

Your Comments

Llamas, and why it is so difficult to get someone to buy me one as a pet. All I ask is that it breathes fire. I have simple needs people. Seriously. p.s. this lecture was hard.

did not understand any of this

wTF. so over my head

I have no idea what I am doing :(

Wow! That was cool. There is so much that you learn in high school that you just take as factual and move on. But when you can finally see it derived and shown that it is actually true...WOW!
Pretty awesome.

This was undoubtedly my favorite prelecture of the semester, its so cool!

I was doing fine until we combined ampere's modified law and faraday's law. Also go over the magnetic and electric fields in empty space. Are they generated from the empty space itself, or do they come from some ambient source like stars?

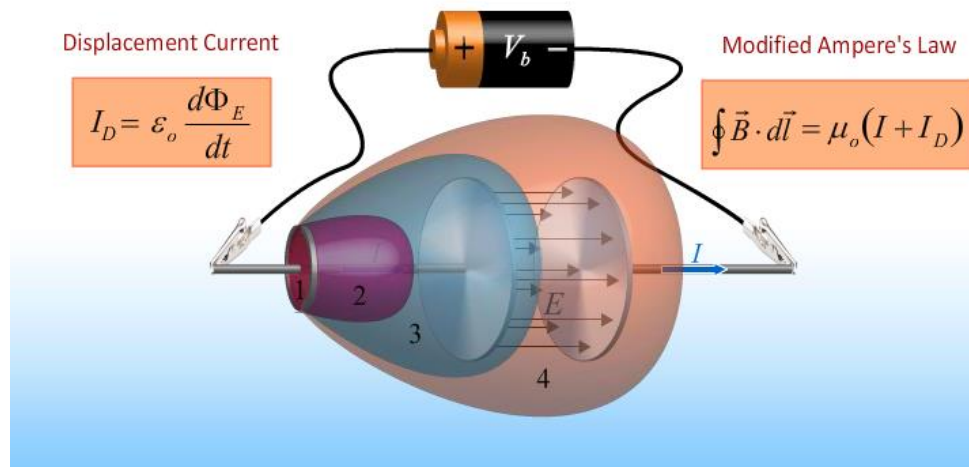
its mindblowing that ϵ_0 and μ_0 are related to the speed of light. Can you calculate the speed of light more accurately through measuring ϵ_0 and μ_0 , or by measuring it directly (with lasers, etc)?

I suspect that people's reactions to electric and magnetic waves that are out of phase will be quite polarized.

Physics 212

Lecture 22

DISPLACEMENT CURRENT and EM WAVES



MAXWELL'S EQUATIONS

Gauss' Law for E Fields

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Gauss' Law for B Fields

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

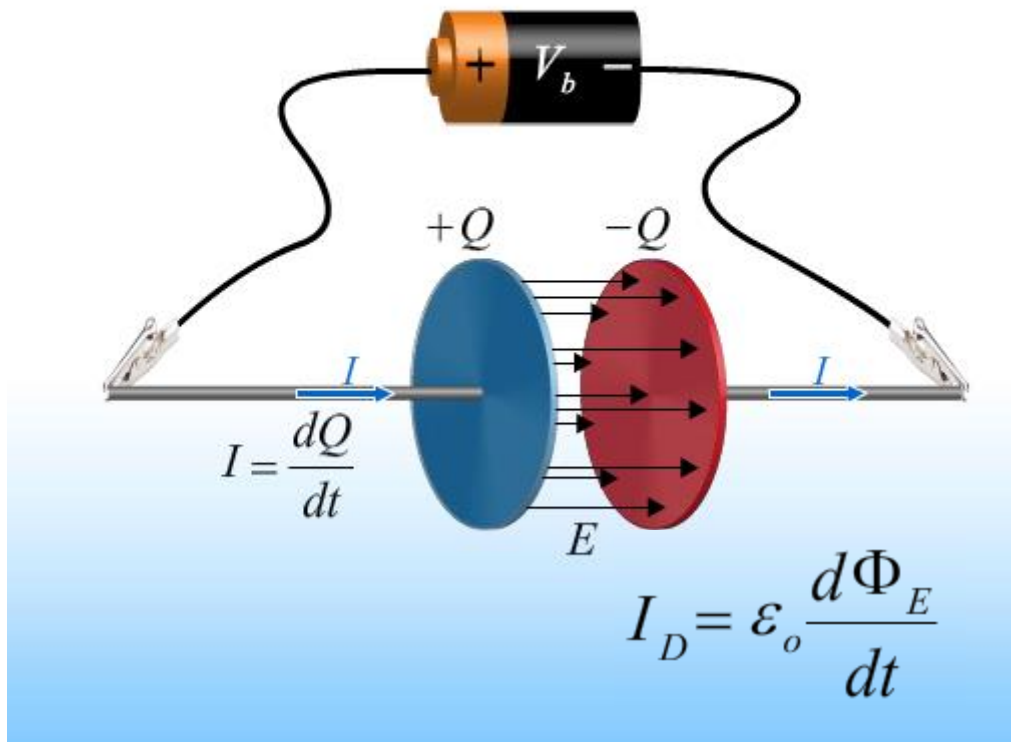
Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

After Prelecture 21: Modify Ampere's Law

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} = \mu_0 (I + I_D)$$



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$



$$\Phi = EA = \frac{Q}{\epsilon_0}$$



$$Q = \epsilon_0 \Phi$$



$$\frac{dQ}{dt} = \epsilon_0 \frac{d\Phi}{dt} \equiv I_D$$

Displacement Current

Real Current:

