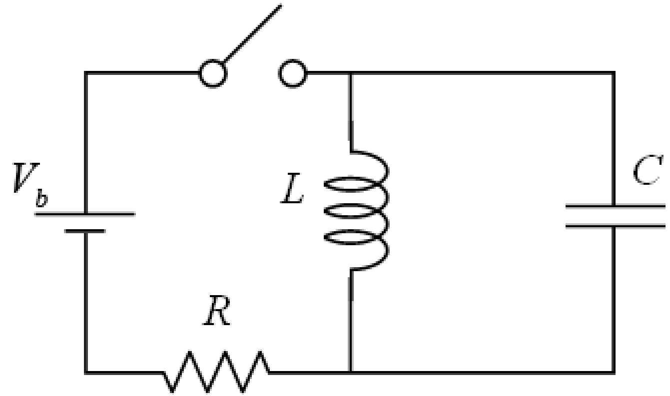
Your Comments

I am not feeling great about this midterm...some of this stuff is really confusing still and I don't know if I can shove everything into my brain in time, especially after spring break. Can you go over which topics will be on the exam Wednesday?

This was a very confusing prelecture. Do you think you could go over thoroughly how the LC circuits work qualitatively?

I may have missed something simple, but in question 1 during the prelecture why does the charge on the capacitor have to be 0 at t=0? I feel like that bit of knowledge will help me with the test wednesday



I remember you mentioning several weeks ago that there was one equation you were going to add to the 2013 equation sheet... which formula was that? Thanks!

Some Exam Stuff

Exam Wed. night (March 27th) at 7:00

- Covers material in Lectures 9 – 18
- Bring your ID: Rooms determined by discussion section (see link)

Don't forget:

- Worked examples in homeworks (the optional questions)
- Other old exams

For most people, taking old exams is most beneficial

Final Exam dates are now posted

The Big Ideas L9-18

Kirchoff's Rules

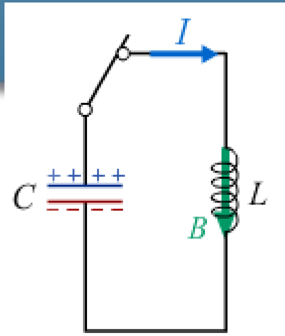
- Sum of voltages around a loop is zero
- Sum of currents into a node is zero
- Kirchoff's rules with capacitors and inductors
  - In RC and RL circuits: charge and current involve exponential functions with time constant: "charging and discharging"
  - E.g. $$IR + \frac{Q}{C} = R\frac{dQ}{dt} + \frac{Q}{C} = V$$
- Capacitors and inductors store energy

Magnetic fields

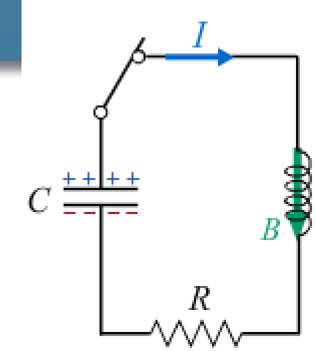
- Generated by electric currents (no magnetic charges)
- Magnetic forces only on charges in motion $\vec{F}_{mag} = q\vec{v}x\vec{B}$
- Easiest to calculate with Ampere's Law $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enclosed}$
- Changing magnetic fields can generate electric fields! FARADAY'S LAW

$$\int \vec{E}\cdot d\vec{\ell} = EMF = \Delta V = -\frac{d}{dt}\int B\cdot dA = -\frac{d\phi_{mag}}{dt}$$

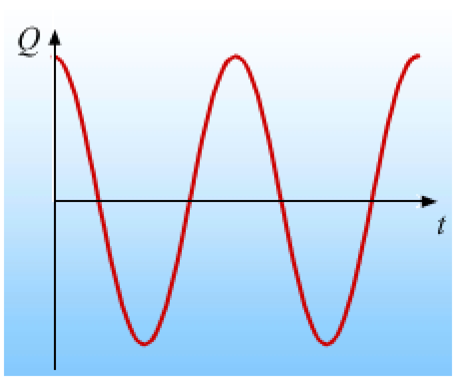
Physics 212
Lecture 19



RLC Circuit



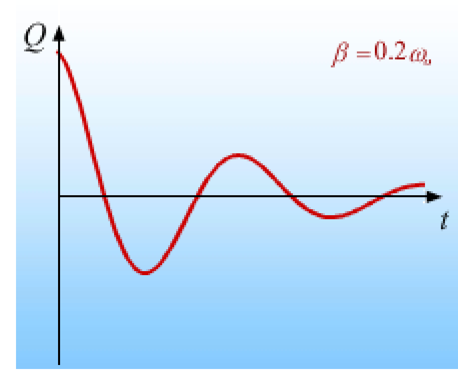
Today's Concepts:



