Your Comments

That's really interesting stuff! And it doesn't actually look all that intimidating.

Will you discuss how dangerous discharging multi-Farad capacitors charged with a few Volts can be? And how you can probably find a few big ones in old electronics like CRTs and radios?

Is it possible for you to assign all the homeworks and prelectures right now ? (keeping the due dates the same ofcourse).

I'm still having a lot of trouble understanding Electric potential, and adding capacitance to the mix is just making it worse. I feel like I'm guessing and using intuition a lot more than I'm actually learning anything right now, and I also feel like it wasn't well explained in the prelecture what happens when you put the third conducting plate in between the two plates of the capacitor.

Why, for the love of all that is sacred, do people keep saying that a given positive charge 'moves?' I feel like this is something that should not be as common as it is yet I hear it almost everyday from classmates. I think it would be good to bring up and clarify, if nothing else, for the sake of my sanity.

I am very, very tired and will probably be quite crabby in lecture tomorrow. Sorry in advance.

Yo broski, what the hecks this noise about no lecture of just working through a mock exam before our midterm? Seriously, though...

Physics 212
Lecture 7

Today's Concept: (Applications of Gauss, E and V)

A) Conductors

B) Capacitance

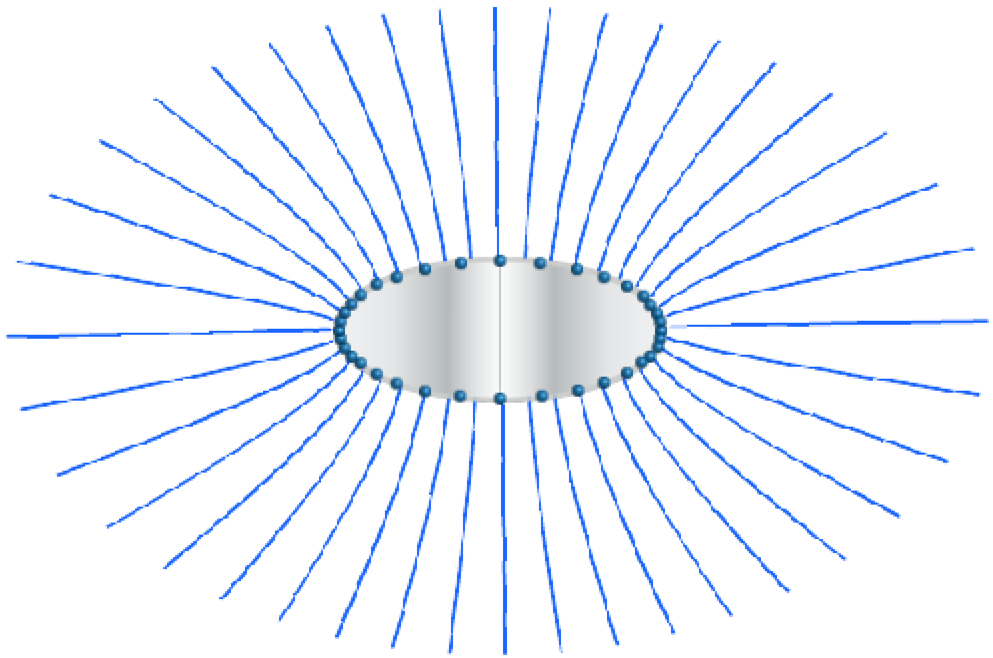
Exam Logistics

1) EXAM 1: WED February 13th at 7pm
   - Sign Up in Gradebook for Conflict Exam at 5:15pm if desired
   - If you have double conflict please email Prof. Aksimentiev
   - MATERIAL: Lectures 1 - 8

2) EXAM 1 PREPARATION
   - Old Exams are a good way to assess what you need to know
   - Prelecture of Fall 2010 solutions available

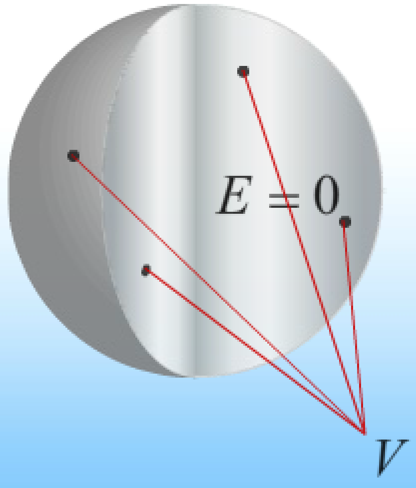
3) Extra Office Hours (Tuesday/ Wednesday next week in 276 Loomis)

Main Point 1: (Conductors)