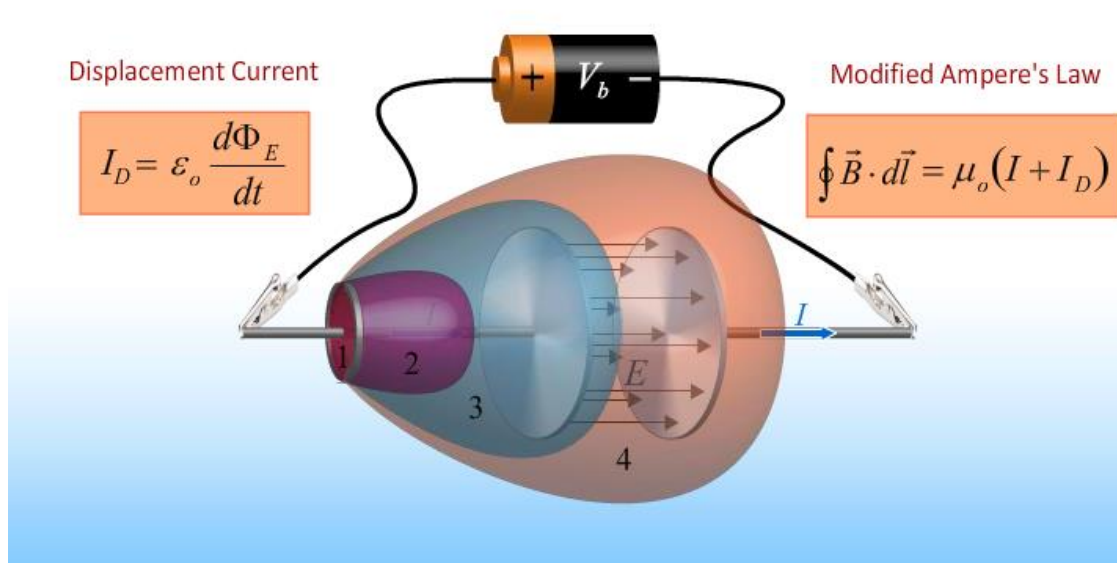
Charge Q passes through area A in time t :

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area A changes in time

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$



Modified Ampere's Law

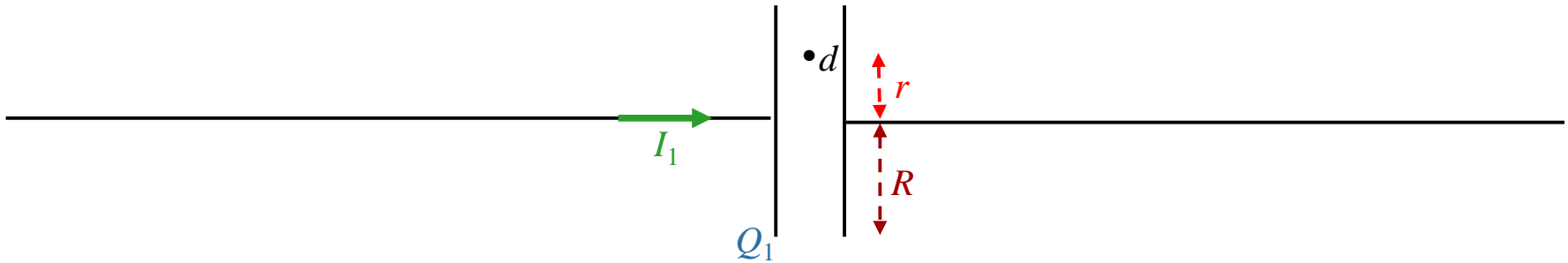
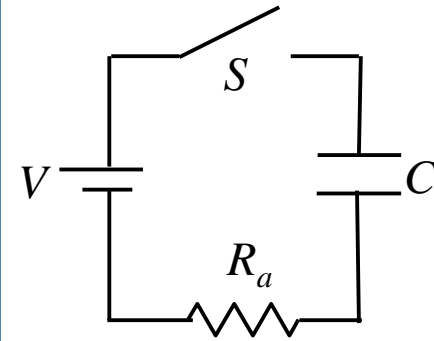
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Free space

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .

At time t_1 , what is the magnetic field B_1 at a radius r (point d) in between the plates of the capacitor?



Conceptual and Strategic Analysis

Charge Q_1 creates electric field between the plates of C

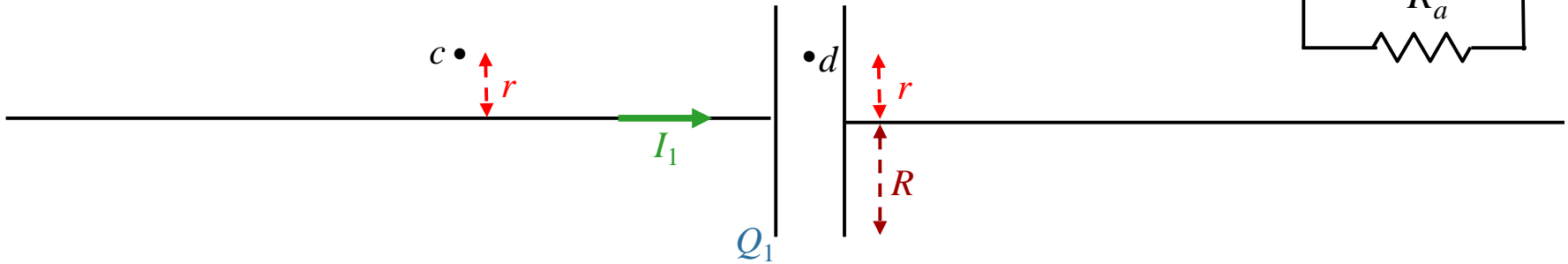
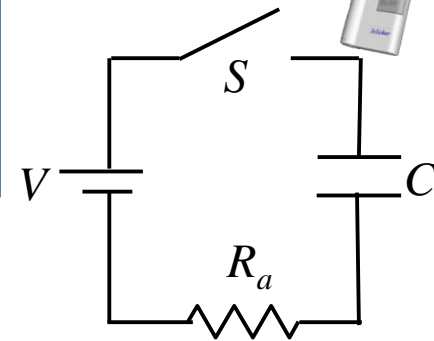
Charge Q_1 changing in time gives rise to a changing electric flux between the plates

Changing electric flux gives rise to a displacement current I_D in between the plates

Apply (modified) Ampere's law using I_D to determine B

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



Compare the magnitudes of the B fields at points c and d .

A) $B_c < B_d$

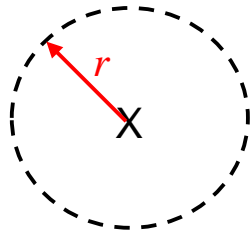
B) $B_c = B_d$

C) $B_c > B_d$

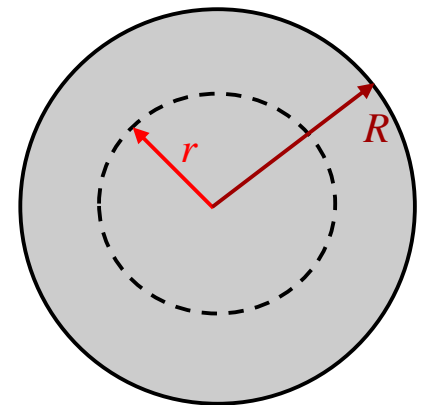
What is the difference?

