Your Comments

I work as an audio technician, and I've seen Q before as a quantity you adjust in variable equalizers! If you want to eliminate a broad spectrum of sound centered around a certain frequency you dial back the Q value; and vice-versa for cutting out a very specific frequency. So in reality I'm just dialing back the resistance in an LCR circuit!

Can you explain why power is stepped up for transmission? My old man tried to explain it to me once but it's been quite some time since his days as an undergrad.

I keep falling asleep during lecture whenever you give us permission and then I feel like I miss so much when I go to do the homework. Please don't give us permission to fall asleep anymore.

In lecture, do you think you could give a more qualitative representation of relative voltages and currents at resonance? If I can reason a rough estimate of how a circuit works without applying equations, I feel I get a much better grasp of the concept.

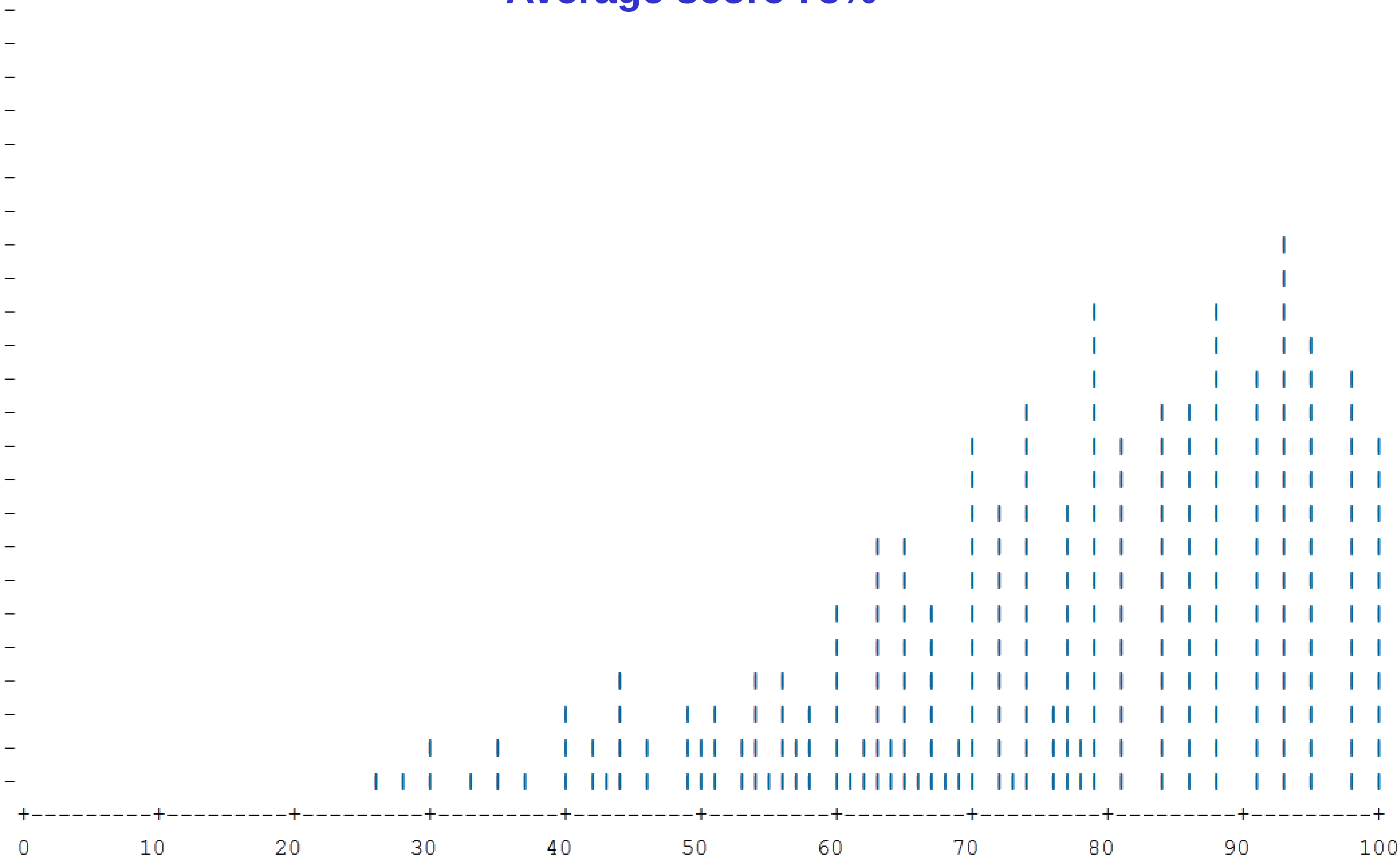
I feel that this lecture on transformers would be more exciting if it were directed by Michael Bay. At least try and fit an explosion or giant robot or something into the lecture.

Wow this was ridiculous. Just lots of equations being thrown at me. Thank you.

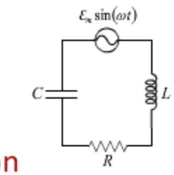
something different needs to happen for this unit. I have always been throughly impressed by how much the Physics department cares about our understanding of the material. You do a phenomenal job, every week, with this well thought out plan for our better understanding. this set of lessons, however, with all the phases and everything, is still just really really confusing. I think that something special should happen in discussion or something (maybe it will, I have it tomorrow morning)

Nice work on hour exam 2

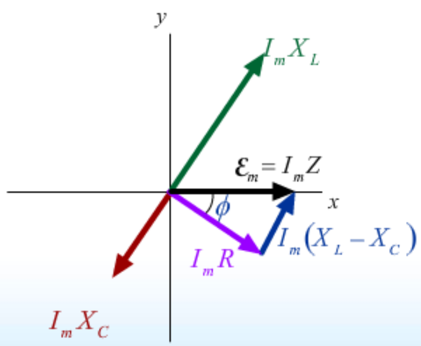
Average score 75%



Physics 212
Lecture 21



Voltage Phasor Diagram



Phase Relation

$$\tan\phi = \frac{X_L - X_C}{R}$$

Impedance

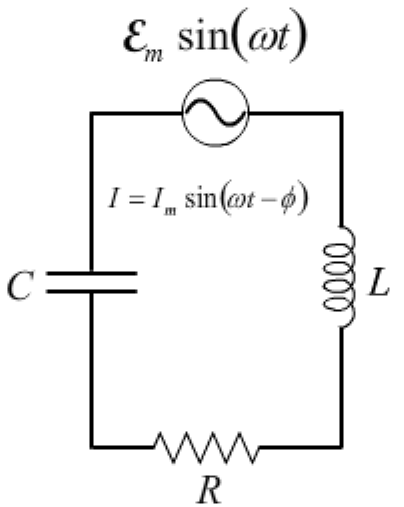
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Maximum Current

$$I_m = \frac{\mathcal{E}_m}{Z}$$

Looks intimidating, but isn't bad!

