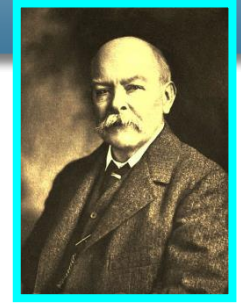


Your Comments



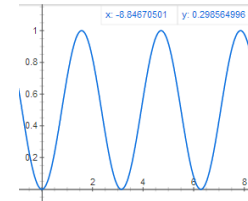
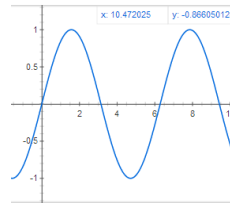
"Poynting vector? not pointing vector? did the one guy in the history of physics with the last name "poynting" really have to get a direction vector named after himself?"

MIT made a little mini-game based on being near the speed of light. The goal is you have to pick up these little balls which magically make the speed of light lower. There's time dilation, the doppler effect, the searchlight effect, Lorentz transformation, and the runtime effect. I don't know if you want to show this to the class or anything, but it's a lot of fun to experience.
<http://gamelab.mit.edu/games/a-slower-speed-of-light/>

We definitely need to test out the "running towards the i-clicker" question from the prelecture.

I'm sure I speak for the whole class when I say that I LOVED only having 1 I.E. and only 3 problems sets for homework this past week. Since mom's weekend is this weekend it would be AWESOME for the HW to be that short again so we can spend more time with our families and less time worrying about getting homework done. Just a suggestion :)

"Can we please go into some detail about these root mean square things? It all seems so enragingly arbitrary"



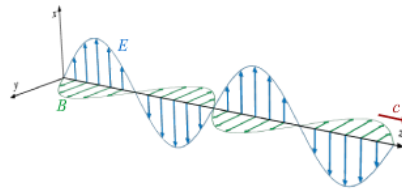
Physics at 2 in the morning is not how most people would choose to start a birthday... But this is actually kinda cool!

I don't care what statistics and research studies say, this SUCKS!!! Every possible way of making you confused, stressed and angry exists within smart physics!!!

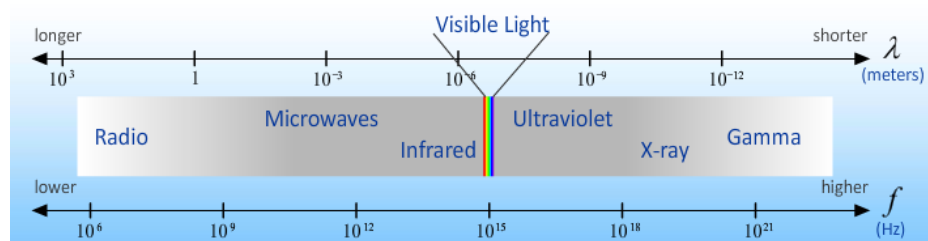
Physics 212

Lecture 23

PROPERTIES of ELECTROMAGNETIC WAVES

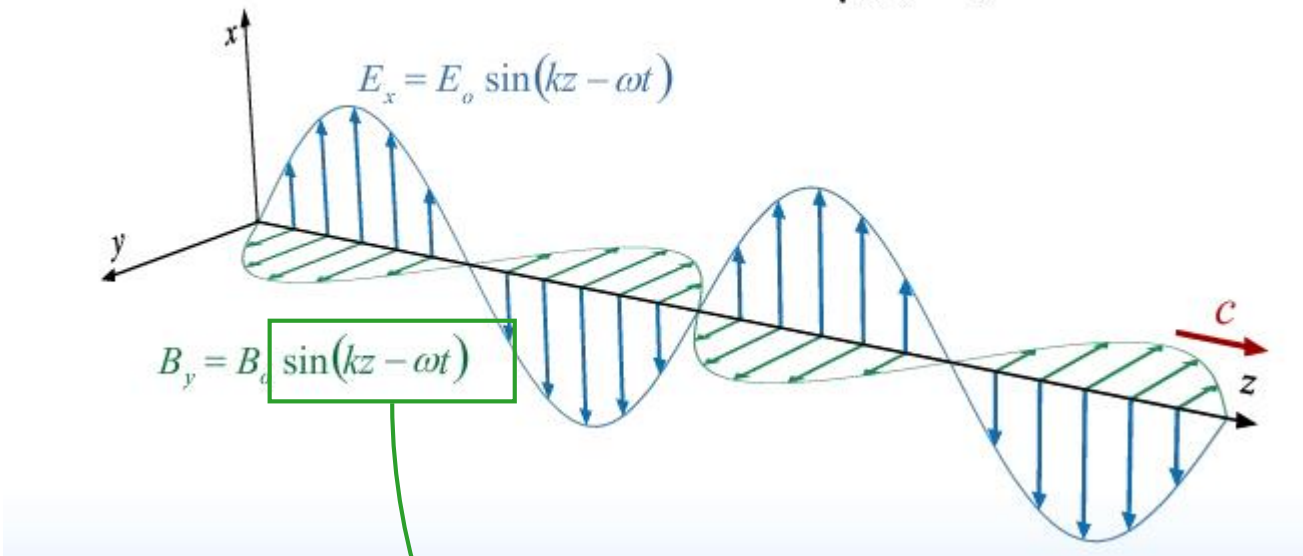


Electromagnetic Spectrum



Plane Waves from Last Time

$$\text{Velocity } c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E_0}{B_0} = 3 \times 10^8 \text{ m/s}$$



