

# Your Comments

I feel like I watch a pre-lecture, and agree with everything said, but feel like it doesn't click until lecture.

Conductors and Insulators with Gauss's law please.....so basically everything!

I don't think we go over enough actual problems during lecture that will help with our homework.

Hi professor. I am really sorry for my extremely unstudious physics prelecture attitude during this prelecture. You see, today is my birthday and I spent it calling home and enjoying the simple joys of life... a little too simple to comprehend physics prelectures today, though physics is a joy.

Explain how the E-field at a point an arbitrary distance from an infinite plane of charge can be constant. It doesn't make intuitive sense.

I guessed on most of the checkpoints... Does that mean that physics is just not for me?

Can you explain induced charges on conductors more thoroughly?

Can you try to incorporate more modern/interesting physics topics into your lectures? Even if it's just a quick blip at the start of the lecture, it would be nice to think about how what we're learning now applies to more advanced physics and/or engineering applications.

Why do we talk about infinitely long planes and cylinders? Are we using it as a stepping stone to more real-world situations or are there actually cases in which there is something like an infinitely long charged cylinder?

# *Electricity & Magnetism*

## *Lecture 4*

### Today's Concepts:

- A) Conductors
- B) Using Gauss' Law

“Electromagnetism is so easy! (Said no one ever...)”

# Conductors and Insulators

**Conductors = charges free to move**

e.g. metals



**Insulators = charges fixed**

e.g. glass



# Define: Conductors = Charges Free to Move

Claim:  $E = 0$  inside any conductor at equilibrium

Charges in conductor move to make  $E$  field zero inside. (Induced charge distribution).

If  $E \neq 0$ , then charge feels force and moves!

Claim: Excess charge on conductor only on surface at equilibrium

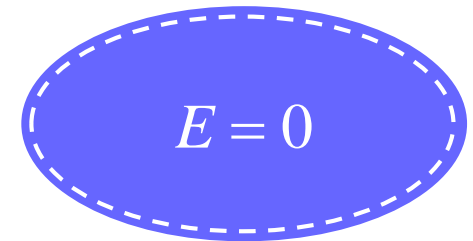
Why?

➤ Apply Gauss' Law

➤ Take Gaussian surface to be just inside conductor surface

➤  $E = 0$  everywhere inside conductor  $\rightarrow \oint_{\text{surface}} \vec{E} \cdot d\vec{A} = 0$

➤ Gauss' Law:  $\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \rightarrow Q_{\text{enc}} = 0$



[SIMULATION 2](#)

# Gauss' Law + Conductors + Induced Charges

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

ALWAYS TRUE!

If choose a **Gaussian surface** that is entirely in metal, then  $E = 0$  so  $Q_{enclosed}$  must also be zero!

$$E = \frac{Q_{enc}}{A\epsilon_0}$$

How Does This Work?

Charges in conductor move to surfaces to make  $Q_{enclosed} = 0$ .

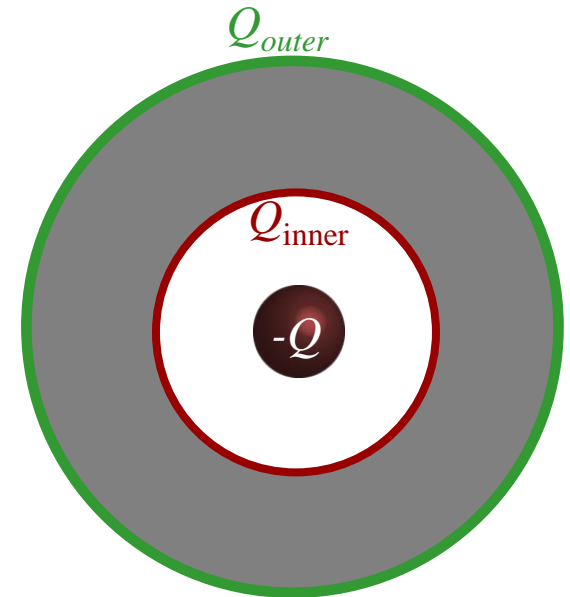
We say charge is induced on the surfaces of conductors

# Charge in Cavity of Conductor



A particle with charge  $-Q$  is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?