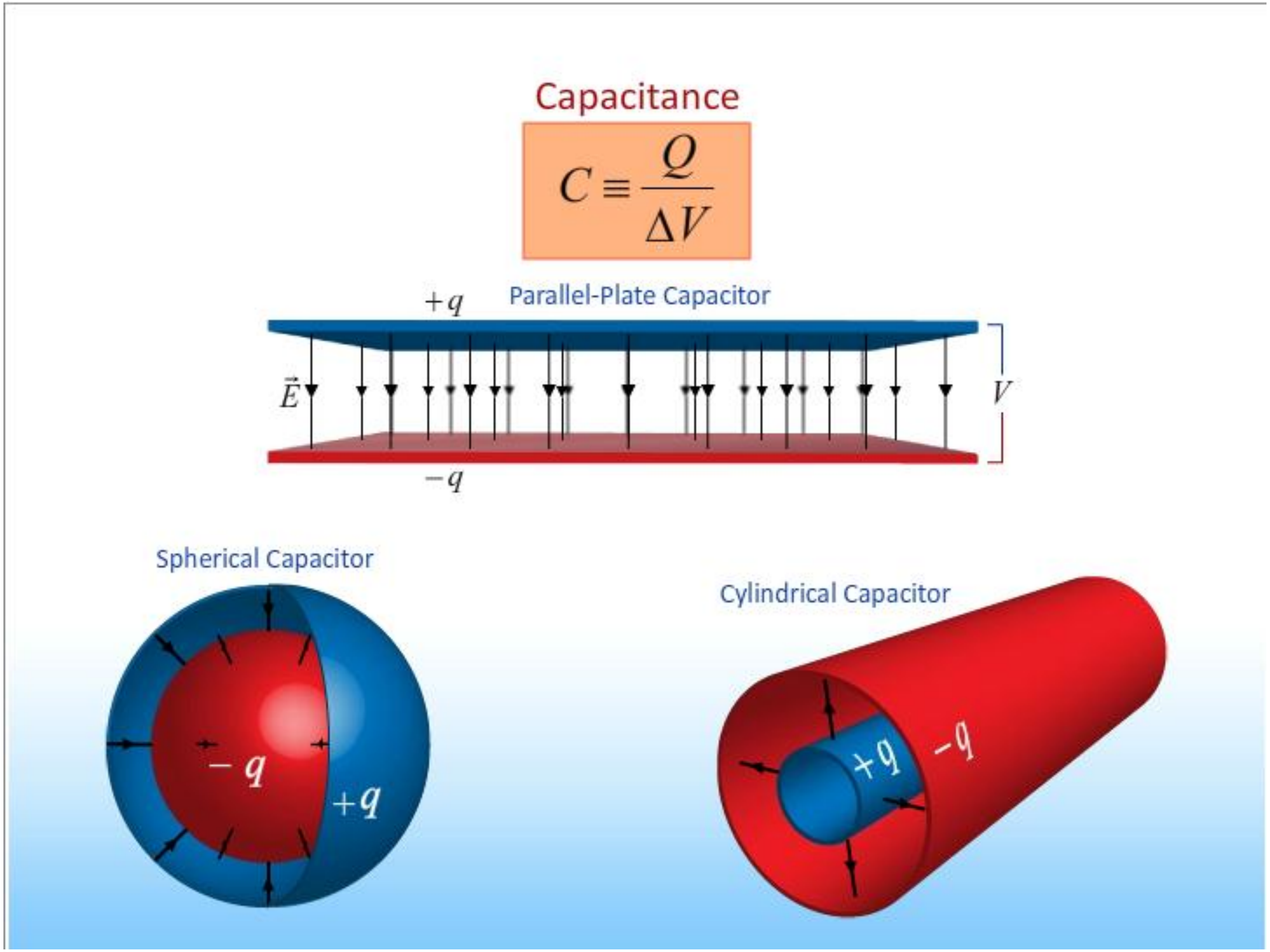


- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

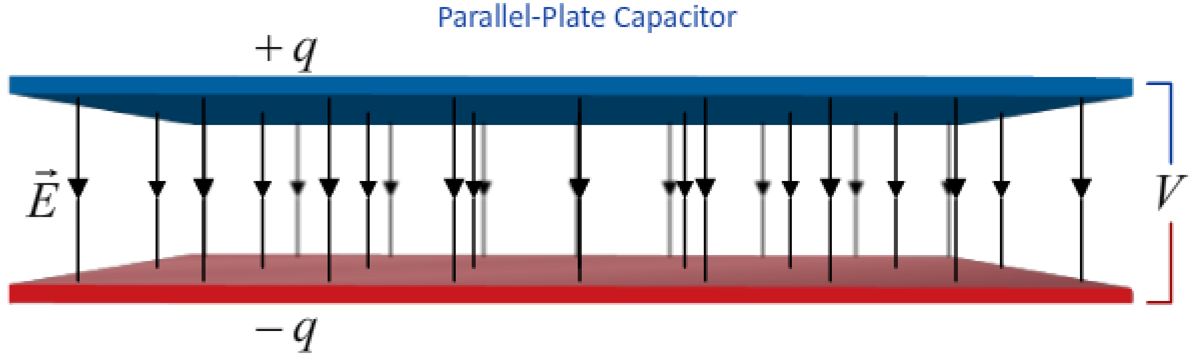
Conducting Sphere



Main Point 2: Capacitance = Q/V



Main Point 3: Capacitors Store Energy in E



$$u = \frac{1}{2}\varepsilon_o E^2$$ Energy Density

Energy Stored in Capacitors

$$U = \frac{1}{2}QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2}CV^2$$

Conductors

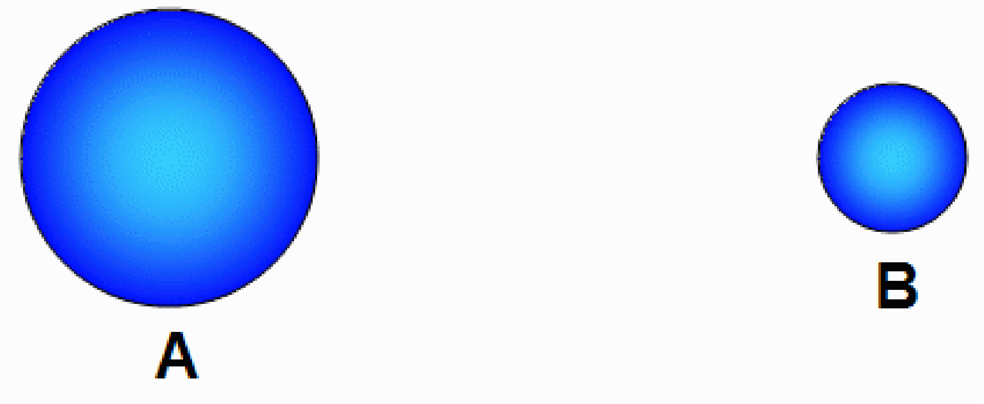
You did well on the questions on charge distributions on conductors

The Main Points

- Charges free to move
- $E = 0$ in a conductor
- Surface $=$ Equipotential
- E at surface perpendicular to surface

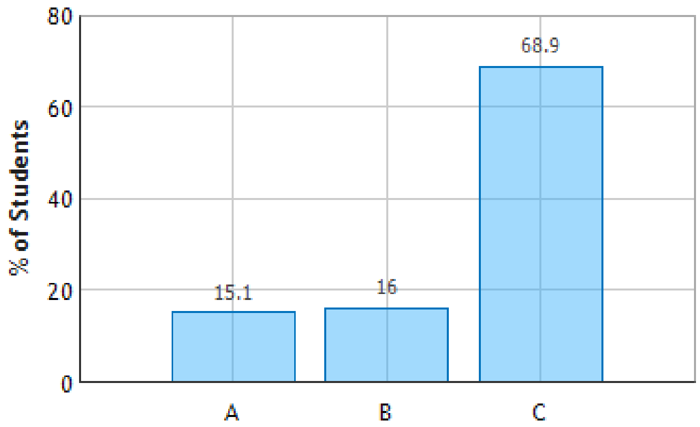
CheckPoint 1a

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B



Compare the potential on surface A with the potential on surface B

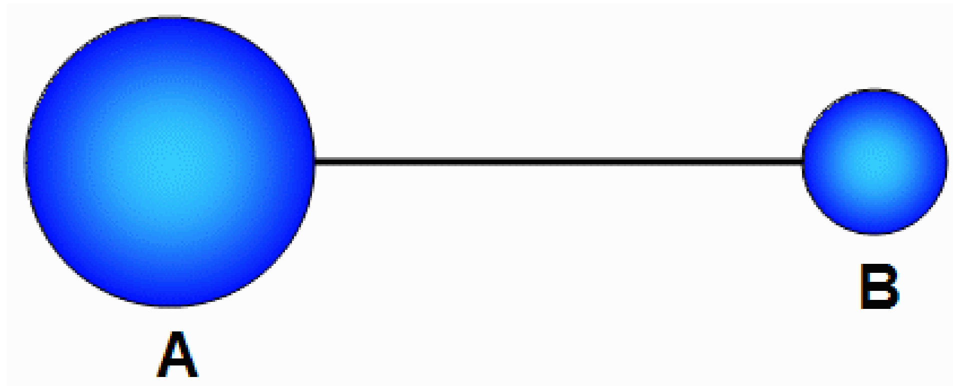
A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$



They both replicate the electric field due to a point charge, and since the surface of A is farther away from its center than B, it will have a weaker electric field and therefore have less potential.

