

# Comments: Yours and Ours

Please go over the integrations one more time and please explain how much mathematical knowledge is expected in this class.

Will we need to know how to write an integral like the one shown in the prelecture or in the Question in the prelecture? If so, can we focus a bit more on what each piece means physically?

The integrals, they went by too quickly

I found infinite lines of charge to be slightly confusing in terms of the integration involved.

the balloon popping thing was awesome!!! also what the heck is a pre-flight? i know what a pre-lecture is, we had plenty of those in 211, but pre-flight? is that the same as Checkpoint or something?

The idea of "force per unit charge" is a little hard to get used to, especially in the idea of which  $q$  is which when we write " $F/q = K*(q/r^2)$ ". Also, what is the  $r$  with the pointy accent thing over it at the end of the force equation? It isn't mentioned and doesn't seem to do anything...

Electric fields are pretty powerful things. You should go in to more detail about how awesome they are.

1. Please respect your fellow students - please operate only your own clicker
2. Quite a bit of homework this week and building will be closed Sun/Mon ☹️
3. Don't panic!.....Don't be intimidated by integrals!

# *Electricity & Magnetism*

## *Lecture 2*

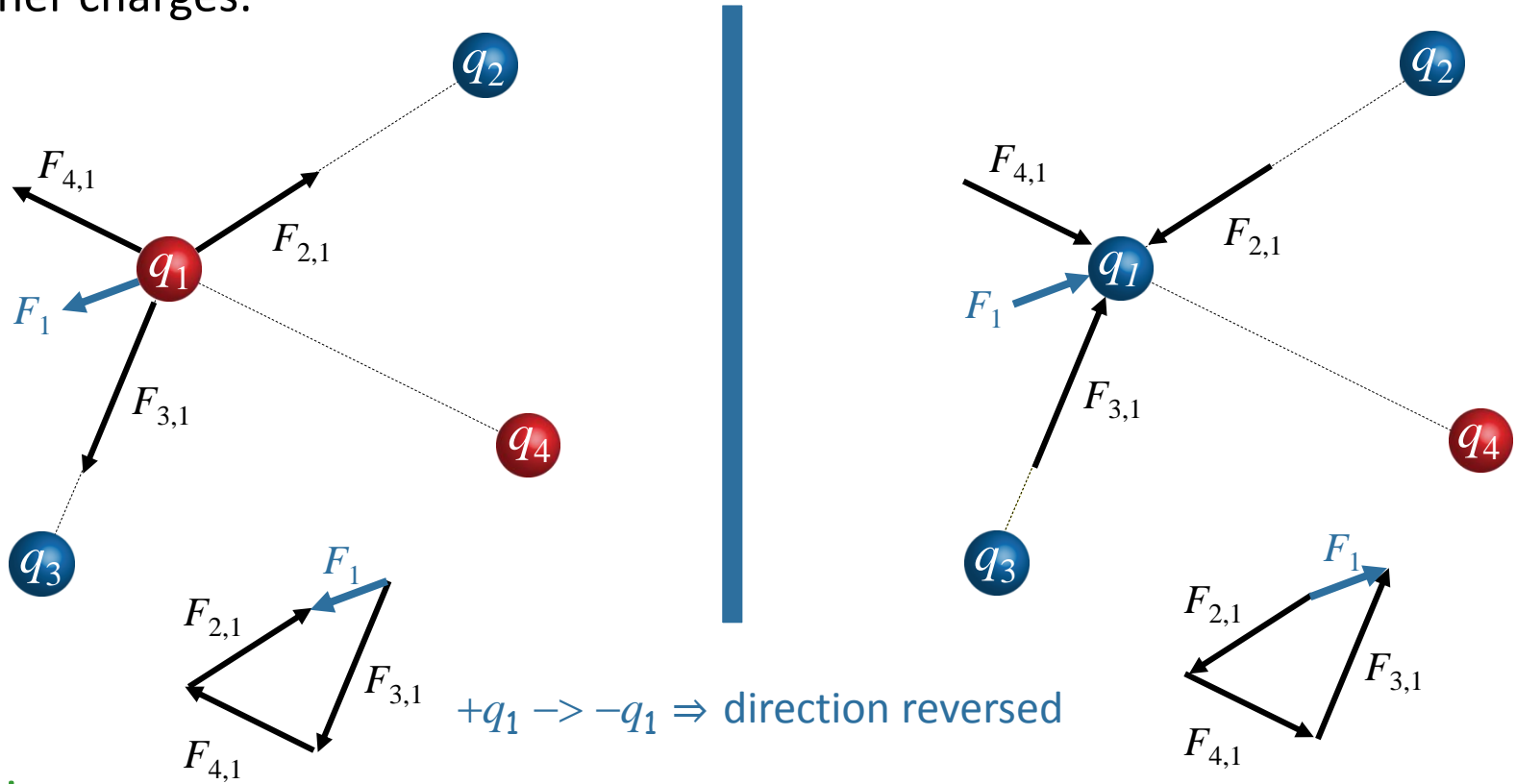
### Today's Concepts:

- A) The Electric Field
- B) Continuous Charge Distributions

I find that relating everything to integrals made me very confused. The concept of electric fields also baffles me - is it simply just a method of quantifying the force from a charge of a specified amount onto another a specified distance away?