The Driven LCR Circuit



Frequency Dependence of Maximum Current

$$I_m = \frac{\mathcal{E}_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{(x^2-1)^2}{x^2}}}$$

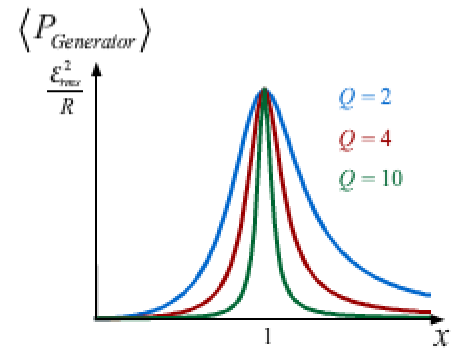
Quality Factor

$$Q \equiv 2\pi \left[\frac{U_{max}}{\Delta U}\right]_{cycle} \xrightarrow{\text{evaluate at}} \omega = \omega_o$$

Average Power per Cycle

$$\langle P_{Generator} \rangle = \frac{\mathcal{E}_{rms}^2}{R} \frac{x^2}{x^2 + Q^2 (x^2-1)^2}$$

where $x \equiv \frac{\omega}{\omega_o}$ & $Q^2 = \frac{L}{R^2 C}$



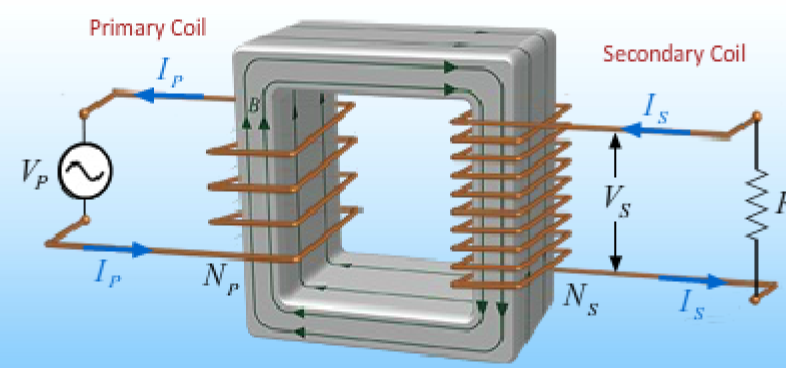
Transformers

Voltage Relation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Current Relation

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$



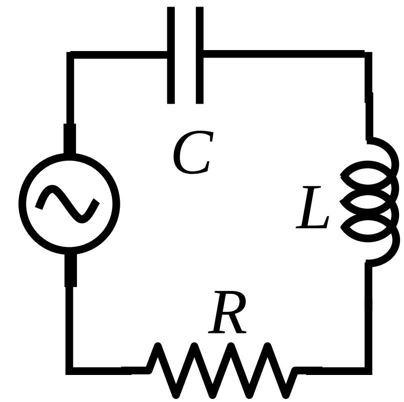
Peak AC Problems

"Ohms" Law for each element

NOTE: Good for PEAK values only)

$$V_{gen} = I_{max} Z$$
$$V_{Resistor} = I_{max} R$$
$$V_{inductor} = I_{max} X_L$$
$$V_{Capacitor} = I_{max} X_C$$

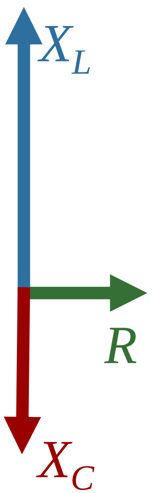
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$X_L = \omega L$$
$$X_C = \frac{1}{\omega C}$$



Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

$$X_L = \omega L = 200\Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 122\Omega$$
$$X_C = \frac{1}{\omega C} = 100\Omega$$
$$I_{max} = \frac{V_{gen}}{Z} = 0.13A$$



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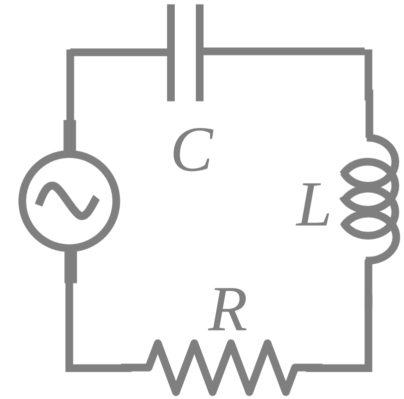


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

Which element has the largest peak voltage across it?

A) Generator

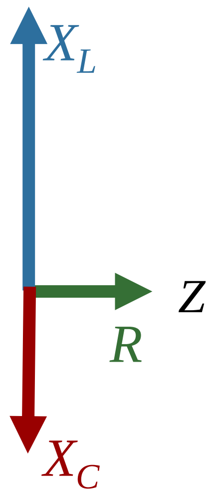
B) Inductor

C) Resistor

D) Capacitor

E) All the same.

$$V_{max} = I_{max} X$$



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Peak AC Problems

"Ohms" Law for each element

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$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L$$

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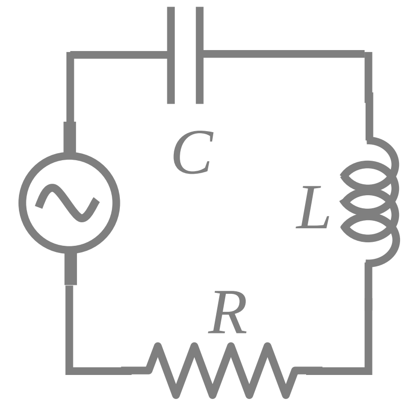


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

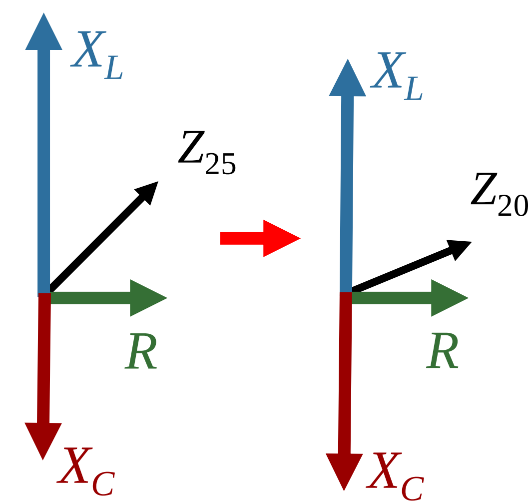
What happens to the impedance if we decrease the angular frequency to 20 rad/sec?