- A) inner =  $-Q$ , outer =  $+Q$
- B) inner =  $-Q/2$ , outer =  $+Q/2$
- C) inner = 0, outer = 0
- D) inner =  $+Q/2$ , outer =  $-Q/2$
- E) inner =  $+Q$ , outer =  $-Q$



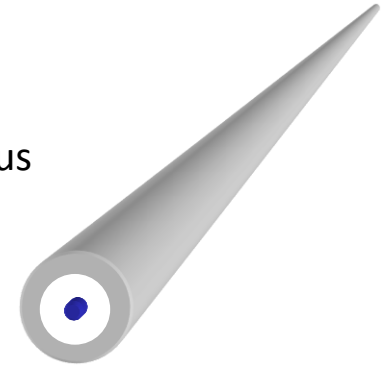
Since  $E = 0$  in conductor

➤ Gauss' Law: 
$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

# Infinite Cylinders

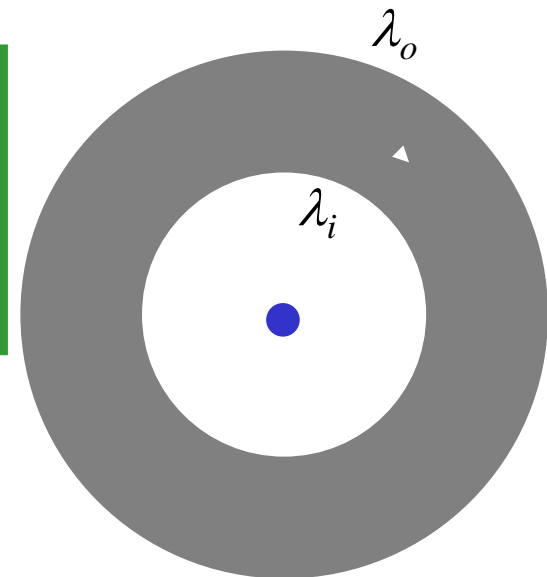
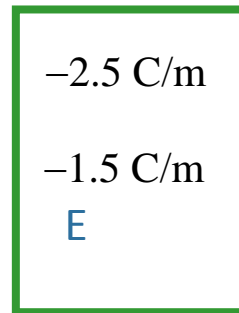


A long thin wire has a uniform positive charge density of  $2.5 \text{ C/m}$ . Concentric with the wire is a long thick conducting cylinder, with inner radius  $3 \text{ cm}$ , and outer radius  $5 \text{ cm}$ . The conducting cylinder has a net linear charge density of  $-4 \text{ C/m}$ .



What is the linear charge density of the induced charge on the inner surface of the conducting cylinder ( $\lambda_i$ ) and on the outer surface ( $\lambda_o$ )?

|              |          |        |        |          |          |
|--------------|----------|--------|--------|----------|----------|
| $\lambda_i:$ | +2.5 C/m | -4 C/m | 0      | -2.5 C/m | -2.5 C/m |
| $\lambda_o:$ | -6.5 C/m | 0      | -4 C/m | +2.5 C/m | -1.5 C/m |
|              | A        | B      | C      | D        | E        |



# Gauss' Law

I'm confused with how to determine which gaussian surface is best suited to calculate an electric field

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull  $E$  outside and get  $E = \frac{Q_{enc}}{A\epsilon_0}$

In General, integral to calculate flux is difficult.... and not useful!

To use **Gauss' Law** to calculate  $E$ , need to choose surface carefully!

1) Want  $E$  to be constant and equal to value at location of interest

OR

2) Want  $E \cdot A = 0$  so doesn't add to integral

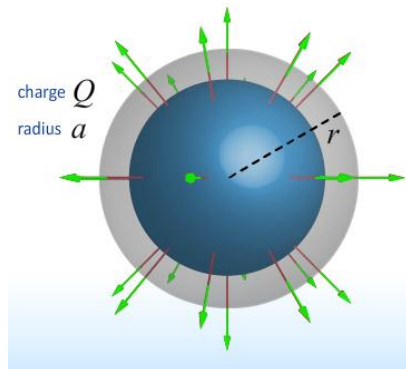
# Gauss' Law Symmetries

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull  $E$  outside and get  $E = \frac{Q_{enc}}{A\epsilon_0}$