E and B are perpendicular and in phase

Oscillate in time and space

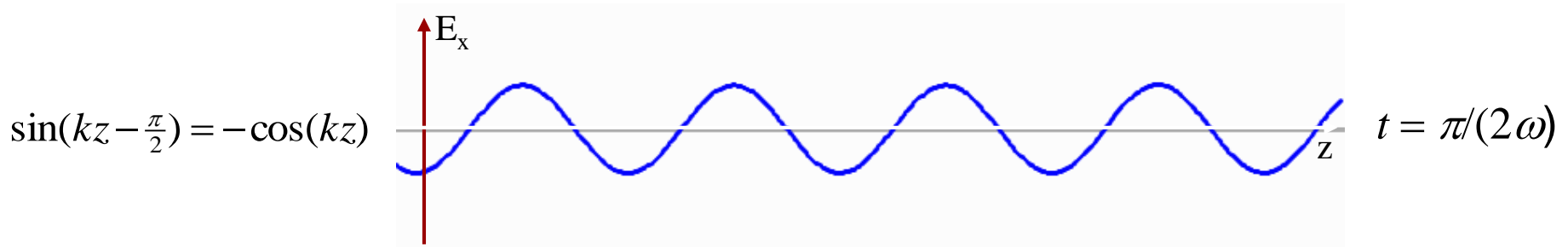
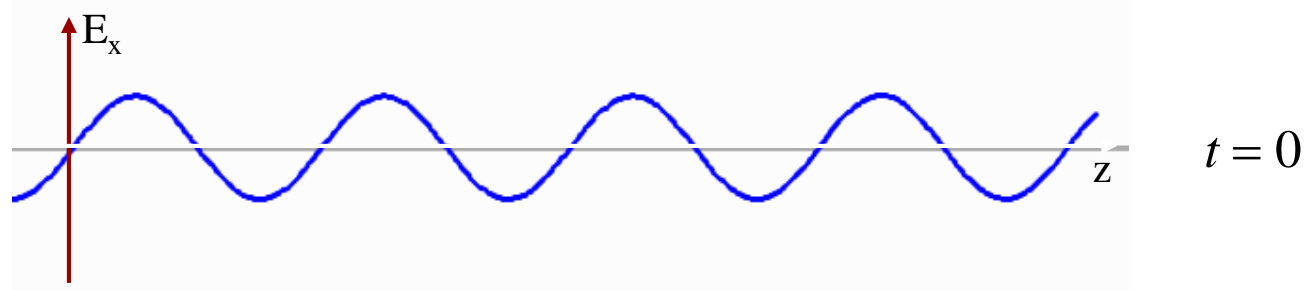
Direction of propagation given by $E \times B$

$$E_0 = cB_0$$

Argument of \sin/\cos gives direction of propagation

Understanding the speed and direction of the wave

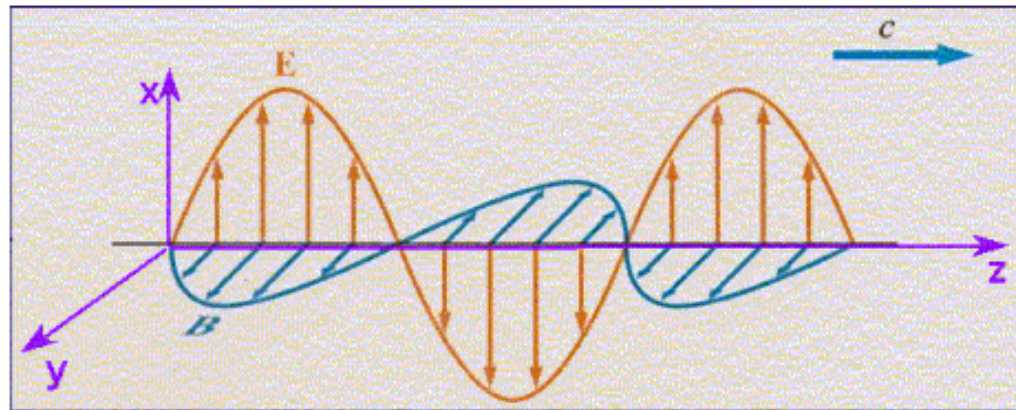
$$E_x = E_0 \sin(kz - \omega t)$$



What has happened to the wave form in this time interval?

It has **MOVED TO THE RIGHT** by $1/4 \lambda$

CheckPoint 1a



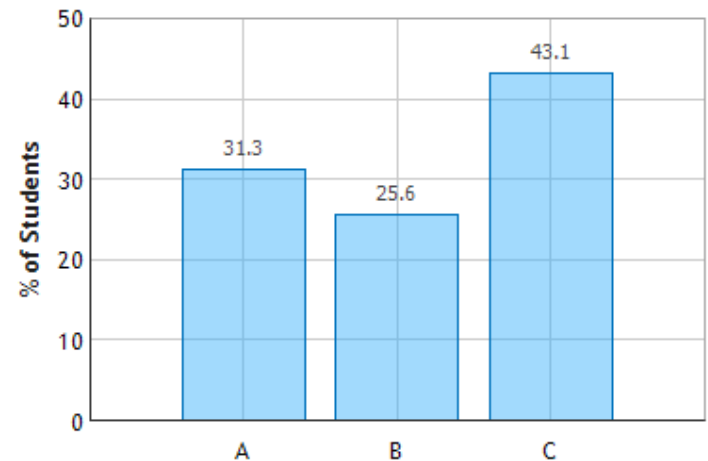
Which equation correctly describes this electromagnetic wave?

$E_x = E_0 \sin(kz \oplus \omega t)$ No – moving in the minus z direction

$E_y = E_0 \sin(kz - \omega t)$ No – has E_y rather than E_x

$B_y = B_0 \sin(kz - \omega t)$

Electromagnetic Waves: Question 1 (N = 792)



CheckPoint 2a



Your iclicker operates at a frequency of approximately 900 MHz (900×10^6 Hz).
What is the approximately wavelength of the EM wave produced by your iclicker?

- 0.03 meters
- 0.3 meters
- 3.0 meters
- 30. meters

$$C = 3.0 \times 10^8 \text{ m/s}$$

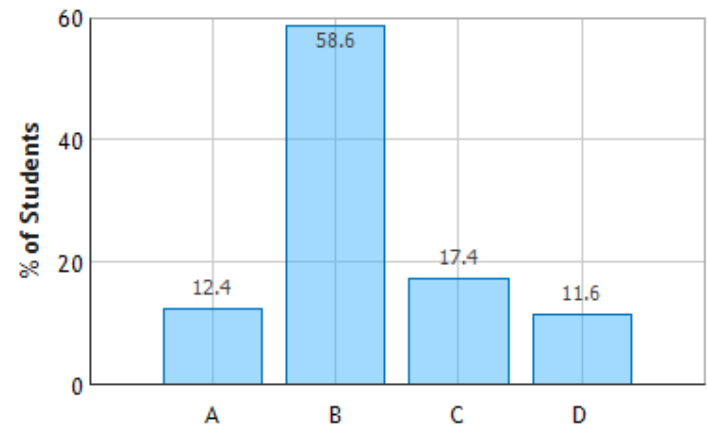
Wavelength is equal to the speed of light divided by the frequency.