Apply (modified) Ampere's Law

point c :
 $I(\text{enclosed}) = I_1$

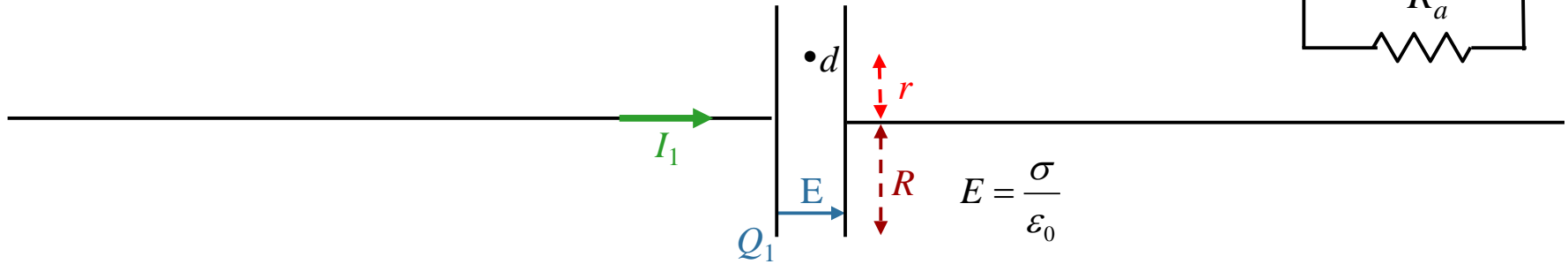
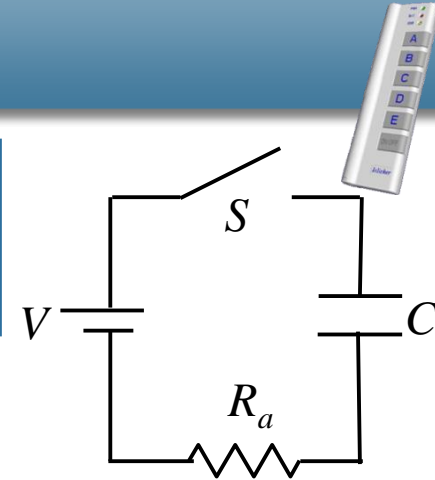


point d :
 $I_D(\text{enclosed}) < I_1$



Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



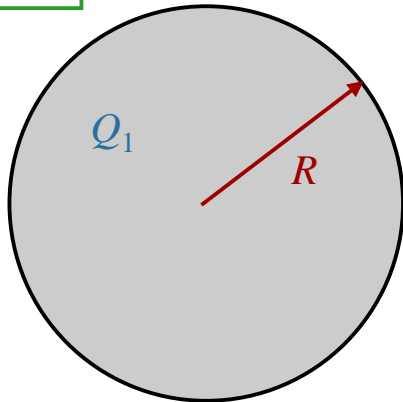
What is the magnitude of the electric field between the plates?

A) $E = \frac{Q_1}{\pi R^2 \epsilon_0}$

B) $E = \frac{Q_1 \pi R^2}{\epsilon_0}$

C) $E = \frac{Q_1}{\epsilon_0}$

D) $E = \frac{Q_1}{r}$

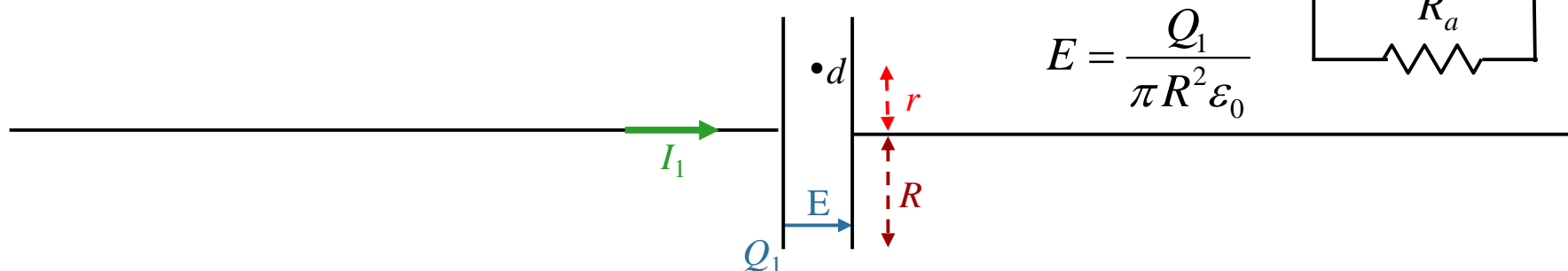
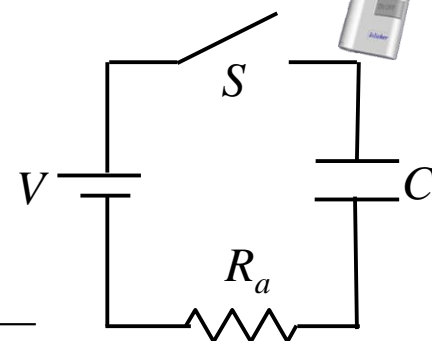


$$E = \frac{\sigma}{\epsilon_0} \rightarrow \sigma = \frac{Q_1}{A} = \frac{Q_1}{\pi R^2} \rightarrow E = \frac{Q_1}{\epsilon_0 \pi R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



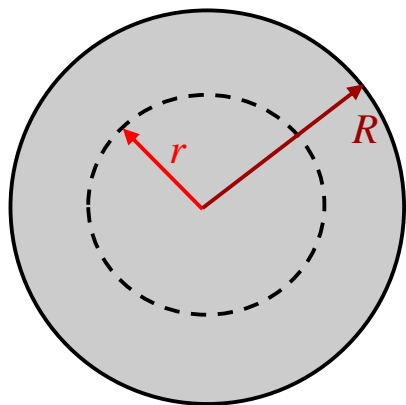
What is the electric flux through a circle of radius r in between the plates?