# Coulomb's Law (from last time)

If there are more than two charges present, the total force on any given charge is just the **vector sum** of the forces due to each of the other charges:



MATH:

$$\vec{F}_1 = \frac{kq_1q_2}{r_{12}^2} \hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2} \hat{r}_{13} + \frac{kq_1q_4}{r_{14}^2} \hat{r}_{14} \quad \rightarrow \quad \vec{E} = \frac{\vec{F}_1}{q_1} = \frac{kq_2}{r_{12}^2} \hat{r}_{12} + \frac{kq_3}{r_{13}^2} \hat{r}_{13} + \frac{kq_4}{r_{14}^2} \hat{r}_{14}$$

# Electric Field

“Can you explain the derivations of the equations for electric fields? “

“What is the essence of an electric field? “

The electric field  $E$  at a point in space is simply the force per unit charge at that point.

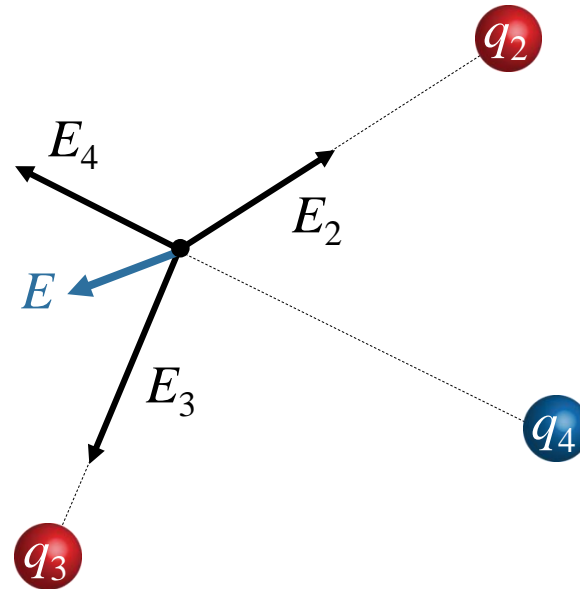
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k \frac{Q}{r^2} \hat{r}$$

Superposition 
$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

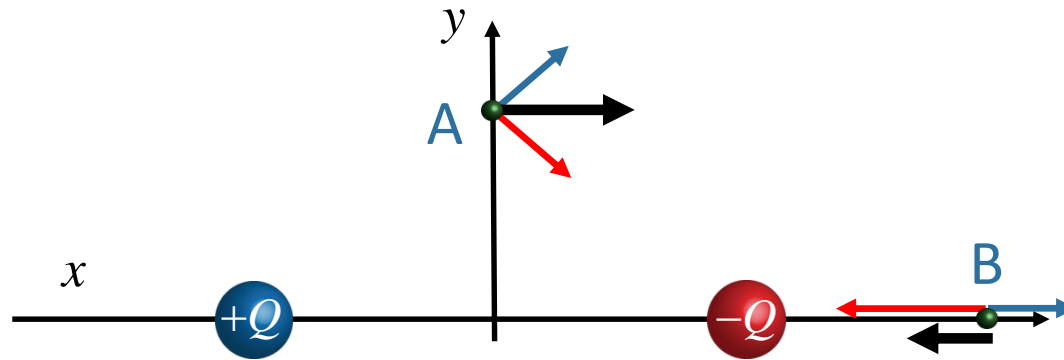
Field points toward negative and  
Away from positive charges.



# Checkpoint



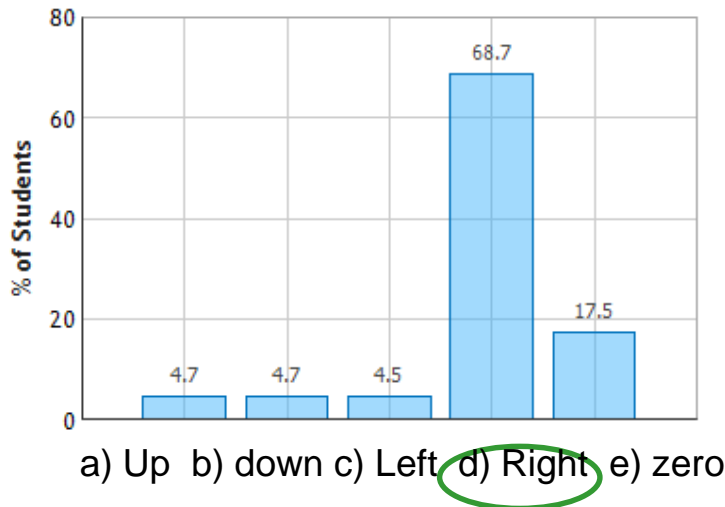
"I can't figure out the field directions at all."



Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

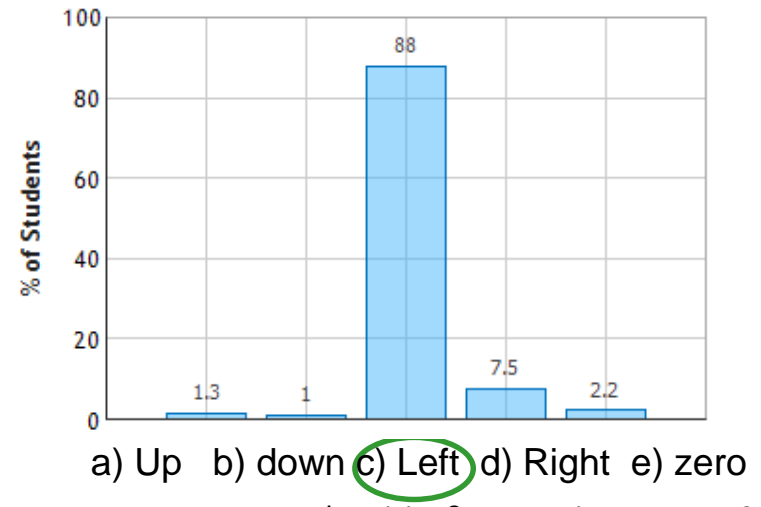
What is direction at point A

Field Directions: Question 1 (N = 817)

