Electricity & Magnetism
Lecture 3

Today's Concepts:

A) Electric Flux

B) Field Lines

→ Gauss' Law

Your Comments

"Is Eo (epsilon knot) a fixed number? " "epsilon 0 seems to be a derived quantity. Where does it come from?"

IT'S JUST A CONSTANT

$$\vec{E} = k\frac{q}{r^2}\hat{r} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_o r^2}\frac{q}{r^2}\hat{r}$$

$$k \equiv \frac{1}{4\pi\varepsilon_o}$$

$k = 9 \text{ x } 10^9 \text{ N m}^2 / \text{C}^2$

$\varepsilon_o = 8.85 \; x \; 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2$

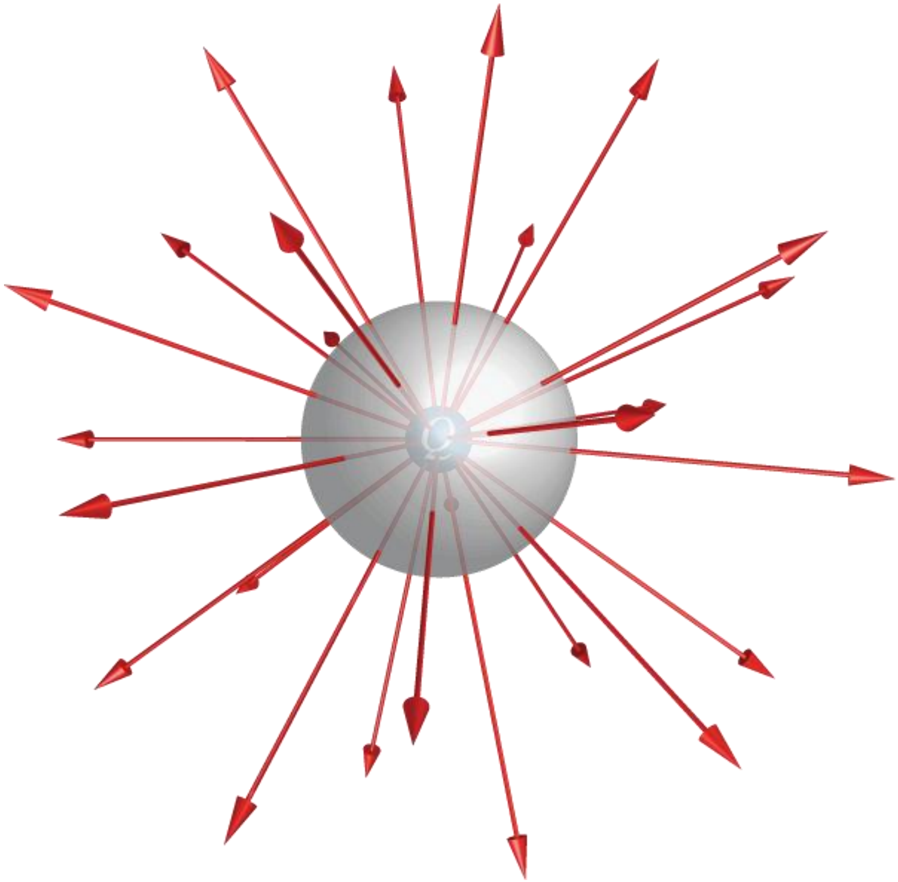
"I fluxing love physics!"

"My only point of confusion is this: if we can represent any flux through any surface as the total charge divided by a constant, why do we even bother with the integral definition? is that for non-closed surfaces or something?

These concepts are a lot harder to grasp than mechanics.

WHEW.....this stuff was quite abstract.....it always seems as though I feel like I understand the material after watching the prelecture, but then get many of the clicker questions wrong in lecture...any idea why or advice?? Thanks

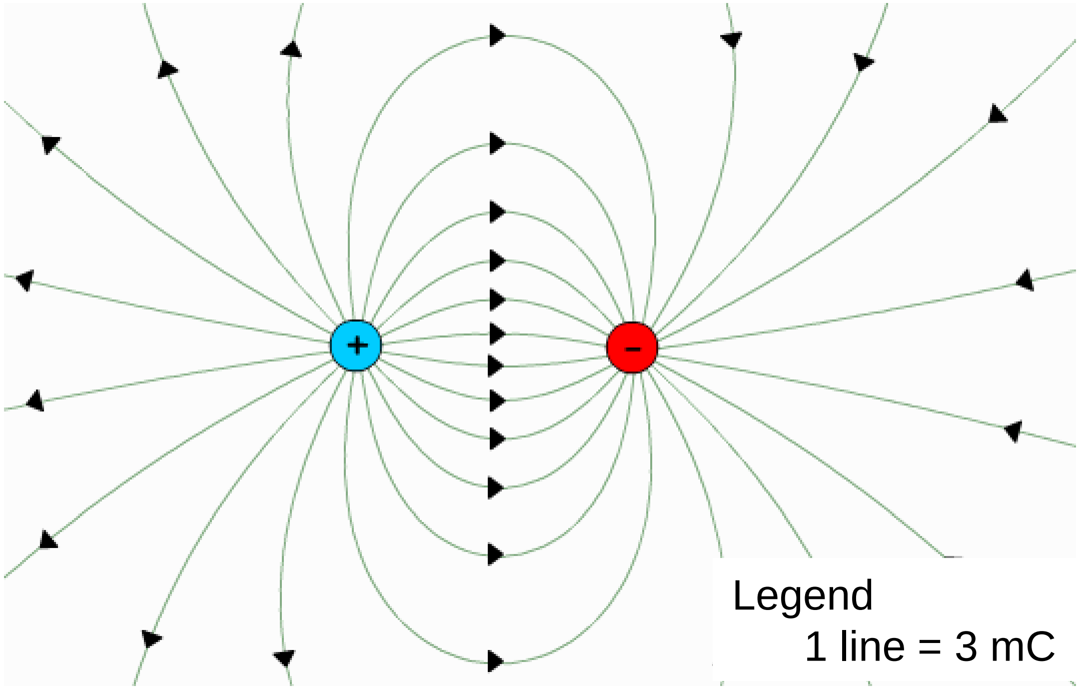
Electric Field Lines



Direction & Density of Lines
represent
Direction & Magnitude of E

Point Charge:
Direction is radial
Density $\propto 1/R^2$

Electric Field Lines

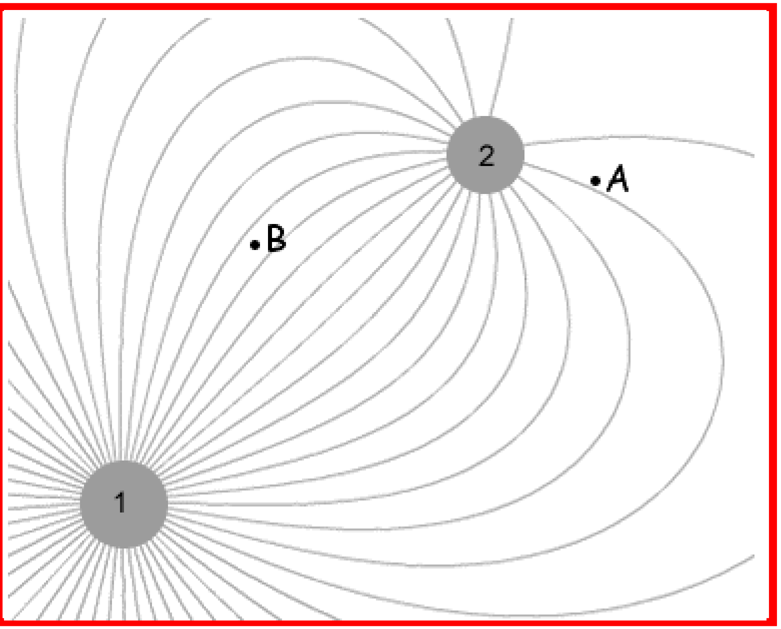


Dipole Charge Distribution:
Direction & Density
much more interesting.