$$\lambda = \frac{c}{f} = \frac{300,000,000}{900,000,000} = \frac{1}{3}$$

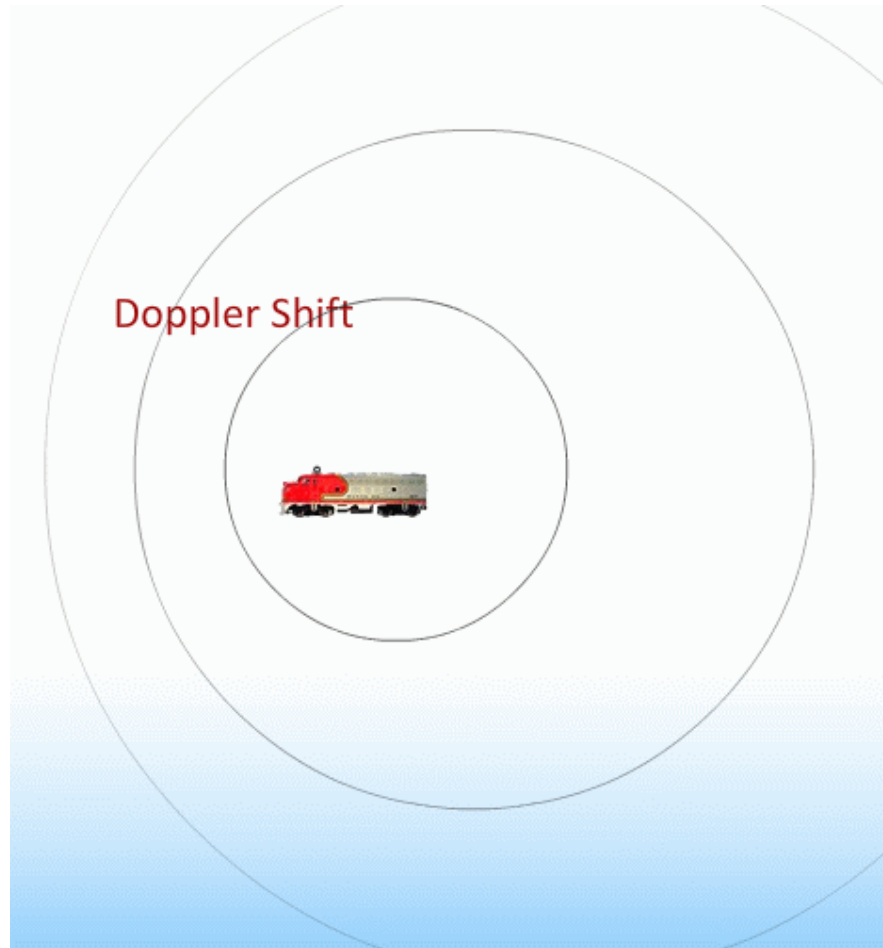
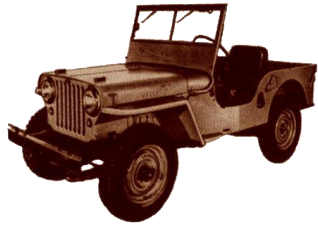
Check:

Look at size of antenna on base unit

EM waves from an iclicker: Question 1 (N = 792)



Doppler Shift



The Big Idea

As source approaches:
Wavelength decreases
Frequency Increases

Doppler Shift for E-M Waves

What's Different from Sound or Water Waves ?

Sound /Water Waves :

You can calculate (no relativity needed)

BUT

Result is somewhat complicated: is source or observer moving wrt medium?

Electromagnetic Waves :

You need relativity (time dilation) to calculate

BUT

Result is simple: only depends on relative motion of source & observer

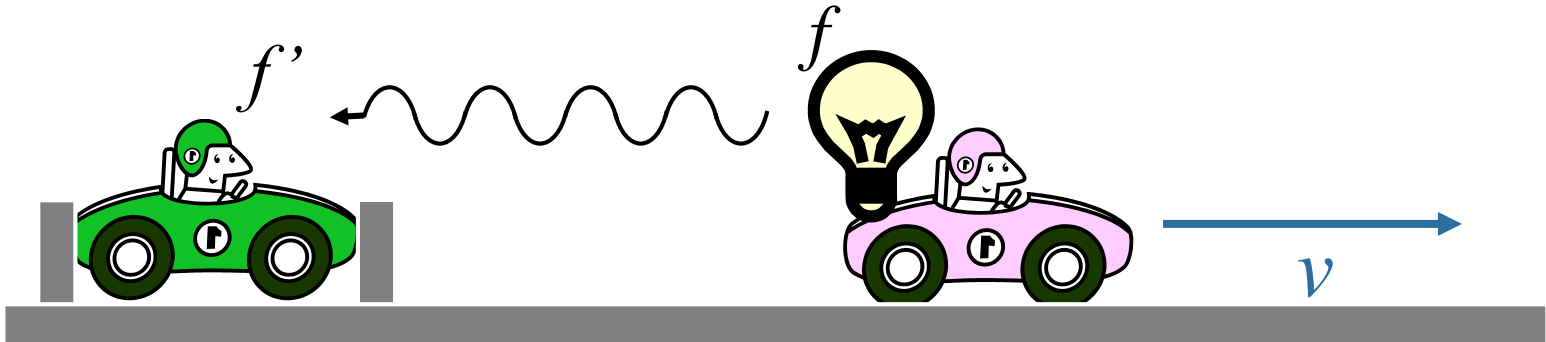
$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$$

$$\beta = v/c$$

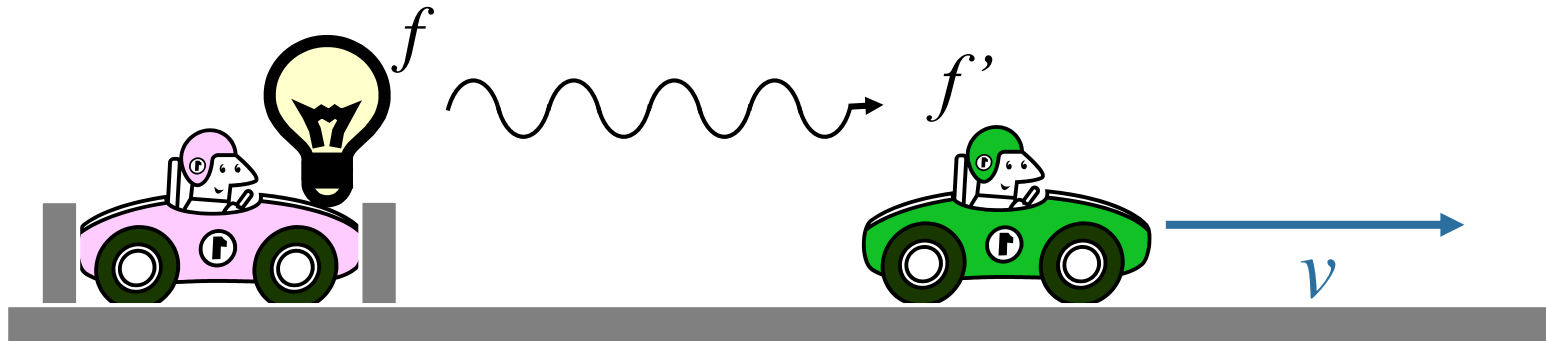
$\beta > 0$ if source & observer are approaching

$\beta < 0$ if source & observer are separating

Doppler Shift for E-M Waves



or



The Doppler Shift is the SAME for both cases!

f'/f only depends on the relative velocity

$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$$

Doppler Shift for E-M Waves

A Note on Approximations

$$f' = f \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}} \quad \xrightarrow{\beta \ll 1} \quad f' \approx f(1 + \beta)$$

why?