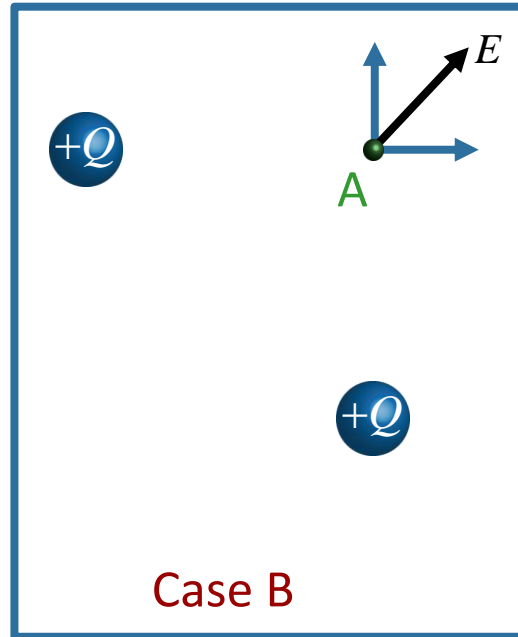
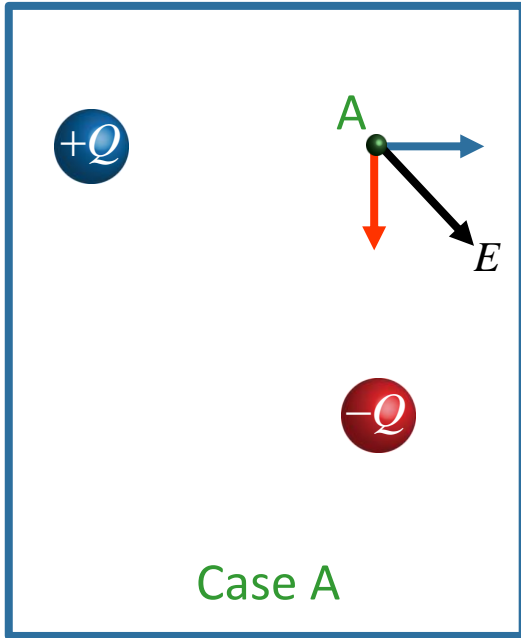


What is direction at point B

Field Directions: Question 3 (N = 815)



# Checkpoint

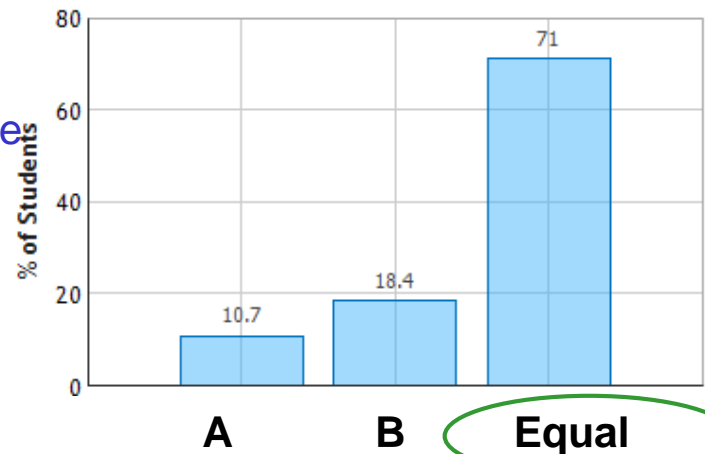


In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest? (Select C if you think they are equal)

“Same because the forces are not canceling each other out”

“in case B, the two charges carry same charges, they will counteract in the line parallel to the line connecting them”

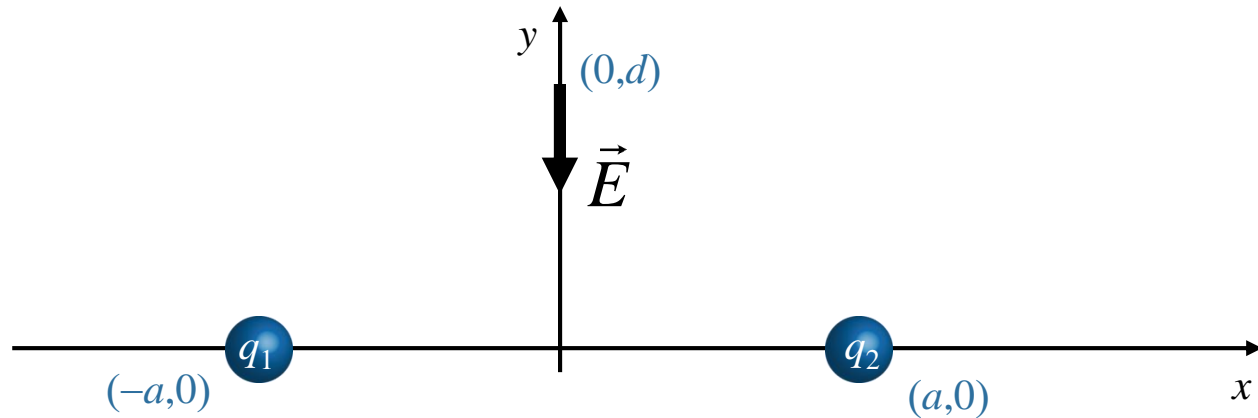
“The electric field of the point charge is  $kq/r^2$  so if two point charges are the same sign the field will be greater”