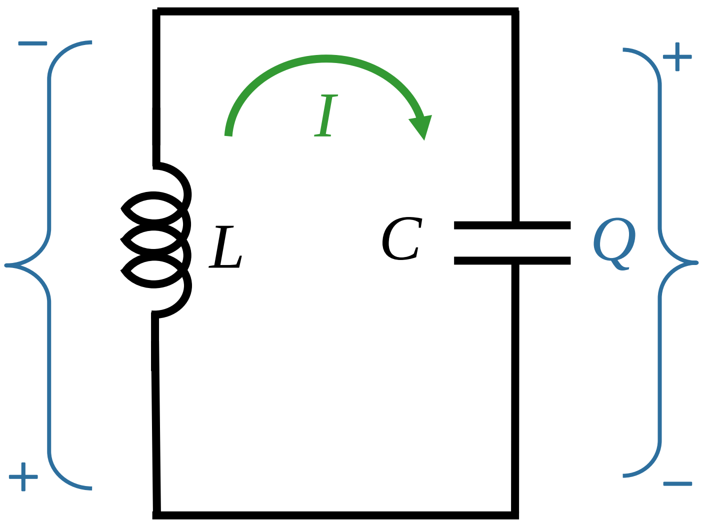
A) Oscillation Frequency

B) Energy

C) Damping



LC Circuit



$$V_L = L\frac{dI}{dt}$$

$$V_C = \frac{Q}{C}$$

Circuit Equation: $\frac{Q}{C} + L\frac{dI}{dt} = 0$

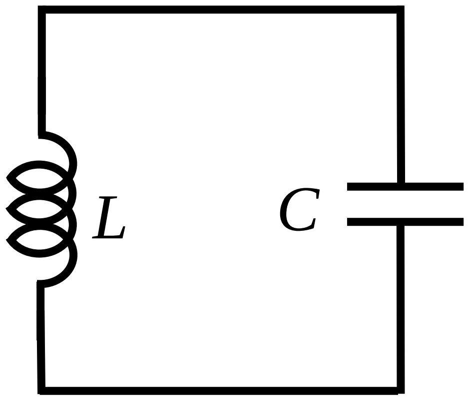
$$I = \frac{dQ}{dt} \quad \rightarrow \quad \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \quad \rightarrow \quad \frac{d^2Q}{dt^2} = -\omega^2 Q$$

where

$$\omega = \frac{1}{\sqrt{LC}}$$

CheckPoint 1a

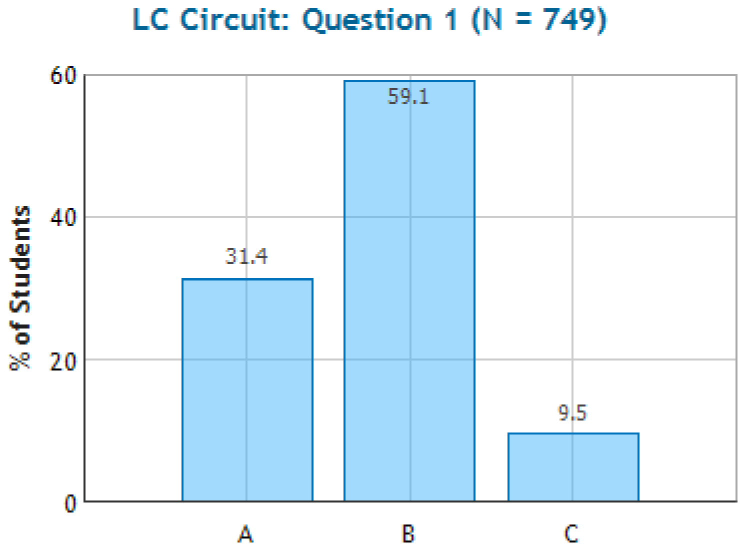
At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



What is the potential difference across the inductor at $t = 0$?

A) $V_L = 0$
B) $V_L = Q_{max}/C$
C) $V_L = Q_{max}/2C$

since $V_L = V_C$



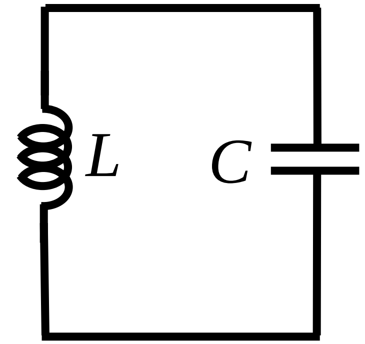
The voltage across the capacitor is Q_{max}/C Kirchhoff's Voltage Rule implies that must also be equal to the voltage across the inductor

Pendulum.

LC Circuits analogous to mass on spring

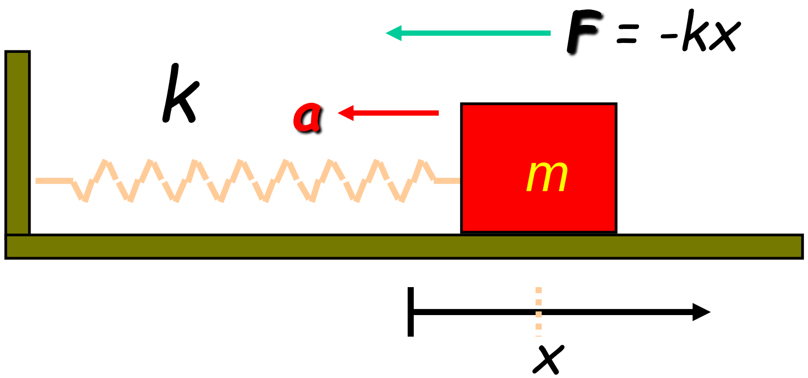
$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

$$\omega = \frac{1}{\sqrt{LC}}$$



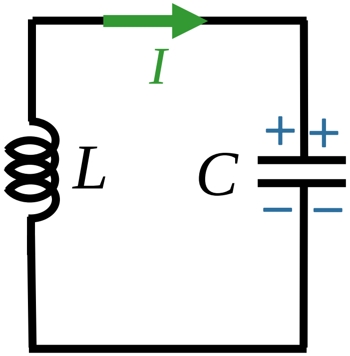
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

