Your Comments

This seemed very graphical and diagram based. What kind of problems would we see from this topic?

This was one of the most confusing things I've ever seen. I did AP C in high school but I don't think we did this topic? Or am I forgetting something?

What is reactance?

Can we discuss qualitatively why the current and voltage are out of phase for capacitors and inductors? It's difficult to remember which is which without understanding why it works the way it does.

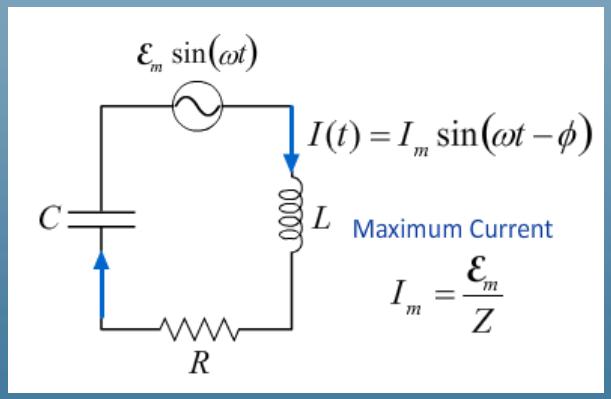
I don't like having to do these prelectures the day of an exam...

This is the first time I didn't understand the prelecture at all. Please explain this!! What does the phasor diagram do??

always wanted to learn about AC. Speaking of AC, can you play "Epic Rap Battles of History: Thomas Edison vs Nikola Tesla" before class? http://www.youtube.com/watch?v=gJ1Mz7kGVf0. Tesla is awesome!

THIS IS UNREAL @@.. who came up with the idea of a phasor diagram?

Set phasors to stun!



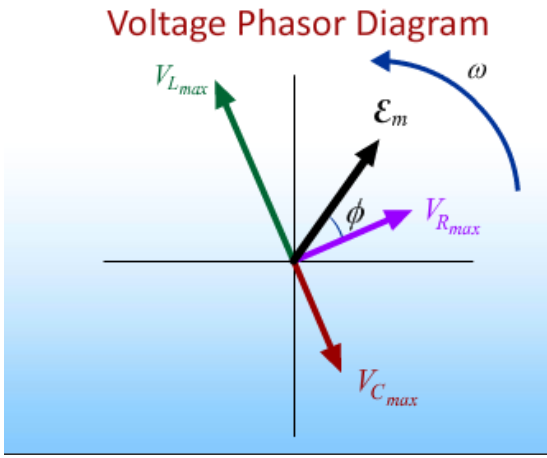
Physics 212 Lecture 20

Today's Concept:

AC Circuits

Maximum currents & voltages

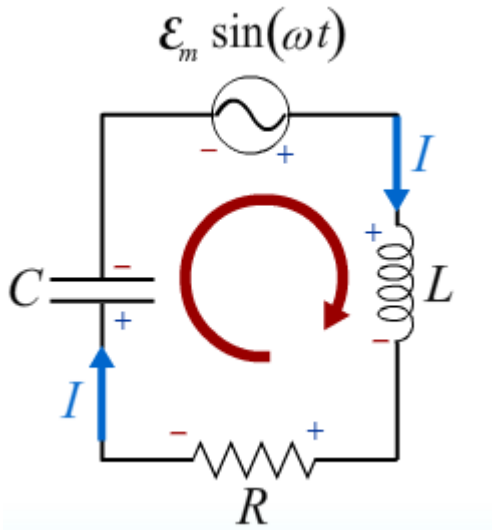
Phasors: A Simple Tool



Big Idea

KVR

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}-\mathcal{E}_m\sin(\omega t)=0$$



Maximum Values (easy V=IR)

$$I_{max}=\mathcal{E}_{max}/Z$$

$$V_{Rmax}=I_{max}R$$

$$V_{Lmax}=I_{max}X_L$$

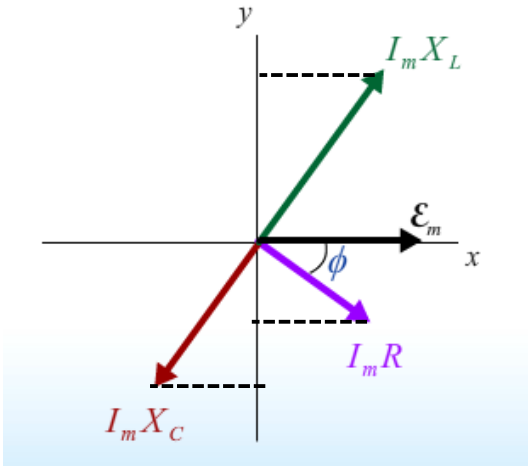
$$V_{Cmax}=I_{max}X_C$$

Value at specific time (phasors)