A) *Z* increases

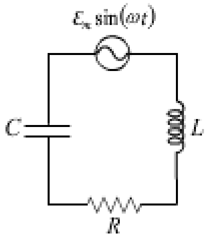
B) *Z* remains the same

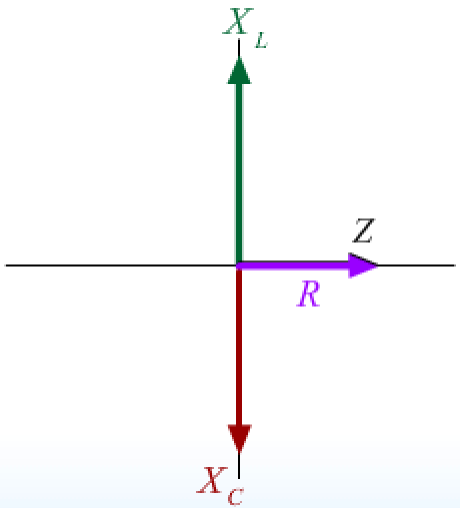
C) *Z* decreases

$(X_L - X_C)$: (200 – 100) -> (160 – 125)



Resonance





Resonance

I_m is a maximum $\longrightarrow$ $I_m = \dfrac{\mathcal{E}_m}{R}$

$\omega = \omega_o$

Z minimized $\longrightarrow$ $X_L = X_C$

$\phi = 0^o$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Light-bulb Demo

Resonance

Frequency at which voltage across inductor and capacitor cancel

R is independent of ω

X_L increases with ω

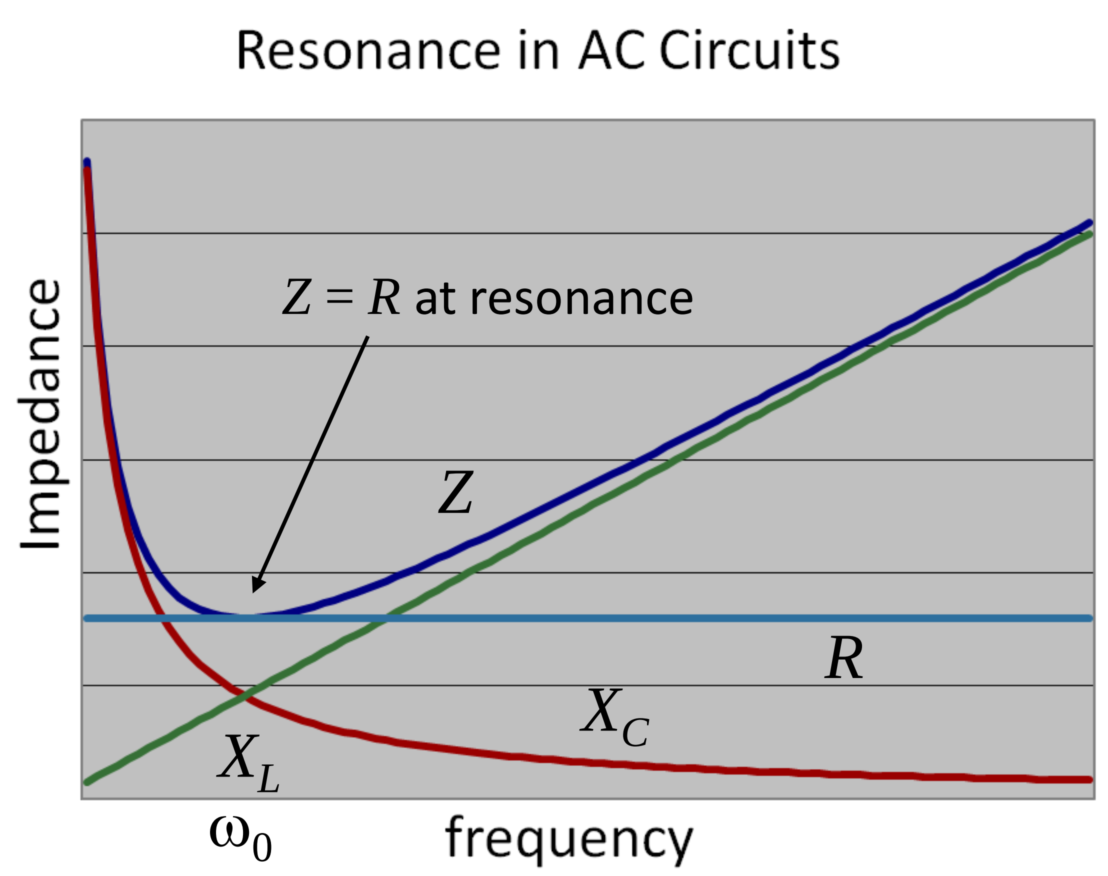
$$X_L = \omega L$$

X_C increases with $1/\omega$

$$X_C = \frac{1}{\omega C}$$

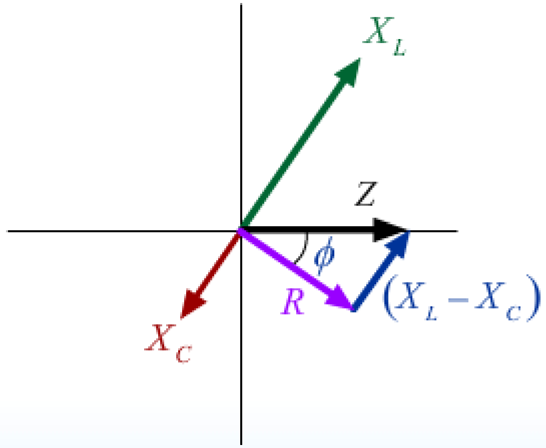
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is minimum at resonance



Resonance: $X_L = X_C$ $\omega_0 = \frac{1}{\sqrt{LC}}$

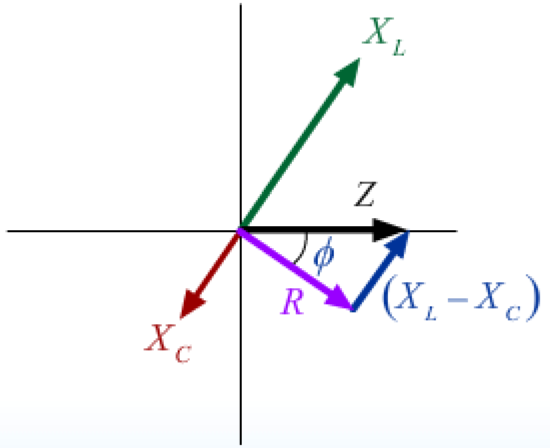
Off Resonance



$$I_m = \frac{\mathcal{E}_m}{Z}$$

$$I_m = \frac{\mathcal{E}_m}{R} \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Z