A) $\Phi_E = \frac{Q_1}{\epsilon_0} \pi r^2$

B) $\Phi_E = \frac{Q_1}{\epsilon_0} \pi R^2$

C) $\Phi_E = \frac{Q_1 r^2}{\epsilon_0 R^2}$

D) $\Phi_E = \frac{Q_1 \pi r^2}{\epsilon_0 R^2}$

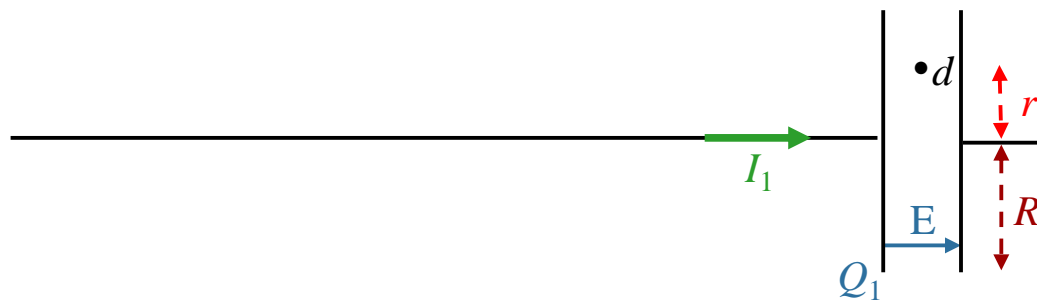
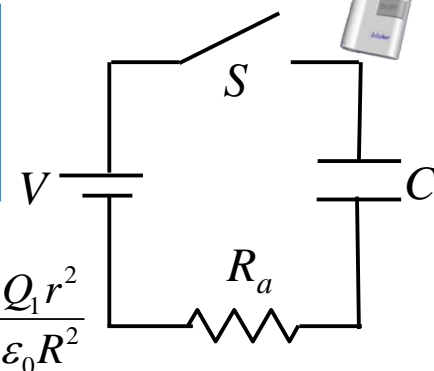


$$\Phi_E = \vec{E} \cdot \vec{A} \quad \longrightarrow \quad \Phi_E = \frac{Q_1}{\epsilon_0 \pi R^2} \pi r^2 \quad \longrightarrow \quad \Phi_E = \frac{Q_1}{\epsilon_0} \frac{r^2}{R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



$$\Phi_E = \frac{Q_1 r^2}{\epsilon_0 R^2}$$

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

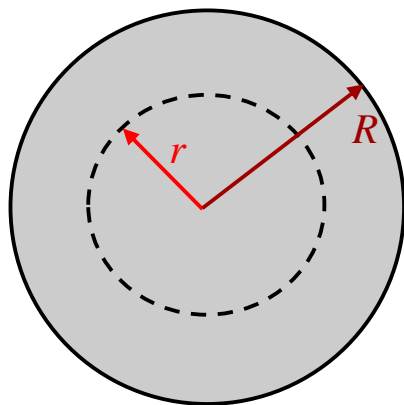
What is the displacement current enclosed by circle of radius r ?

A) $I_D = I_1 \frac{R^2}{r^2}$

B) $I_D = I_1 \frac{r}{R}$

C) $I_D = I_1 \frac{r^2}{R^2}$

D) $I_D = I_1 \frac{R}{r}$



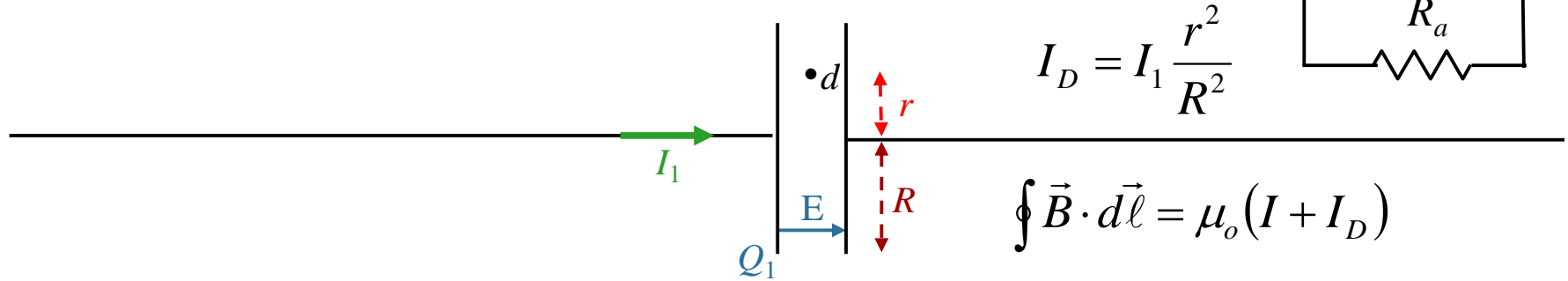
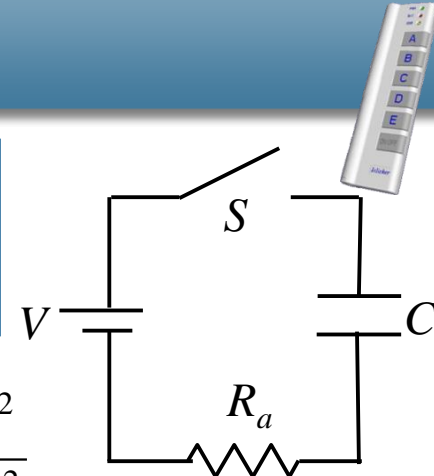
$$I_D = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ_1}{dt} \frac{r^2}{R^2} = I_1 \frac{r^2}{R^2}$$

→ $I_D = I_1 \frac{r^2}{R^2}$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed.

Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .



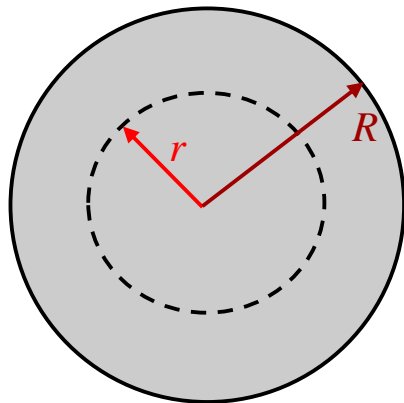
What is the magnitude of the B field at radius r ?

A) $B = \frac{\mu_0 I_1}{2\pi R}$

B) $B = \frac{\mu_0 I_1}{2\pi r}$

C) $B = \frac{\mu_0 I_1 R}{2\pi r^2}$

D) $B = \frac{\mu_0 I_1 r}{2\pi R^2}$



Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_D)$

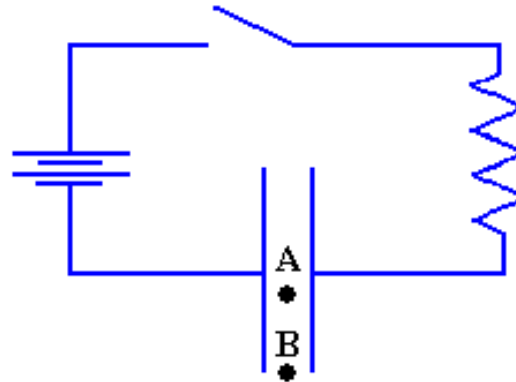
$\rightarrow B \cdot 2\pi r = \mu_0 \left(0 + I_1 \frac{r^2}{R^2} \right)$

$\rightarrow B = \frac{\mu_0 I_1 r}{2\pi R^2}$

Checkpoint 1B



At time $t=0$ the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.