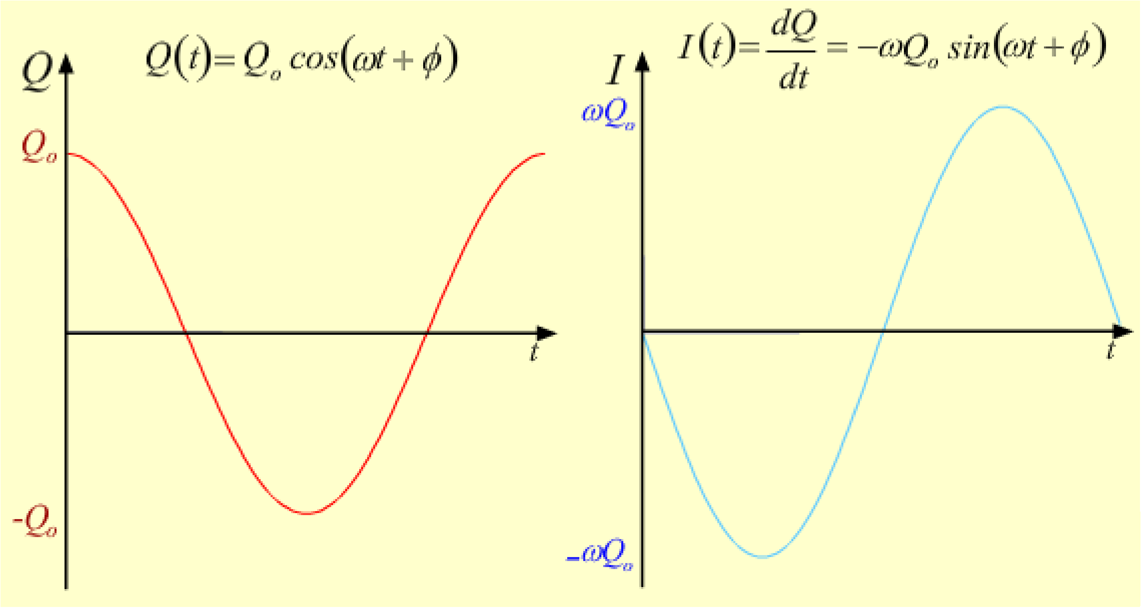
$$\omega = \sqrt{\frac{k}{m}}$$



Same thing if we notice that $k \leftrightarrow \frac{1}{C}$ and $m \leftrightarrow L$

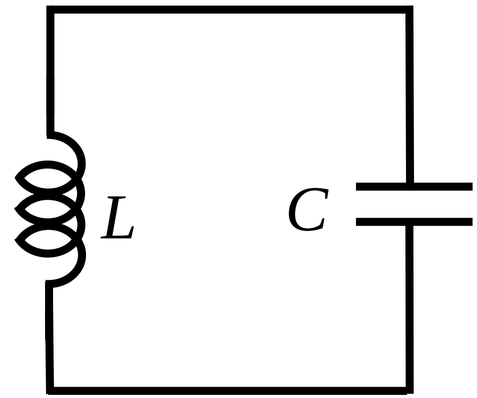
Time Dependence





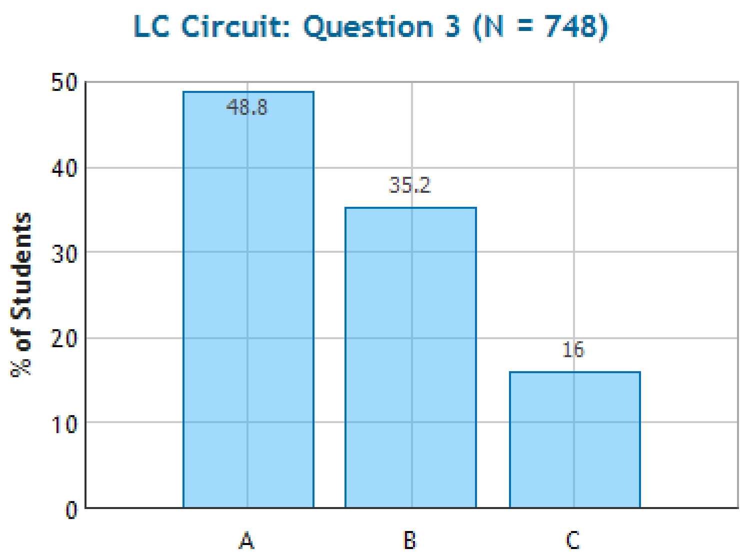
CheckPoint 1b

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



What is the potential difference across the inductor at when the current is maximum?