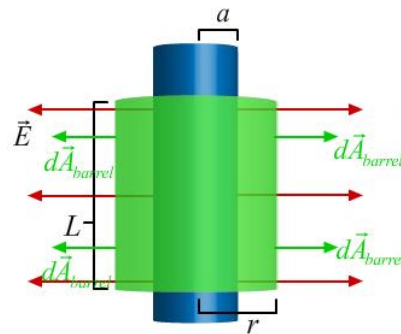
## Spherical



$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi r^2 \epsilon_0}$$

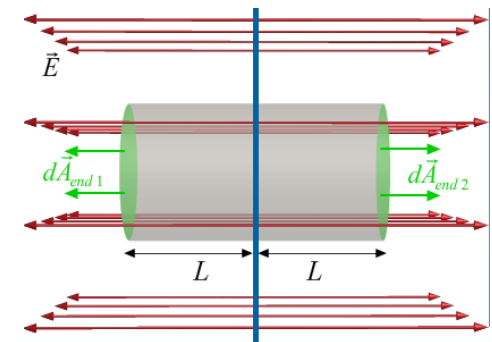
## Cylindrical



$$A = 2\pi rL$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

## Planar



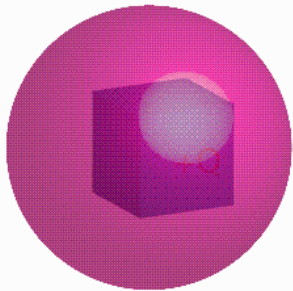
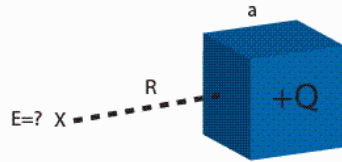
$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\epsilon_0}$$

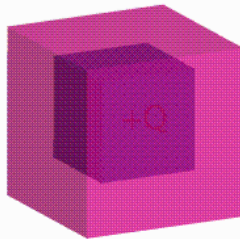
# Checkpoint 1



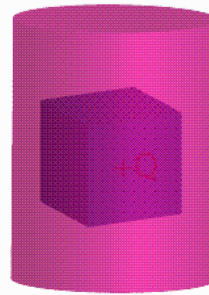
2) You are told to use Gauss' Law to calculate the electric field at a distance  $R$  away from a charged cube of dimension  $a$ . Which of the following Gaussian surfaces is best suited for this purpose?



(A)



(B)



(C)