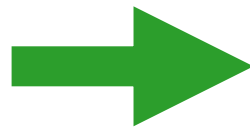
Taylor Series: Expand $F(\beta) = \left(\frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}}$ around $\beta = 0$

$$F(\beta) = F(0) + \frac{F'(0)}{1!} \beta + \frac{F''(0)}{2!} \beta^2 + \dots$$

Evaluate:

$$F(0) = 1$$

$$F'(0) = 1$$



$$F(\beta) \approx 1 + \beta$$

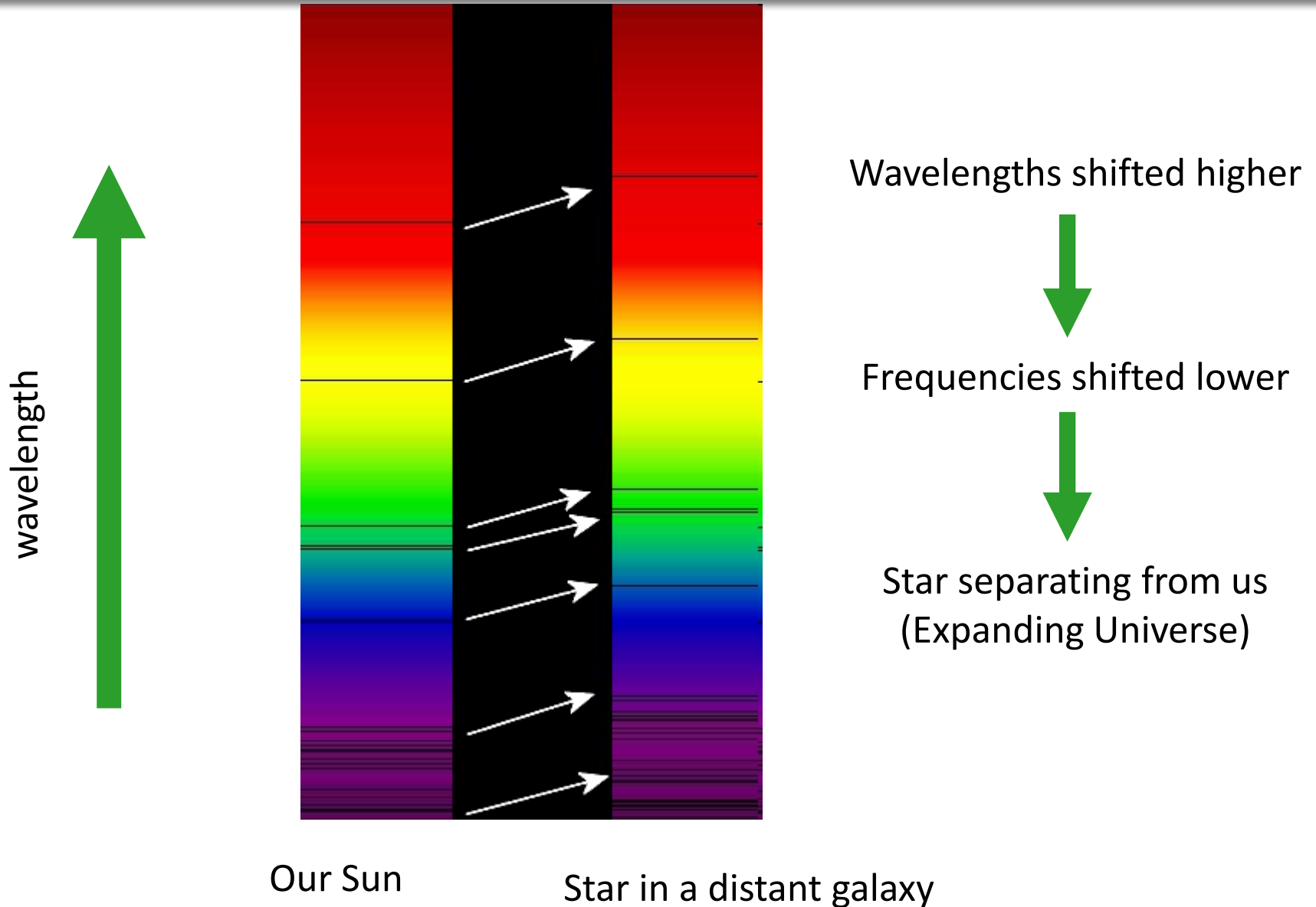
NOTE:

$$F(\beta) = (1 + \beta)^{1/2}$$

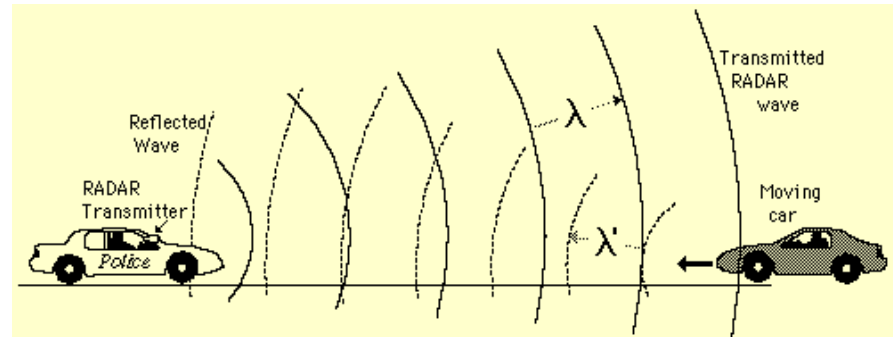


$$F(\beta) \approx 1 + \frac{1}{2} \beta$$

Red Shift



Example



Police radars get twice the effect since the EM waves make a round trip:

$$f' \approx f(1 + 2\beta)$$

If $f = 24,000,000,000 \text{ Hz}$ (k-band radar gun)

$c = 300,000,000 \text{ m/s}$

v	β	f'	$f' - f$
30 m/s (67 mph)	1.000×10^{-7}	$24,000,004,800$	4800 Hz
31 m/s (69 mph)	1.033×10^{-7}	$24,000,004,959$	4959 Hz

CheckPoint 2b



If you wanted to see the EM wave produced by the iclicker with your eyes, which of the following would work? (Note: Your eyes are sensitive to EM waves w/ frequency around 10^{14} Hz)

A) Run away from the iclicker when it is voting.

B) Run toward the iclicker when it is voting.