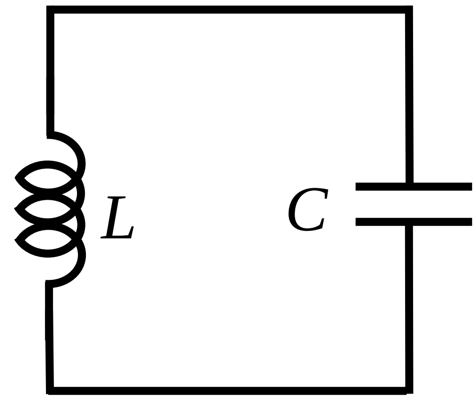
A) $V_L = 0$
B) $V_L = Q_{max}/C$
C) $V_L = Q_{max}/2C$



dI/dt is zero when current is maximum

CheckPoint 1c

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.

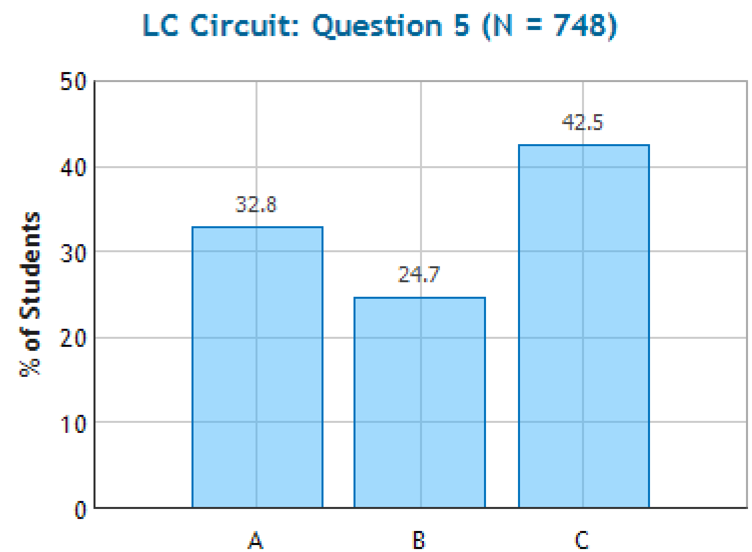


How much energy is stored in the capacitor when the current is a maximum ?

A) $U = Q_{max}^2/(2C)$
B) $U = Q_{max}^2/(4C)$
C) $U = 0$

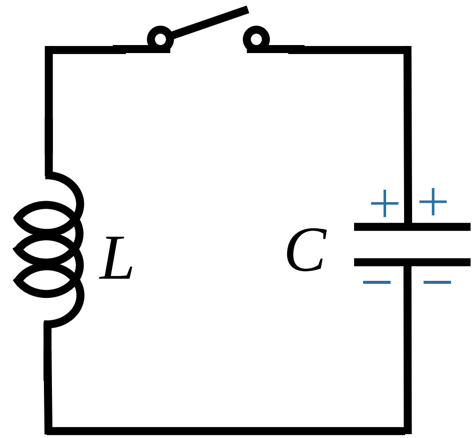
Total Energy is constant!

$U_{Lmax} = ½\, LI_{max}^2$
$U_{Cmax} = Q_{max}^2/2C$
$I = max$ when $Q = 0$



CheckPoint 2a

The capacitor is charged such that the top plate has a charge $+Q_0$ and the bottom plate $-Q_0$. At time $t = 0$, the switch is closed and the circuit oscillates with frequency $\omega = 500$ radians/s.



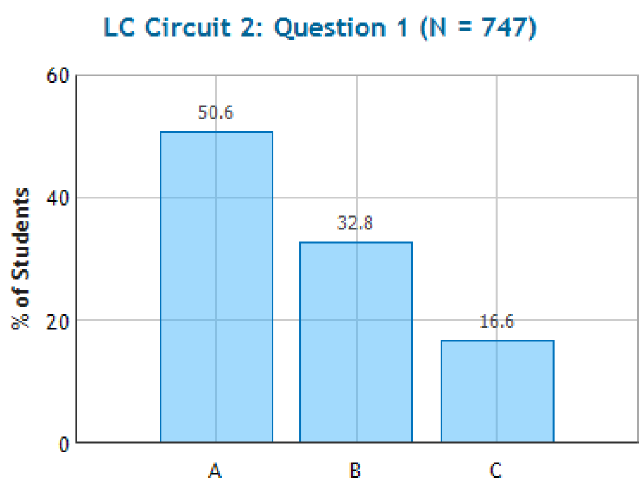
$L = 4 \text{ x } 10^{-3}\ H$

$\omega = 500$ rad/s

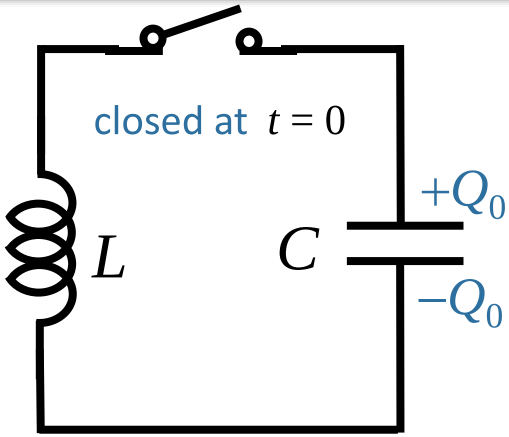
What is the value of the capacitor C?

