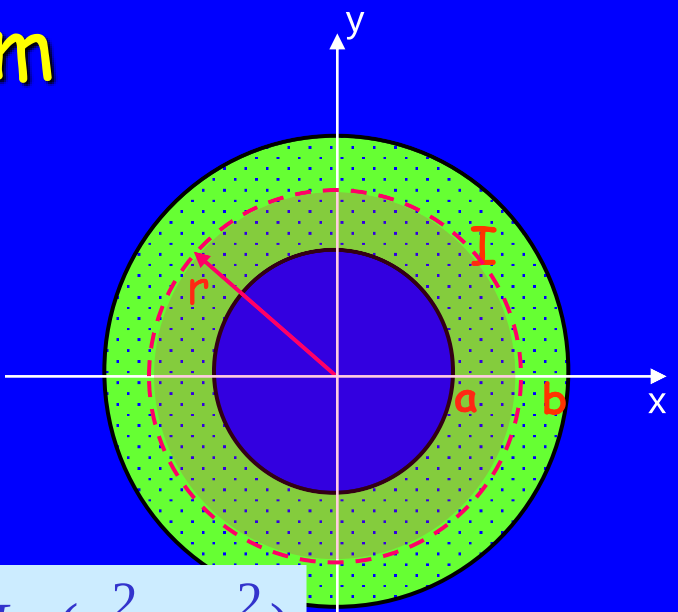


(C)

Example Problem

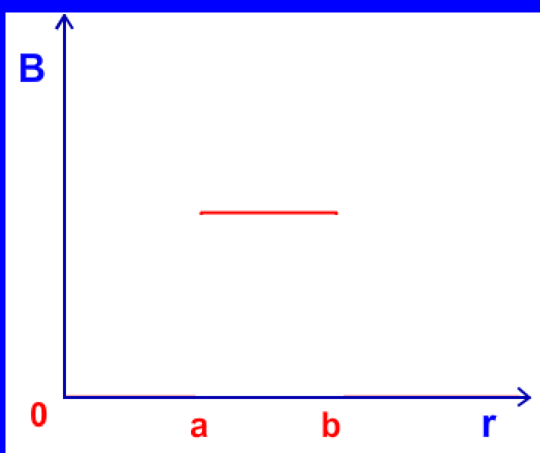
What does |B| look like for a < r < b ?

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \quad \Rightarrow \quad B \cdot 2\pi r = \mu_o \cdot j A_{enc}$$

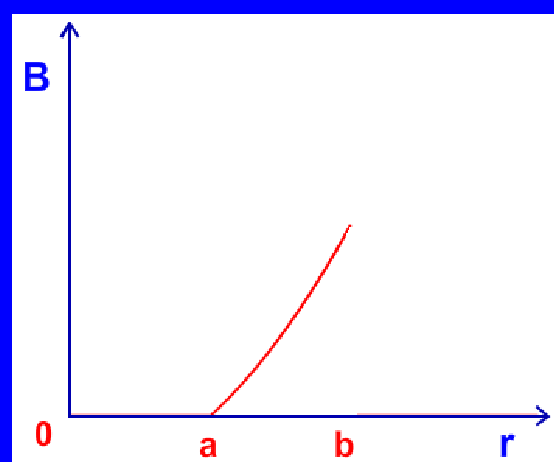


$$B \cdot 2\pi r = \mu_o \cdot \frac{I}{\pi(b^2 - a^2)} \cdot \pi(r^2 - a^2) \quad \Rightarrow \quad B = \frac{\mu_o I}{2\pi r} \cdot \frac{(r^2 - a^2)}{(b^2 - a^2)}$$

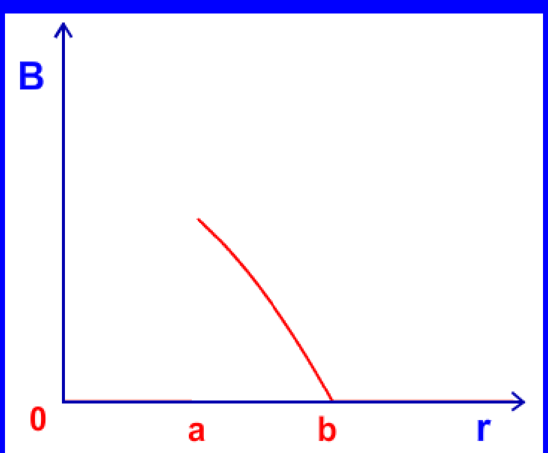
Starts at 0 and increases almost linearly