# Two Charges

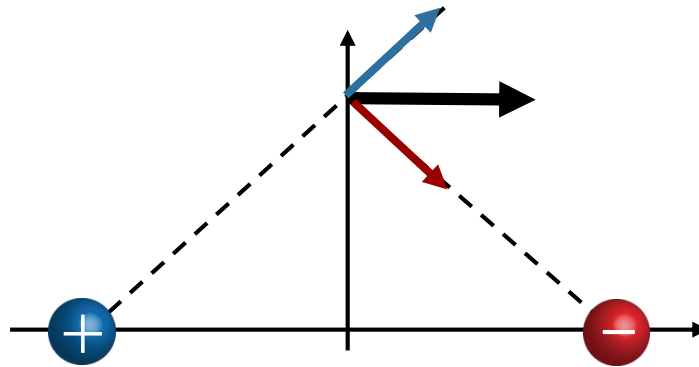
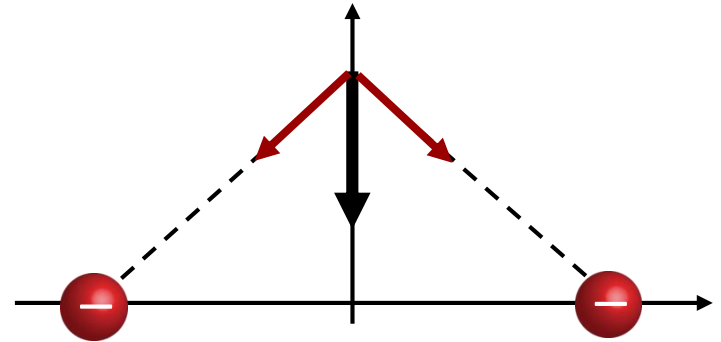
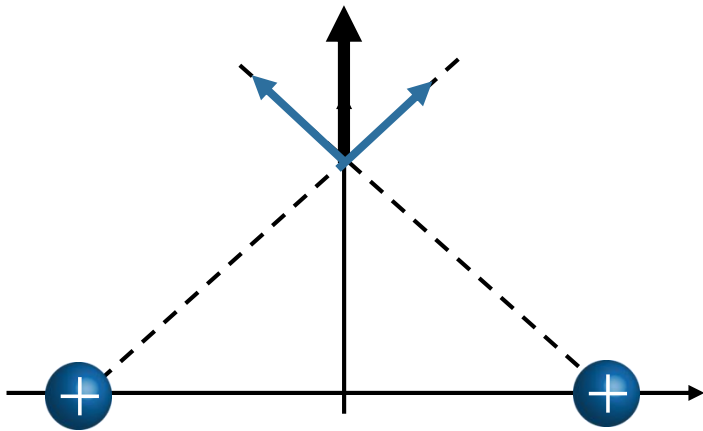


Two charges  $q_1$  and  $q_2$  are fixed at points  $(-a,0)$  and  $(a,0)$  as shown. Together they produce an electric field at point  $(0,d)$  which is directed along the negative  $y$ -axis.



Which of the following statements is true:

- A) Both charges are negative
- B) Both charges are positive
- C) The charges are opposite
- D) There is not enough information to tell how the charges are related



# Checkpoint



A positive test charge  $q$  is released from rest at distance  $r$  away from a charge of  $+Q$  and a distance  $2r$  away from a charge of  $+2Q$ . How will the test charge move immediately after being released?

8) How will the test charge move immediately after being released?

- to the left    to the right    stay still    other

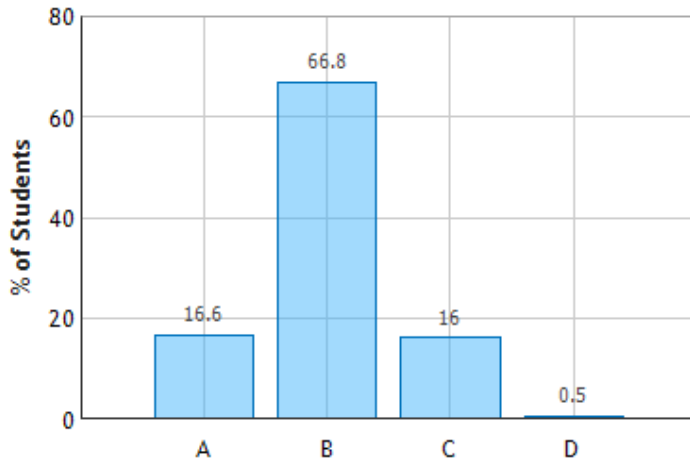
A   B   C   D

INTERESTING: statement is correct, but given in support of “to the left” !!



“(A LEFT) In Coulomb's law, the distance is squared, so a doubling of distance is more significant than a doubling of charge.”