A) $C = 1 \text{ x } 10^{-3}\ F$
B) $C = 2 \text{ x } 10^{-3}\ F$
C) $C = 4 \text{ x } 10^{-3}\ F$

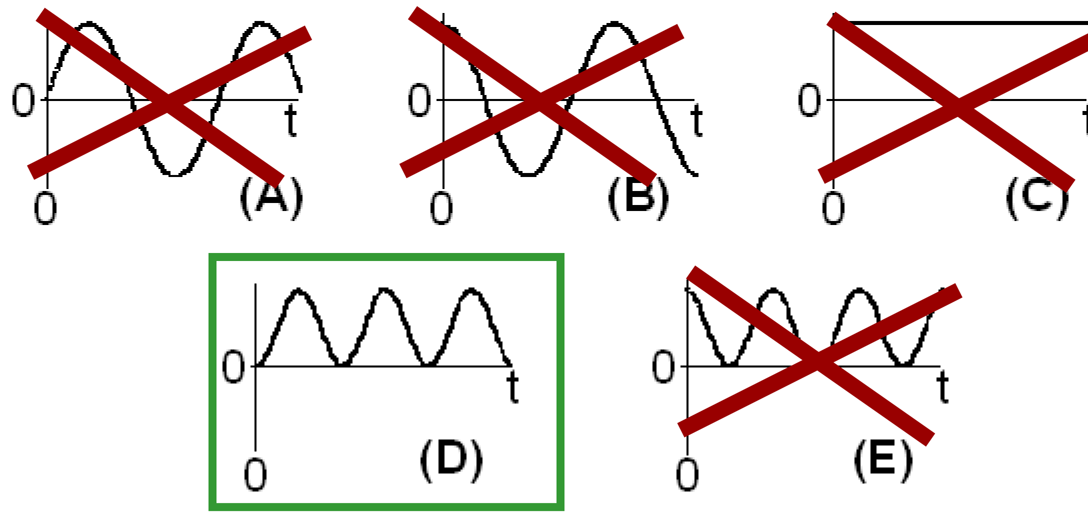
$$\omega = \frac{1}{\sqrt{LC}} \quad \longrightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{(25\times10^4)(4\times10^{-3})} = 10^{-3}$$



CheckPoint 2b



Which plot best represents the energy in the inductor as a function of time starting just after the switch is closed?

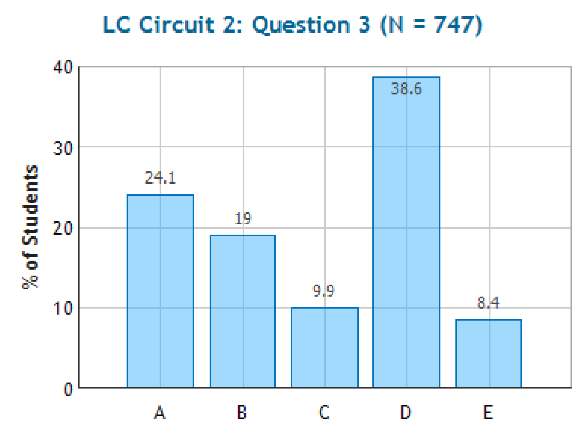


$$U_L = \frac{1}{2} L I^2$$

Energy proportional to $I^2 \Rightarrow C$ cannot be negative

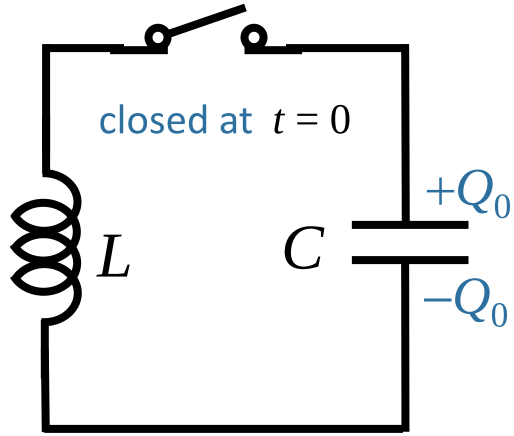
Current is changing $\Rightarrow U_L$ is not constant

Initial current is zero

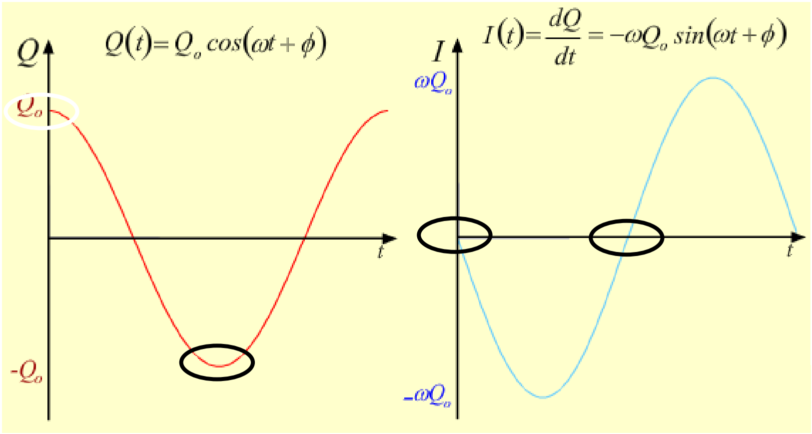


CheckPoint 2c

When the energy stored in the capacitor reaches its maximum again for the first time after $t = 0$, how much charge is stored on the top plate of the capacitor?



