Your Comments

"Wow." <- Best part of the prelecture

Could you explain what exactly an equipotential is? And also how the del operator illustrates the realationship between the electric potential and the electric field?

Do we need to learn the spherical and cylindrical representations of the potential gradient that were given in the prelecture? Because those formulas seem like they could get very tricky.

i don't understand how you get the direction of the electric field from the graph of the electric potential. the magnitude makes sense, that's just the slope, but where does the direction come from...?

I just really want to get on the screen at the beginning of lecture...it's my goal in life

So what's the difference between electric potential energy and electric potential?

More examples and hopefully less homework this week!

But overall it seems easier than Gauss's law.

Physics 212
Lecture 6

Today's Concept:

Electric Potential

(Defined in terms of Path Integral of Electric Field)

Big Idea

Last time we defined the electric potential energy of charge q in an electric field:

$$\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

The only mention of the particle was through its charge q.

We can obtain a new quantity, the electric potential, which is a PROPERTY OF THE SPACE, as the potential energy per unit charge.

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Note the similarity to the definition of another quantity which is also a PROPERTY OF THE SPACE, the electric field.

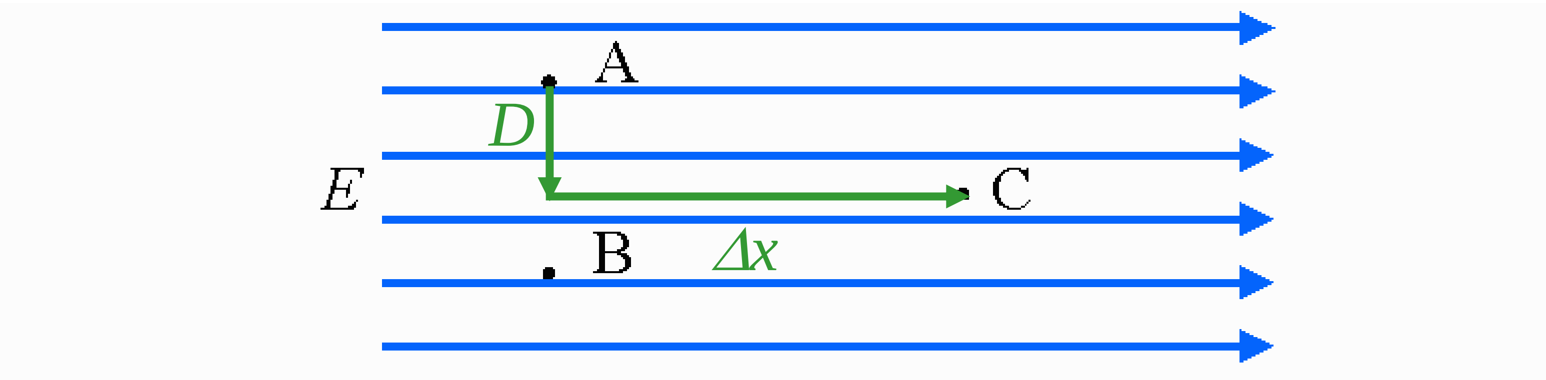
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric Potential is like Height (E points down hill for positive charge)

While I understand the logic of the math here, I'm having some trouble understanding electric potential intuitively. Could you help explain what exactly the value shows us?

Electric Potential from E field

Consider the three points A, B, and C located in a region of constant electric field as shown.



What is the sign of $\Delta V_{AC} = V_C - V_A$?

A) $\Delta V_{AC} < 0$ B) $\Delta V_{AC} = 0$ C) $\Delta V_{AC} > 0$

E points down hill

Remember the definition: $\Delta V_{A\to C} = -\int_A^C \vec{E}\cdot d\vec{l}$

Choose a path (any will do!)

$$\Delta V_{A\to C} = -\int_A^D \vec{E}\cdot d\vec{l} - \int_D^C \vec{E}\cdot d\vec{l} \quad \longrightarrow \quad \Delta V_{A\to C} = 0 - \int_D^C \vec{E}\cdot d\vec{l} = -E\Delta x < 0$$

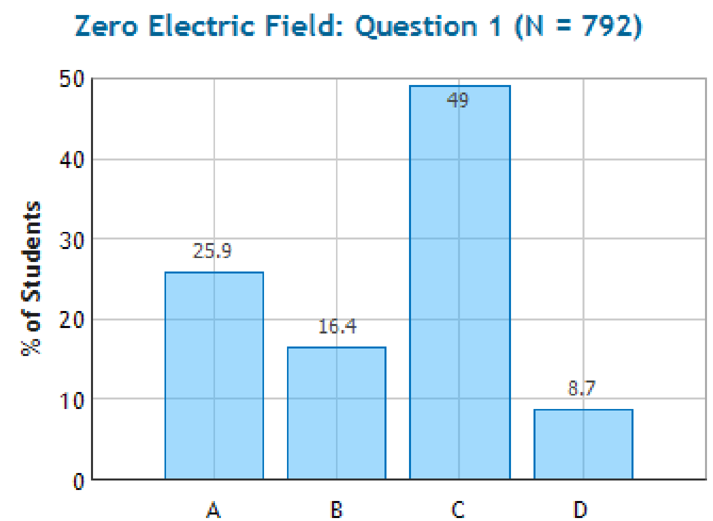
CheckPoint 2

If the electric field is zero in a region of space, what does that tell you about the electric potential in that region?