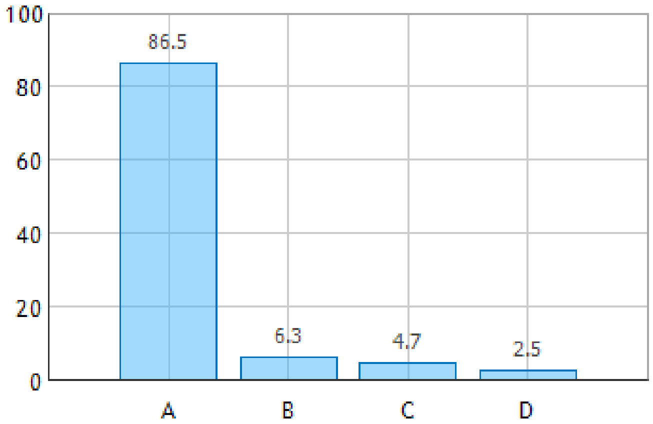
Simulation

"Please discuss further how the number of field lines is determined. Is this just convention? I feel as if the "number" of field lines is arbitrary, because the field is a vector field and is defined for all points in space. Therefore, it is possible to draw an infinite number of lines following this field between the charges. Thanks!"

CheckPoint 3.1



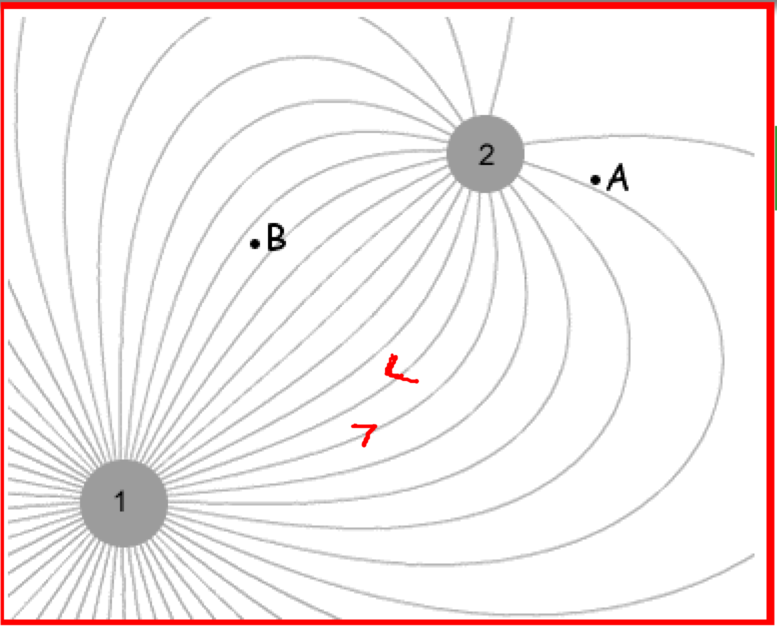
A. $|Q_1| > |Q_2|$
B. $|Q_1| = |Q_2|$
C. $|Q_1| < |Q_2|$
D. Not enough info



"Q1 has more field lines than Q2, so Q1 has a charge with greater magnitude..."

Simulation

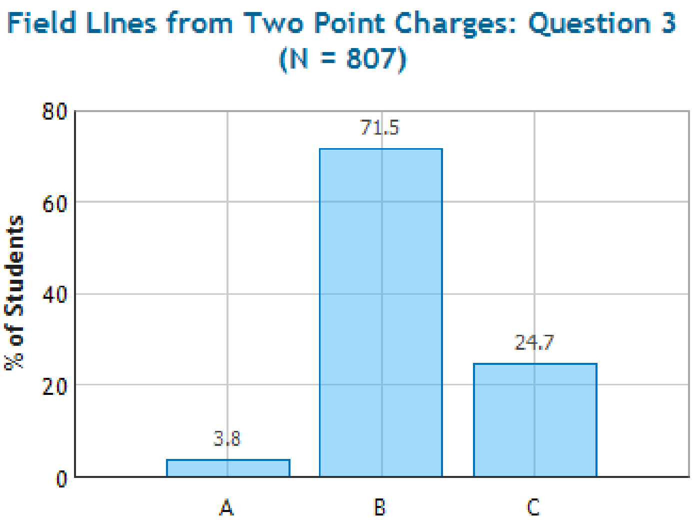
CheckPoint 3.3



A. Q_1 and Q_2 have the same sign

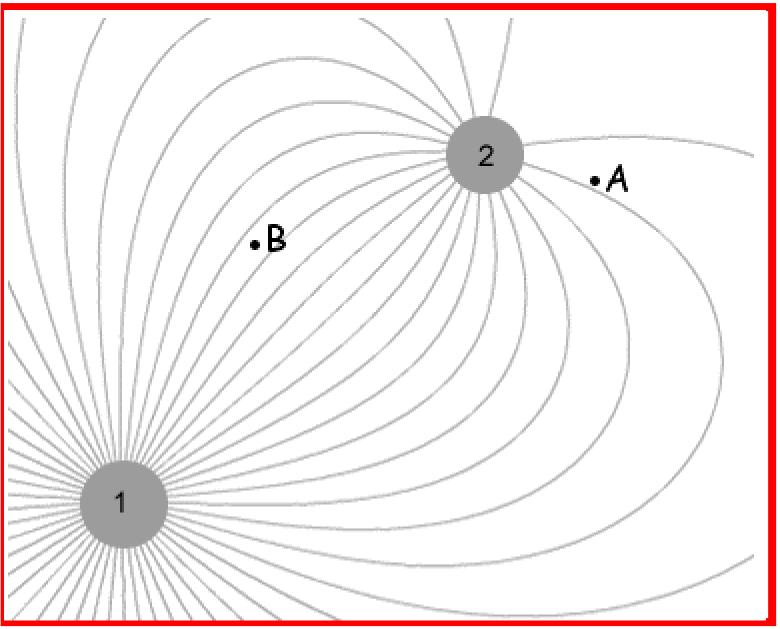
B. Q_1 and Q_2 have opposite signs

C. C. Not enough info

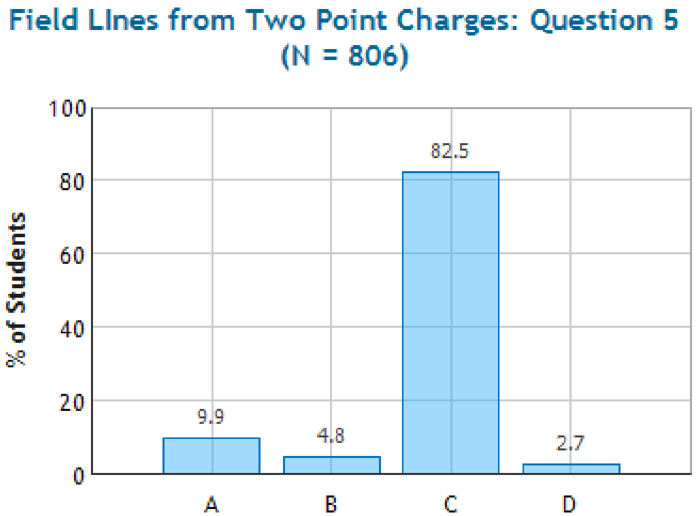


"They are opposite because the field lines connect the two charges."