(A)



(B)

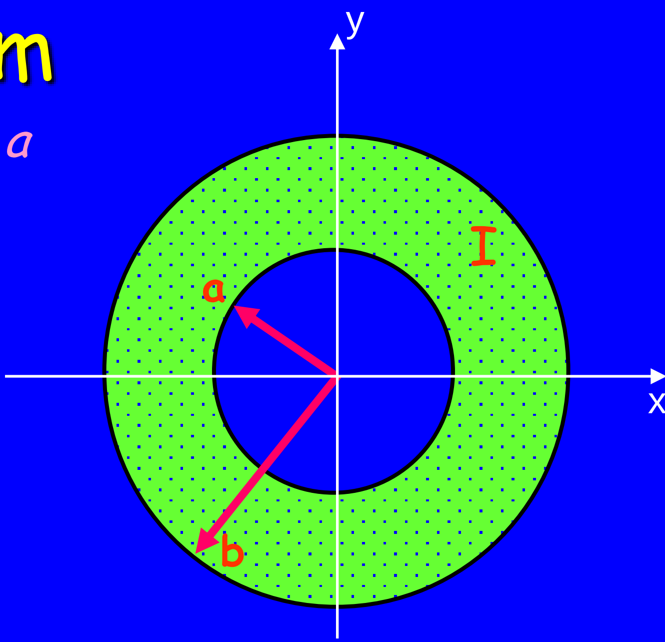


(C)

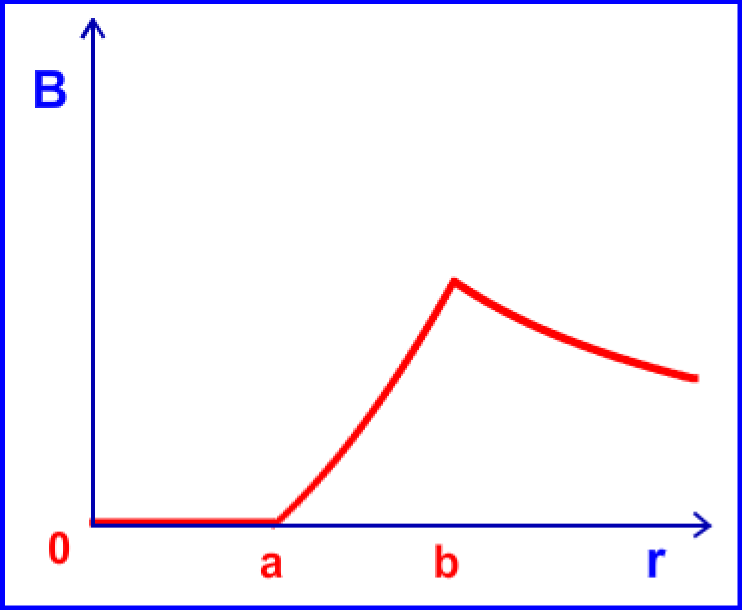
:45

Example Problem

An infinitely long cylindrical shell with inner radius *a* and outer radius *b* carries a uniformly distributed current I ***out of the screen***.



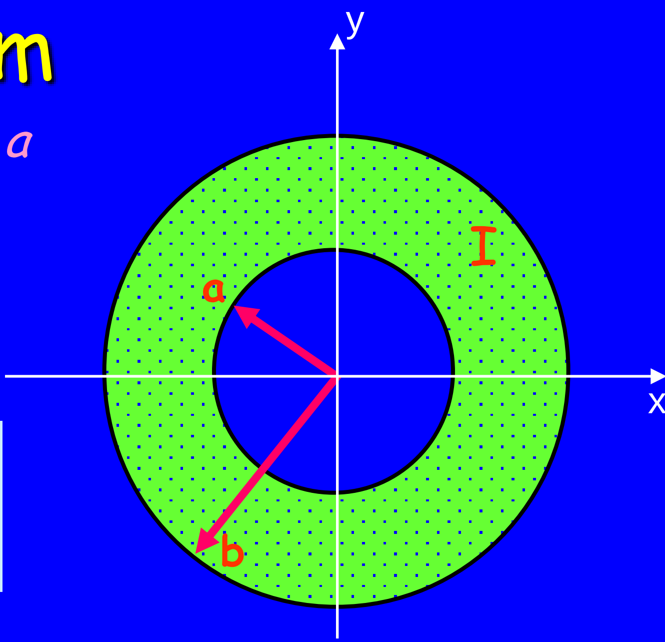
Sketch |B| as a function of *r*.