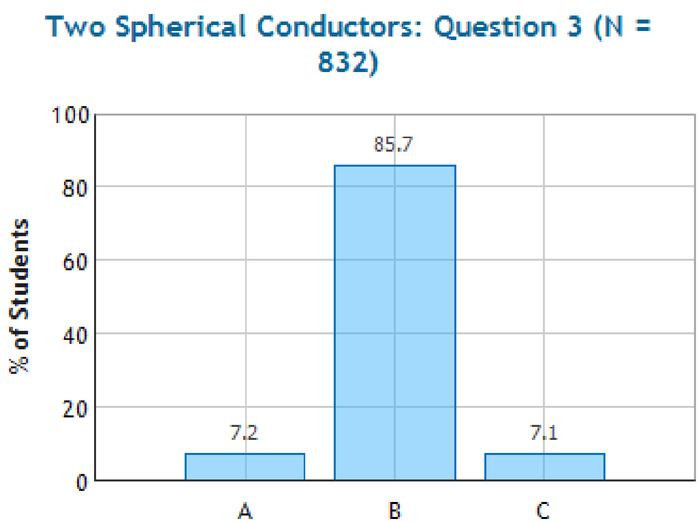
CheckPoint 1b

The two conductors are now attached by a conducting wire.



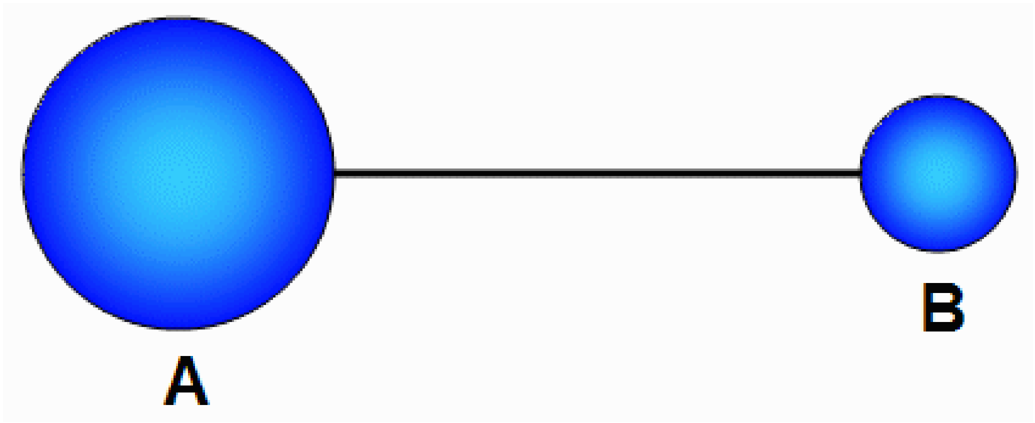
Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$



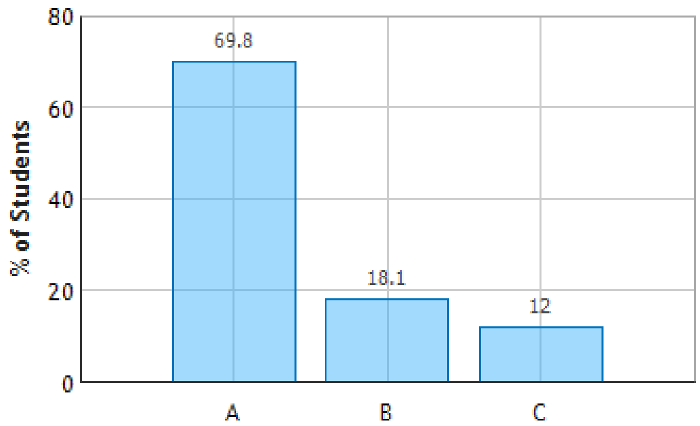
"When the conductors are connected by a string they become one conductor. Since conductors are equipotentials, the electric potentials at the conductor surfaces are the same."

CheckPoint 1c



What happens to the charge on sphere A when the wire is attached

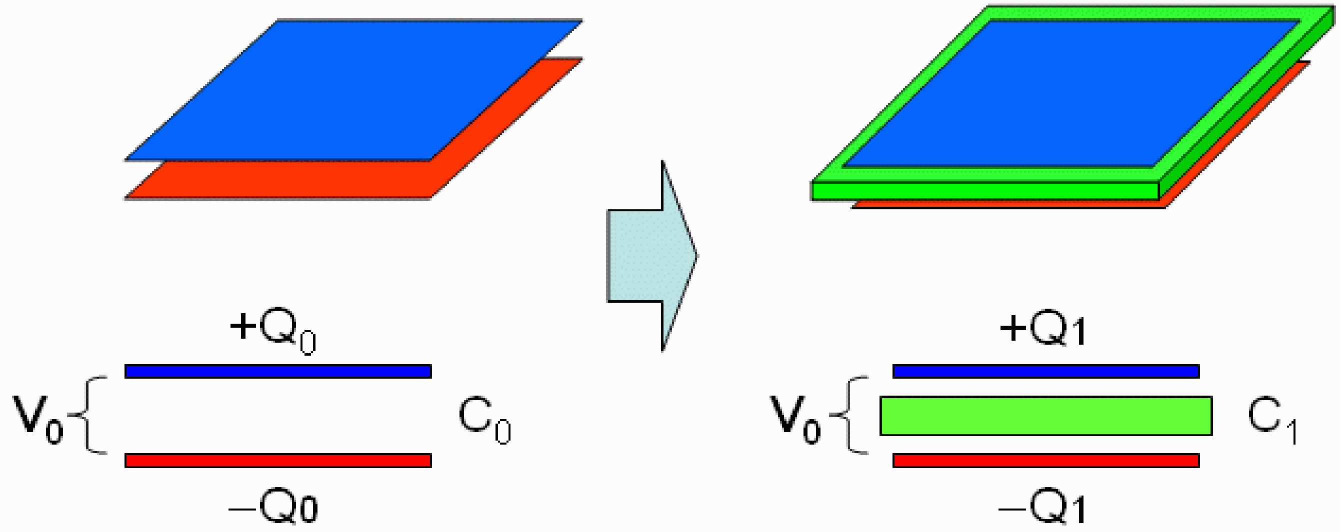
A) Q_A increases B) Q_A decreases C) Q_A does not change



"The electric potential was lower at A , which means positive charges will travel towards A"

Parallel Plate Capacitor

Two parallel plates of area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one.The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same as before.



THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We'll work through the example in the prelecture and then do the preflight questions.

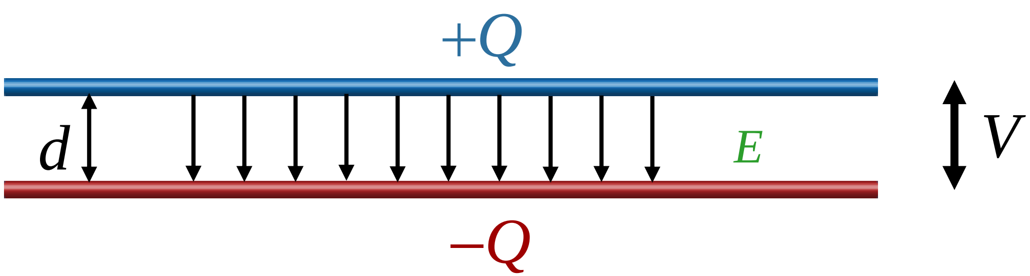
Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

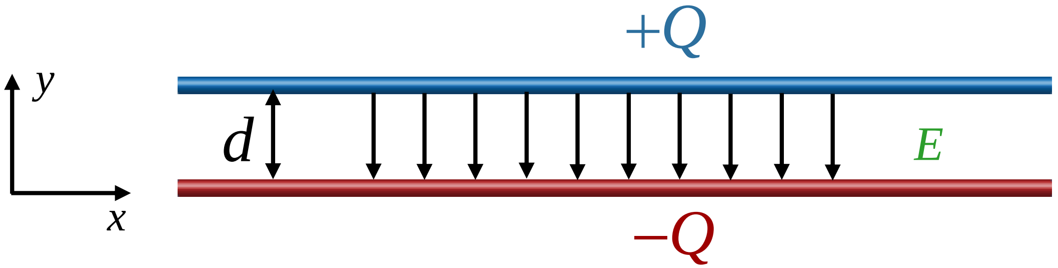
How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to Q !
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Example (done in Prelecture 7)



What is σ ?

$$E = \frac{\sigma}{\varepsilon_o} \qquad \sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

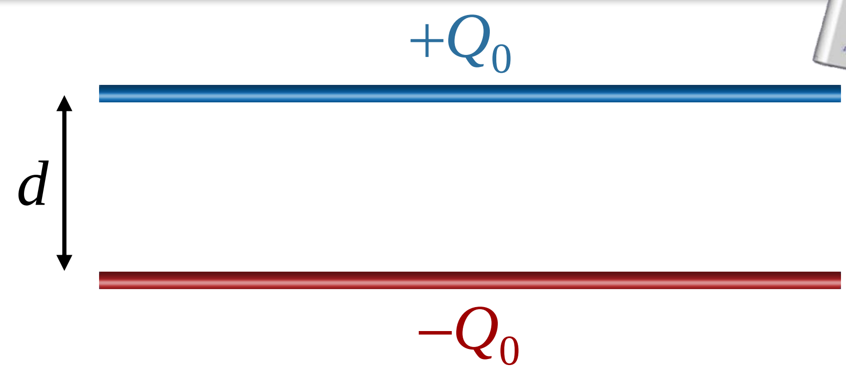
$$V = -\int_0^d \vec{E} \cdot d\vec{y} \longrightarrow V = -\int_0^d (-E dy) = E\int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

As promised, V is proportional to Q !

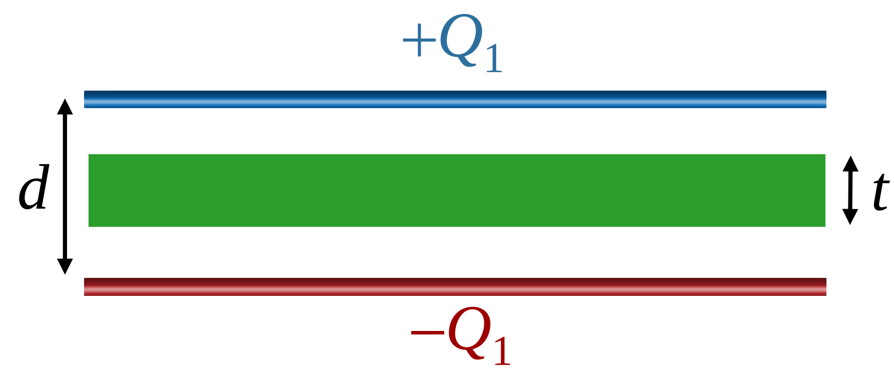
$$C \equiv \frac{Q}{V} = \frac{Q}{Qd/\varepsilon_o A} \longrightarrow C = \frac{\varepsilon_0 A}{d}$$

C determined by geometry !

Question Related to CheckPoint



Initial charge on capacitor $= Q_0$



Insert uncharged conductor

Charge on capacitor now $= Q_1$

How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$

B) $Q_1 = Q_0$

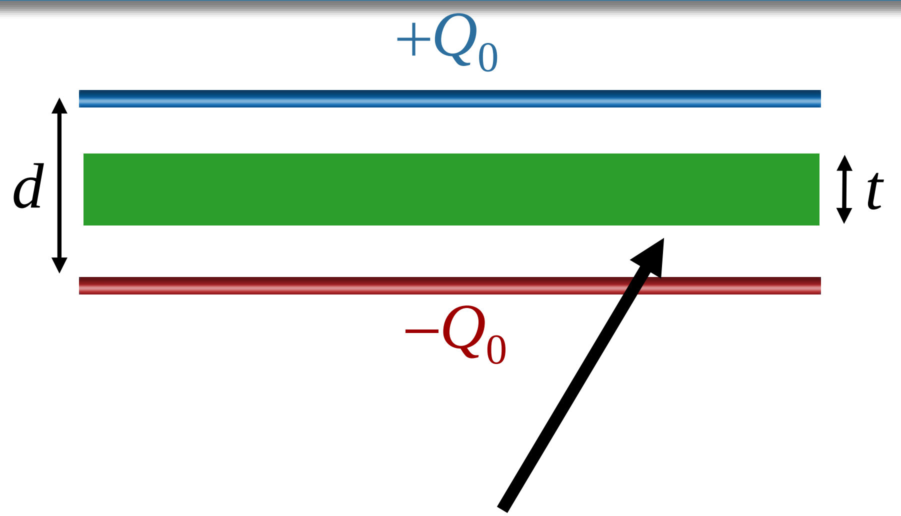
C) $Q_1 > Q_0$

Plates not connected to anything

↓

CHARGE CANNOT CHANGE !