CheckPoint 3.5



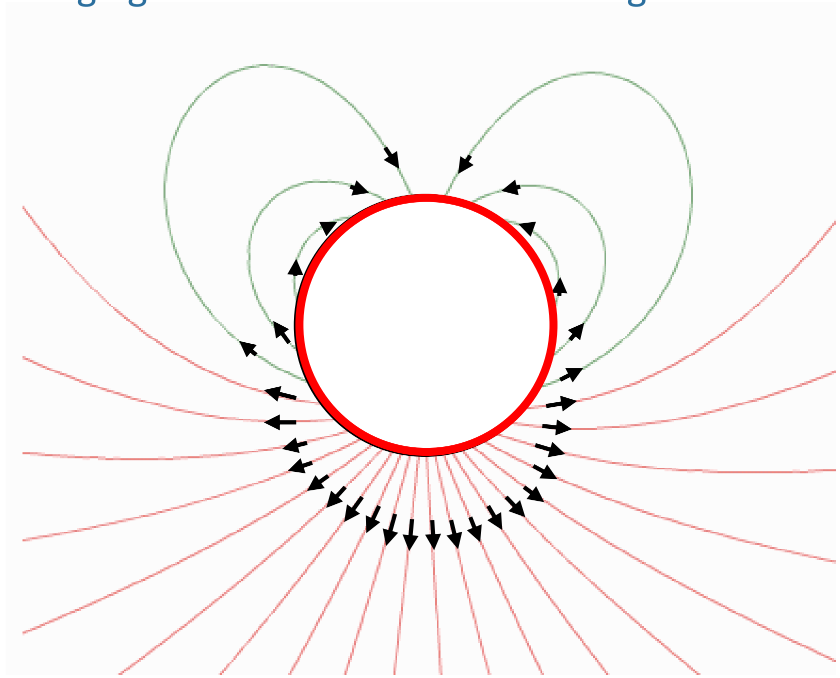
A. $|E_A| > |E_B|$
B. $|E_A| = |E_B|$
C. $|E_A| < |E_B|$
D. Not enough info



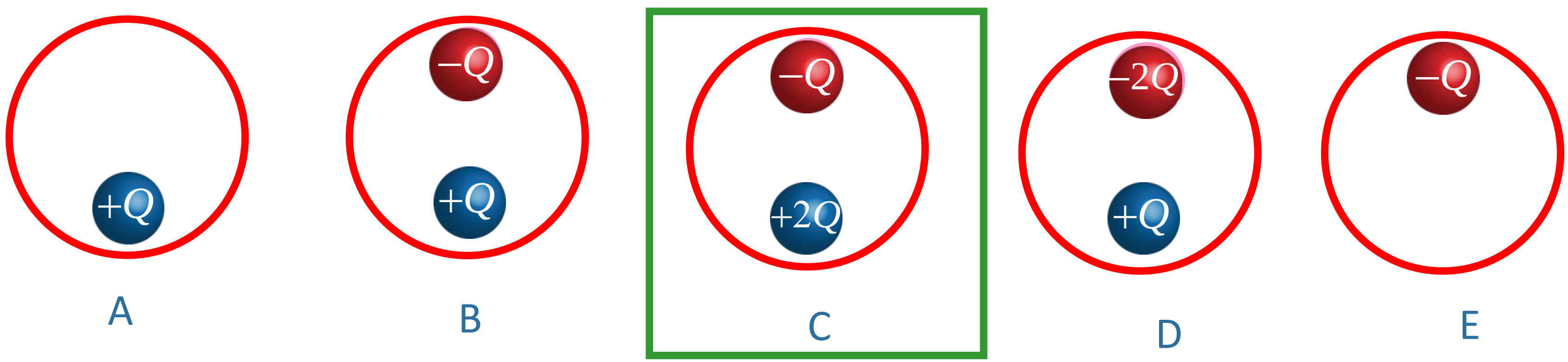
"the lines are closer together at point B"

Point Charges

"Telling the difference between positive and negative charges while looking at field lines. Does field line density from a certain charge give information about the sign of the charge?"