Compare the magnitudes of the magnetic fields at points A and B just after the switch is closed

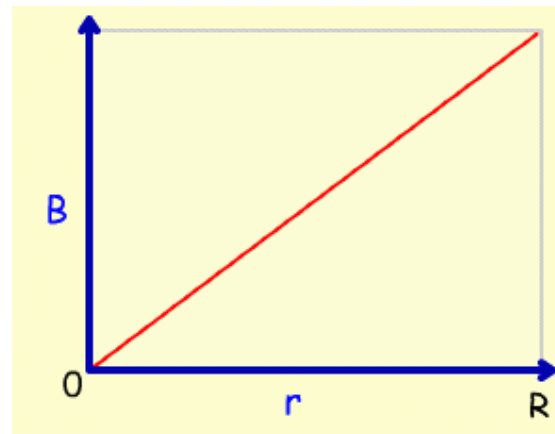
A. $B_A < B_B$

B. $B_A = B_B$

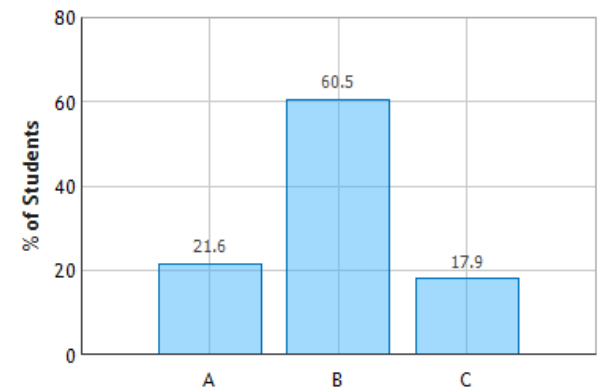
C. $B_A > B_B$

From the calculation we just did:

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$



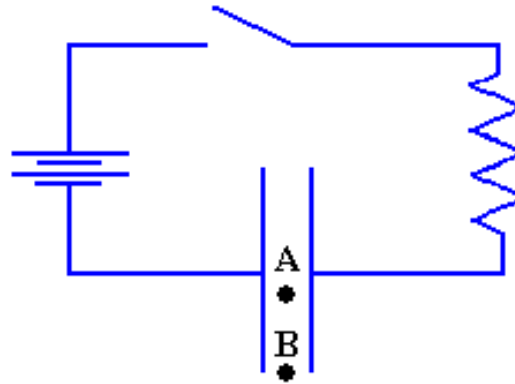
Displacement Current: Question 3 (N = 731)



Checkpoint 1a



At time $t=0$ the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.



After the switch is closed, there will be a magnetic field at point A which is proportional to the current in the circuit:

A. True

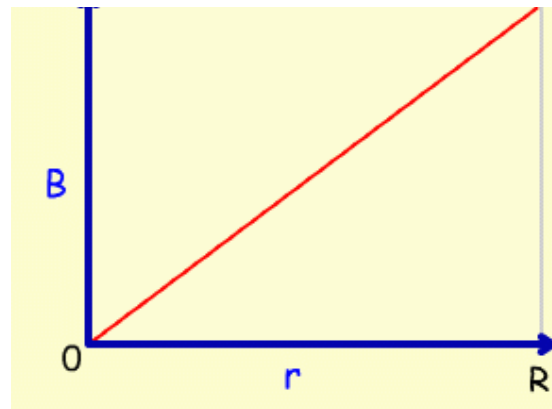
B. False

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

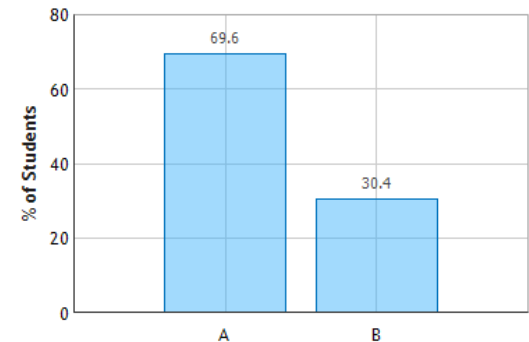
B is proportional to I

but

At A, $B = 0$!!



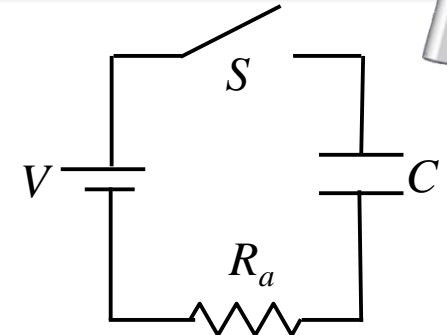
Displacement Current: Question 1 (N = 731)