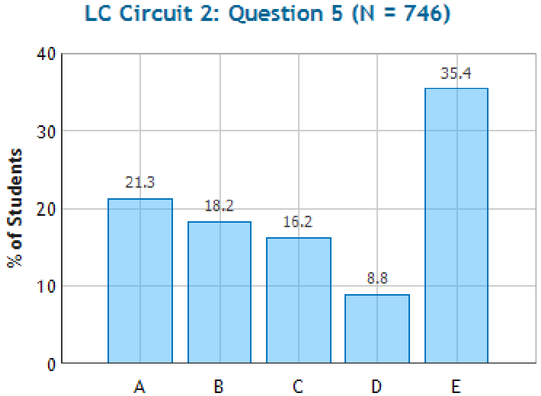
A) $+Q_0$
B) $+Q_0\,/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$



Q is maximum when current goes to zero

$$I = \frac{dQ}{dt}$$

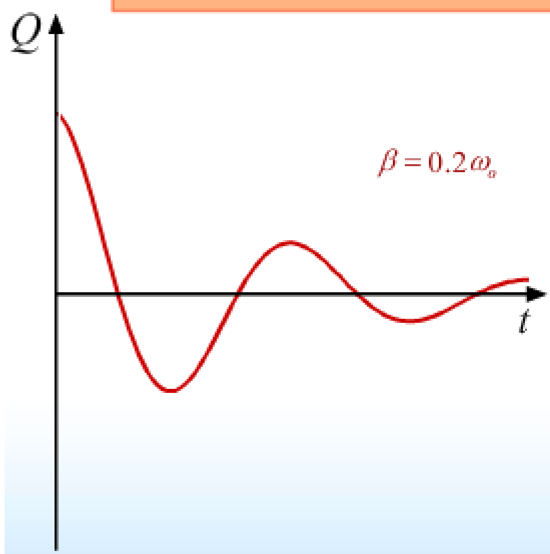
Current goes to zero twice during one cycle

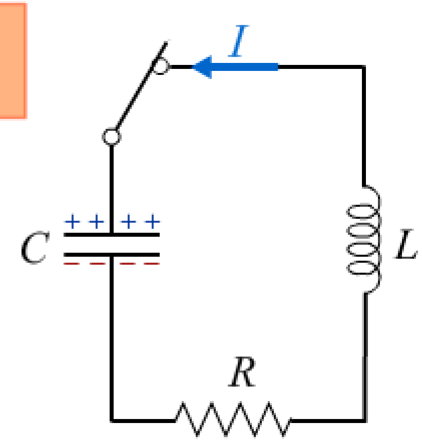


Add R: Damping

Just like LC circuit but energy but the oscillations get smaller because of R

$$Q(t) = Q_o e^{-\beta t} \cos(\omega' t + \phi)$$





Damping Factor $\beta = \dfrac{R}{2L}$

Oscillation Frequency $\omega'^2 = \omega_o^2 - \beta^2$

Natural Frequency $\omega_o = \dfrac{1}{\sqrt{LC}}$

Concept makes sense… …but answer looks kind of complicated

Physics Truth #1:

Even though the answer sometimes looks complicated...

$$Q(t) = Q_o \cos(\omega t - \phi)$$

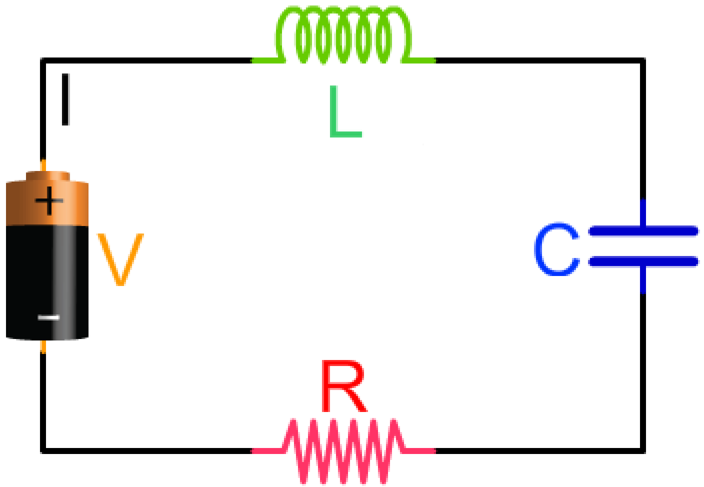
the physics under the hood is still very simple!