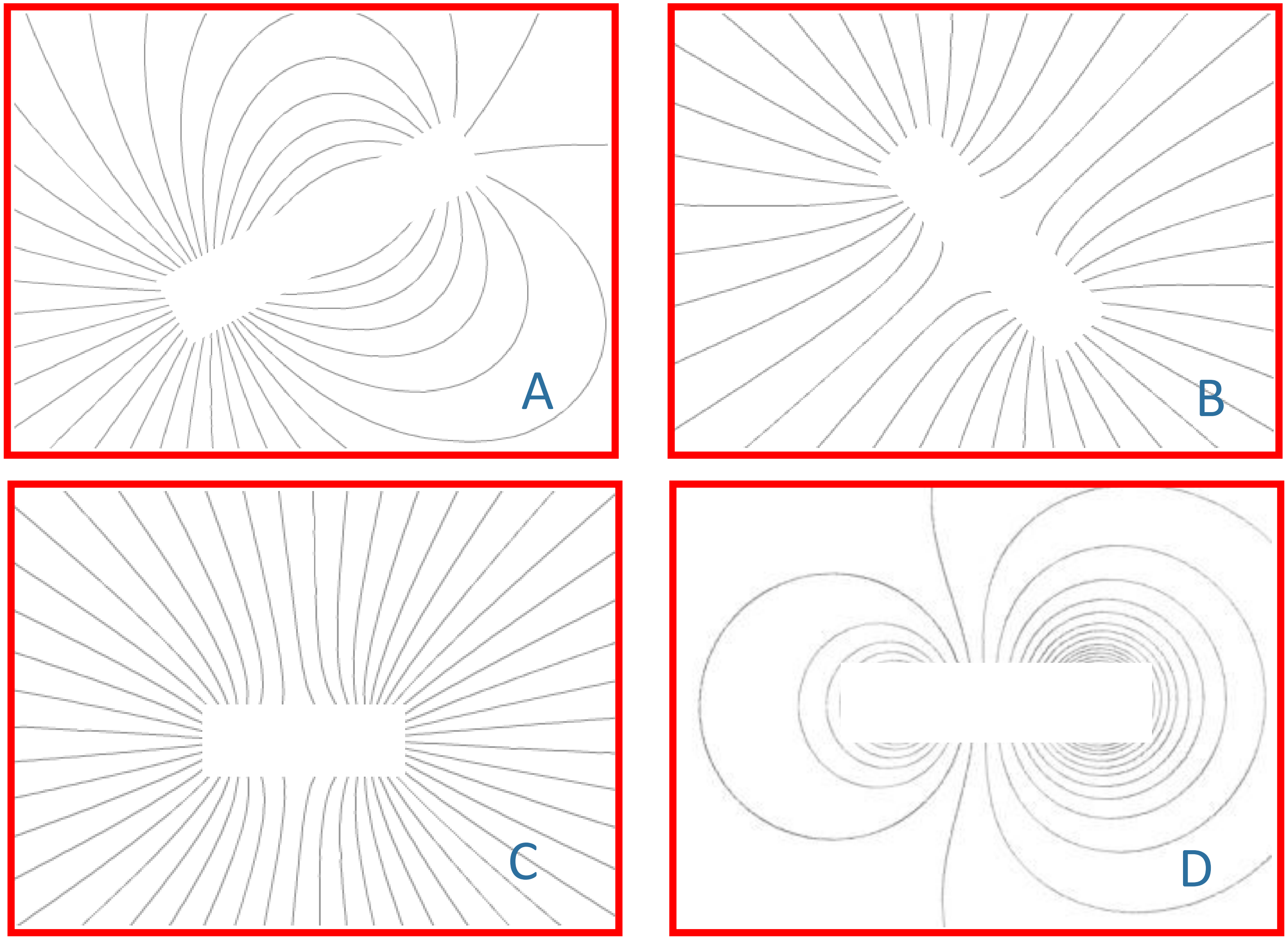


What charges are inside the red circle?



Electric Field lines

Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?



Simulation

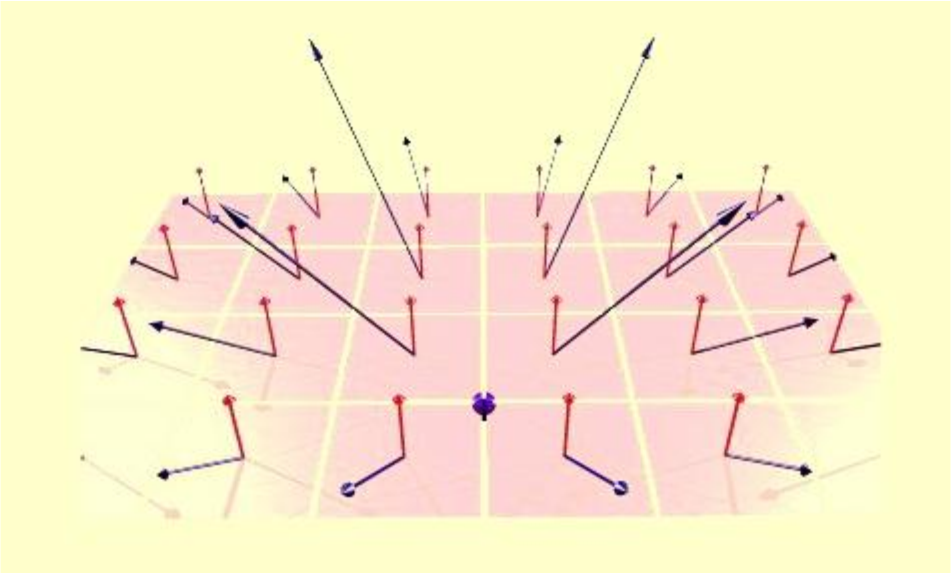
Electric Flux "Counts Field Lines"

"I'm very confused by the general concepts of flux through surface areas. please help"

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A}$$

Flux through surface S

Integral of $\vec{E} \cdot d\vec{A}$ on surface S

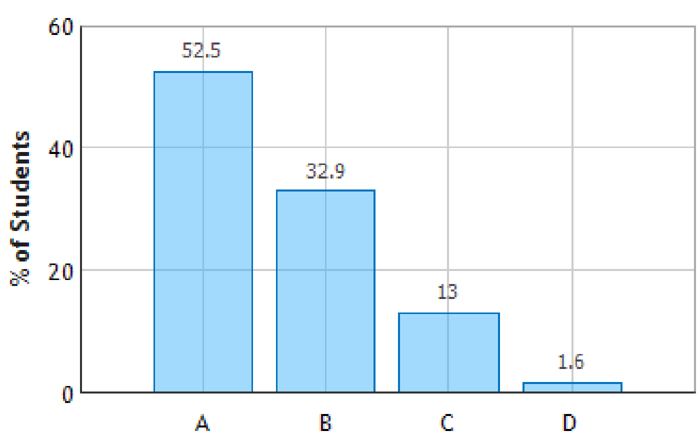


CheckPoint 1

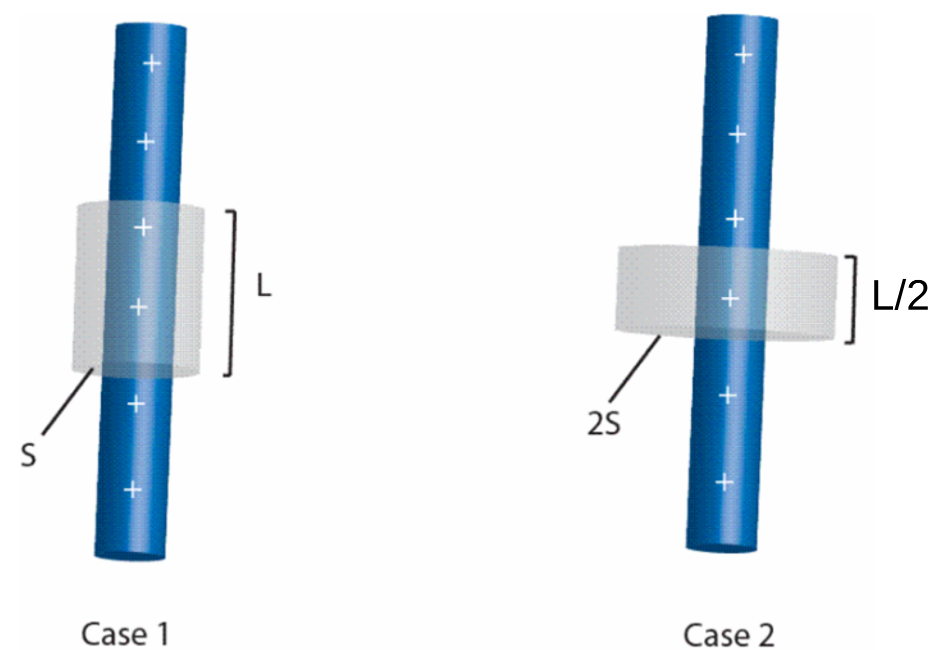
"Case 1 has twice as much charge enclosed as case 2, so the flux will be twice as big in case 1."