y component gives voltage

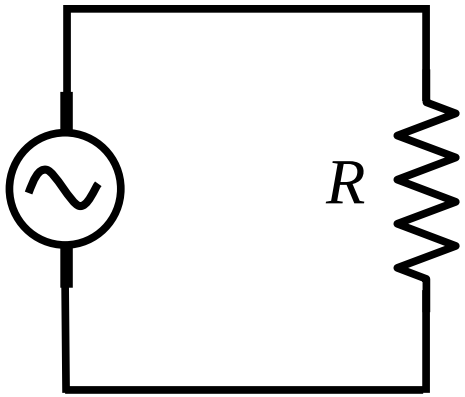
V_{Inductor} Leads current

$V_{\text{Capacitor}}$ Lags current



Resistors

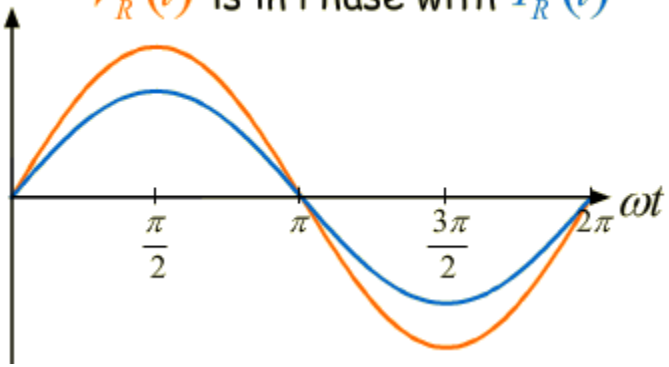
$\mathcal{E} = V_{max}\sin(\omega t)$



$I = V_R/R = V_{max}/R \sin(\omega t)$

Amplitude = V_{max}/R

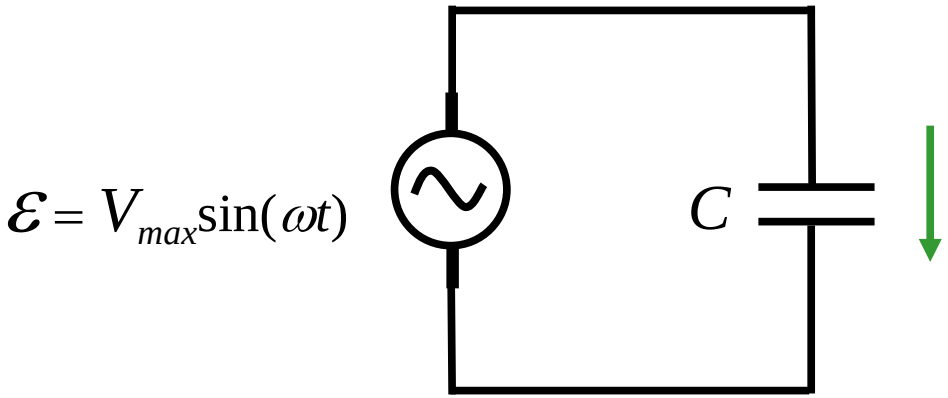
$V_R(t)$ is in Phase with $I_R(t)$



Capacitors

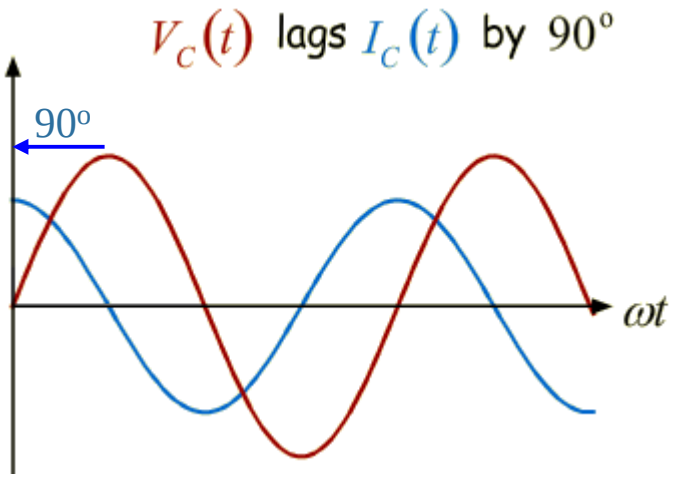
$$Q = CV = CV_{max}\sin(\omega t)$$

$$I = dQ/dt$$



$$I = V_{max}\omega C\cos(\omega t)$$

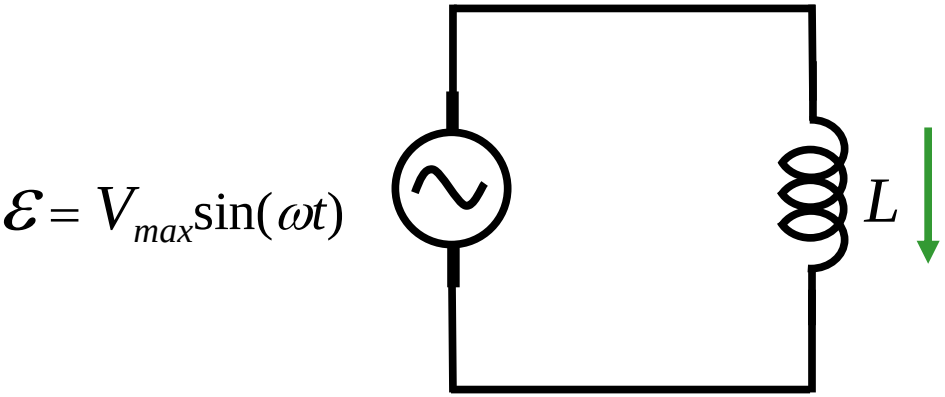
Amplitude $= V_{max}/X_C$



where $X_C = 1/\omega C$
is like the "resistance"
of the capacitor
X_C depends on ω