**D) The field cannot be calculated using Gauss' Law**

E) None of the above

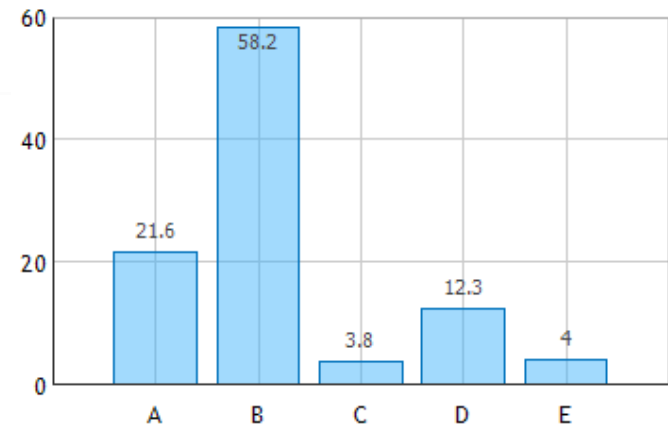
**THE CUBE HAS NO GLOBAL SYMMETRY!**

THE FIELD AT THE FACE OF THE CUBE

**IS NOT**

PERPENDICULAR OR PARALLEL

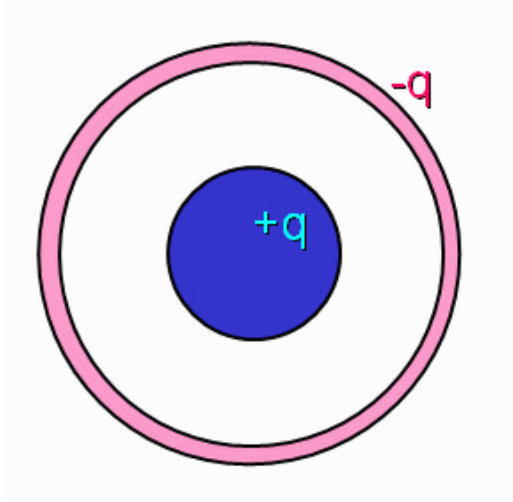
|    |       |   |             |
|----|-------|---|-------------|
| 3D | POINT | → | SPHERICAL   |
| 2D | LINE  | → | CYLINDRICAL |
| 1D | PLANE | → | PLANAR      |



# CheckPoint 3.1



4) A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.



What is direction of field between blue and red spheres?

- The field point radially outward    The field point radially inward    The field is zero

A) Outward

B) Inward

C) Zero

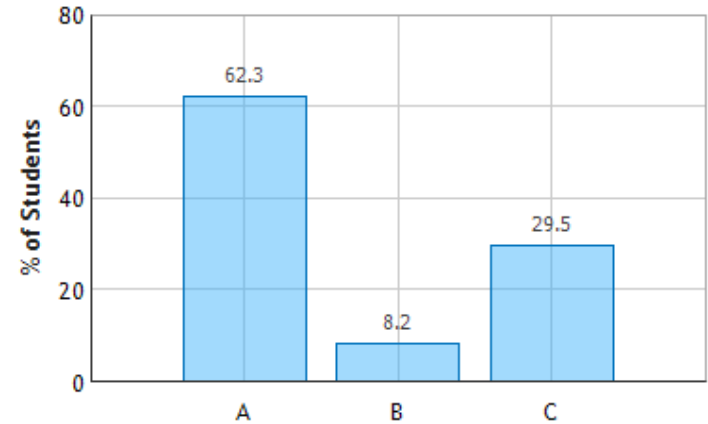
“The electric lines origin from  $+q$  and terminate at  $-q$ ”

“negative charge is pulled towards positive charge”

“No electric field within a conductor”

**Careful:** what does **inside** mean?  
This is always true for a solid conductor  
(within the material of the conductor)  
Here we have a charge “inside”

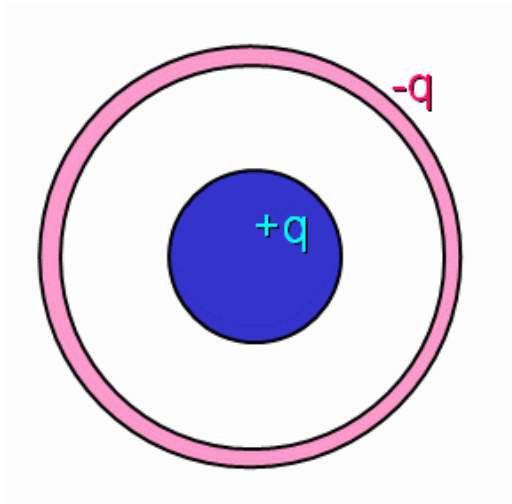
Charged Conducting Sphere and Spherical Shell: Question 1 (N = 816)



# Checkpoint 3.3



4) A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same.



What is direction of field OUTSIDE the red sphere?

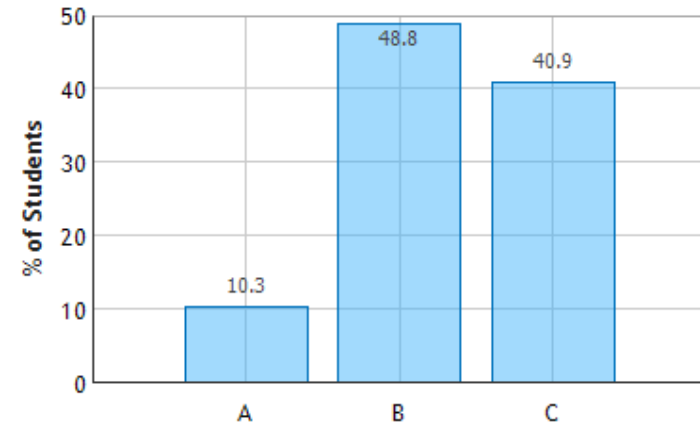
- The field point radially outward     The field point radially inward     The field is zero