C) Neither will work, moving relative to the iclicker won't change the frequency reaching your eyes.

$$f_{\text{iclicker}} = 900 \text{ MHz}$$

Need to shift frequency UP



Need to approach iclicker ($\beta > 0$)

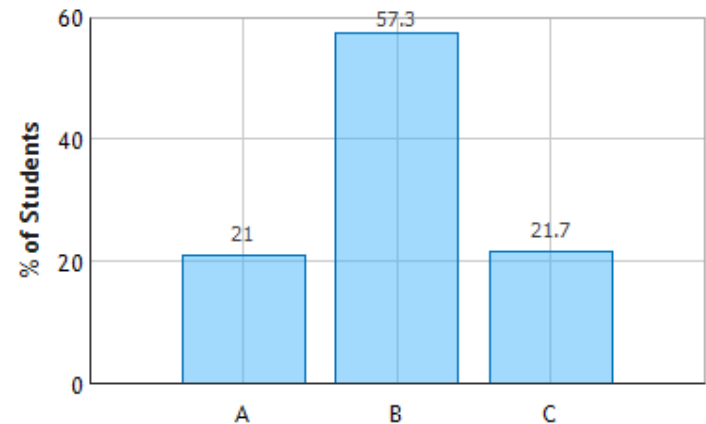
How fast would you need to run to see the iclicker radiation?

$$\frac{f'}{f} = \frac{10^{14}}{10^9} = 10^5 = \left(\frac{1+\beta}{1-\beta} \right)^{1/2}$$



$$10^{10} = \left(\frac{1+\beta}{1-\beta} \right) \rightarrow \beta = \frac{10^{10} - 1}{10^{10} + 1} = \frac{1 - 10^{-10}}{1 + 10^{-10}}$$

Approximation Exercise: $\beta \approx 1 - (2 \times 10^{-10})$



Waves Carry Energy

Total Energy Density

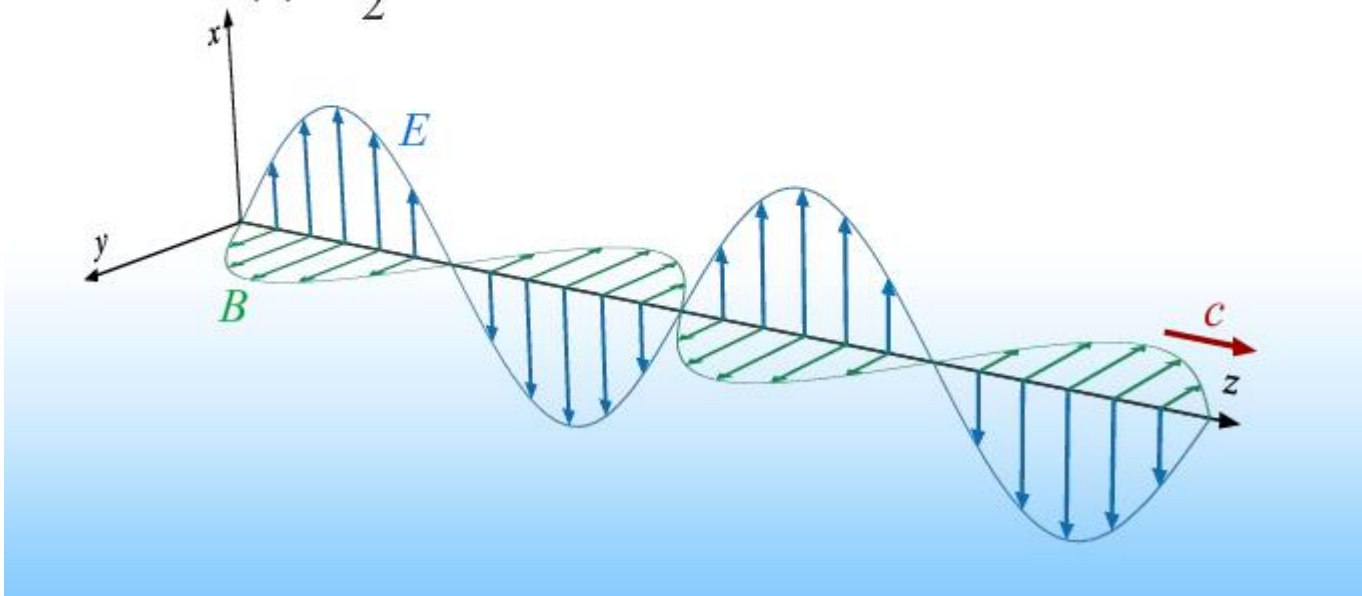
$$u = \epsilon_0 E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2$$

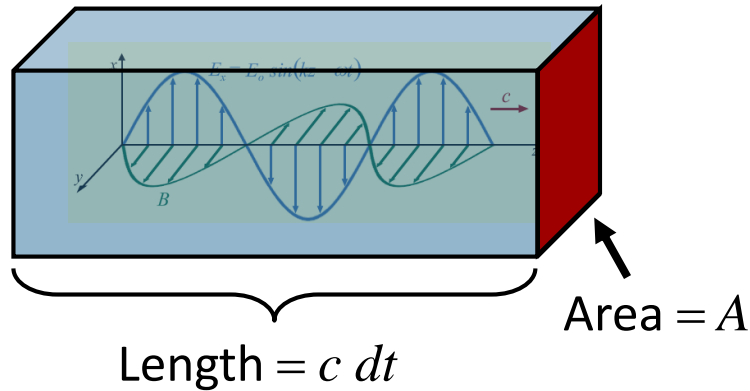
Intensity

$$I = \frac{1}{2} c \epsilon_0 E_0^2 = c \langle u \rangle$$



Intensity

Intensity = Average energy delivered per unit time, per unit area



$$\rightarrow I \equiv \frac{1}{A} \left\langle \frac{dU}{dt} \right\rangle$$

$$\rightarrow \langle dU \rangle = \langle u \rangle \cdot \text{volume} = \langle u \rangle A c dt$$

Total Energy Density

$$u = \epsilon_0 E^2$$

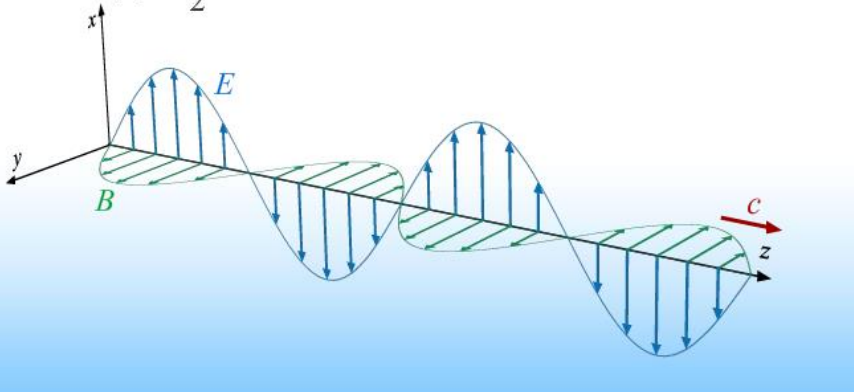
Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2$$