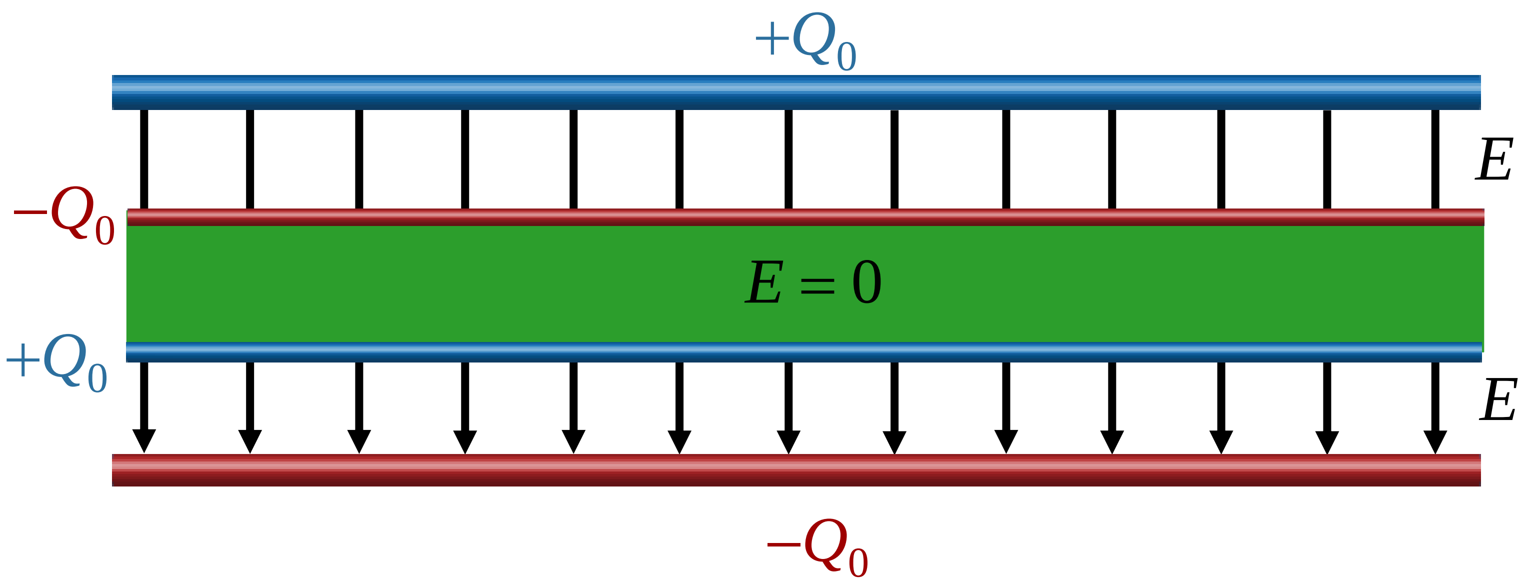
Where to Start ?



What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$
B) $+Q_0/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$

Why ?



WHAT DO WE KNOW ?

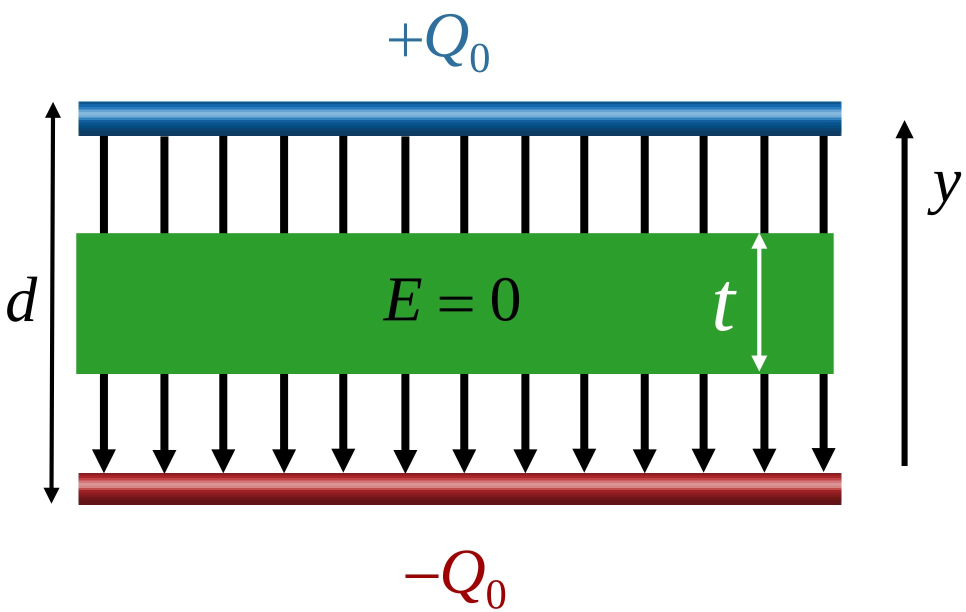
E must be $= 0$ in conductor !

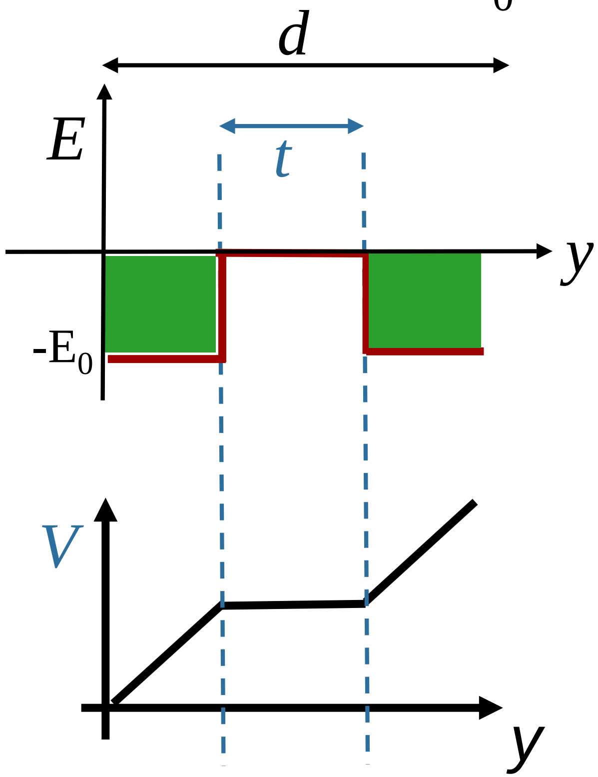
Charges inside conductor move to cancel E field from top & bottom plates.