Motion of Test Charge: Question 1 (N = 817)



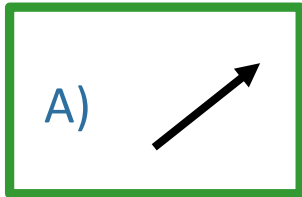
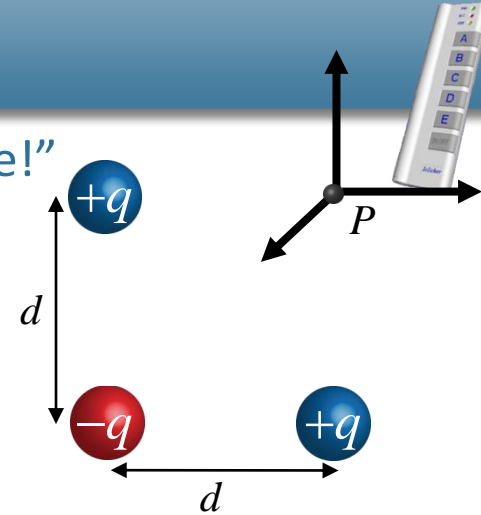
“(B RIGHT) The distance is squared, so the charge on the right hand side would need to be 4 times as large for the particle to remain still..”

“(C Still) The  $+2Q$  is  $2r$  away from the  $q$  so the 2 will cancel out and just be  $+Q$  and  $r$  which is the same as on the left.”

# Example

“Show me more electric field examples, please!”

What is the direction of the electric field at point  $P$ , the unoccupied corner of the square?



C)  $E = 0$

D) Need to know  $d$

E) Need to know  $d$  &  $q$

Calculate  $E$  at point  $P$ .

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

$$E_x = \frac{1}{4\pi\epsilon_o} \left( \frac{q}{d^2} - \frac{q}{(\sqrt{2}d)^2} \cos \frac{\pi}{4} \right)$$

$$E_y = \frac{1}{4\pi\epsilon_o} \left( \frac{q}{d^2} - \frac{q}{(\sqrt{2}d)^2} \sin \frac{\pi}{4} \right)$$

# Continuous Charge Distributions

“I don't understand the whole  $dq$  thing and  $\lambda$ .”

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

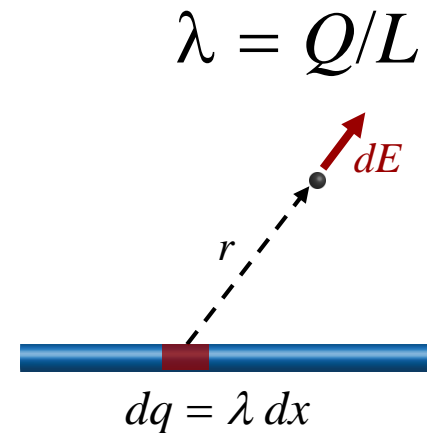
WHAT DOES THIS MEAN ?

Integrate over all charges ( $dq$ )

$r$  is vector from  $dq$  to the point at which  $E$  is defined

Linear Example:

pt for  $E$  •



charges •••••••

# Charge Density



“What are the units for charge density (lambda)? .”

Linear ( $\lambda = Q/L$ ) Coulombs/meter  
Surface ( $\sigma = Q/A$ ) Coulombs/meter<sup>2</sup>  
Volume ( $\rho = Q/V$ ) Coulombs/meter<sup>3</sup>

## Some Geometry

$$A_{sphere} = 4\pi R^2$$

$$A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \frac{4}{3}\pi R^3$$

$$V_{cylinder} = \pi R^2 L$$

What has more net charge?.

- A) A sphere w/ radius 2 meters and volume charge density  $\rho = 2 \text{ C/m}^3$
- B) A sphere w/ radius 2 meters and surface charge density  $\sigma = 2 \text{ C/m}^2$
- C) Both A) and B) have the same net charge.

$$Q_A = \rho V = \rho \frac{4}{3}\pi R^3$$

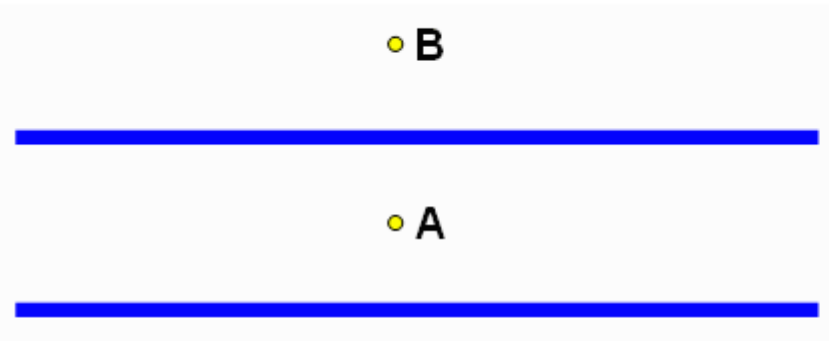
$$Q_B = \sigma A = \sigma 4\pi R^2$$



$$\frac{Q_A}{Q_B} = \frac{\rho \frac{4}{3}\pi R^3}{\sigma 4\pi R^2} = \frac{1}{3} \frac{\rho}{\sigma} R$$

# Checkpoint

Two infinite lines of charge are shown below.



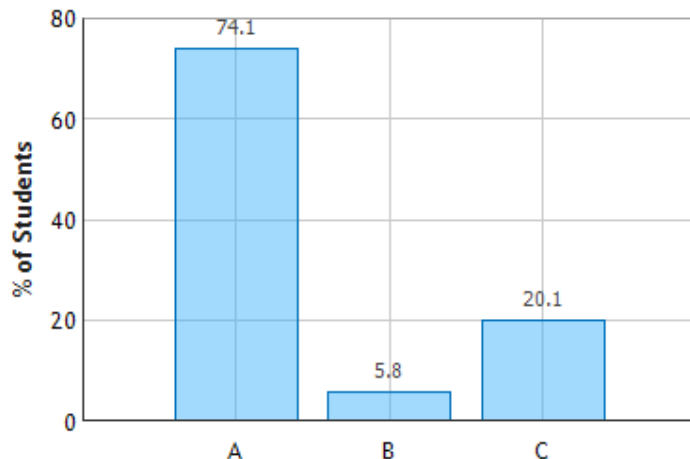
Both lines have identical charge densities  $+\lambda$  C/m. Point A is equidistant from both lines and Point B is located above the top line as shown. How does  $E_A$ , the magnitude of the electric field at point A, compare to  $E_B$ , the magnitude of the electric field at point B?