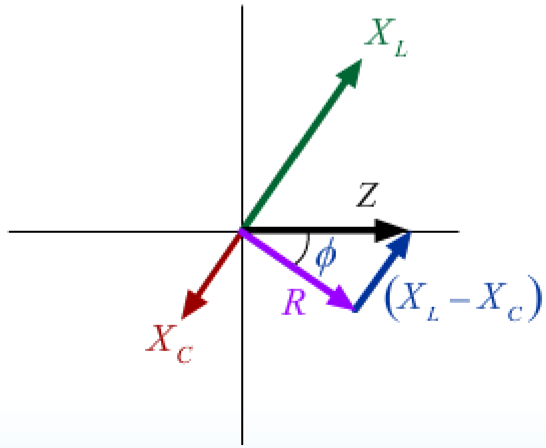


$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C} \rightarrow Q \equiv 2\pi \frac{U_{\max}}{\Delta U}$$

$$I_m = \frac{\mathcal{E}_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{(x^2 - 1)^2}{x^2}}}$$

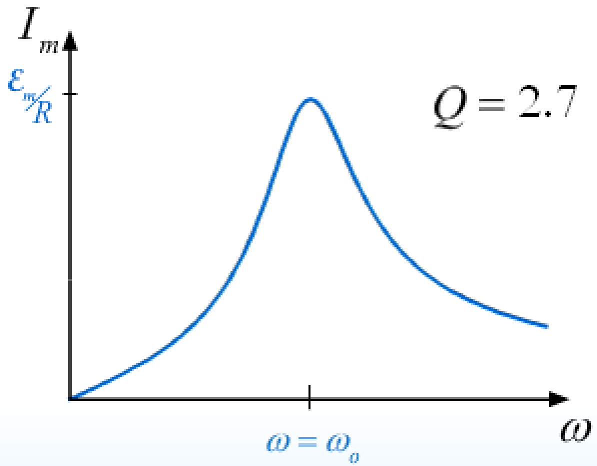
U_{max} = max energy stored
ΔU = energy dissipated
in one cycle at resonance

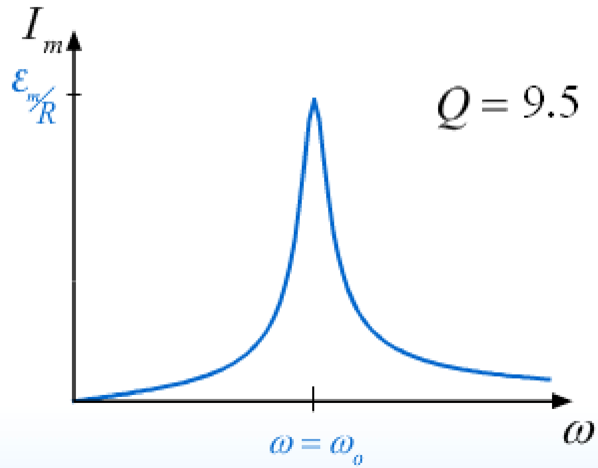
Off Resonance



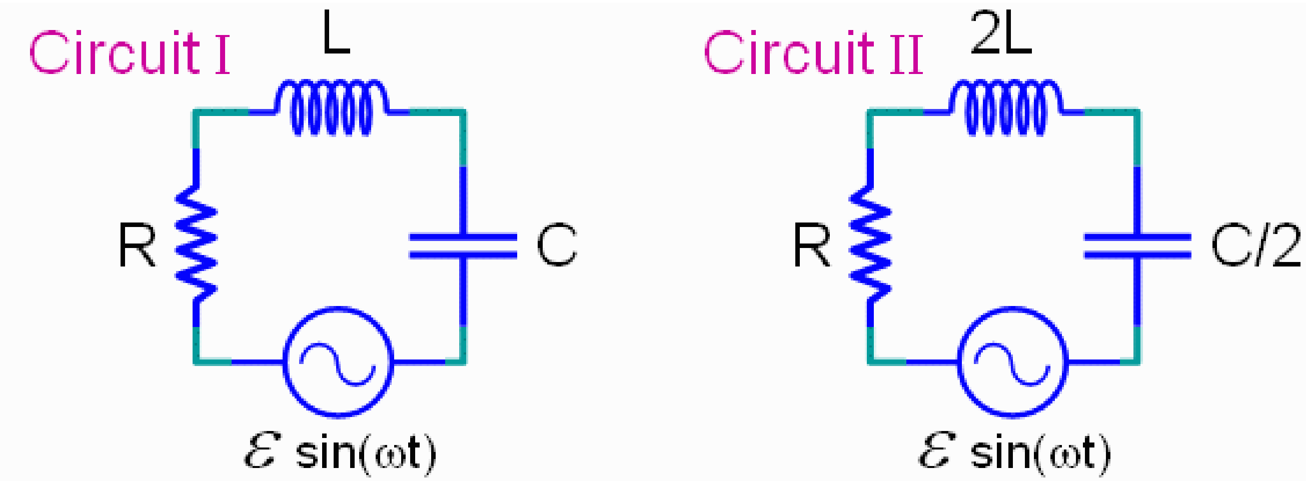
$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C}$$

$$I_m = \frac{\varepsilon_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{\left(x^2 - 1\right)^2}{x^2}}}$$





CheckPoint 1a



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

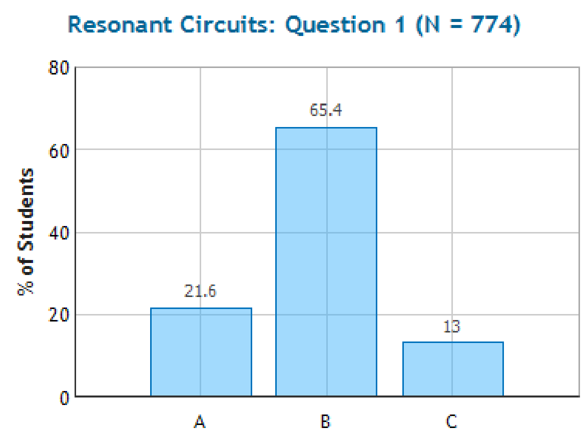
Compare the peak voltage across the resistor in the two circuits