Intensity

$$I = \frac{1}{2} c \epsilon_0 E_0^2 = c \langle u \rangle$$

$$\rightarrow I = c \langle u \rangle$$



Sunlight on Earth:

$$I \sim 1000 \text{ J/s/m}^2$$

$$\sim 1 \text{ kW/m}^2$$

Waves Carry Energy

Total Energy Density

$$u = \epsilon_0 E^2$$

Intensity

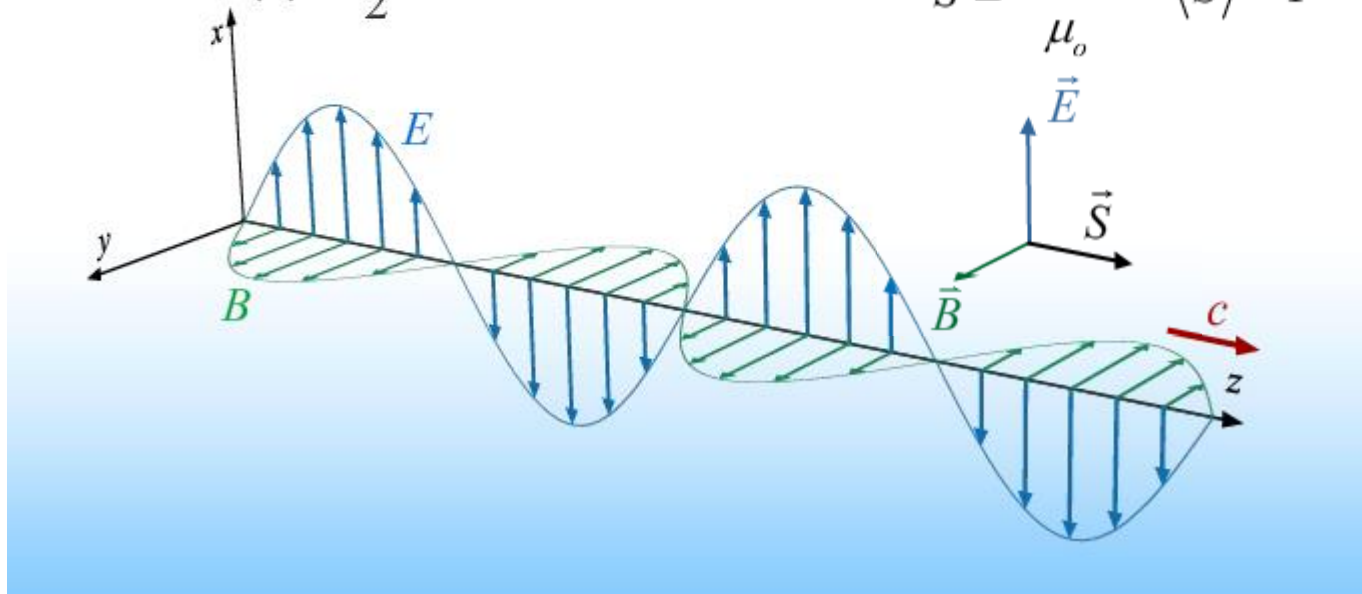
$$I = \frac{1}{2} c \epsilon_0 E_0^2 = c \langle u \rangle$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_0} \quad \langle S \rangle = I$$

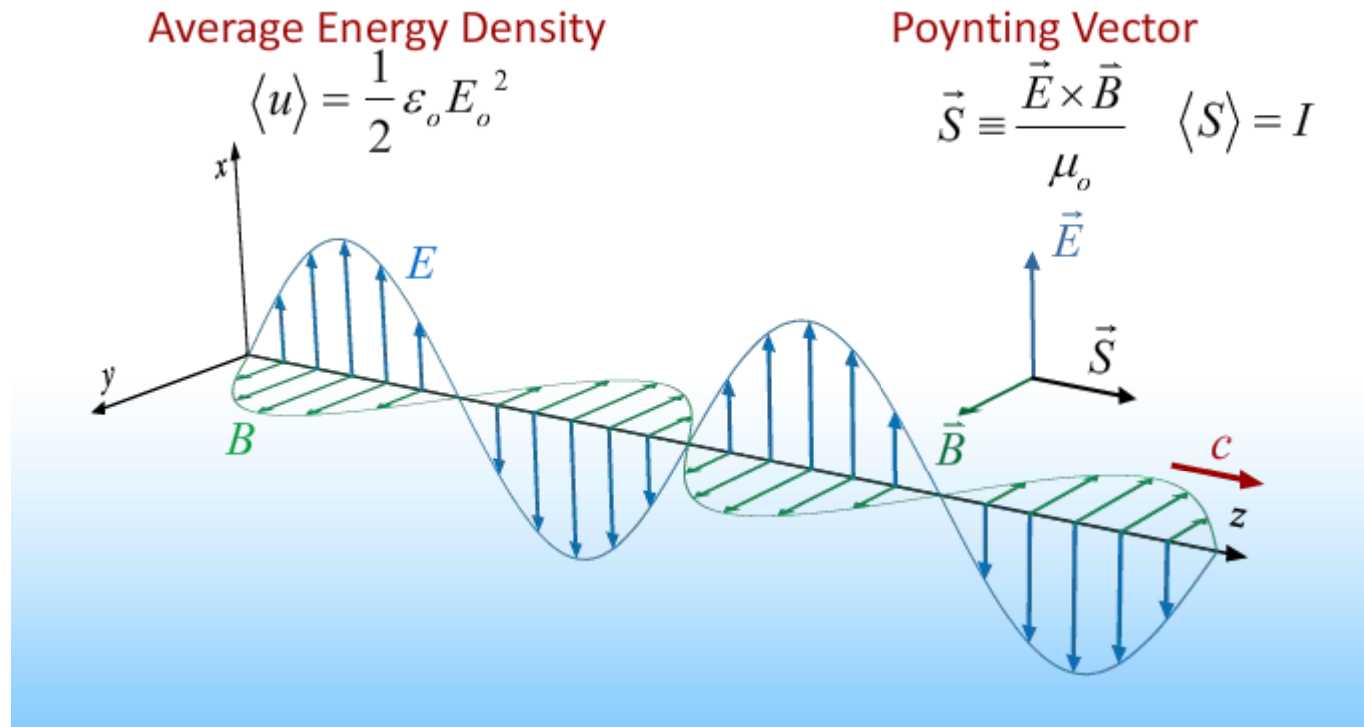


Comment on Poynting Vector

Just another way to keep track of all this:

Its magnitude is equal to I

Its direction is the direction of propagation of the wave



Power in EM Waves: Example

A cell phone tower has a transmitter with a power of 100 W. What is the magnitude of the peak electric field a distance 1500 m (~ 1 mile) from the tower? Assume the transmitter is a point source.