- $E_A < E_B$
- $E_A = E_B$
- $E_A > E_B$

A) ( $E_A < E_B$ ) “Electric Field at point **A** cancels out to be zero and electric field at point **B** experiences **E** field from both line to move upward.”

C) ( $E_A > E_B$ ) “**A** gets more of the field because it is close to both lines of charge. **B** gets less of the field because it is close to one and far away from the other.”

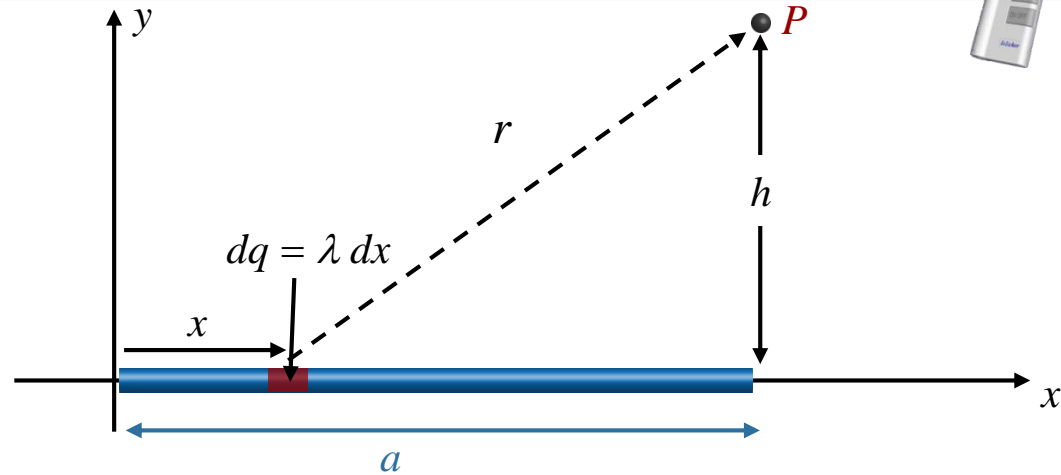
Two Lines of Charge: Question 1 (N = 815)



# Calculation

“How is the integration of  $dE$  over  $L$  worked out, step by step?”

Charge is uniformly distributed along the  $x$ -axis from the origin to  $x = a$ . The charge density is  $\lambda$  C/m. What is the  $x$ -component of the electric field at point  $P$ :  $(x,y) = (a,h)$ ?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What is  $\frac{dq}{r^2}$  ?

A)  $\frac{dx}{x^2}$

B)  $\frac{dx}{a^2 + h^2}$

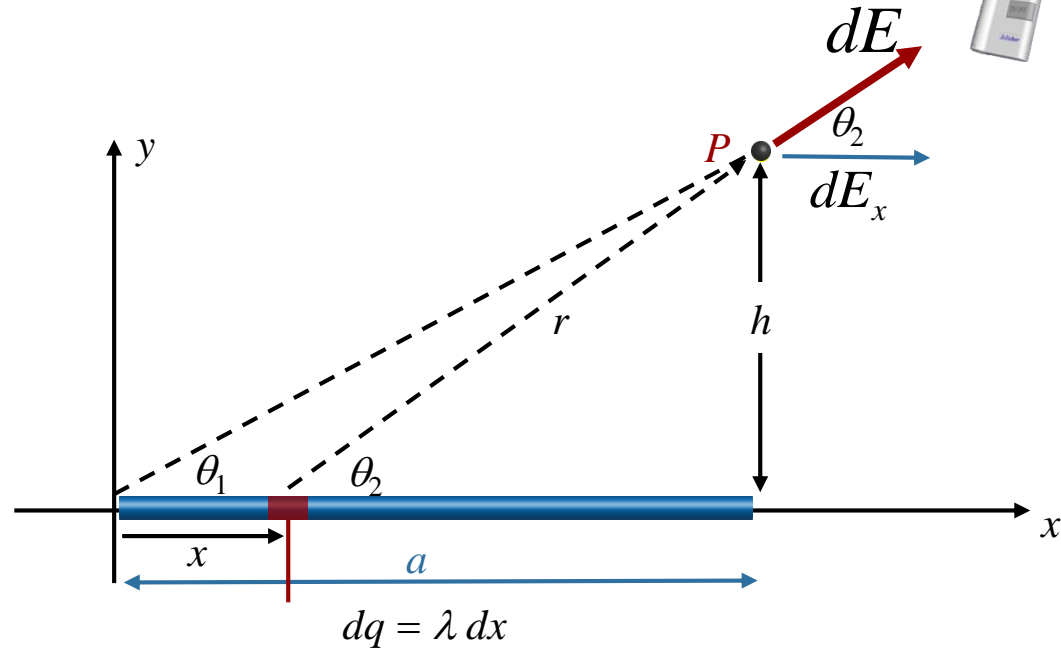
C)  $\frac{\lambda dx}{a^2 + h^2}$

D)  $\frac{\lambda dx}{(a-x)^2 + h^2}$

E)  $\frac{\lambda dx}{x^2}$

# Calculation

Charge is uniformly distributed along the  $x$ -axis from the origin to  $x = a$ . The charge density is  $\lambda$  C/m. What is the  $x$ -component of the electric field at point  $P$ :  $(x,y) = (a,h)$ ?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

We want:

$$E_x = \int dE_x$$

What is  $dE_x$ ?