A) The electric potential is zero everywhere in this region.
B) The electric potential is zero at at least one point in this region.
C) The electric potential is constant everywhere in this region.
D) There is not enough information given to distingush which of the above answers is correct.

Remember the definition

$$\Delta V_{A\to B} = -\int_A^B \vec{E}\cdot d\vec{l}$$



$\vec{E} = 0$ → $\Delta V_{A\to B} = 0$ → V is constant!

E from V

If we can get the potential by integrating the electric field:

$$\Delta V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{l}$$

We should be able to get the electric field by differentiating the potential?

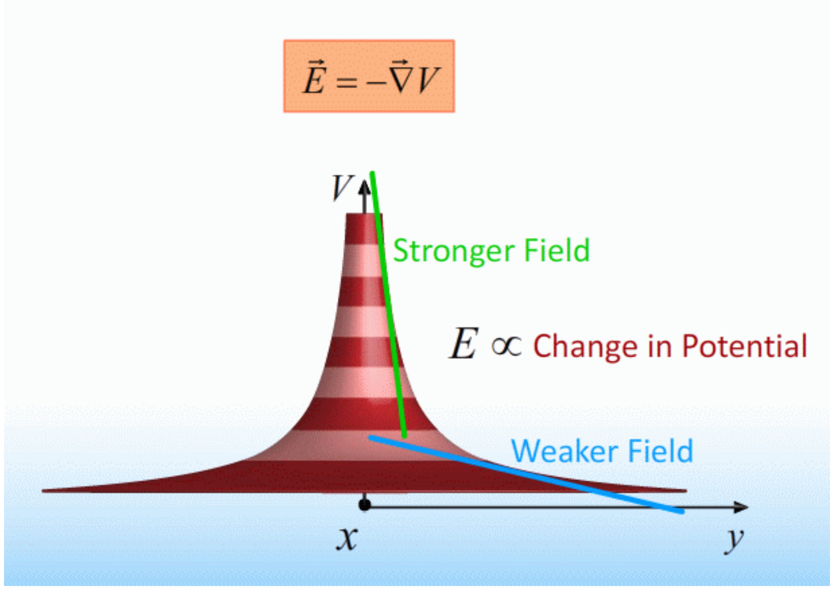
$$\vec{E} = -\vec{\nabla}V$$

In Cartesian coordinates:

$$E_x = -\frac{\partial V}{dx}$$

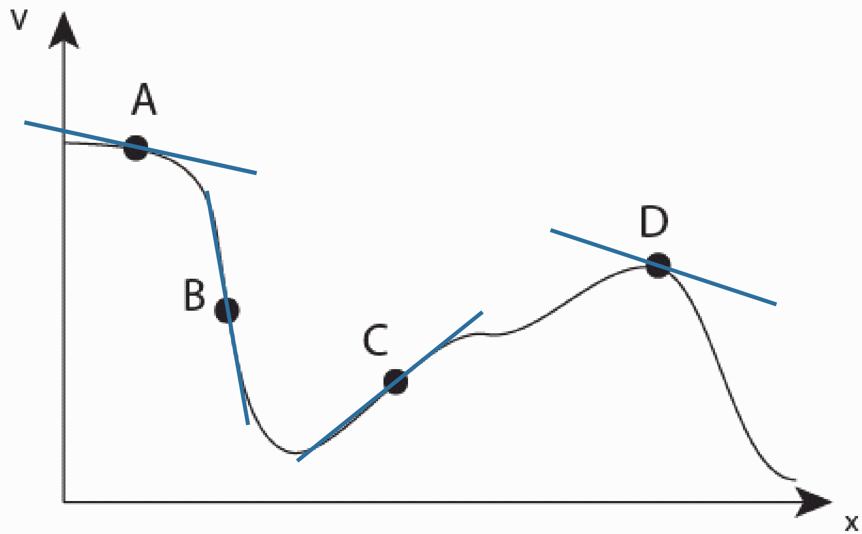
$$E_y = -\frac{\partial V}{dy}$$

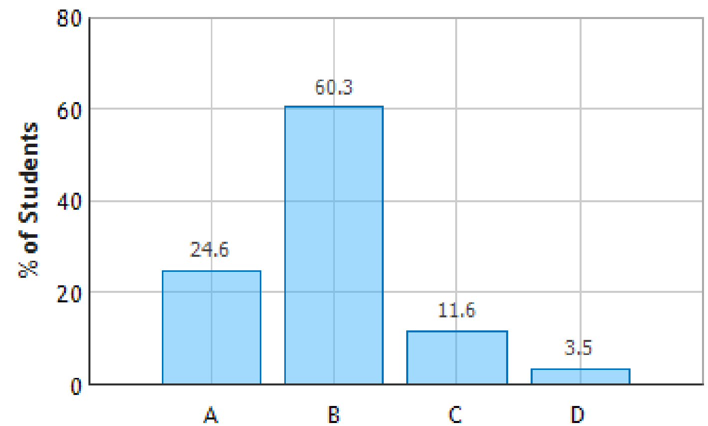
$$E_z = -\frac{\partial V}{dz}$$



CheckPoint 1a

2) The electric potential in a certain region is plotted in the following graph





At which point is the magnitude of the electric field greatest?