A) Outward

B) Inward

C) Zero

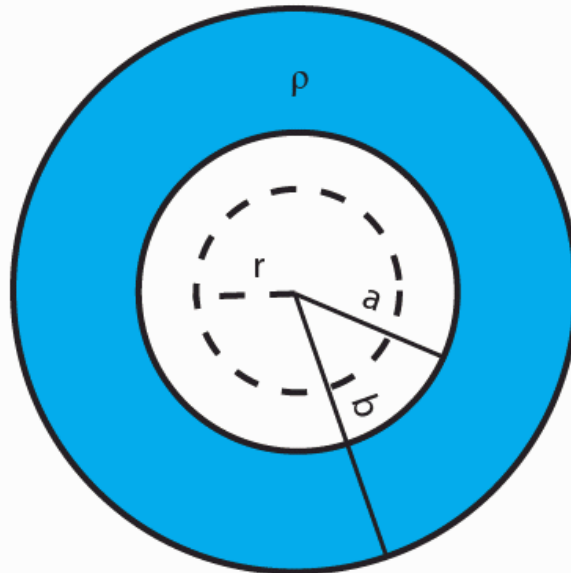
Charged Conducting Sphere and Spherical Shell: Question 3 (N = 815)



# Checkpoint 2

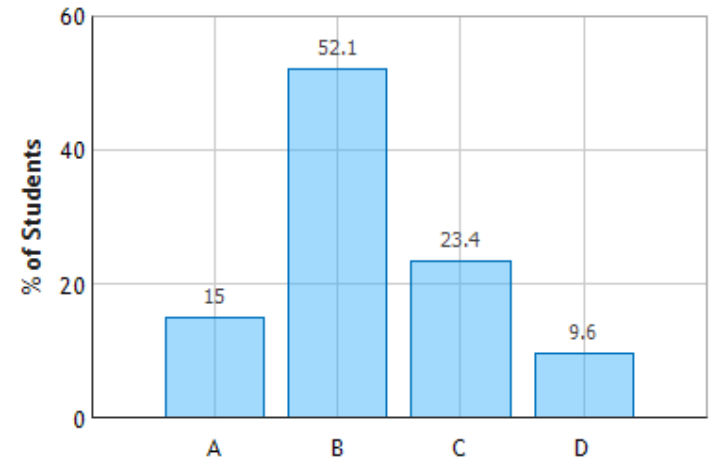


4) A charged spherical insulator shell has inner radius  $a$  and outer radius  $b$ . The charge density on the shell is  $\rho$ .



What is magnitude of  $E$  at dashed line ( $r$ )?

Charged Spherical Shell: Question 1 (N = 816)



A)  $\frac{\rho}{\epsilon_0}$  "Plug into the Gauss' Law equation "

B) **Zero** "There is no charge enclosed in the sphere of radius  $r$ "

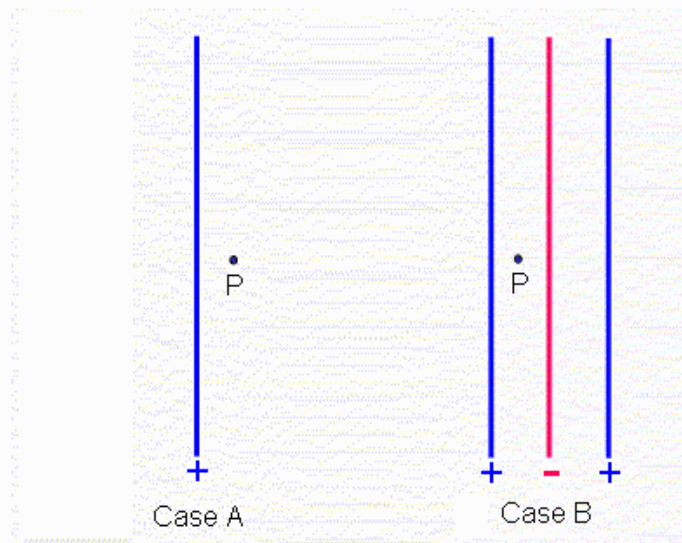
C)  $\frac{\rho(b^3 - a^3)}{3\epsilon_0 r^2}$  "E(4pir^2)=p(pi)(4/3)(b^3-a^3)/e0 and the pi\*4 cancels."