A)  $dE \cos \theta_1$

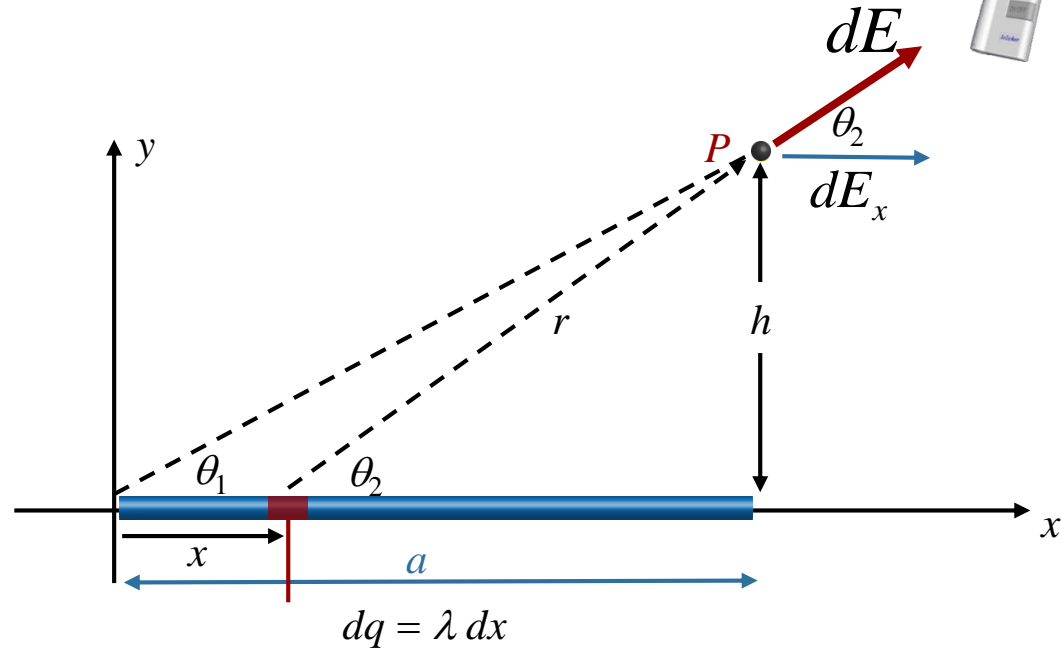
**B)  $dE \cos \theta_2$**

C)  $dE \sin \theta_1$

D)  $dE \sin \theta_2$

# Calculation

Charge is uniformly distributed along the  $x$ -axis from the origin to  $x = a$ . The charge density is  $\lambda$  C/m. What is the  $x$ -component of the electric field at point  $P$ :  $(x,y) = (a,h)$ ?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x = \int dE \cos \theta_2$$

What is  $E_x$  ?

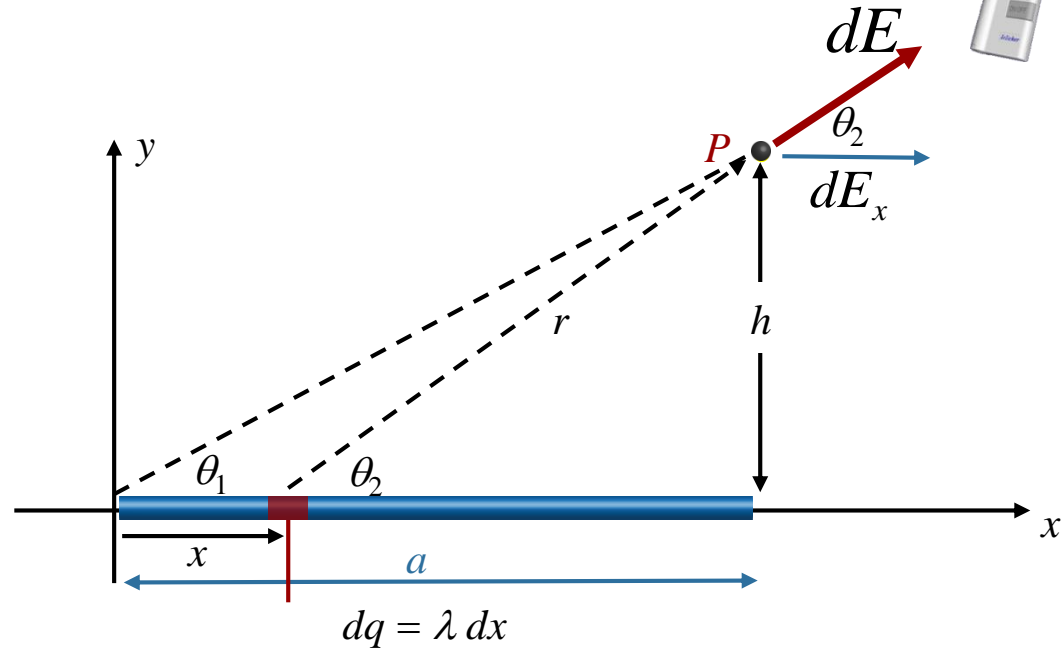
$$\text{A) } \int_0^a \frac{k\lambda \cos \theta_2 dx}{(a-x)^2 + h^2}$$

$$\text{B) } \lambda k \cos \theta_2 \int_0^a \frac{dx}{(a-x)^2 + h^2}$$

C) A and B are both OK       $\cos \theta_2$  **DEPENDS ON**  $x$ !