“A) The The higher the potential the higher the electric field at that point “

“B) The slope of the line is the steepest at point B.”

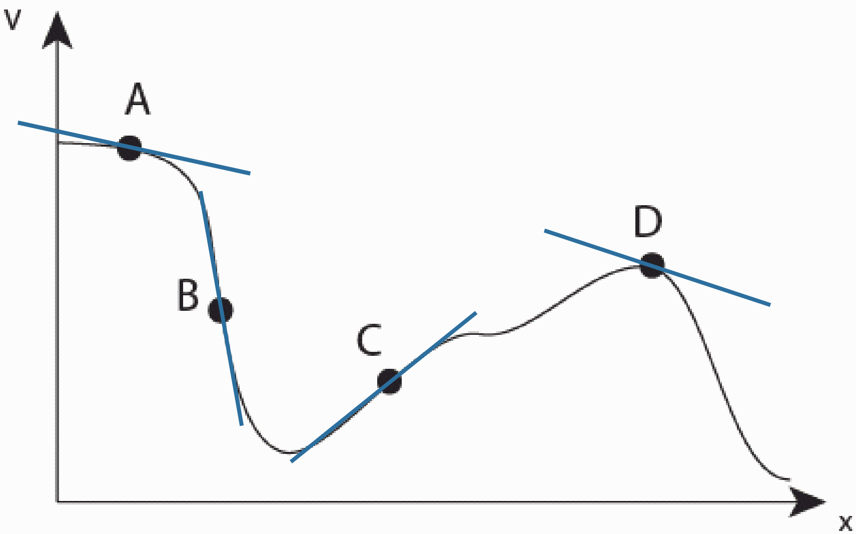
“C) C has the greatest slope “

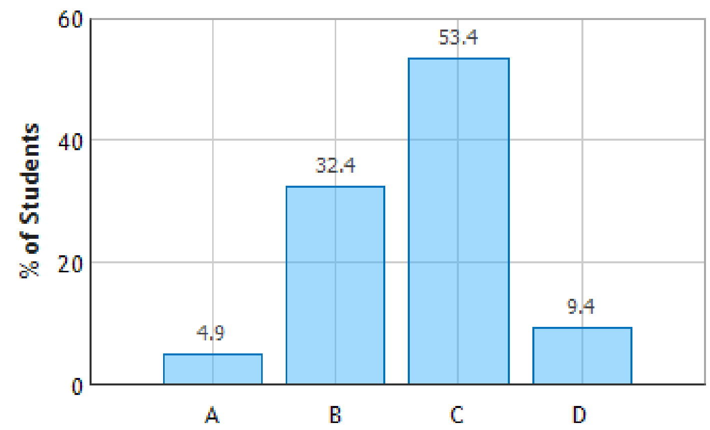
How do we get E from V?

$$\vec{E} = -\vec{\nabla} V \quad \rightarrow \quad E_x = -\frac{\partial V}{dx} \quad \rightarrow \quad \text{Look at slopes!}$$

CheckPoint 1b

2) The electric potential in a certain region is plotted in the following graph





At which point is the electric field pointing in the negative x direction?

"B) Negative slope of the line "

"C) There is a positive slope. "

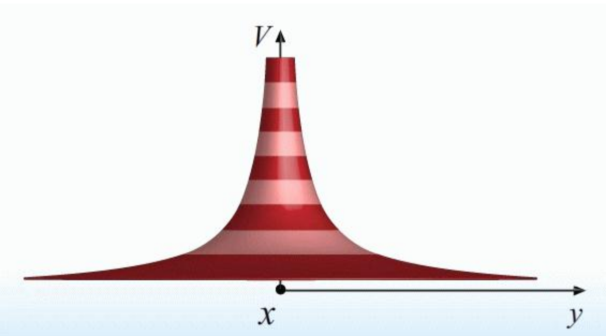
How do we get E from V?

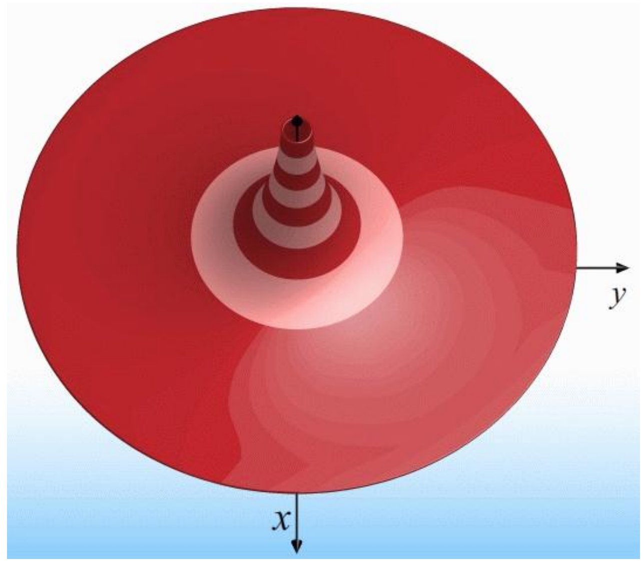
$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow$$