D) None of above

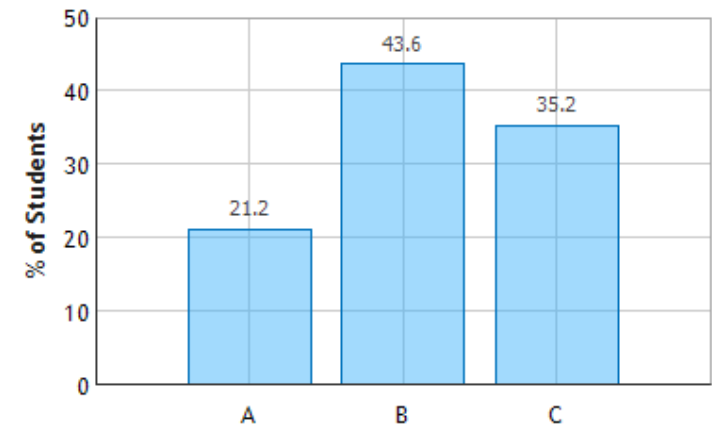
# Checkpoint 4



10) In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same.



Infinite Sheets of Charge: Question 1 (N = 816)



In which case is  $E$  at point  $P$  the biggest?

- A) A      B) B      **C) the same**

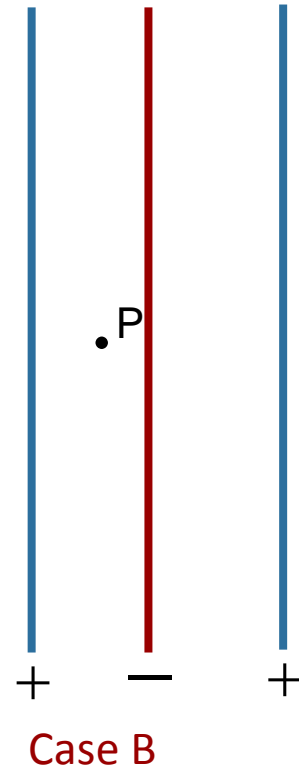
“There is an overall right E field in Case A, in Case B the negative and positive planes cancel out. “

“The sum of the fields is greater in case B. The negative is closer than the rightmost positive so it has a greater effect. “

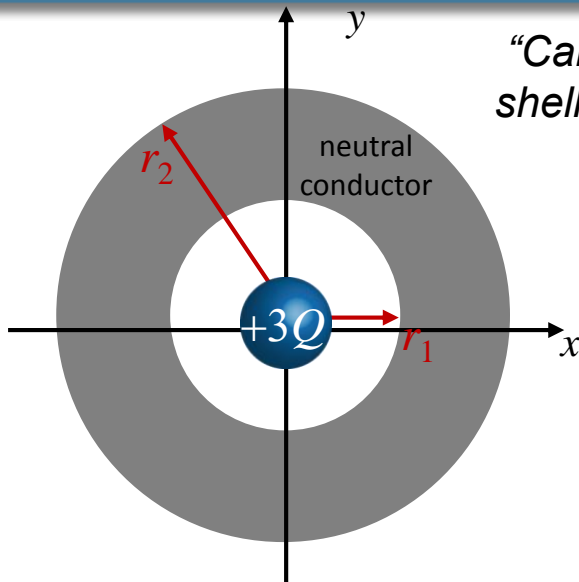
“Charged infinite planes create constant electric fields. In Case A, there is one positive field, and in Case B there are two positive and one negative. In Case B, one negative and one positive cancel out, leaving only one positive electric field. “

# Superposition:

Lets do calculation!



# Calculation



“Can we please go over the conducting sphere with the conducting shell around it?”

Point charge  $+3Q$  at center of neutral conducting shell of inner radius  $r_1$  and outer radius  $r_2$ .

a) What is  $E$  everywhere?

First question: Do we have enough symmetry to use Gauss' Law to determine  $E$ ?

Yes, Spherical Symmetry (what does this mean???)