:48

Example Problem

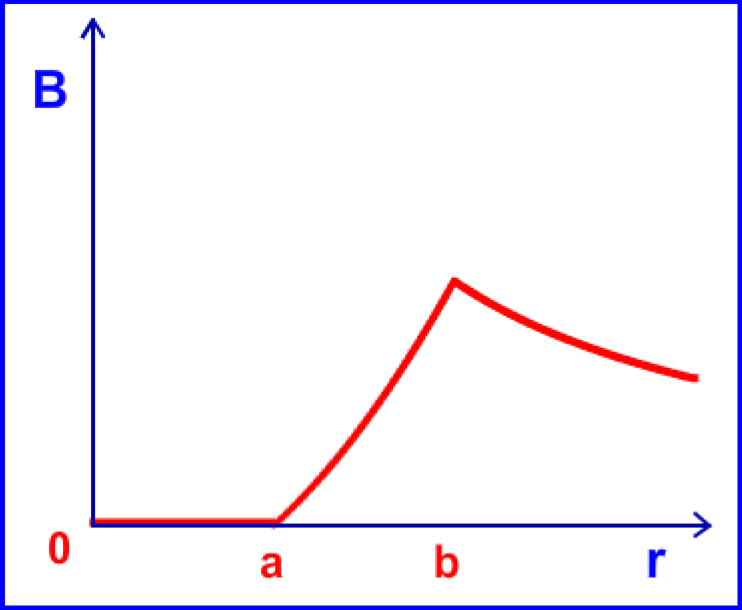
An infinitely long cylindrical shell with inner radius *a* and outer radius *b* carries a uniformly distributed current I ***out of the screen***.



Sketch |B| as a function of *r*.

How big is B at *r* = *b*?

$$B = \frac{\mu_o I}{2\pi r} \cdot \frac{(r^2 - a^2)}{(b^2 - a^2)}$$



Suppose I = 10 A, b = 1 mm

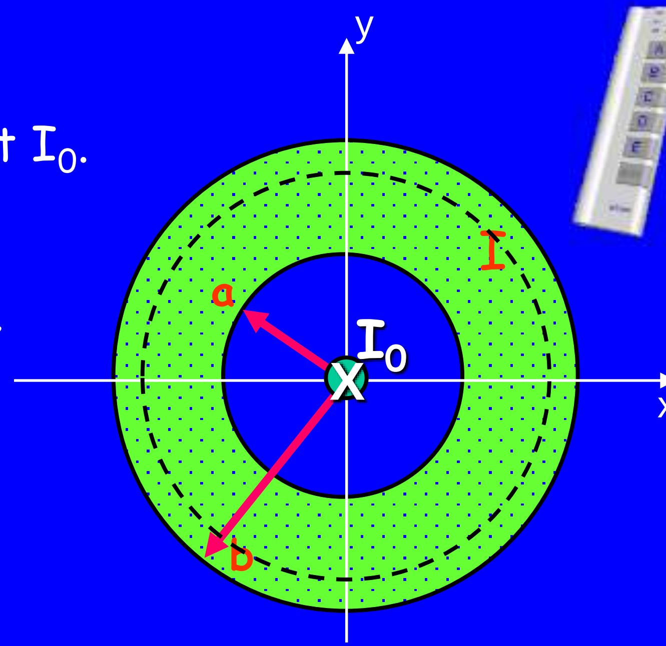
$$B(b) = \frac{\mu_o I}{2\pi b}$$

$$= \frac{4\pi x10^{-7}\ \mathrm{Tm/A} \cdot 10\mathrm{A}}{2\pi \cdot 0.001\,\mathrm{m}}$$

$$= 2x10^{-3}\ \mathrm{T}$$

Follow-Up

Add an infinite wire along the z axis carrying current I_0.



What must be true about I_0 such that there is some value of r, $a < r < b$, such that $B(r) = 0$?

A) $|I_0| > |I|$ AND I_0 into screen

B) $|I_0| > |I|$ AND I_0 out of screen

C) $|I_0| < |I|$ AND I_0 into screen

D) $|I_0| < |I|$ AND I_0 out of screen

E) There is no current I_0 that can produce B = 0 there

B will be zero if total current enclosed = 0