$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

The elements of a circuit are very simple:

$$V_L = L\frac{dI}{dt}$$

$$V = V_L + V_C + V_R$$



$$V_C = \frac{Q}{C}$$

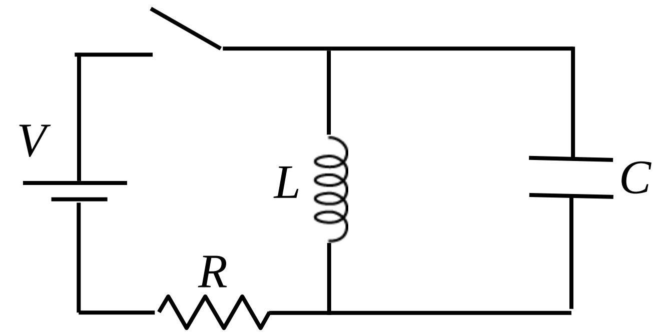
$$I = \frac{dQ}{dt}$$

$$V_R = IR$$

This is all we need to know to solve for anything!

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is Q_{MAX}, the maximum charge on the capacitor?

Conceptual Analysis

Once switch is opened, we have an LC circuit

Current will oscillate with natural frequency ω_0

Strategic Analysis

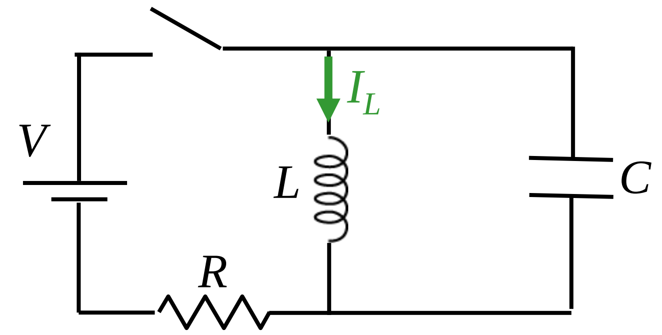
Determine initial current

Determine oscillation frequency ω_0

Find maximum charge on capacitor

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is I_L, the current in the inductor, immediately after the switch is opened? Take positive direction as shown.

A) $I_L < 0$ B) $I_L = 0$ C) $I_L > 0$

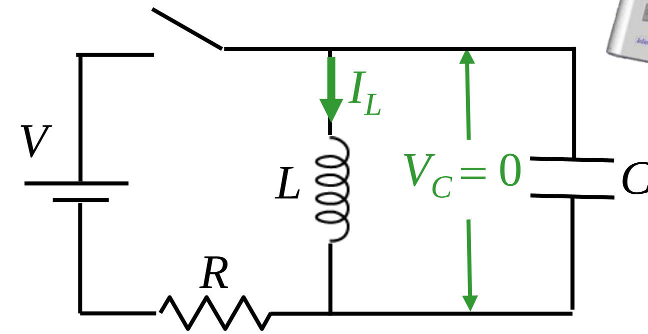
Current through inductor immediately after switch is opened
is the same as
the current through inductor immediately before switch is opened

before switch is opened:

all current goes through inductor in direction shown

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$

The energy stored in the capacitor immediately after the switch is opened is zero.

A) TRUE B) FALSE

before switch is opened:

$dI_L/dt \sim 0 \Rightarrow V_L = 0$

BUT: $V_L = V_C$

since they are in parallel

$\longrightarrow V_C = 0$

after switch is opened:

V_C cannot change abruptly

$\longrightarrow V_C = 0$

$\longrightarrow U_C = ½\, CV_C^2 = 0$!

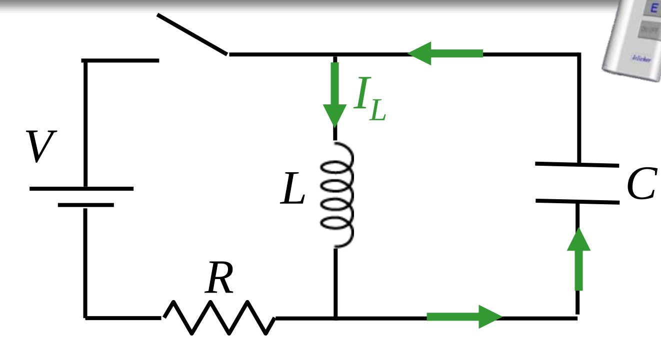
IMPORTANT: NOTE DIFFERENT CONSTRAINTS AFTER SWITCH OPENED

CURRENT through INDUCTOR cannot change abruptly

VOLTAGE across CAPACITOR cannot change abruptly

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$ $\quad$ $V_C(t = 0+) = 0$

What is the direction of the current immediately after the switch is opened?

A) clockwise

B) counterclockwise

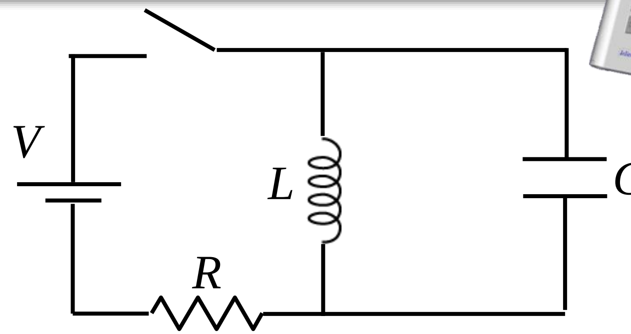
Current through inductor immediately after switch is opened
is the same as
the current through inductor immediately before switch is opened

Before switch is opened: Current moves down through L

After switch is opened: Current continues to move down through L

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$ $\qquad$ $V_C(t = 0+) = 0$

What is the magnitude of the current right after the switch is opened?