Magnitude of  $E$  depends only on  $R$

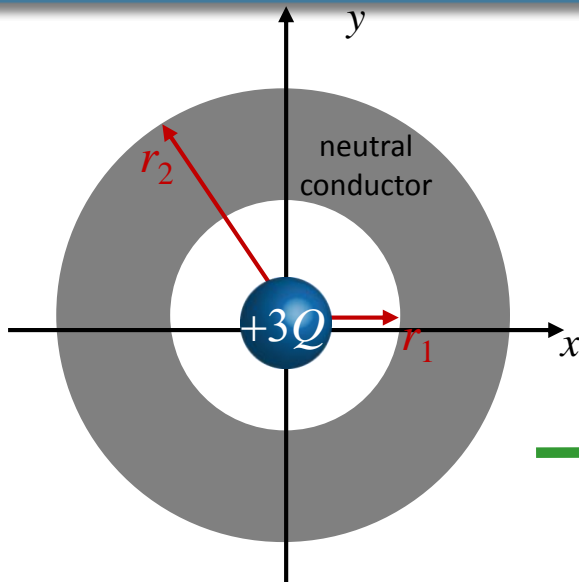
A) Direction of  $E$  is along  $\hat{x}$

B) Direction of  $E$  is along  $\hat{y}$

C) Direction of  $E$  is along  $\hat{r}$

D) None of the above

# Calculation



Point charge  $+3Q$  at center of neutral conducting shell of inner radius  $r_1$  and outer radius  $r_2$ .

A) What is  $E$  everywhere?

We know:

magnitude of  $E$  is *fcn* of  $r$   
direction of  $E$  is along  $\hat{r}$

We can use **Gauss' Law** to determine  $E$

Use **Gaussian surface** = sphere centered on origin

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$r < r_1$

$$\int E dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{+3Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$r_1 < r < r_2$

$$A) E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$B) E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r_1^2}$$

$$C) E = 0$$

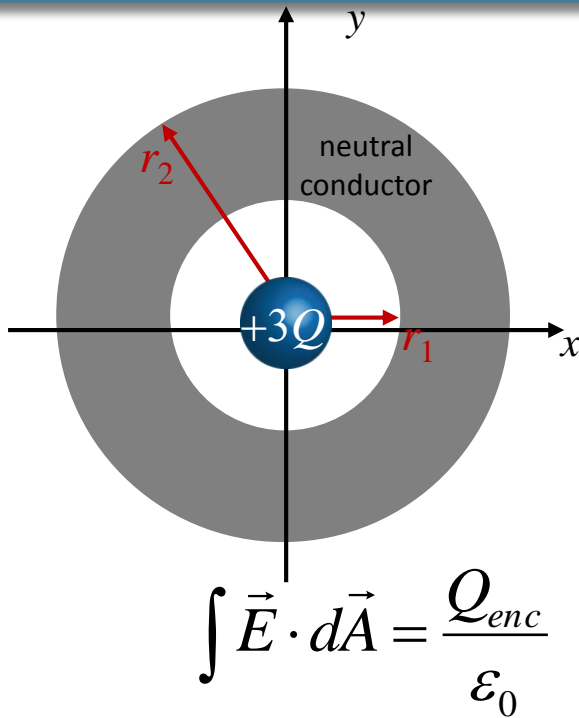
$r > r_2$

$$A) E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$B) E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{(r - r_2)^2}$$

$$C) E = 0$$

# Calculation



Point charge  $+3Q$  at center of neutral conducting shell of inner radius  $r_1$  and outer radius  $r_2$ .

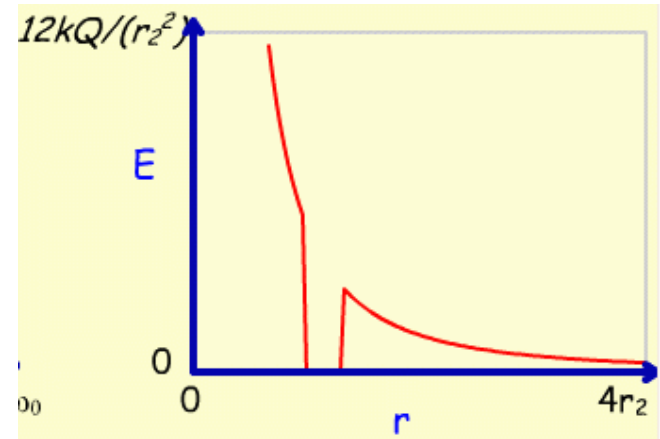
A) What is  $E$  everywhere?