What is the intensity of the wave 1500 m from the tower?

A) 1.5 nW/m²

B) 3.5 μW/m²

C) 6 mW/m²

$$I = \frac{P}{4\pi r^2} = \frac{100 \text{ W}}{4\pi (1500 \text{ m})^2} = 3.5 \frac{\mu\text{W}}{\text{m}^2}$$

What is the peak value of the electric field?

$$I = \langle \langle |\vec{S}| \rangle \rangle = \left\langle \left\langle \frac{|\vec{E} \times \vec{B}|}{\mu_0} \right\rangle \right\rangle = \left\langle \left\langle \frac{E}{\mu_0} \frac{E}{c} \right\rangle \right\rangle = \frac{1}{\mu_0 c} \frac{E_0^2}{2} \Rightarrow E_0 = \sqrt{2\mu_0 c I}$$

$$E_0 = \left(2 \cdot 4\pi \times 10^{-7} \cdot 3 \times 10^8 \cdot 3.5 \times 10^{-6} \right)^{1/2} = 51 \frac{\text{mV}}{\text{m}}$$

Checkpoint 1 b



Which of the following actions will increase the energy carried by an electromagnetic wave?

A. Increase E keeping ω constant

C. Both of the above will increase the energy

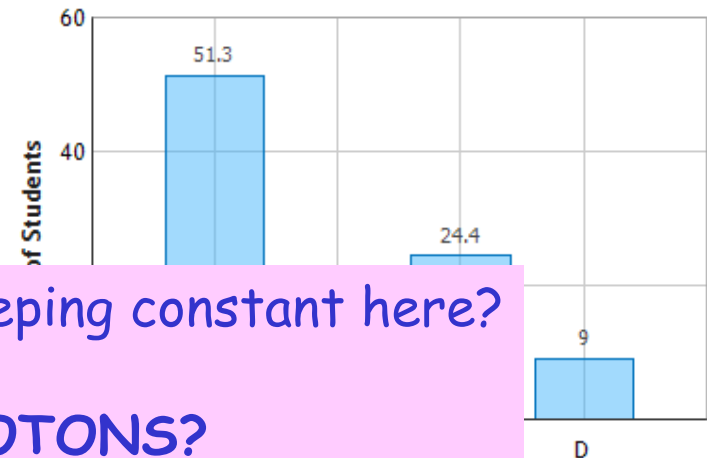
B. Increase ω keeping E constant

D. Neither of the above will increase the energy

Intensity

$$I = \frac{1}{2} c \epsilon_0 E_0^2$$

Electromagnetic Waves: Question 3 (N = 792)



But then again, what are we keeping constant here?

WHAT ABOUT PHOTONS?

The energy of one photon is

$$\mathcal{E}_{\text{photon}} = hf = h\omega/2\pi$$

$$U_{\text{wave}} = N_{\text{photons}} \times \mathcal{E}_{\text{photon}}$$

$$\mathcal{E}_{\text{photon}} / \text{Volume} = 1/2 \epsilon_0 E_0^2$$

Photons

We believe the energy in an e-m wave is carried by photons

Question: What are Photons?

Answer: Photons are Photons.

Photons possess both wave and particle properties

Particle:

Energy and Momentum localized

Wave:

They have definite frequency & wavelength ($f\lambda = c$)

Connections seen in equations:

$$E = hf$$

$$p = h/\lambda$$

Planck's constant

$$h = 6.63e^{-34} \text{ J} \cdot \text{s}$$

Question: How can something be both a particle and a wave?

Answer: It can't (when we observe it)

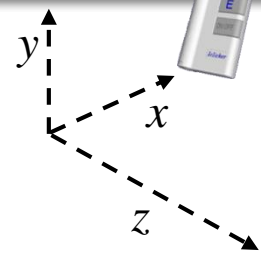
What we see depends on how we choose to measure it!

The mystery of quantum mechanics: More on this in PHYS 214

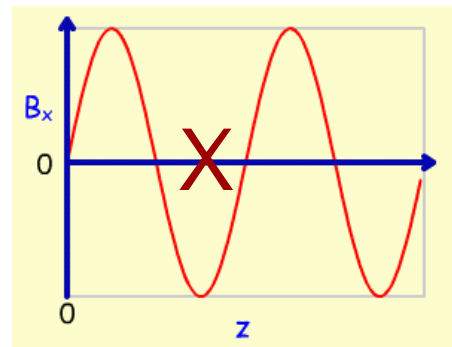
Exercise



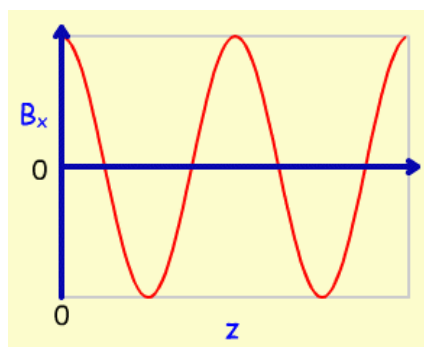
An electromagnetic wave is described by: $\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$
 where \hat{j} is the unit vector in the $+y$ direction.



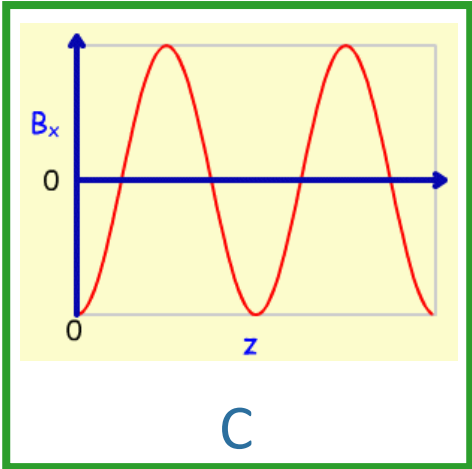
Which of the following graphs represents the z - dependence of B_x at $t = 0$?