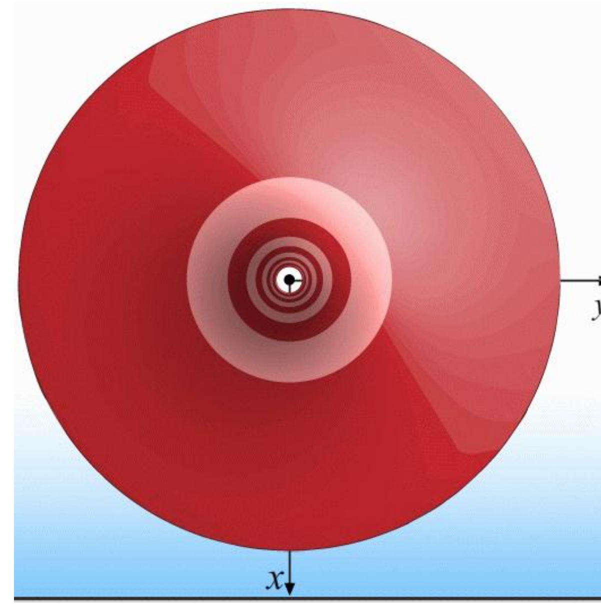
Look at slopes!

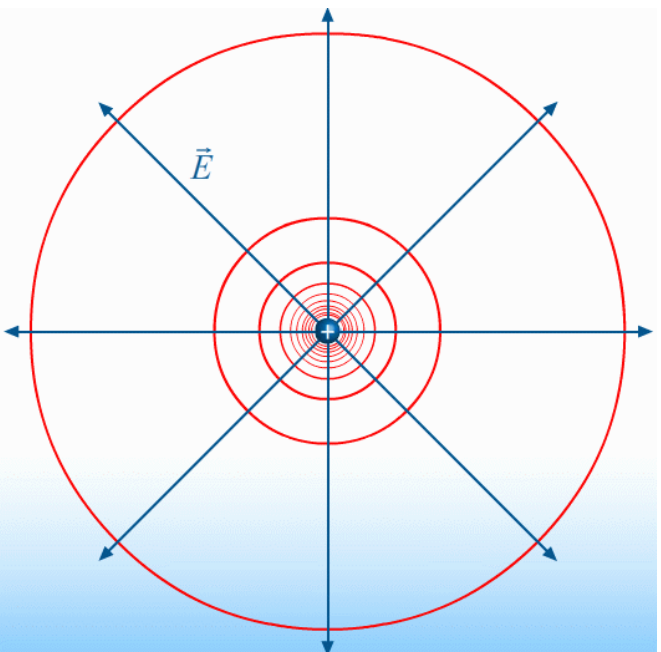
Equipotentials

Equipotentials are the locus of points having the same potential.







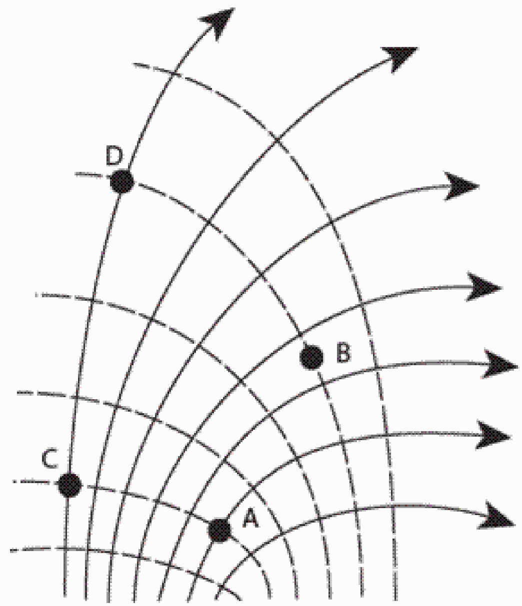


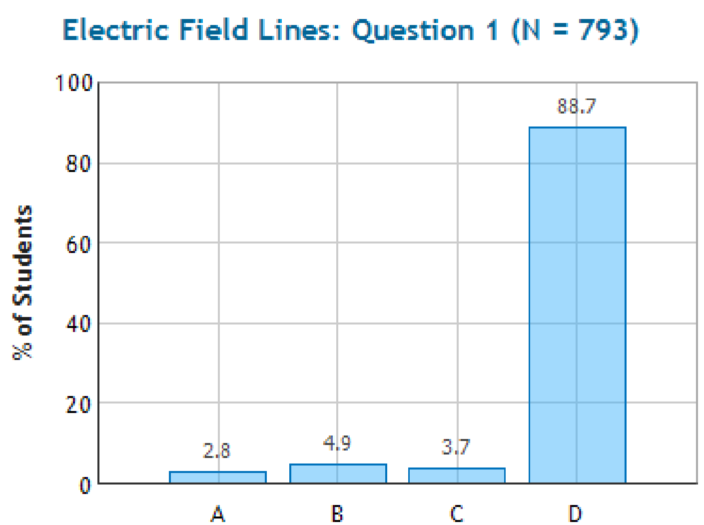
Equipotentials are
ALWAYS
perpendicular to the electric field lines.