# Calculation

Charge is uniformly distributed along the  $x$ -axis from the origin to  $x = a$ . The charge density is  $\lambda$  C/m. What is the  $x$ -component of the electric field at point  $P$ :  $(x,y) = (a,h)$ ?



$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

We know:

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x = \int dE \cos \theta_2$$

What is  $\cos \theta_2$  ?

A)  $\frac{x}{\sqrt{a^2 + h^2}}$

B)  $\frac{a-x}{\sqrt{(a-x)^2 + h^2}}$

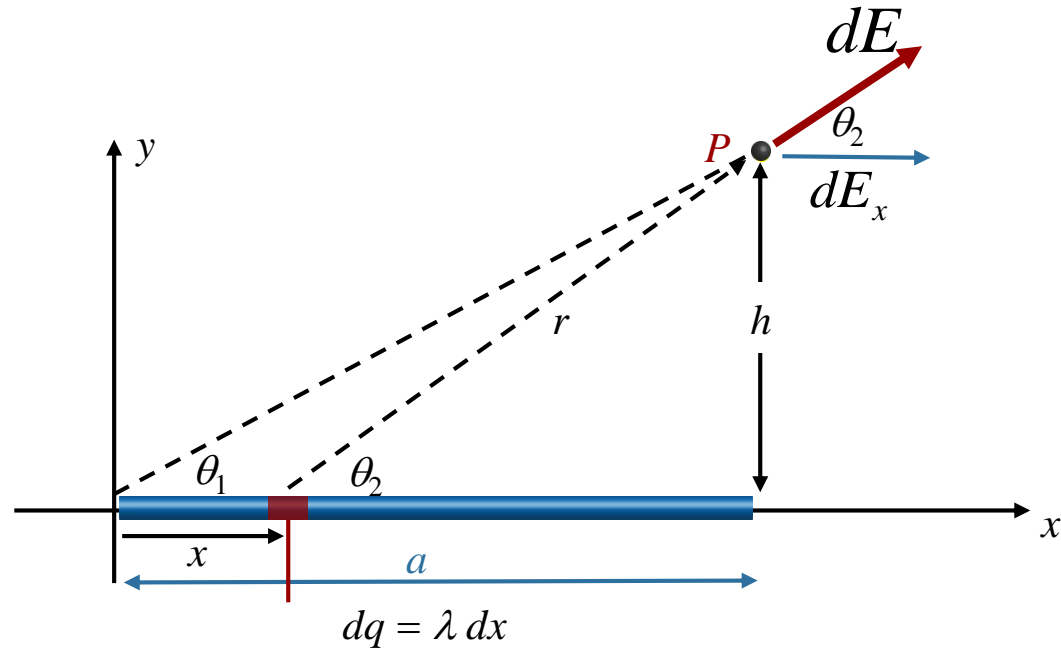
C)  $\frac{a}{\sqrt{a^2 + h^2}}$

D)  $\frac{a}{\sqrt{(a-x)^2 + h^2}}$



# Calculation

Charge is uniformly distributed along the  $x$ -axis from the origin to  $x = a$ . The charge density is  $\lambda$  C/m. What is the  $x$ -component of the electric field at point  $P$ :  $(x,y) = (a,h)$ ?



We know: 
$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2} \quad E_x = \int dE_x = \int dE \cos \theta_2 \quad \cos \theta_2 = \frac{a-x}{\sqrt{(a-x)^2 + h^2}}$$

What is  $E_x(P)$  ?

$$E_x(P) = \lambda k \int_0^a dx \frac{a-x}{((a-x)^2 + h^2)^{3/2}} \quad \longrightarrow \quad E_x(P) = \frac{\lambda k}{h} \left( 1 - \frac{h}{\sqrt{h^2 + a^2}} \right)$$

# Homework Problem

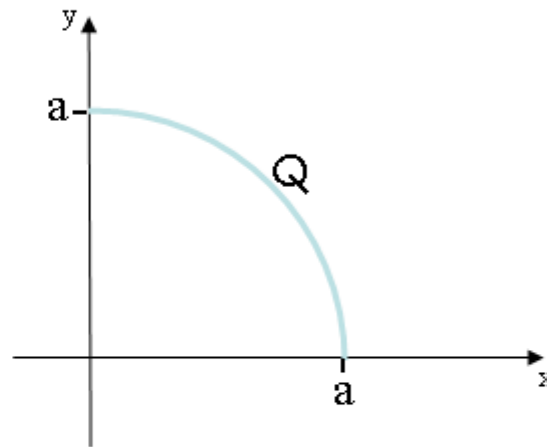
## Homework: Coulomb's Law

Deadline: 100% until Tuesday, January 22 at 8:00 AM

### Electric Field from Arc of Charge

1 2 3 4 5

A total charge  $Q = -4.2 \mu\text{C}$  is distributed uniformly over a quarter circle arc of radius  $a = 7.7 \text{ cm}$  as shown.



1) What is  $\lambda$  the linear charge density along the arc?

C/m

2) What is  $E_x$ , the value of the x-component of the electric field at the origin  $(x,y) = (0,0)$ ?

N/C