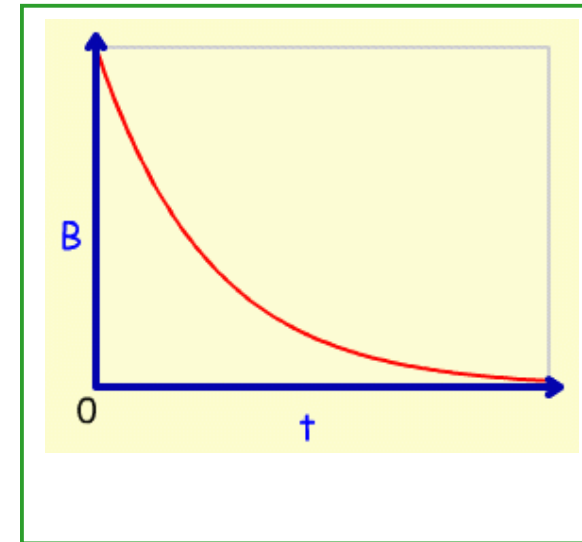
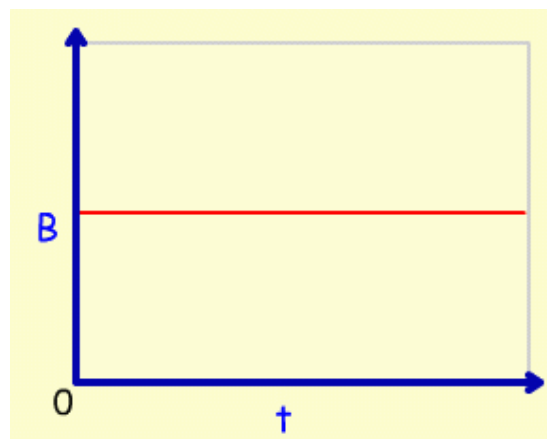
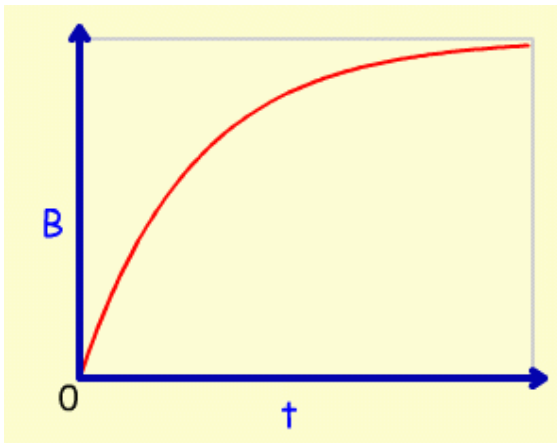
Follow-Up

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R . At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1 .

What is the time dependence of the magnetic field B at a radius r between the plates of the capacitor?



$$B_1 = \frac{\mu_0 I_1 r}{2\pi R^2}$$



B at fixed r is proportional to the current I

Close switch: $V_C = 0 \Rightarrow I = V/R_a$ (maximum)

I exponentially decays with time constant $\tau = R_a C$

Follow-Up 2

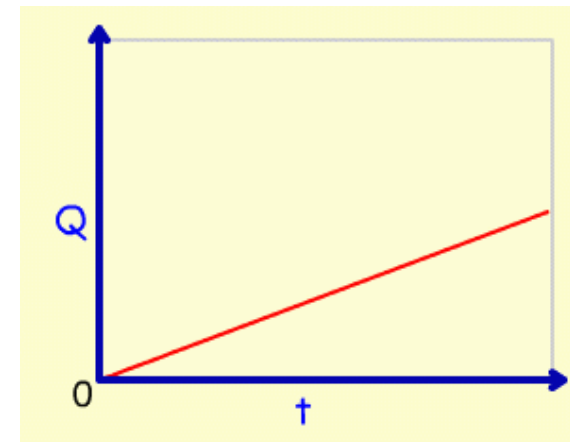
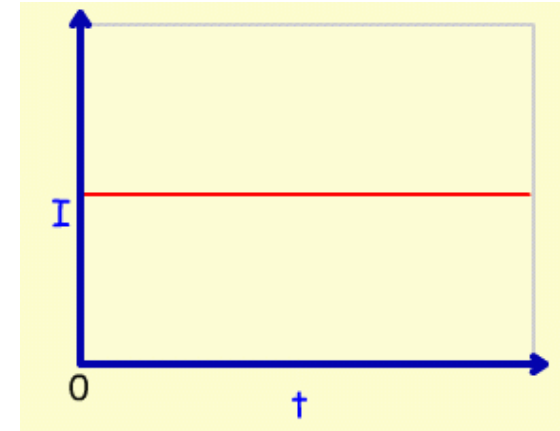


Suppose you were able to charge a capacitor with constant current (does not change in time).

Does a B field exist in between the plates of the capacitor?

A) YES

B) NO



Constant current $\Rightarrow Q$ increases linearly with time

Therefore E increases linearly with time ($E = Q/(A\epsilon_0)$)

dE/dt is not zero \Rightarrow Displacement current is not zero

$\Rightarrow B$ is not zero !