"The flux in both cases is going to equal the electric field times the surface area of the cylinder. In case 1, the surface area is (2pi*s)*L. In case 2, the surface area is (2pi*2S)*L/2, which reduces to (2pi*s)*L."

"Twice the radius squared is factor of 4, but half the length is 4/2=2 times as large for 2 than 1."



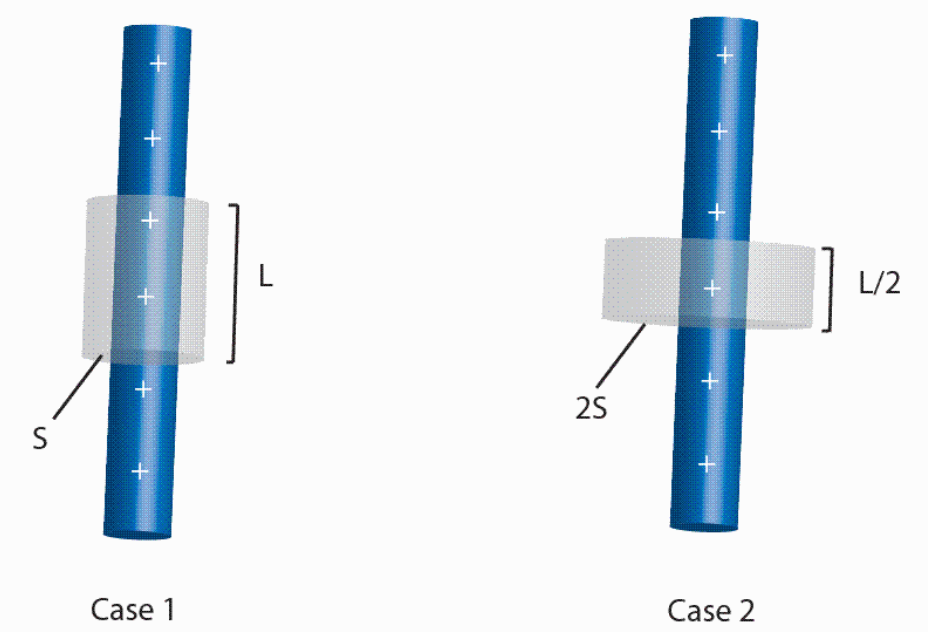
An infinitely long charged rod has uniform charge density λ and passes through a cylinder (gray). The cylinder in Case 2 has twice the radius and half the length compared with the cylinder in Case 1.



(A)	(B)	(C)	(D)
$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none

CheckPoint 2

2) An infinitely long charged rod has uniform charge density of λ, and passes through a cylinder (gray). The cylinder in case 2 has twice the cross sectional area and half the length compared to the cylinder in case 1.



(A)	(B)	(C)	(D)
$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none

Definition of Flux:

$$\Phi \equiv \int_{surface} \vec{E} \cdot d\vec{A}$$

E constant on barrel of cylinder
E perpendicular to barrel surface
(E parallel to dA)

$$\Phi = E \int_{barrel} d\vec{A} = EA_{barrel}$$

Case 1

$$A_{barrel} = 2\pi sL$$

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 s} \rightarrow \Phi_1 = \frac{\lambda L}{\varepsilon_0}$$

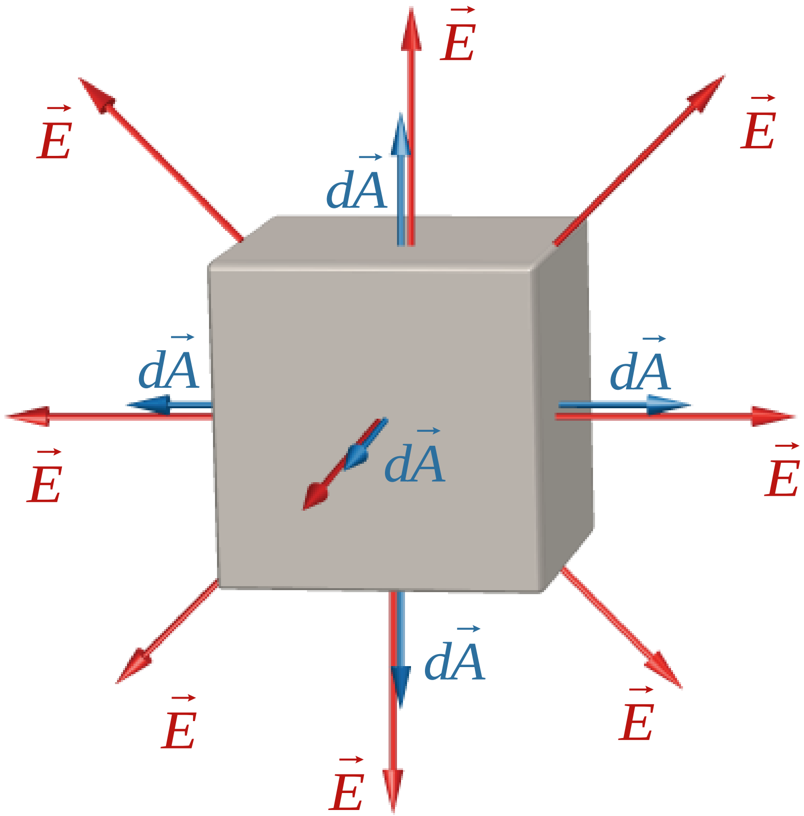
Case 2

$$A_2 = (2\pi(2s))L/2 = 2\pi sL$$

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0(2s)} \rightarrow \Phi_2 = \frac{\lambda(L/2)}{\varepsilon_0}$$

RESULT: GAUSS' LAW
Φ proportional to charge enclosed !

Direction Matters:

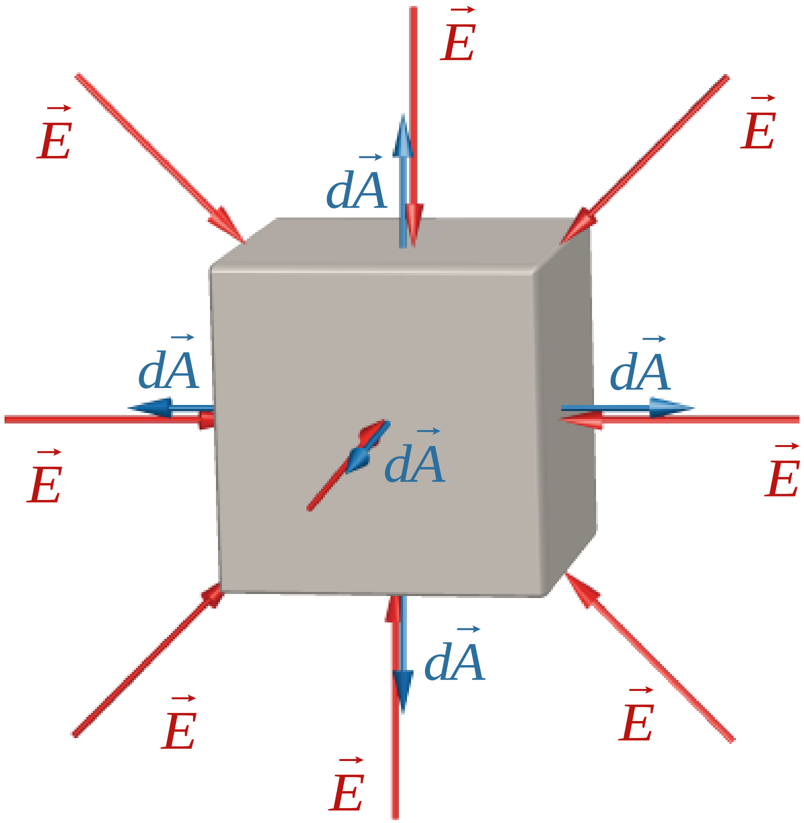


For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

Direction Matters:

Is there such thing as negative flux?