Calculate V

Now calculate V as a function of distance from the bottom conductor.

$$V(y) = -\int_0^y \vec{E} \cdot d\vec{y}$$





What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

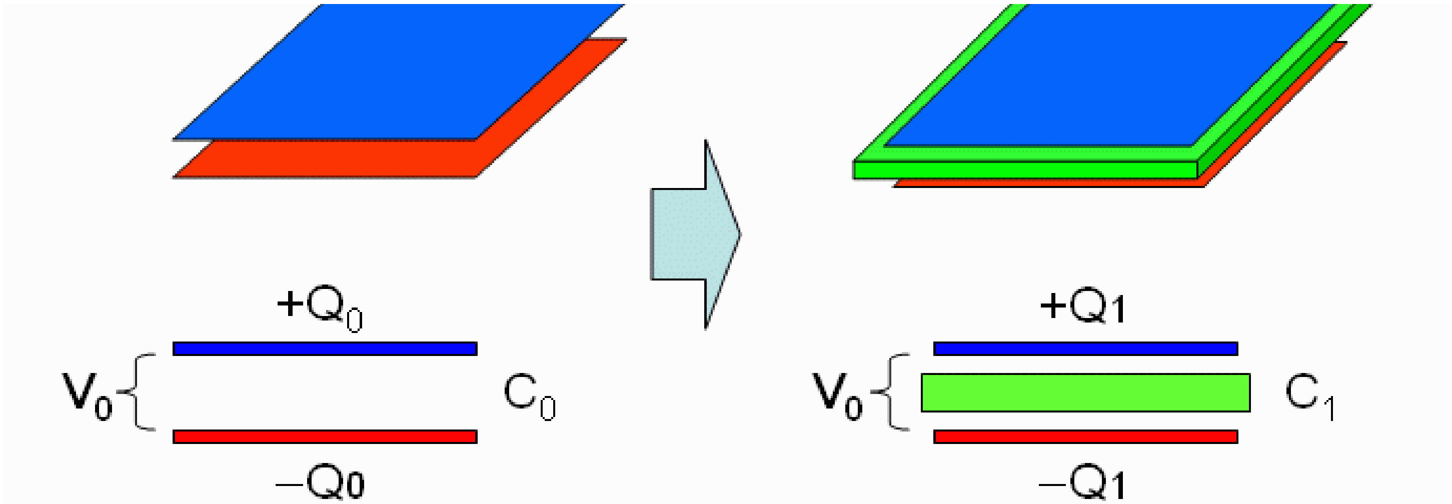
B) $\Delta V = E_0(d - t)$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

Back to CheckPoint 2a

Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, how would charge need to be adjusted to keep same potential difference?

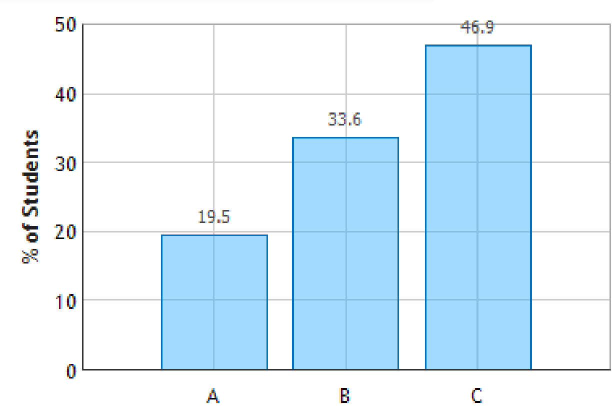


A) $Q_1 < Q_0$ B) $Q_1 = Q_0$ C) $Q_1 > Q_0$

"the plate gets in the way just like in the prelecture "

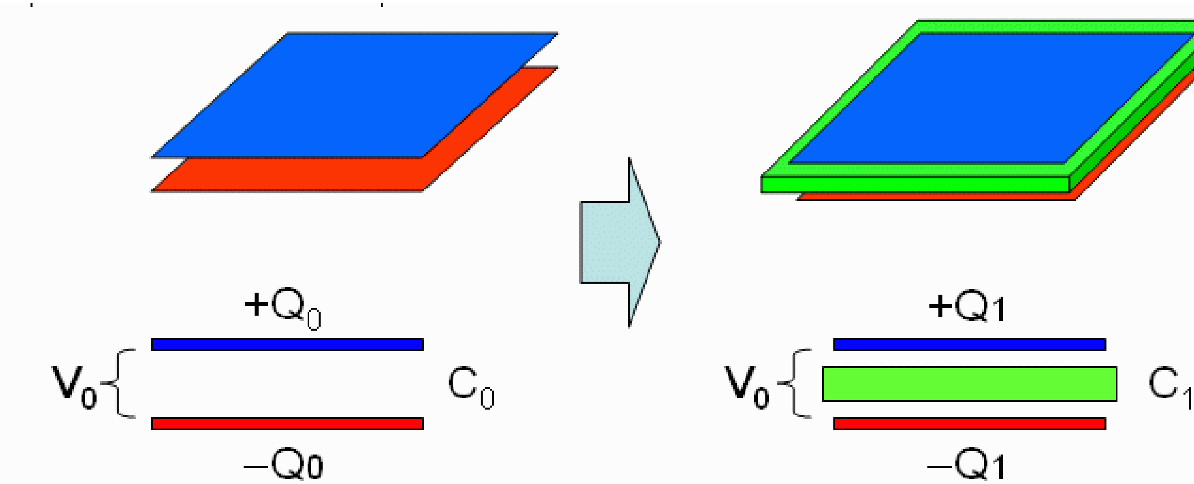
"No charge physically leaves the conductors when another conductor is induced. "

"Because there is no electric field inside the conductor and change in V is Ed, we need a stronger field and therefore stronger charges to make up for the lack of distance.

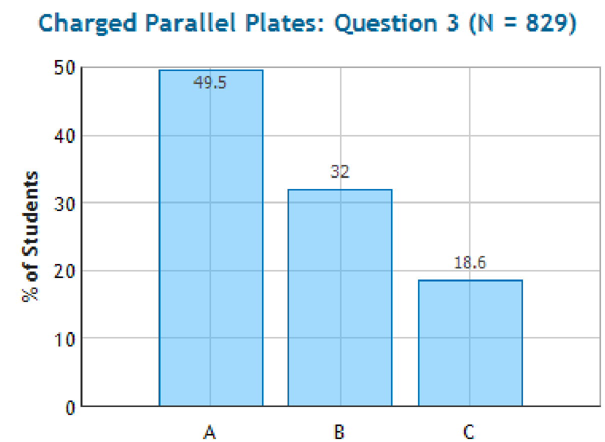


CheckPoint 2b

Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, does C change?