The SPACING of the equipotentials indicates
The STRENGTH of the electric field.

CheckPoint 3a

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



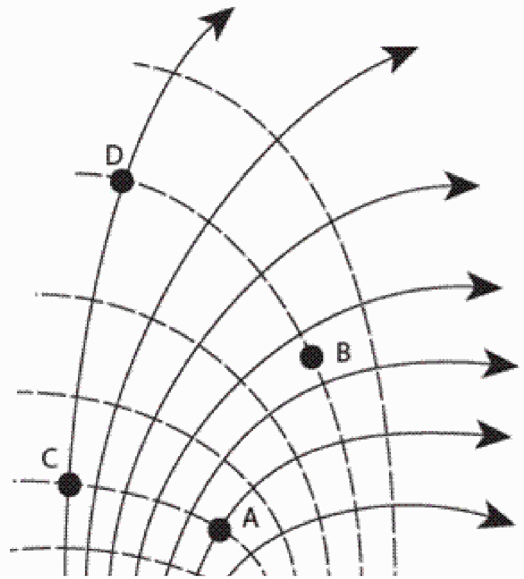


At which point is the magnitude of the electric field the smallest?

"The electric field lines are the least dense at D "

CheckPoint 3b

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



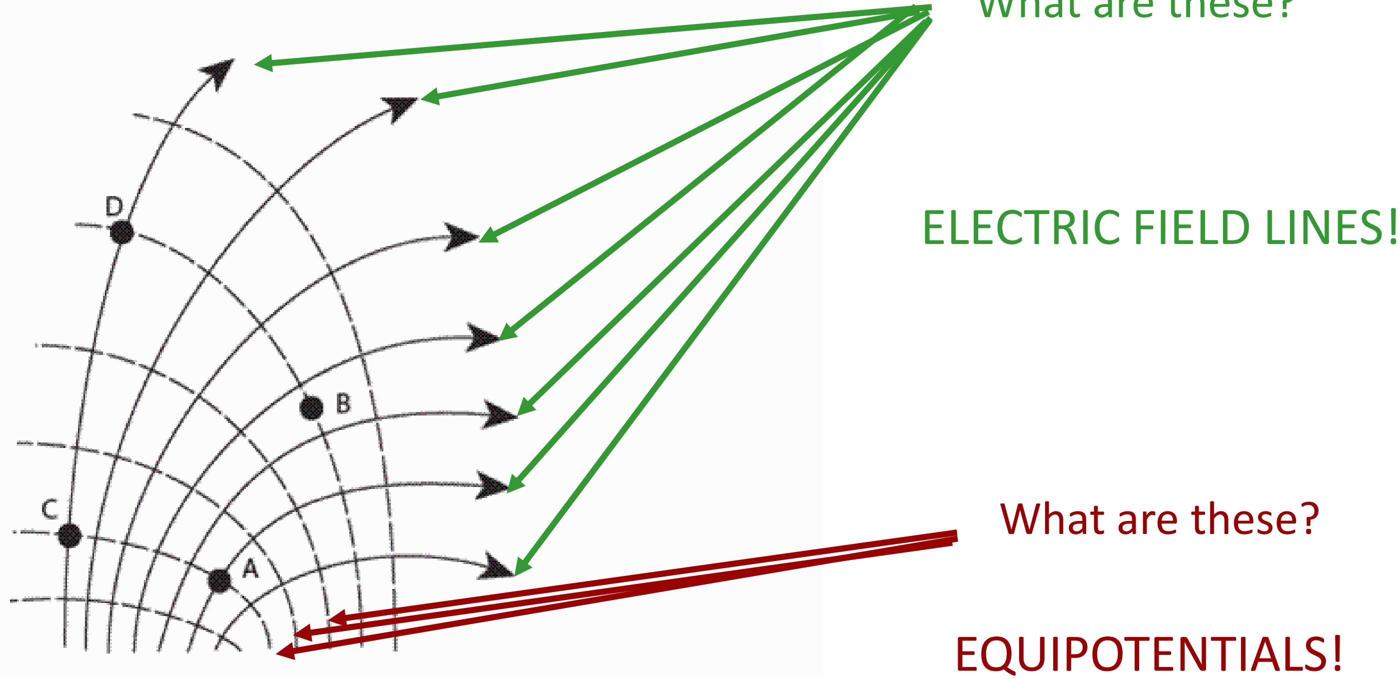
Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

Less than ½ got this correct!

Hint

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



What is the sign of $W_{AC} =$ work done by E field to move negative charge from A to C ?