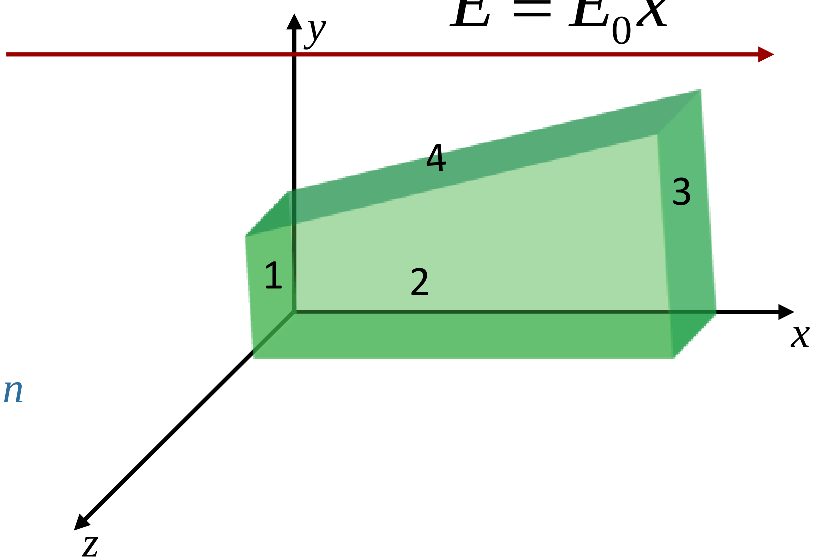


For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} < 0$$

Trapezoid in Constant Field

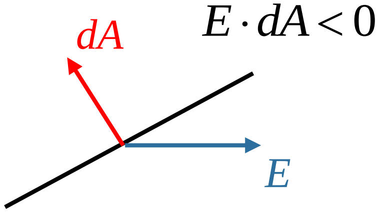
$\vec{E} = E_0 \hat{x}$



Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted

Define Φ_n = Flux through Face n



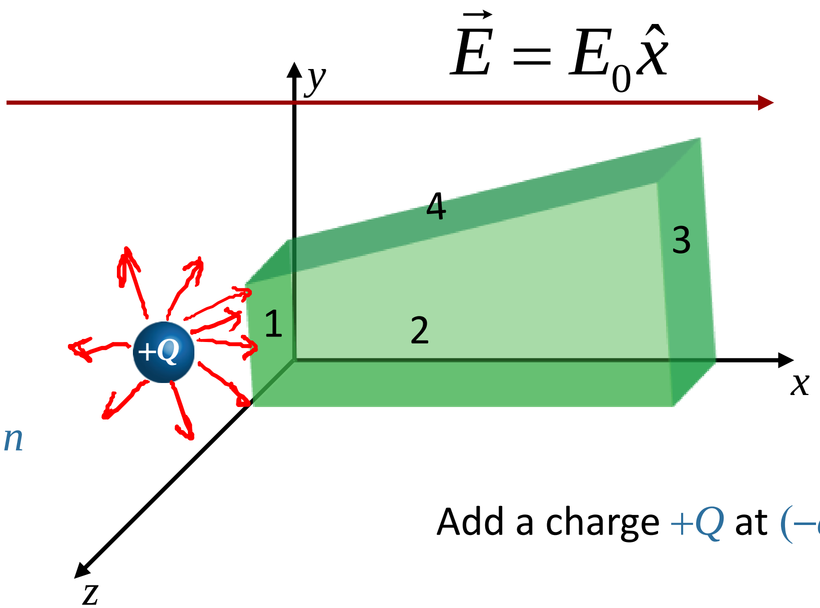
A) $\Phi_1 < 0$
B) $\Phi_1 = 0$
C) $\Phi_1 > 0$

A) $\Phi_2 < 0$
B) $\Phi_2 = 0$
C) $\Phi_2 > 0$

A) $\Phi_3 < 0$
B) $\Phi_3 = 0$
C) $\Phi_3 > 0$

A) $\Phi_4 < 0$
B) $\Phi_4 = 0$
C) $\Phi_4 > 0$

Trapezoid in Constant Field + Q



Label faces:

- 1: $x = 0$
- 2: $z = +a$
- 3: $x = +a$
- 4: slanted

Define Φ_n = Flux through Face n

Φ = Flux through Trapezoid

Add a charge $+Q$ at $(-a, a/2, a/2)$

How does Flux change?