A) $C_1 > C_o$ B) $C_1 = C_o$ C) $C_1 < C_o$



We can determine C from either case

same V (preflight)

same Q (lecture)

C depends only on geometry !

$$E_0 = Q_0 / \varepsilon_0 A$$

Same Q:

$$V_0 = E_0 d \qquad C_0 = Q_0 / E_0 d \qquad C_0 = \varepsilon_0 A / d$$

$$V_1 = E_0 (d - t) \qquad C_1 = Q_0 / (E_0 (d - t)) \qquad C_1 = \varepsilon_0 A / (d - t)$$

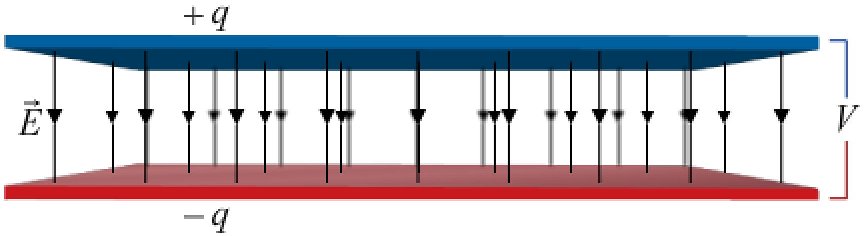
Energy in Capacitors

Energy Stored in Capacitors

$$U = \frac{1}{2} QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2} CV^2$$

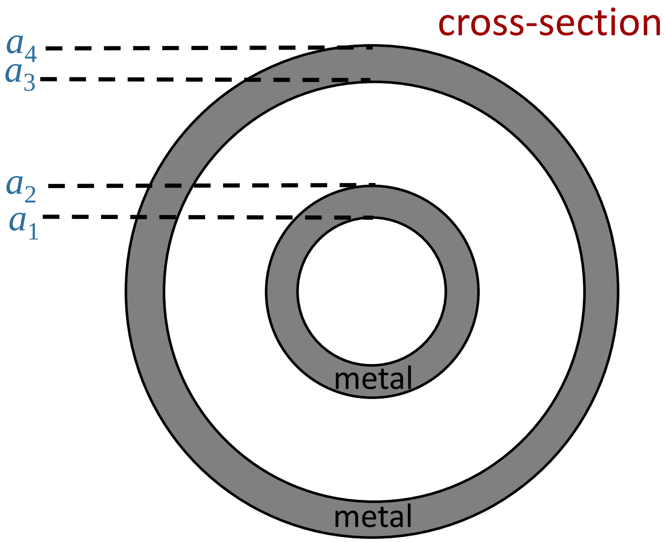
Energy Density

$$u = \frac{1}{2}\varepsilon_o E^2$$



BANG

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

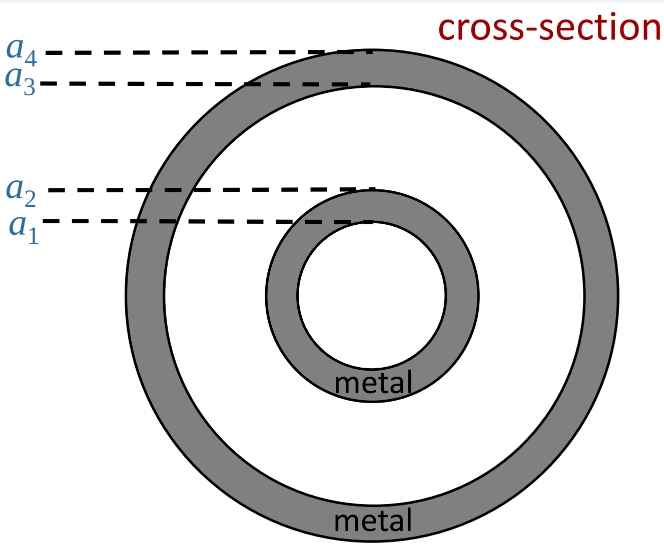
- Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

But what is Q and what is V ? They are not given?

- Important Point: C is a property of the object! (concentric cylinders here)
  - Assume some Q (i.e., $+Q$ on one conductor and $-Q$ on the other)
  - These charges create E field in region between conductors
  - This E field determines a potential difference V between the conductors
  - V should be proportional to Q; the ratio Q/V is the capacitance.

Calculation



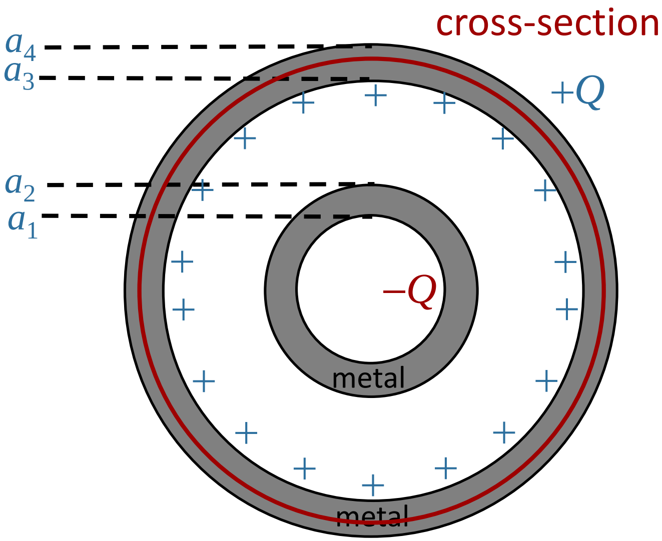
A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

- **Strategic Analysis:**
  - Put $+Q$ on outer shell and $-Q$ on inner shell
  - Cylindrical symmetry: Use Gauss' Law to calculate E everywhere
  - Integrate E to get V
  - Take ratio Q/V: should get expression only using geometric parameters (a_i, L)

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

A) at $r = a_4$ B) at $r = a_3$ C) both surfaces   D) throughout shell

Why?

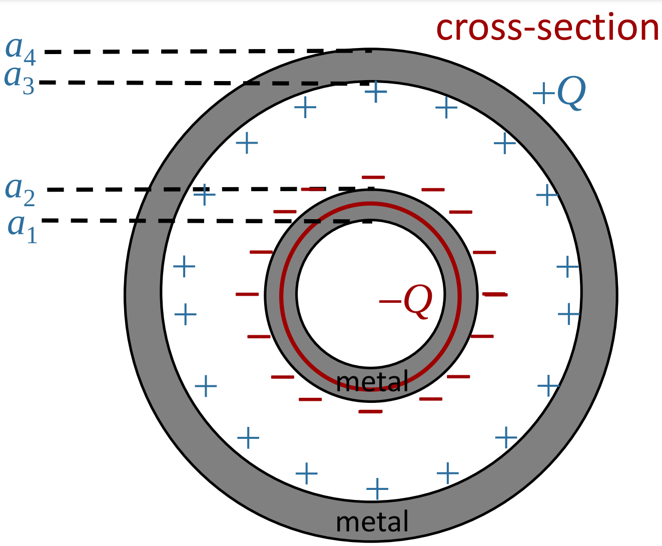
Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$

$\longrightarrow$ $Q_{enclosed} = 0$

We know that $E = 0$ in conductor (between a_3 and a_4)

$Q_{enclosed} = 0$ $\longrightarrow$ $+Q$ must be on inside surface (a_3), so that $Q_{enclosed} = +Q - Q = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $-Q$ on inner conductor located?