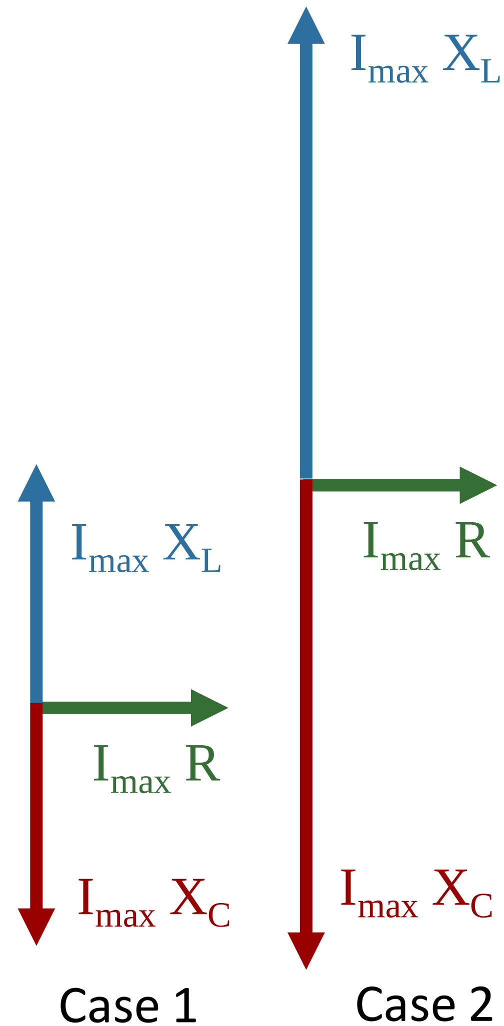
A. $V_I > V_{II}$ **B.** $V_I = V_{II}$ **C.** $V_I < V_{II}$

Resonance: $X_L = X_C$

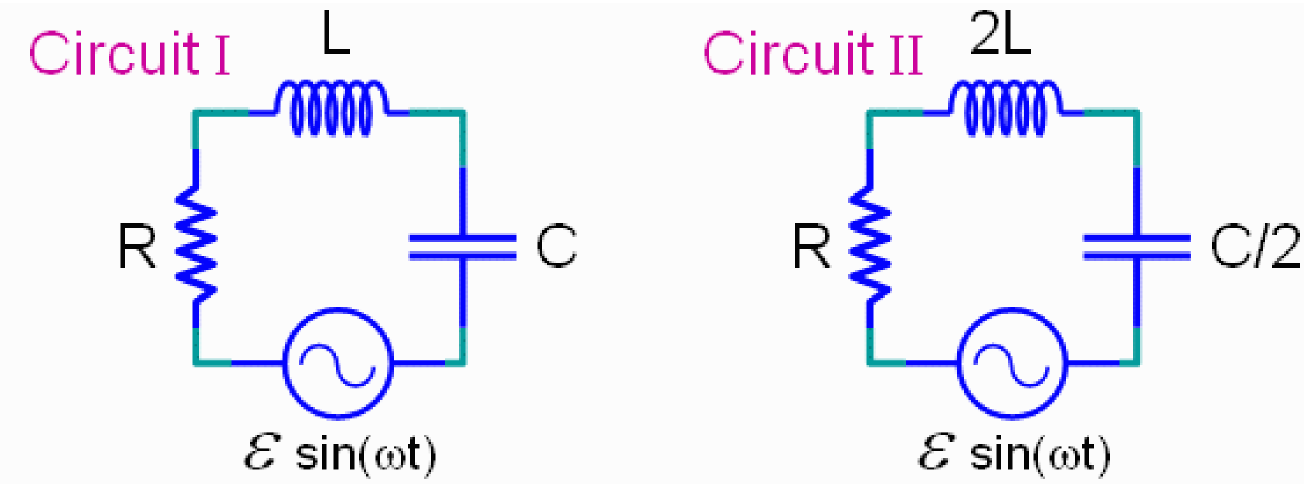
$Z = R$

Same since R doesn't change





CheckPoint 1b

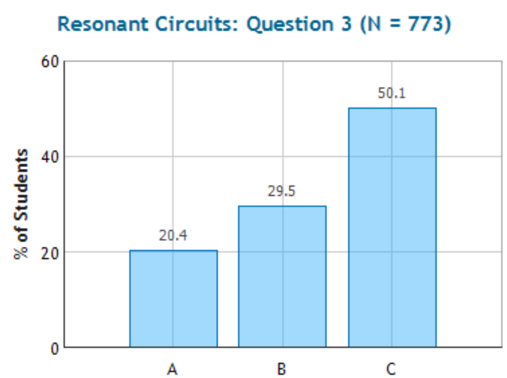


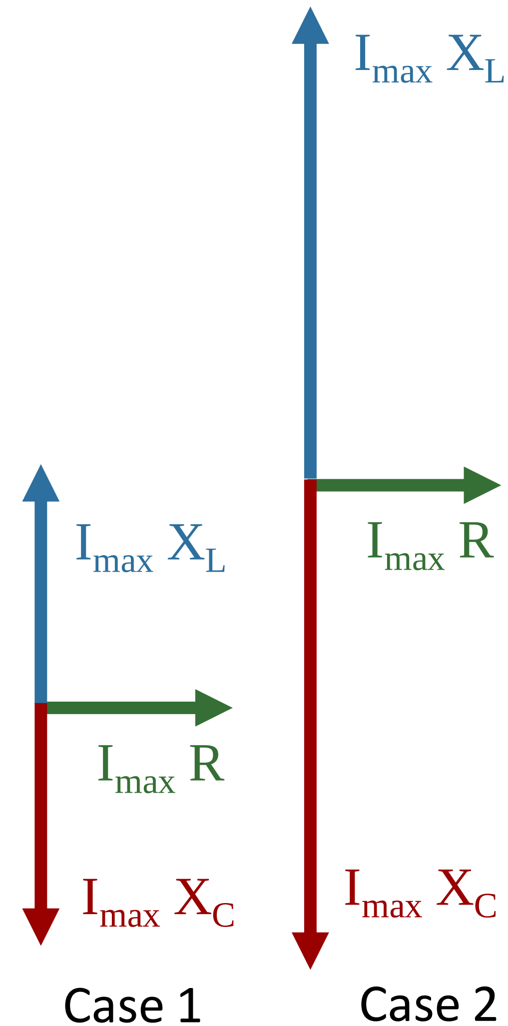
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

Compare the peak voltage across the inductor in the two circuits

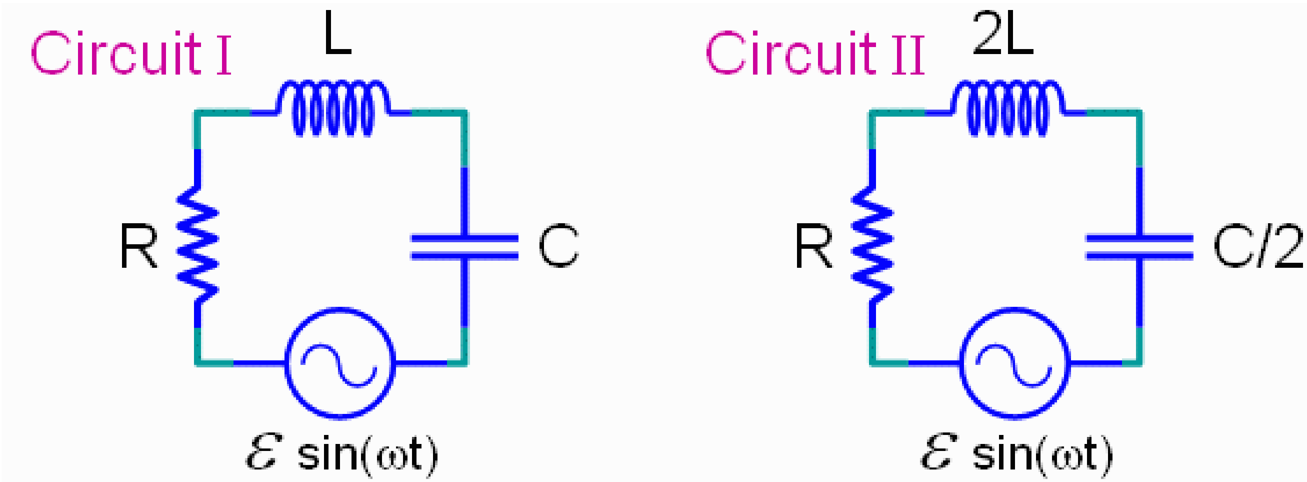
A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ ***C.*** $V_I < V_{II}$