A) $W_{AC} < 0$

B) $W_{AC} = 0$

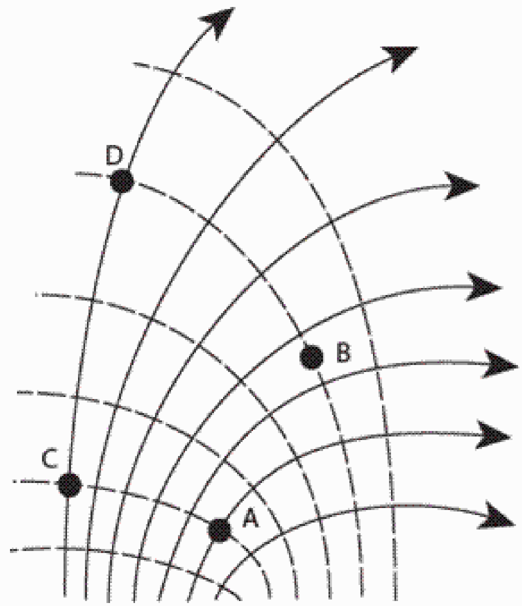
C) $W_{AC} > 0$

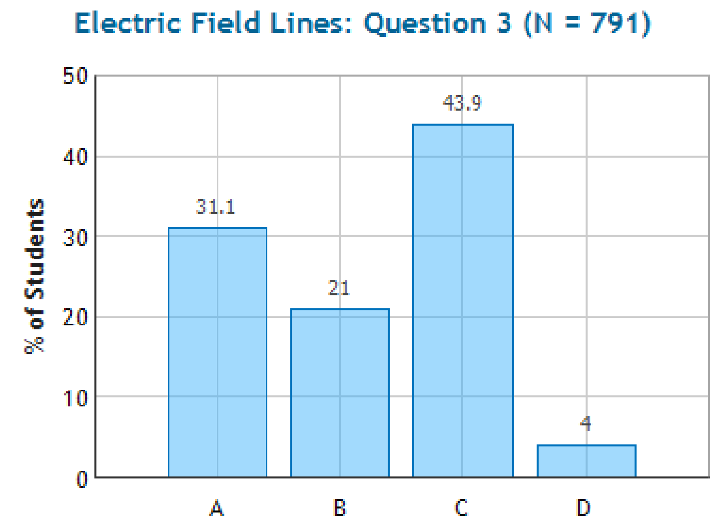
A and C are on the same equipotential → $W_{AC} = 0$

Equipotentials are perpendicular to the E field: No work is done along an equipotential

CheckPoint 3b Again?

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.





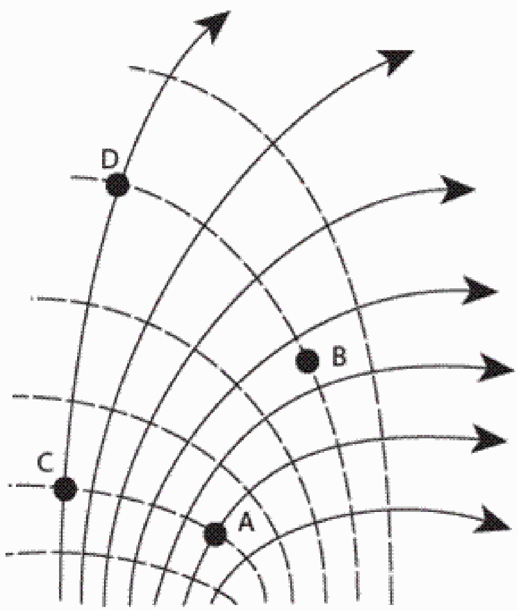
Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

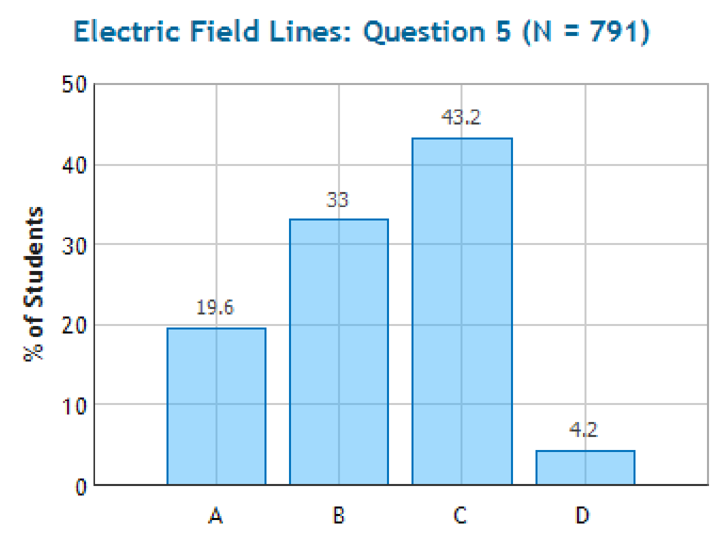
A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

- A and C are on the same equipotential
- B and D are on the same equipotential
- Therefore the potential difference between A and B is the SAME as the potential between C and D

CheckPoint 3c

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.





Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from A to D

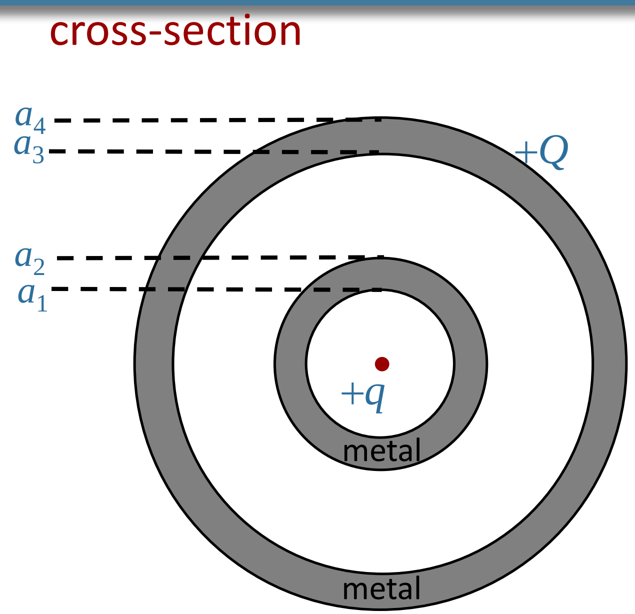
A) More work from A to B

B) More work from D to D

C) Same

D) Can not determine w/o performing calculation

Calculation for Potential



Point charge q at center of concentric conducting spherical shells of radii a_1, a_2, a_3, and a_4. The inner shell is uncharged, but the outer shell carries charge Q.

What is V as a function of r?

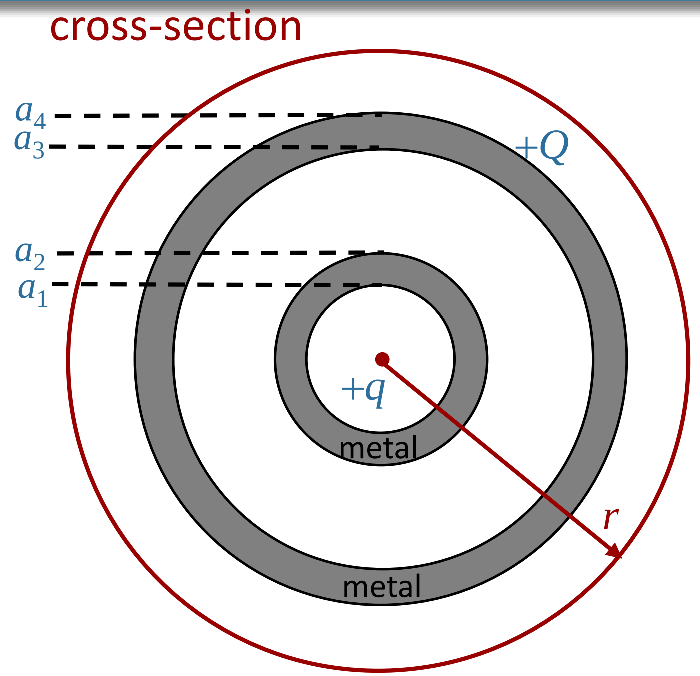
Conceptual Analysis:

- Charges q and Q will create an E field throughout space
- $V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$

Strategic Analysis:

- Spherical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V

Calculation: Quantitative Analysis



$r > a_4$: What is $E(r)$?

A) 0

B) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

C) $\dfrac{1}{2\pi\varepsilon_0}\dfrac{Q+q}{r}$

D) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r^2}$

E) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q-q}{r^2}$

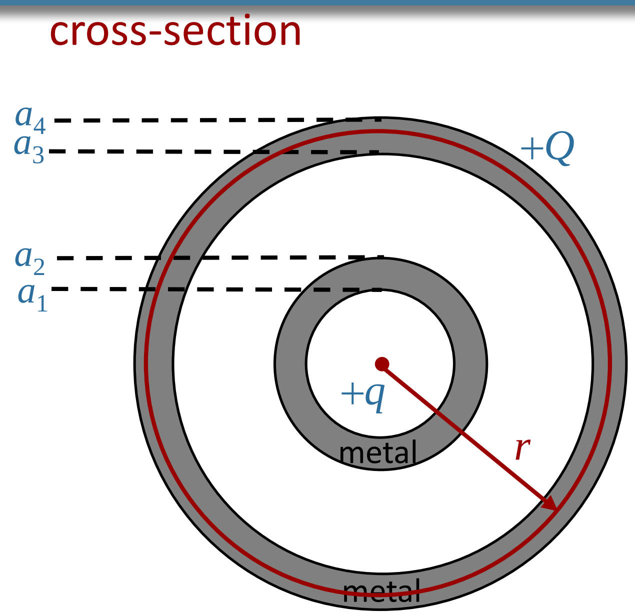
Why?

Gauss' law: $\int \vec{E}\cdot d\vec{A} = \dfrac{Q_{enclosed}}{\varepsilon_0}$

$$E4\pi r^2 = \frac{Q+q}{\varepsilon_0}$$

$$\rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}$$

Calculation: Quantitative Analysis



$a_3 < r < a_4$: What is $E(r)$?

A) 0 B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ C) $\frac{1}{2\pi\varepsilon_0}\frac{q}{r}$

D) $\frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}$ E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Applying Gauss' law, what is $Q_{enclosed}$ for red sphere shown?