A) $I_o = V\sqrt{\frac{C}{L}}$

B) $I_o = \frac{V}{R^2}\sqrt{\frac{L}{C}}$

C) $I_o = \frac{V}{R}$

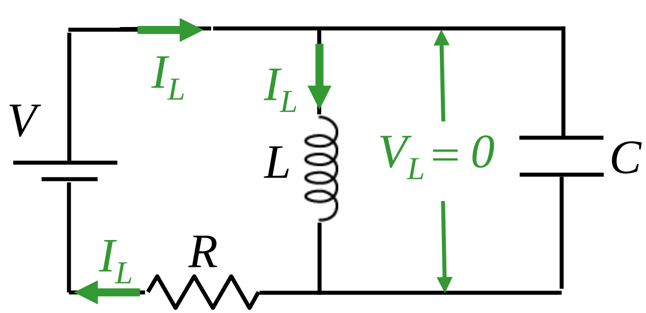
D) $I_o = \frac{V}{2R}$

Current through inductor immediately after switch is opened

is the same as

the current through inductor immediately before switch is opened

Before switch is opened:



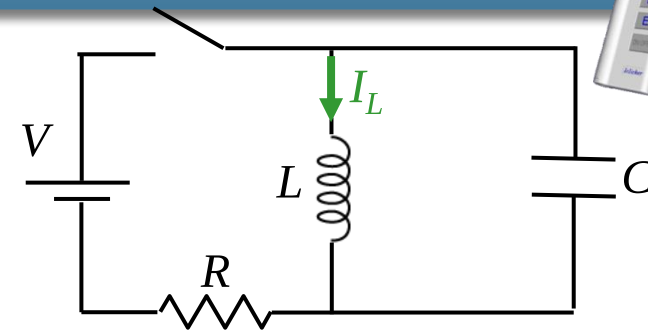
$V_L = 0$

$\downarrow$

$V = I_L R$

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.

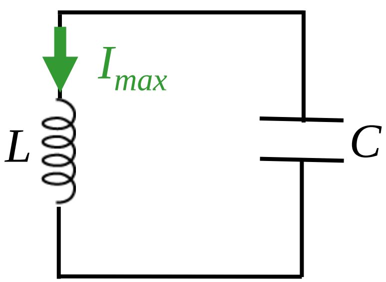


Hint: Energy is conserved

$I_L(t = 0+) = V/R$ $\quad V_C(t = 0+) = 0$

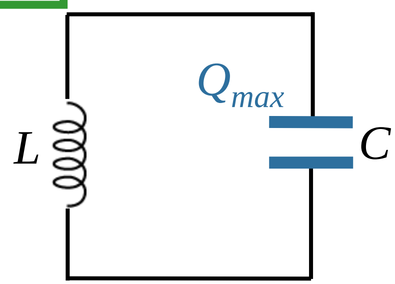
What is Q_{max}, the maximum charge on the capacitor during the oscillations?

A) $Q_{\max} = \frac{V}{R}\sqrt{LC}$ B) $Q_{\max} = \frac{1}{2}CV$ C) $Q_{\max} = CV$ D) $Q_{\max} = \frac{V}{R\sqrt{LC}}$



When I is max
(and Q is 0)

$$U = \frac{1}{2}LI^2$$



When Q is max
(and I is 0)

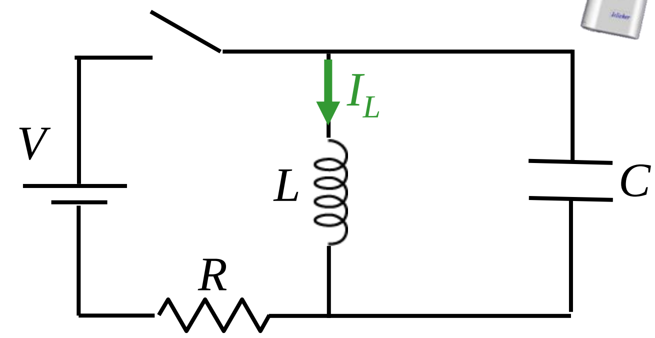
$$U = \frac{1}{2}\frac{Q_{\max}^2}{C}$$

$$\frac{1}{2}LI^2 = \frac{1}{2}\frac{Q_{\max}^2}{C}$$

$$Q_{\max} = I_{\max}\sqrt{LC} = \frac{V}{R}\sqrt{LC}$$

Follow-Up

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



Is it possible for the maximum voltage on the capacitor to be greater than V ?

A) YES

B) NO

$$I_{max} = V/R \qquad Q_{max} = \frac{V}{R}\sqrt{LC}$$

$$Q_{max} = \frac{V}{R}\sqrt{LC} \;\rightarrow\; V_{max} = \frac{V}{R}\sqrt{\frac{L}{C}} \;\rightarrow\;$$

V_{max} can be greater than V IF: $\sqrt{\frac{L}{C}} > R$

We can rewrite this condition in terms of the resonant frequency:

$$\omega_0 L > R \quad \text{OR} \quad \frac{1}{\omega_0 C} > R$$

We will see these forms again when we study AC circuits!