Voltage in second circuit will be twice that of the first because of the $2L$ compared to L.





CheckPoint 1c

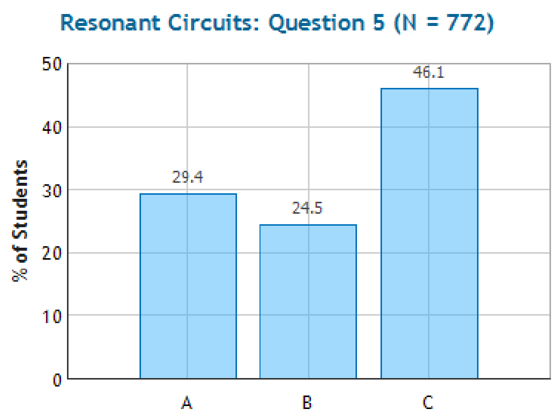


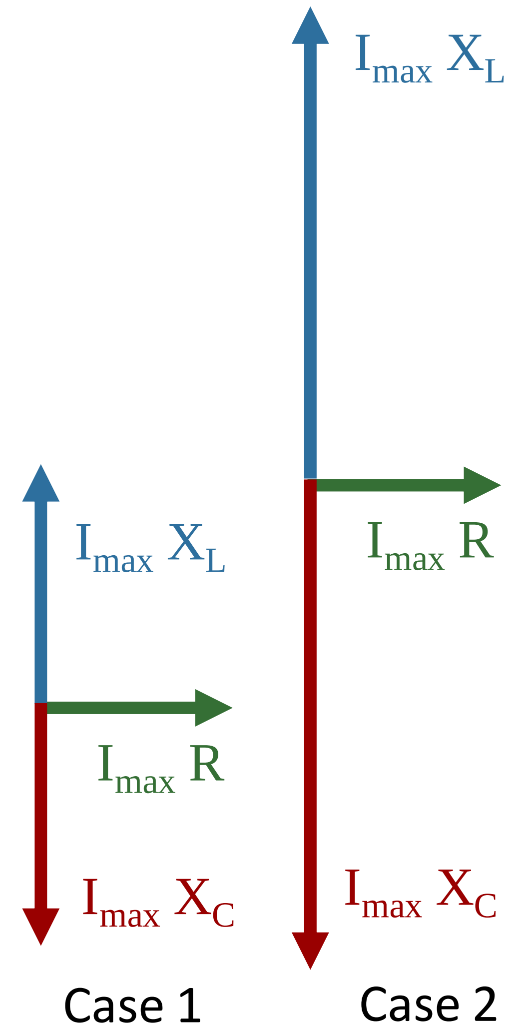
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

Compare the peak voltage <u>across the inductor</u> in the two circuits

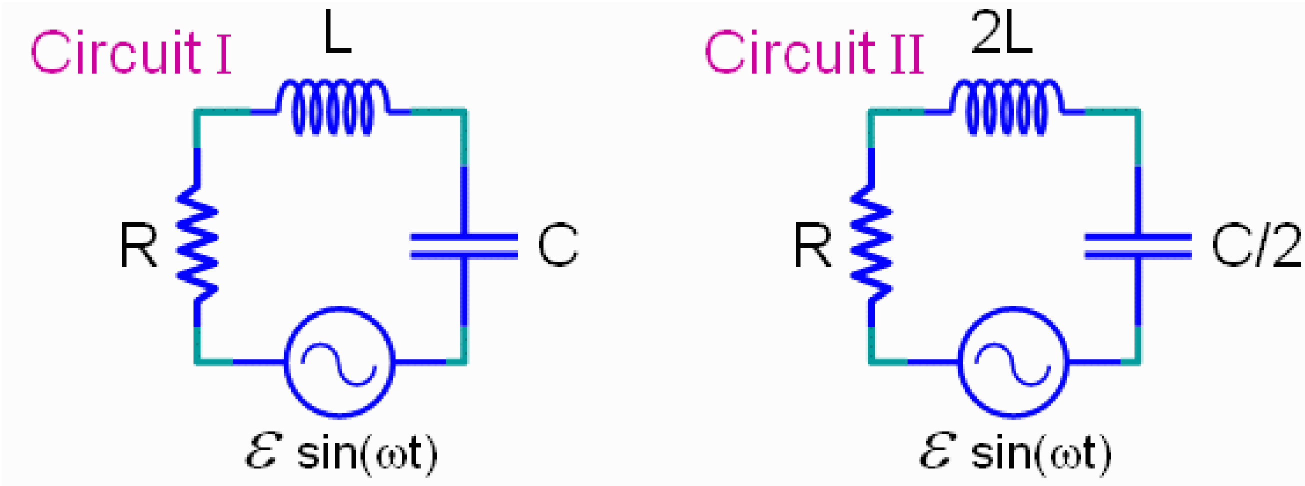
A. $V_I > V_{II}$ ***B.*** $V_I = V_{II}$ ***C.*** $V_I < V_{II}$

The peak voltage will be greater in circuit 2 because the value of X_C doubles.