A) at $r = a_2$ B) at $r = a_1$ C) both surfaces D) throughout shell

Why?

Gauss' law: $$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

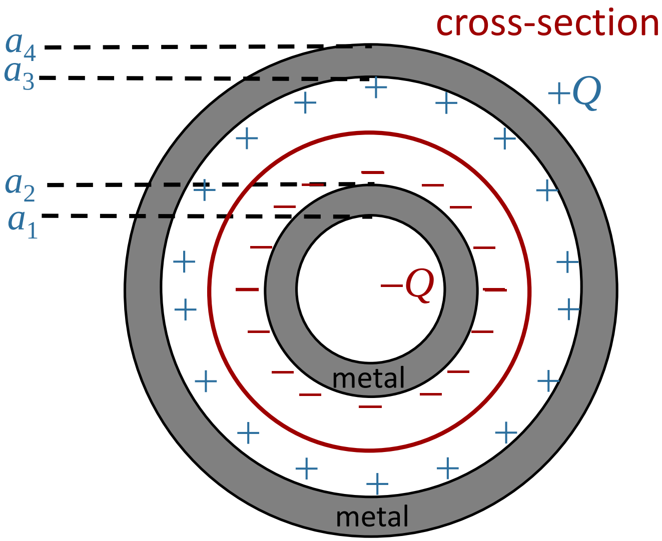
We know that $E = 0$ in conductor (between a_1 and a_2)

$$\longrightarrow Q_{enclosed} = 0$$

$$Q_{enclosed} = 0 \longrightarrow$$

$+Q$ must be on outer surface (a_2), so that $Q_{enclosed} = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is $E(r)$?

A) 0

B) $\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_o}\frac{Q}{Lr}$

D) $\frac{1}{2\pi\varepsilon_o}\frac{2Q}{Lr}$

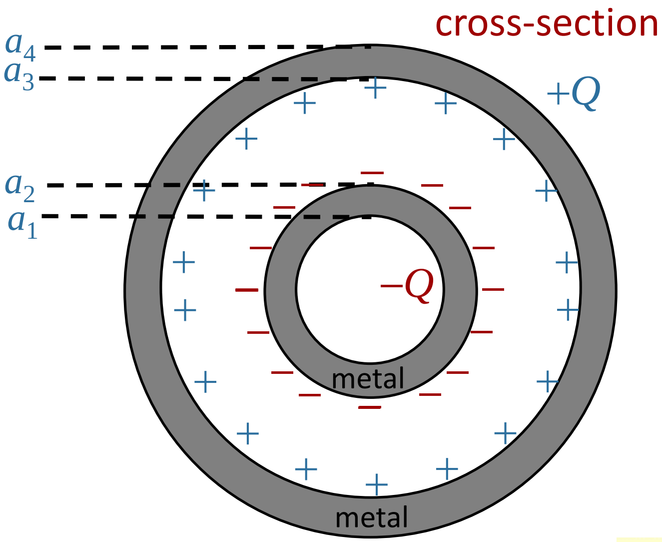
E) $\frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2}$

Why?

Gauss' law: $\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ → $E \cdot 2\pi r L = \frac{Q}{\varepsilon_o}$ → $E = \frac{1}{2\pi\varepsilon_o}\frac{Q}{Lr}$

Direction: Radially In

Calculation

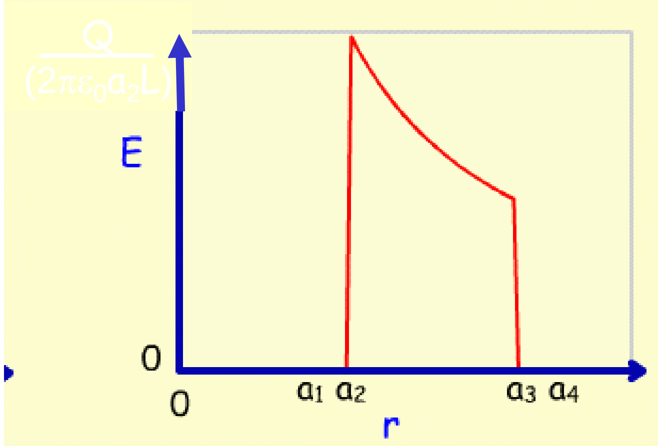


A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

$r < a_2$: $E(r) = 0$
since $Q_{enclosed} = 0$



What is V ?
The potential difference between the conductors.

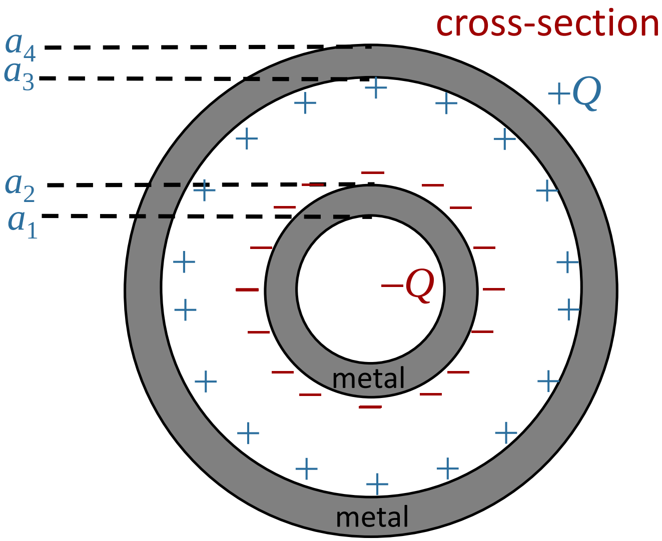
What is the sign of $V = V_{outer} - V_{inner}$?

A) $V_{outer} - V_{inner} < 0$

B) $V_{outer} - V_{inner} = 0$

C) $V_{outer} - V_{inner} > 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

What is $V \equiv V_{outer} - V_{inner}$?

(A) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_1}{a_4}$

(B) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_4}{a_1}$

(C) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_3}{a_2}$

(D) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_2}{a_3}$

$$V = -\int_{a_2}^{a_3}\frac{-Q}{2\pi\varepsilon_o L}\frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\varepsilon_o L}\int_{a_2}^{a_3}\frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{a_3}{a_2}$$

V proportional to Q, as promised

$$\rightarrow \quad C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(a_3/a_2)}$$