A) Φ_1 increases

B) Φ_1 decreases

C) Φ_1 remains same

A) Φ_3 increases

B) Φ_3 decreases

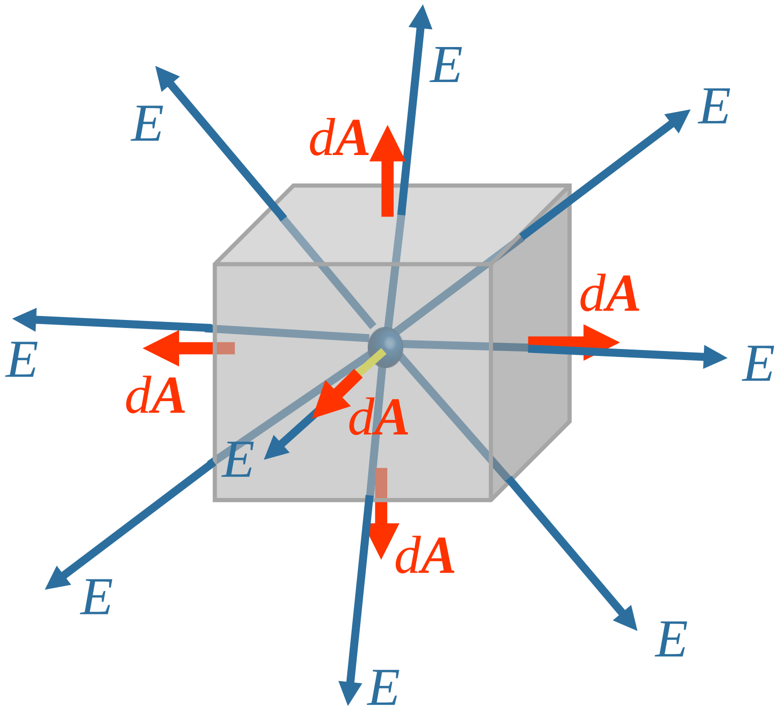
C) Φ_3 remains same

A) Φ increases

B) Φ decreases

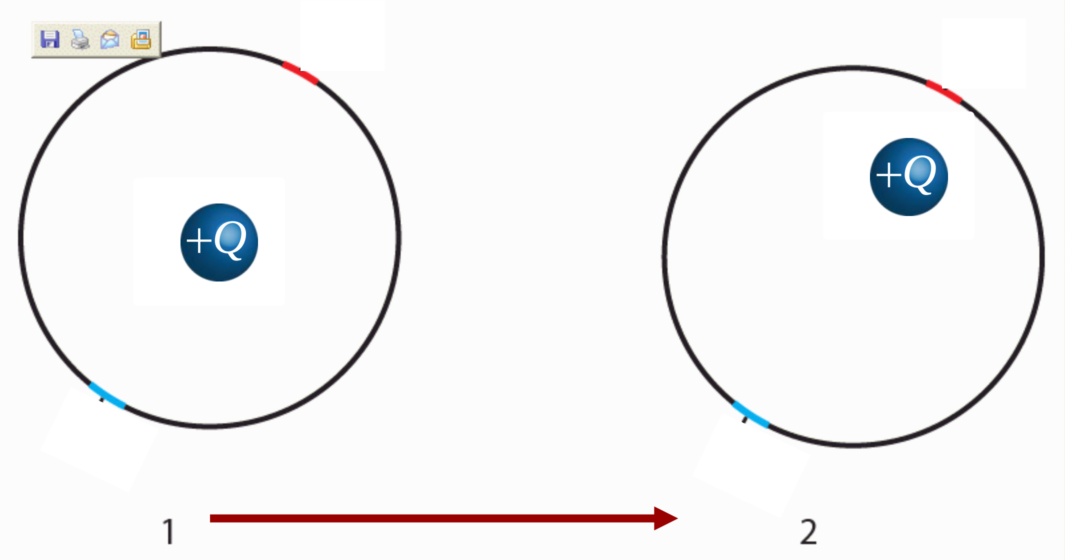
C) Φ remains same

Gauss Law



$$\oint_{\substack{closed\\surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

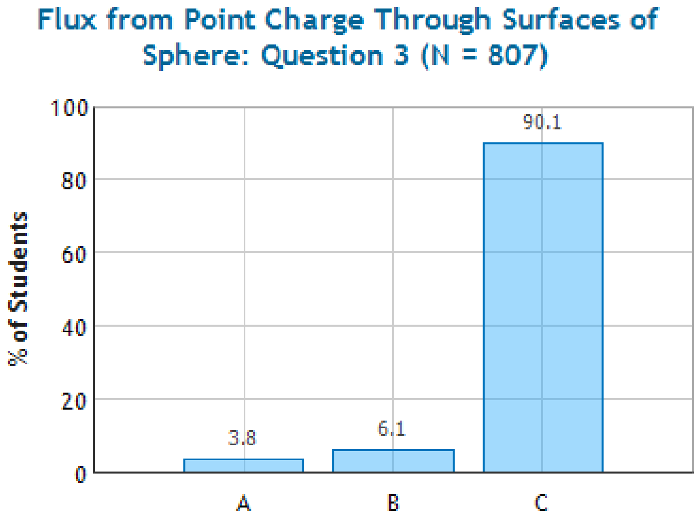
CheckPoint 2.3



A
Φ_E increases

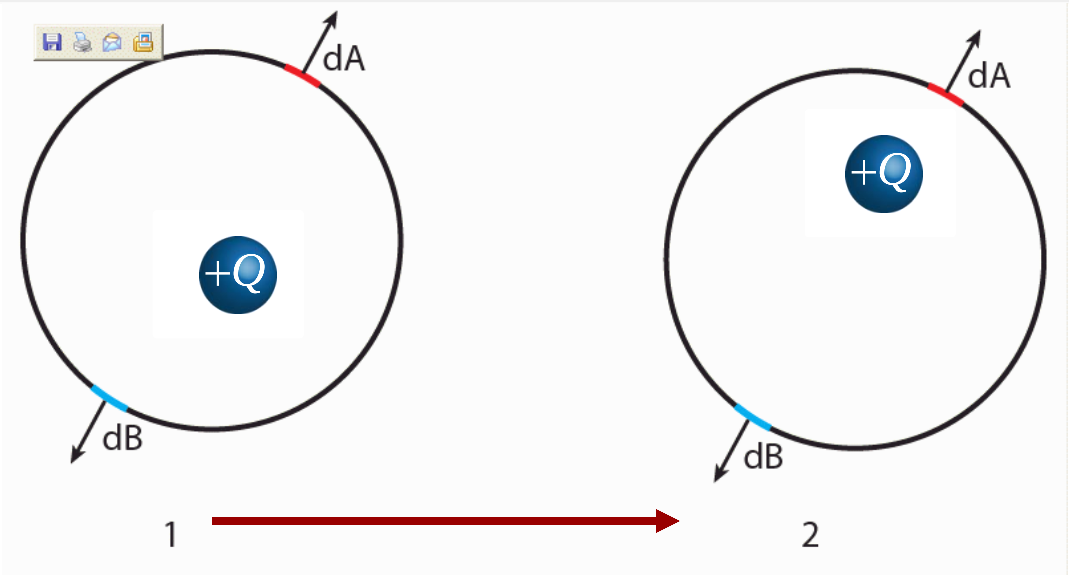
B
Φ_E decreases

C
Φ_E stays same



"Charge enclosed does not change."