CheckPoint 1D



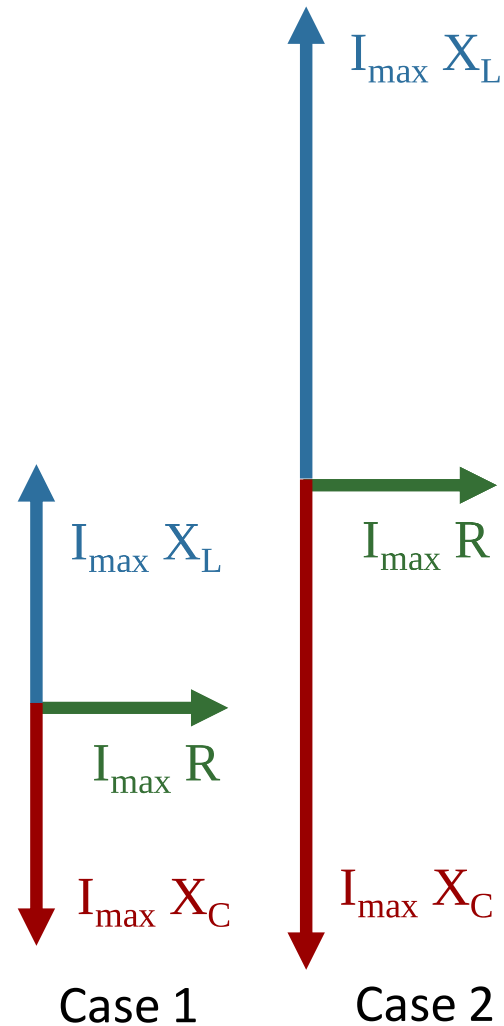
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown

At the resonant frequency, which of the following is true?

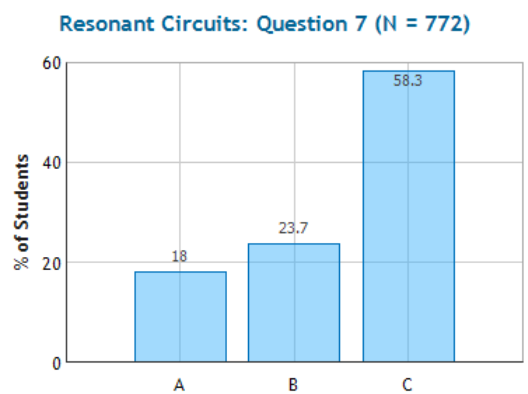
A. Current leads voltage across the generator

B. Current lags voltage across the generator

C. Current is in phase with voltage across the generator



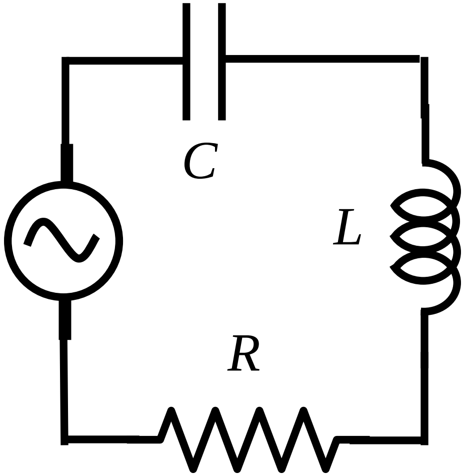
The voltage across the inductor and the capacitor are equal when at resonant frequency, so there is no lag or lead.



Power

$P = IV$ instantaneous always true

- Difficult for Generator, Inductor and Capacitor because of phase
- Resistor I,V are always in phase!



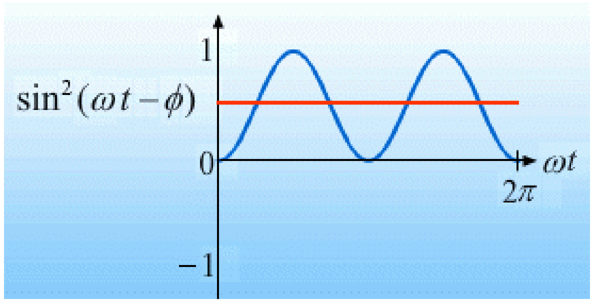
$$P = IV$$
$$= I^2 R$$

Average Power

Inductor and Capacitor $= 0$ $(< \sin(\omega t) \cos(\omega t) > = 0)$

Resistor

$$<I^2R> = <I^2> R = ½\, I^2_{\text{peak}}\, R$$



RMS = Root Mean Square

$I_{\text{peak}} = I_{\text{rms}}\, \text{sqrt}(2)$

$$< I^2 R > = I_{\text{rms}}{}^2 R$$

Power Line Calculation

If you want to deliver 1,500 Watts at 100 Volts over transmission lines w/ resistance of 5 Ohms. How much power is lost in the lines?

- Current Delivered: $I = P/V = 15$ Amps
- Loss $= IV$ (on line) $= I^2 R = 15*15 * 5 = 1{,}125$ Watts!

If you deliver 1,500 Watts at 10,000 Volts over the same transmission lines. How much power is lost?

- Current Delivered: $I = P/V = .15$ Amps
- Loss $= IV$ (on line) $= I^2 R = 0.125$ Watts

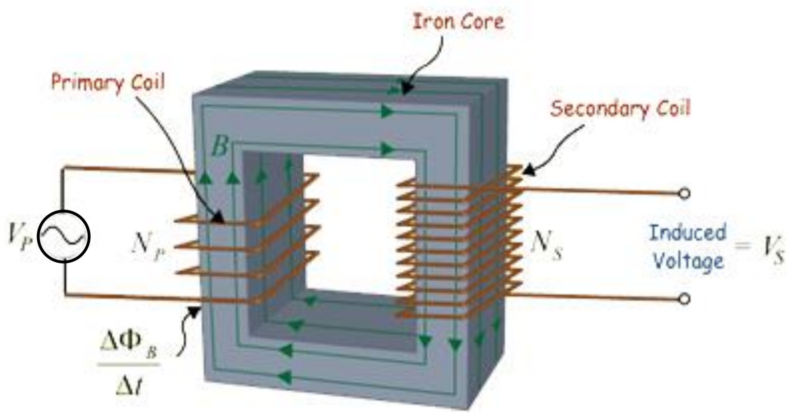
DEMO

Transformers

Application of Faraday's Law

- Changing EMF in Primary creates changing flux
- Changing flux, creates EMF in secondary

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$



Efficient method to change voltage for AC.

Power Transmission Loss $= I^2R$

Power electronics

Demo

Follow-Up from Last Lecture

Consider the harmonically driven series LCR circuit shown.

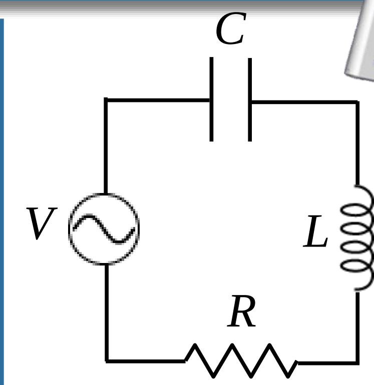
$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^o (cos = sin = 1/sqrt(2))

L and R are unknown.



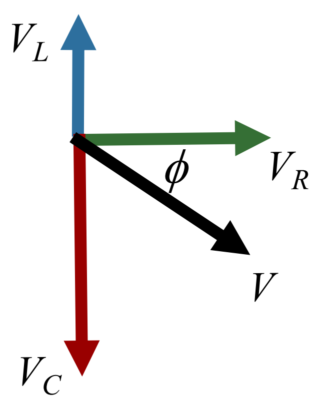
How should we change ω to bring circuit to resonance?