We learned about waves in Physics 211

1-D Wave Equation

$$\frac{d^2 h}{dx^2} = \frac{1}{v^2} \frac{d^2 h}{dt^2}$$

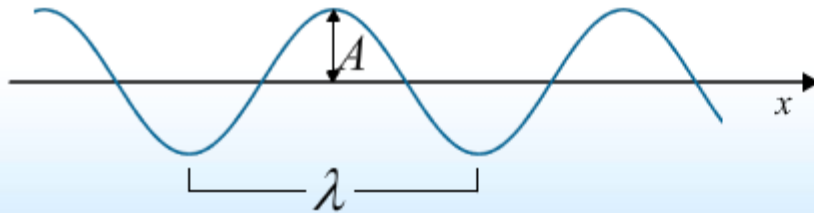


Solution

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

Common Example: Harmonic Plane Wave

$$h(x,t) = A \cos(kx - \omega t)$$



Variable Definitions

Amplitude: A

Wave Number: $k = \frac{2\pi}{\lambda}$

Wavelength: λ

Angular Frequency: $\omega = \frac{2\pi}{T}$

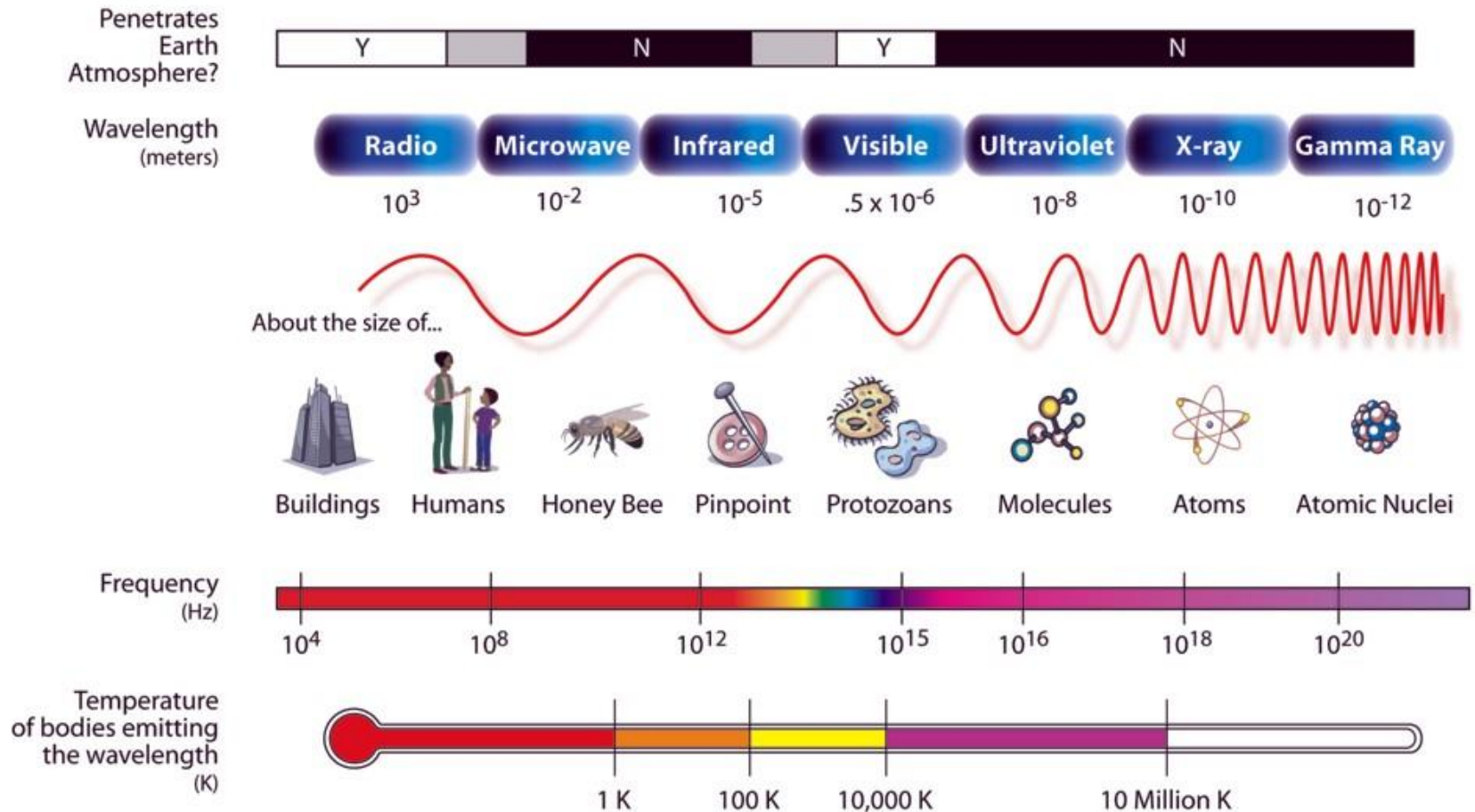
Period: T

Frequency: $f = \frac{1}{T}$

Velocity: $v = \lambda f = \frac{\omega}{k}$

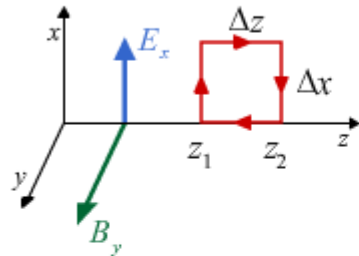
Electromagnetic Spectrum

THE ELECTROMAGNETIC SPECTRUM



Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$



$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z} \frac{\partial B_y}{\partial t}$$

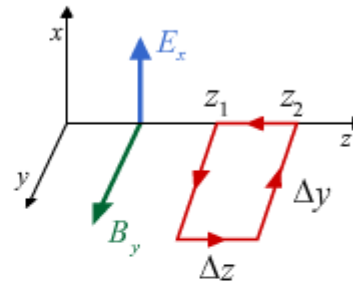
Plane Wave Solution

$$\vec{E} \rightarrow \vec{E}(z,t)$$

$$\vec{B} \rightarrow \vec{B}(z,t)$$

Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$



$$\frac{\partial B_y}{\partial z} = -\mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

$$\frac{\partial}{\partial t} \frac{\partial B_y}{\partial z} = -\mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

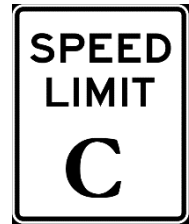
Wave Equation

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

Speed of Electromagnetic Wave

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3.00 \times 10^8 \text{ m/s}$$

Speed of Light !



Special Relativity (1905)

Speed of Light is Constant

Albert Einstein



“How can light move at the same velocity in any inertial frame of reference? That's really trippy.”

see PHYS 225

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

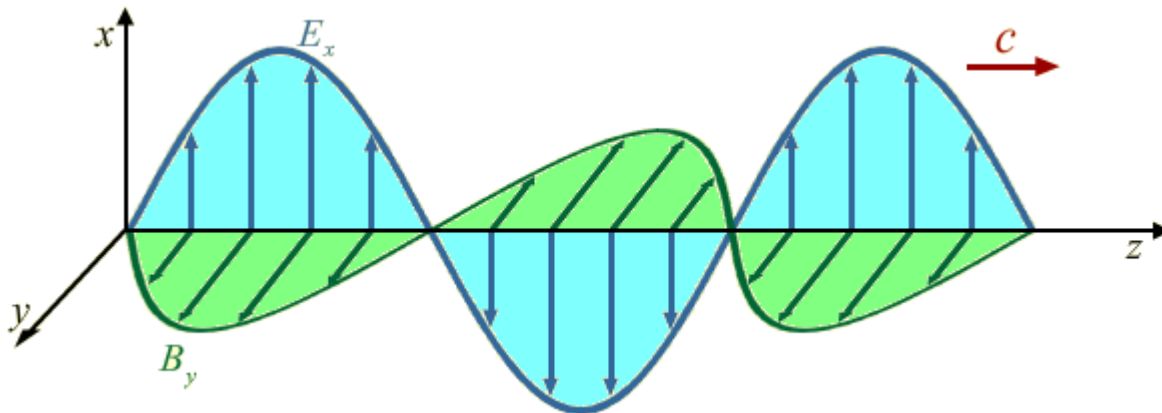
$$\frac{\partial^2 B_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

$$E_x = E_o \cos(kz - \omega t) \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$