CheckPoint 2.1



A

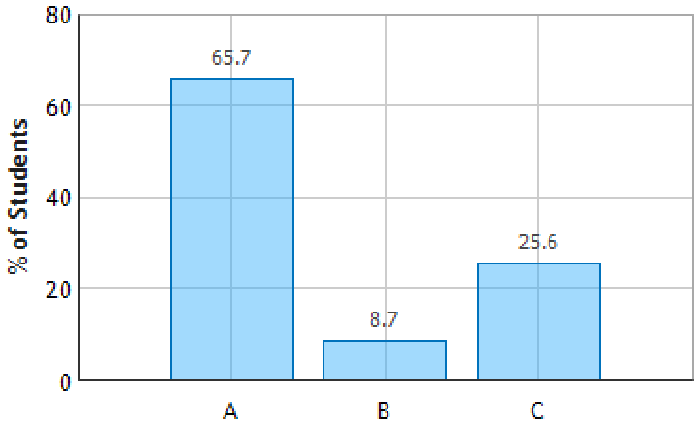
$d\Phi_A$ increases
$d\Phi_B$ decreases

B

$d\Phi_A$ decreases
$d\Phi_B$ increases

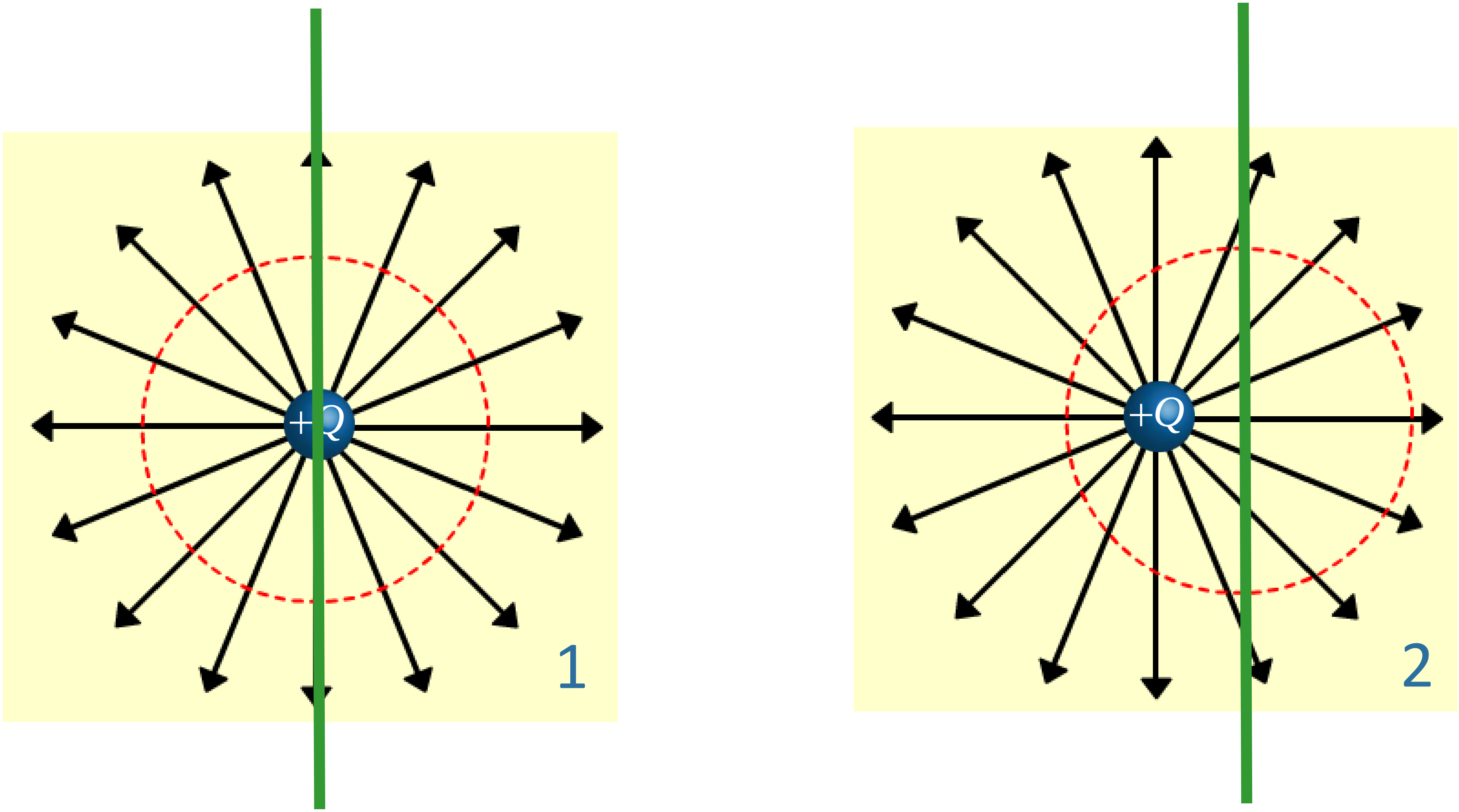
C

$d\Phi_A$ stays same
$d\Phi_B$ stays same



"Flux in this case is equal to the field times the area. While the area stays the same, the field changes such that db flux decreases and da flux increases"

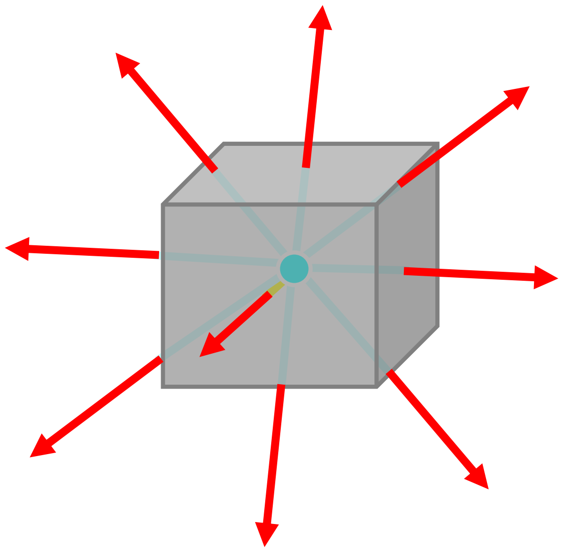
Think of it this way:



The total flux is the same in both cases (just the total number of lines)

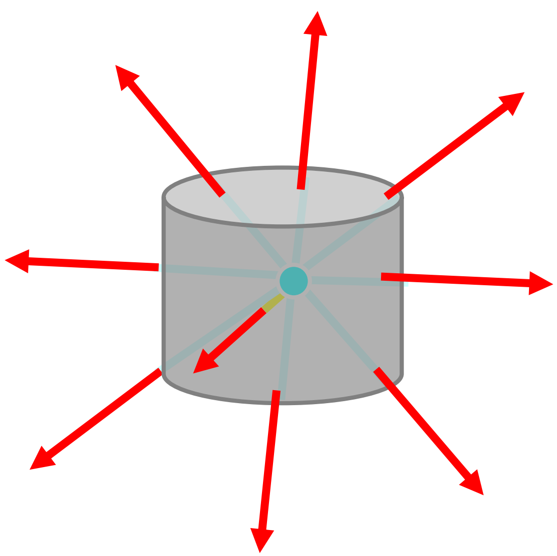
The flux through the right (left) hemisphere is smaller (bigger) for case 2.

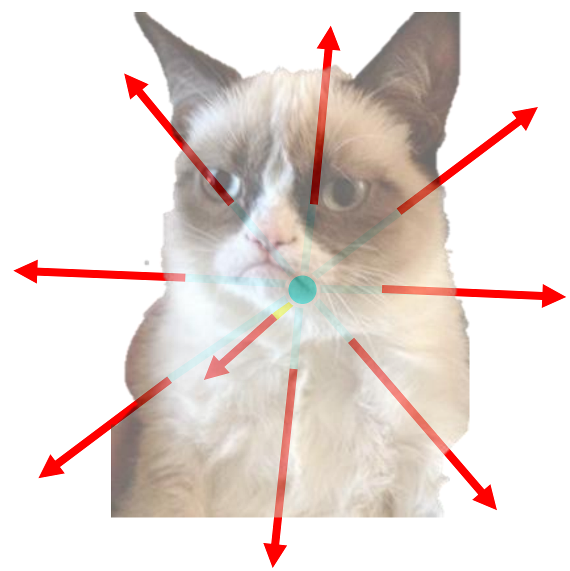
Things to notice about Gauss Law



$$\Phi_S = \int_{\substack{closed \\ surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

If $Q_{enclosed}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.





Things to notice about Gauss Law

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

In cases of high symmetry it may be possible to bring E outside the integral. In these cases we can solve Gauss Law for E

$$E \int dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

So - if we can figure out $Q_{enclosed}$ and the area of the surface A, then we know E!

This is the topic of the next lecture.