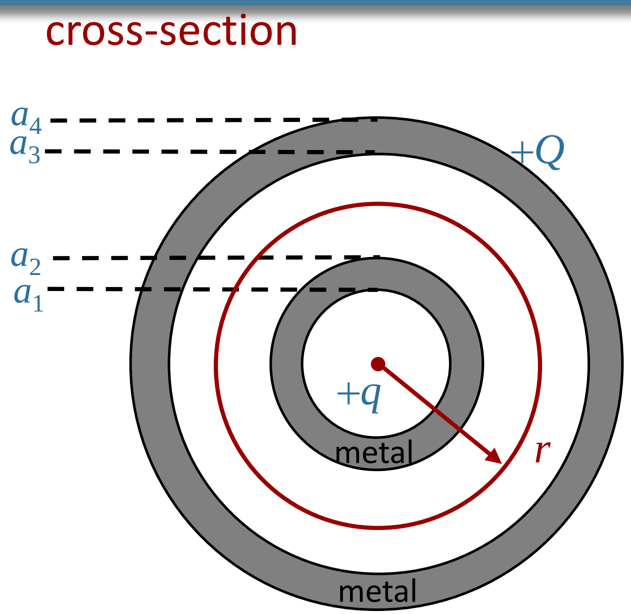
A) q B) $-q$ C) 0

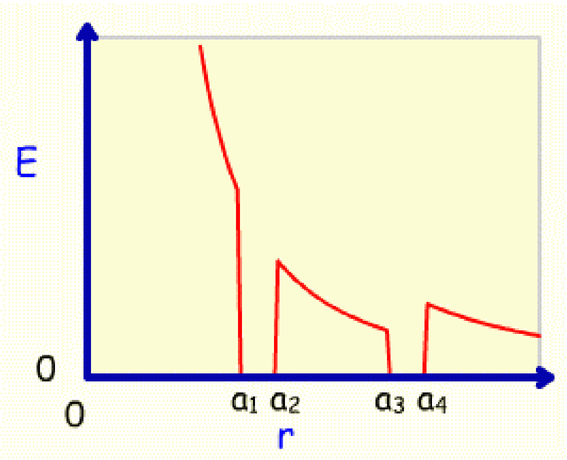
How is this possible?

$-q$ must be induced at $r = a_3$ surface → charge at $r = a_4$ surface $= Q + q$

$$\sigma_3 = \frac{-q}{4\pi a_3^2} \qquad \sigma_4 = \frac{Q+q}{4\pi a_4^2}$$

Calculation: Quantitative Analysis





Continue on in…

$$a_2 < r < a_3: \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$a_1 < r < a_2: \quad E = 0$$

$$r < a_1: \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: $r_0 = \text{infinity}$)
2) Integrate!

$$r > a_4: \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r}$$

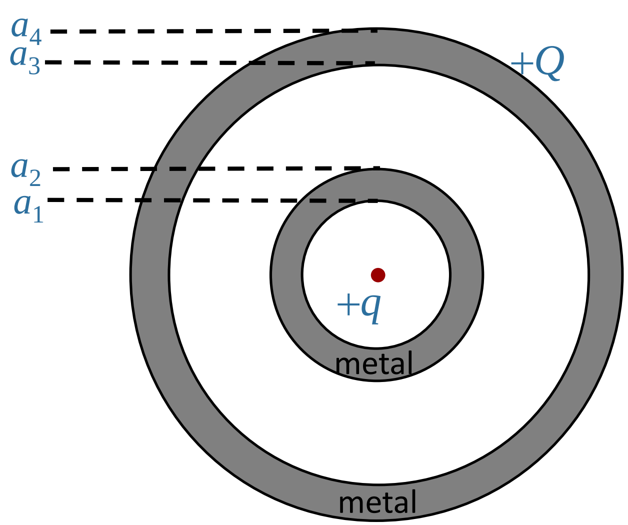
$a_3 < r < a_4$:

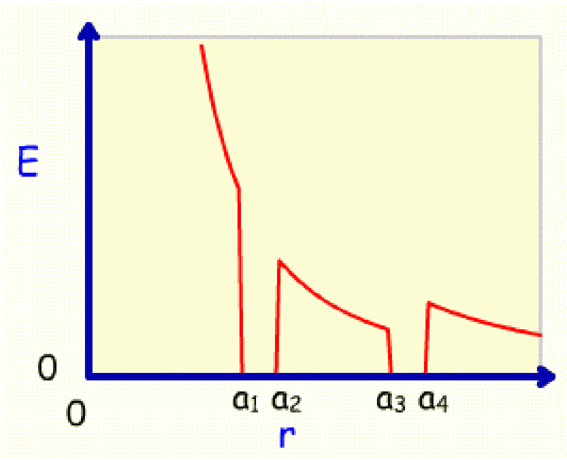
A) $V = 0$

B) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_4}$

C) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_3}$

Calculation: Quantitative Analysis





$r > a_4:\quad V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

$a_3 < r < a_4:\quad V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_4}$

$a_2 < r < a_3:$

$$V(r) = \Delta V(\infty \to a_4) + 0 + \Delta V(a_3 \to r)$$

$$V(r) = \frac{Q+q}{4\pi\varepsilon_0 a_4} + 0 + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a_3}\right) \rightarrow V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{r} - \frac{q}{a_3}\right)$$

$a_1 < r < a_2:$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{a_2} - \frac{q}{a_3}\right)$$

$0 < r < a_1:$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{a_2} - \frac{q}{a_3} + \frac{q}{r} - \frac{q}{a_1}\right)$$