Two Important Features

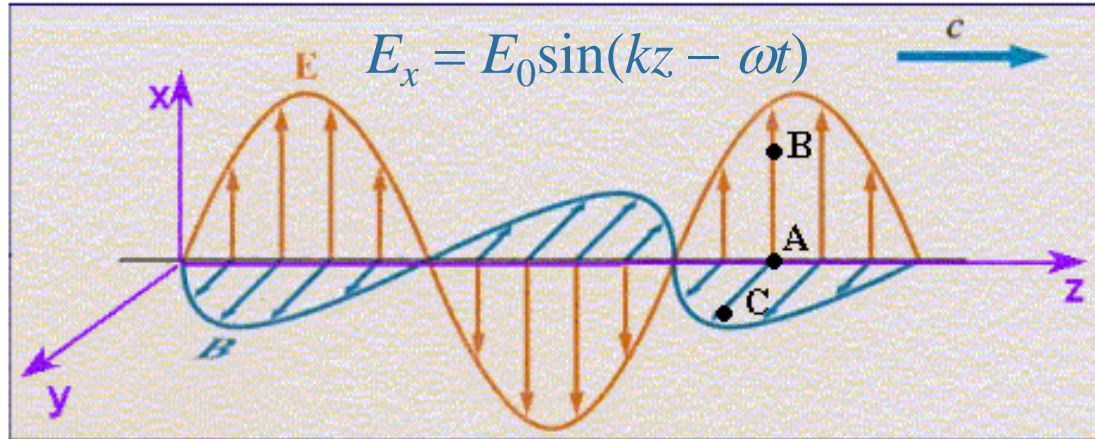
1. B_y is in phase with E_x
2. $B_o = \frac{E_o}{c}$



CheckPoint 2a



An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same z coordinate.



Compare the magnitudes of the electric fields at points A and B

A. $E_A < E_B$

B. $E_A = E_B$

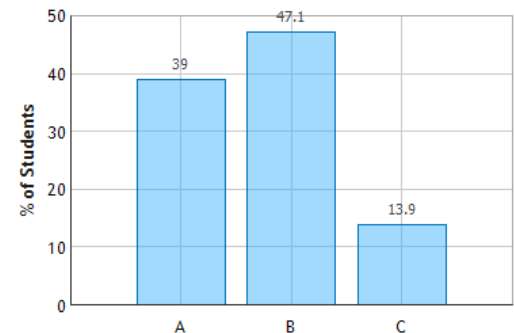
C. $E_A > E_B$

$$E = E_0 \sin(kz - \omega t):$$

E depends only on z coordinate for constant t .

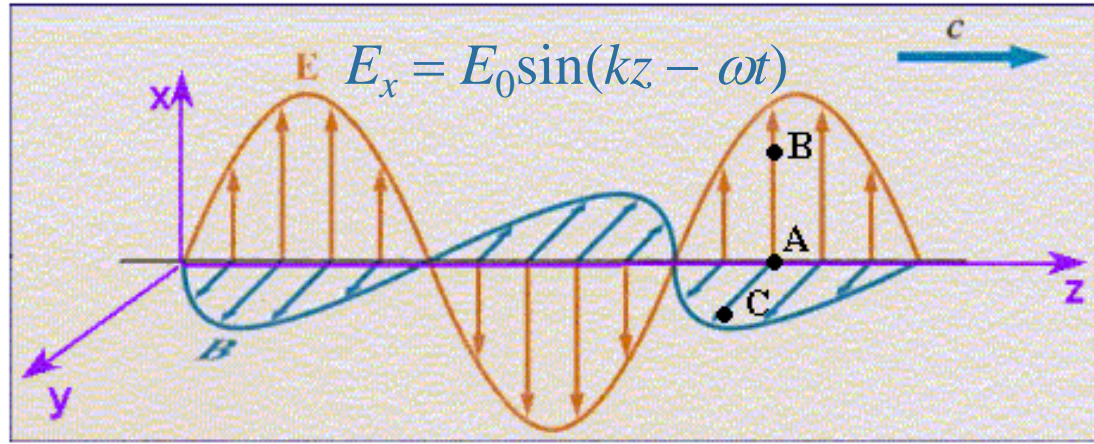
z coordinate is same for A, B, C.

Electromagnetic Waves: Question 1 (N = 728)



CheckPoint 2b

An electromagnetic plane-wave is traveling in the $+z$ direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same z coordinate.



Compare the magnitudes of the electric fields at points A and C

A. $E_A < E_C$

B. $E_A = E_C$

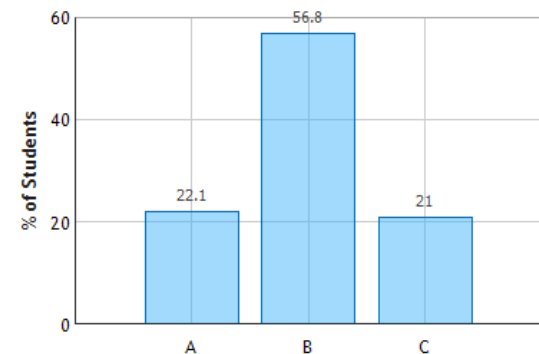
C. $E_A > E_C$

$$E = E_0 \sin(kz - \omega t):$$

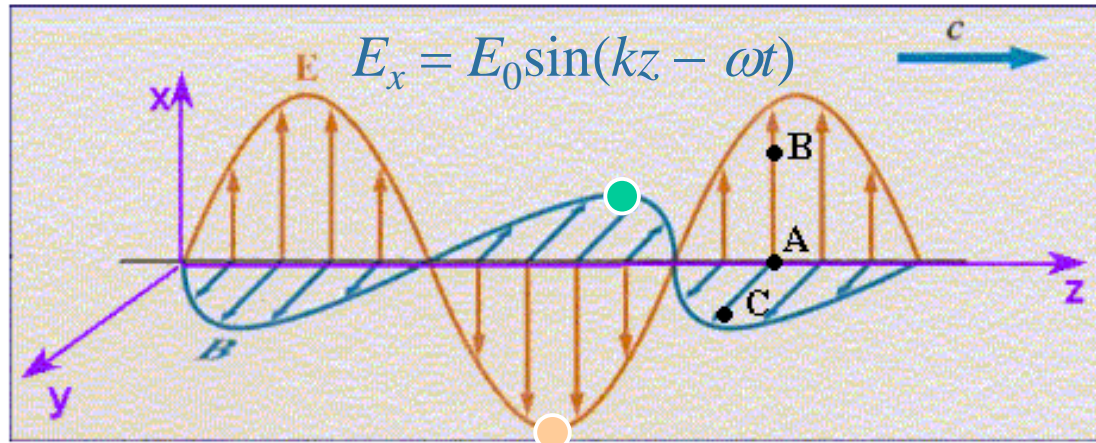
E depends only on z coordinate for constant t .

z coordinate is same for A, B, C.

Electromagnetic Waves: Question 2 (N = 727)



Clicker Question



Consider a point (x,y,z) at time t when E_x is negative and has its maximum value.

At (x,y,z) at time t , what is B_y ?

- A) B_y is positive and has its maximum value
- B) B_y is negative and has its maximum value
- C) B_y is zero
- D) We do not have enough information