A) decrease ω

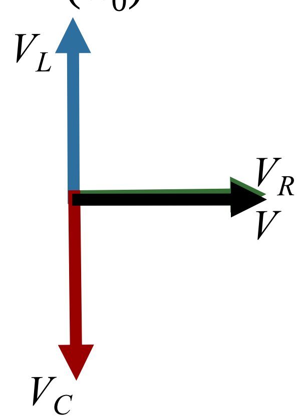
B) increase ω

C) Not enough info

Original ω



At resonance (ω_0)



At resonance

$X_L = X_C$

↓

X_L increases

X_C decreases

→ ω increases

More Follow-Up

Consider the harmonically driven series LCR circuit shown.

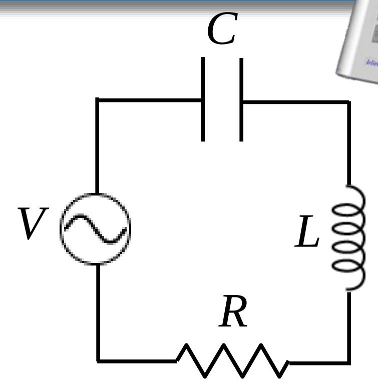
$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2)) → $X_C = 40\sqrt{2}\ k\Omega$

The current leads generator voltage by 45° (cos = sin = 1/sqrt(2))

L and R are unknown.



$R = 25\sqrt{2}\ k\Omega$

$X_L = 15\sqrt{2}\ k\Omega$

By what factor should we increase ω to bring circuit to resonance?
i.e. if $\omega_0 = f\omega$, what is f?

A) $f = \sqrt{2}$

B) $f = 2\sqrt{2}$

C) $f = \sqrt{\frac{8}{3}}$

D) $f = \sqrt{\frac{8}{5}}$

If ω is increased by a factor of f:

X_L increases by factor of f

X_C decreases by factor of f

→

$X_L \to f \cdot 15\sqrt{2}$

$X_C \to (1/f) \cdot 40\sqrt{2}$

At resonance $X_L = X_C$ → $15f = \frac{40}{f}$ → $f^2 = \frac{40}{15}$ → $f = \sqrt{\frac{8}{3}}$

Current Follow-Up

Consider the harmonically driven series *LCR* circuit shown.

$V_{max} = 100\ V$

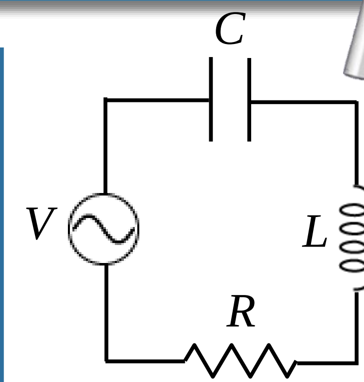
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2)) → $X_C = 40\sqrt{2}\ k\Omega$

The current leads generator voltage by 45° (cos = sin = 1/sqrt(2))

L and *R* are unknown.

What is the maximum current at resonance



$R = 25\sqrt{2}\ k\Omega$

$X_L = 15\sqrt{2}\ k\Omega$

$\omega_0 = \sqrt{\frac{8}{3}}\,\omega$

A) $I_{\max}(\omega_0) = \sqrt{2}\,mA$

B) $I_{\max}(\omega_0) = 2\sqrt{2}\,mA$

C) $I_{\max}(\omega_0) = \sqrt{\frac{8}{3}}\,mA$

At resonance $X_L = X_C$ → $Z = R$ → $I_{\max}(\omega_0) = \frac{V_{\max}}{R} = \frac{100}{25\sqrt{2}} = 2\sqrt{2}\,mA$

Phasor Follow-Up

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

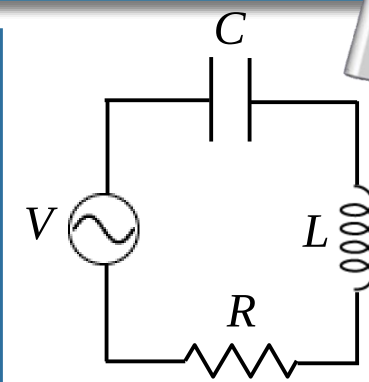
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^{o} (cos = sin = 1/sqrt(2))

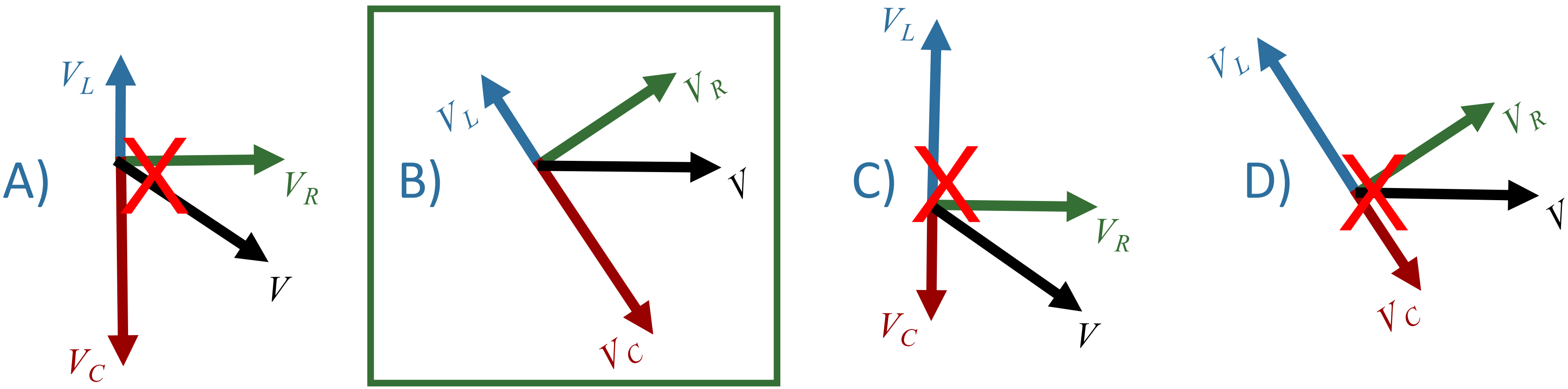
L and R are unknown.

What does the phasor diagram look like at t = 0? (assume $V = V_{max}\sin\omega t$)



$R = 25\sqrt{2}\ k\Omega$

$X_L = 15\sqrt{2}\ k\Omega$



$V = V_{max}\sin\omega t$ -> V is horizontal at $t = 0$ ($V = 0$)

$\vec{V} = \vec{V}_L + \vec{V}_C + \vec{V}_R$ → $V_L < V_C$ if current leads generator voltage