We know:

$$r < r_1 \quad E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$r > r_2$$

$$r_1 < r < r_2 \quad E = 0$$

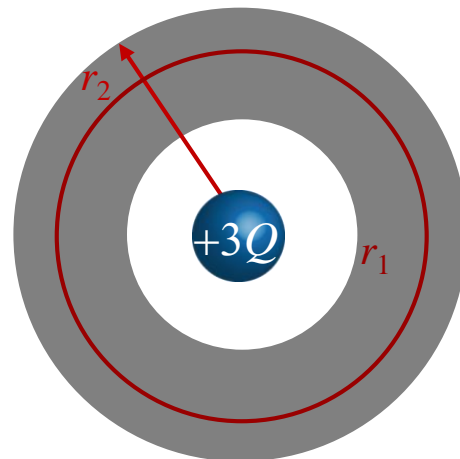


B) What is charge distribution at  $r_1$ ?

A)  $\sigma < 0$

B)  $\sigma = 0$

C)  $\sigma > 0$



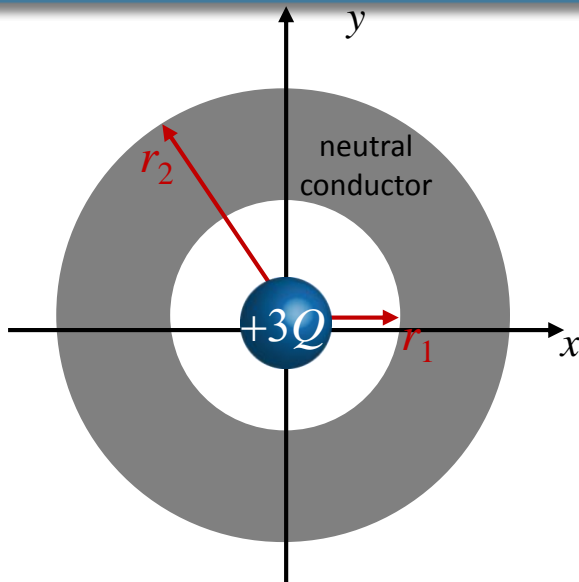
Gauss' Law:

$$E = 0 \rightarrow Q_{enc} = 0 \rightarrow \sigma_1 = \frac{-3Q}{4\pi r_1^2}$$

Similarly:

$$\sigma_2 = \frac{+3Q}{4\pi r_2^2}$$

# Calculation

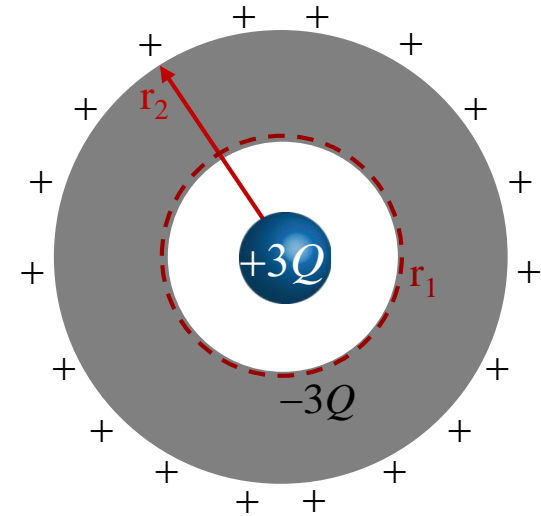


Suppose give conductor a charge of  $-Q$

A) What is  $E$  everywhere?

B) What are charge distributions at  $r_1$  and  $r_2$ ?

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$



$$r < r_1$$

$$\text{A) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$\text{B) } E = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$$

$$\text{C) } E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$r > r_2$$

$$\text{A) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$\text{B) } E = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$$

$$\text{C) } E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$r_1 < r < r_2$$

$$E = 0$$