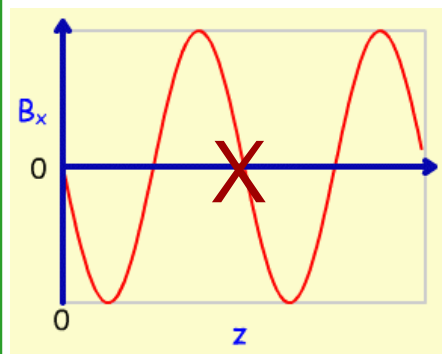
A



B



C

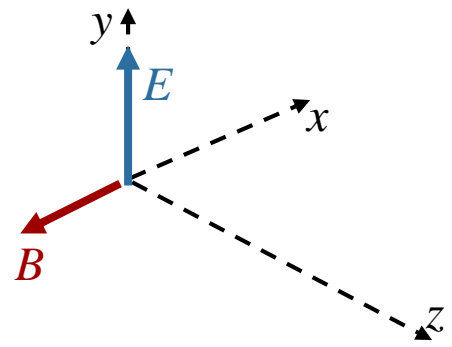


D

E and B are "in phase" (or 180° out of phase)

$\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$ \longrightarrow Wave moves in $+z$ direction

$\vec{E} \times \vec{B}$ Points in direction of propagation



$$\vec{B} = -\hat{i}B_0 \cos(kz - \omega t)$$

Exercise

An electromagnetic wave is described by:

$$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$

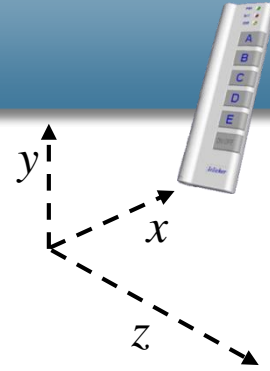
What is the form of B for this wave?

A) $\vec{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

C) $\vec{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

B) $\vec{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

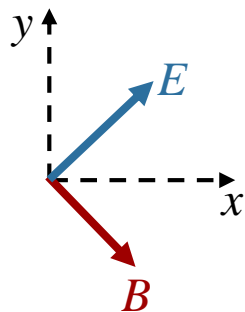
D) $\vec{B} = \frac{-\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$



$$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$$



Wave moves in $-z$ direction



$+z$ points out of screen

$-z$ points into screen

$\vec{E} \times \vec{B}$ Points in direction of propagation

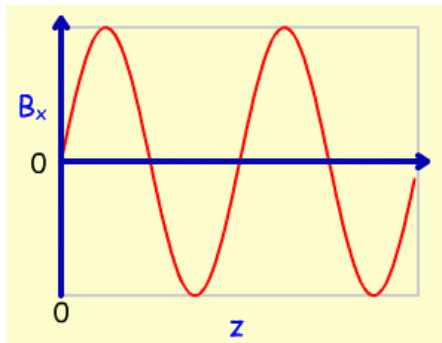
Exercise



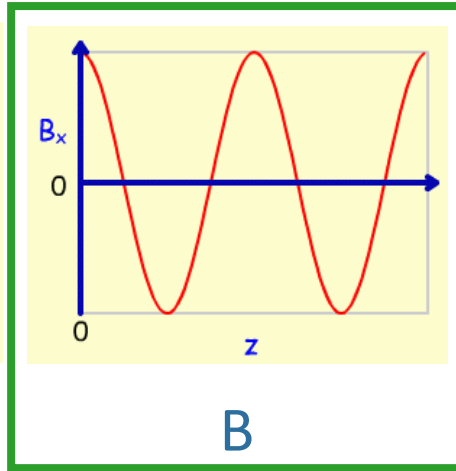
An electromagnetic wave is described by:

$$\vec{E} = \hat{j}E_0 \sin(kz + \omega t)$$

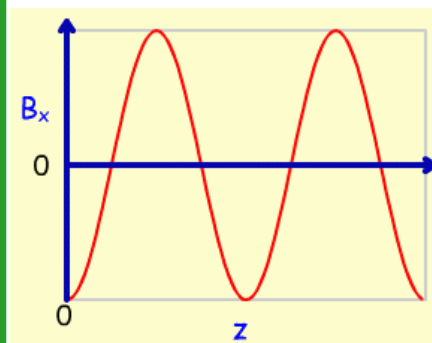
Which of the following plots represents $B_x(z)$ at time $t = \pi/2\omega$?



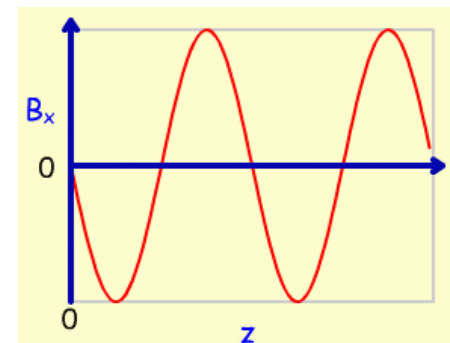
A



B



C

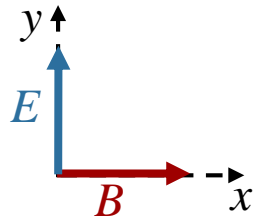


D

Wave moves in negative z – direction



$$\vec{B} = \hat{i}(E_0/c) \sin(kz + \omega t)$$



+ z points out of screen

- z points into screen

at $\omega t = \pi/2$:

$$B_x = (E_0/c) \sin(kz + \pi/2)$$

$$B_x = (E_0/c) \{ \sin kz \cos(\pi/2) + \cos kz \sin(\pi/2) \}$$

$$B_x = (E_0/c) \cos(kz)$$

$\vec{E} \times \vec{B}$ Points in direction of propagation

Exercise



A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

Is it possible that the professor’s argument is correct?

$$(\lambda_{green} = 500 \text{ nm}, \lambda_{red} = 600 \text{ nm})$$

A) YES

B) NO

As professor approaches stoplight, the frequency of its emitted light will be shifted **UP**

The speed of light does not change

Therefore, the wavelength (c/f) would be shifted **DOWN**

If he goes fast enough, he could observe a green light !

Follow-Up



A certain unnamed physics professor was arrested for running a stoplight. He said the light was green. A pedestrian said it was red. The professor then said: “We are both being truthful; you just need to account for the Doppler effect !”

How fast would the professor have to go to see the light as green?

$$(\lambda_{\text{green}} = 500 \text{ nm}, \lambda_{\text{red}} = 600 \text{ nm})$$

- A) 540 m/s B) 5.4×10^4 m/s **C) 5.4×10^7 m/s** D) 5.4×10^8 m/s

Relativistic Doppler effect: $f' = f \sqrt{\frac{1+\beta}{1-\beta}}$

$$\frac{f'}{f} = \frac{600}{500} = \sqrt{\frac{1+\beta}{1-\beta}} \quad \longrightarrow \quad 36(1-\beta) = 25(1+\beta) \quad \longrightarrow \quad \beta = \frac{11}{61} = 0.18$$

Note approximation for small β is not bad: $f' = f(1+\beta) \quad \longrightarrow \quad \beta = \frac{1}{5} = 0.2$

$c = 3 \times 10^8 \text{ m/s} \Rightarrow v = 5.4 \times 10^7 \text{ m/s} \quad \longrightarrow \quad \text{Change the charge to speeding!}$