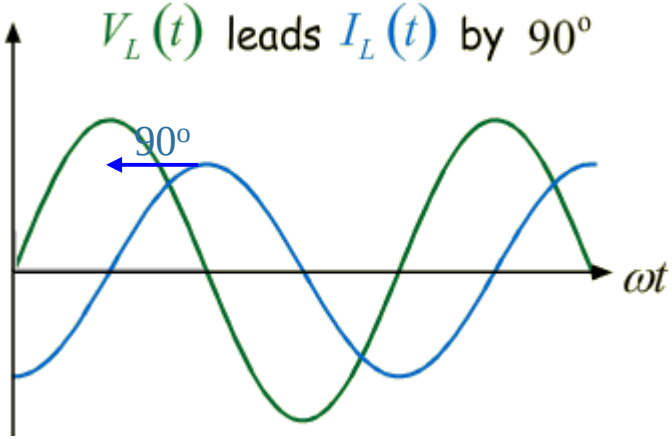
Inductors

$$L\,dI/dt = V_L = V_{max}\sin(\omega t)$$



$$I = -V_{max}/\omega L \cos(\omega t)$$

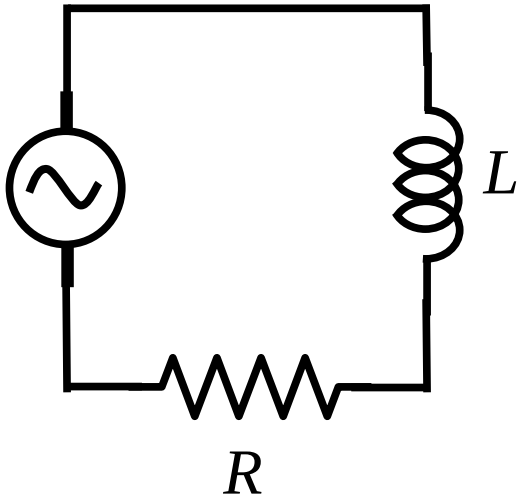
Amplitude $= V_{max}/X_L$



where $X_L = \omega L$
is like the "resistance"
of the inductor
X_L depends on ω

RL Clicker Question

An RL circuit is driven by an AC generator as shown in the figure.



$$X_L = \omega L$$

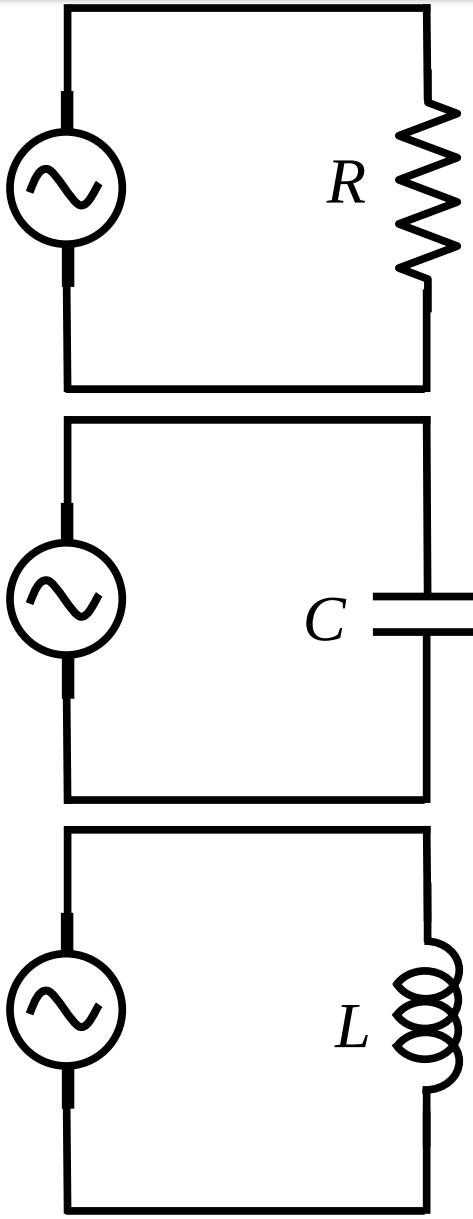
As ω -> 0, so does X_L

As ω -> 0,
resistance of circuit R
current gets bigger

For what driving frequency ω of the generator will the current through the resistor be largest

A) ω large
B) Current through R doesn't depend on ω
C) ω small

Summary



$I_{max} = V_{max}/R$

V_R in phase with I

Because resistors are simple

$I_{max} = V_{max}/X_C$

$X_C = 1/\omega C$

V_C 90° behind I

Current comes first since it charges capacitor

Like a wire at high ω

$I_{max} = V_{max}/X_L$

$X_L = \omega L$

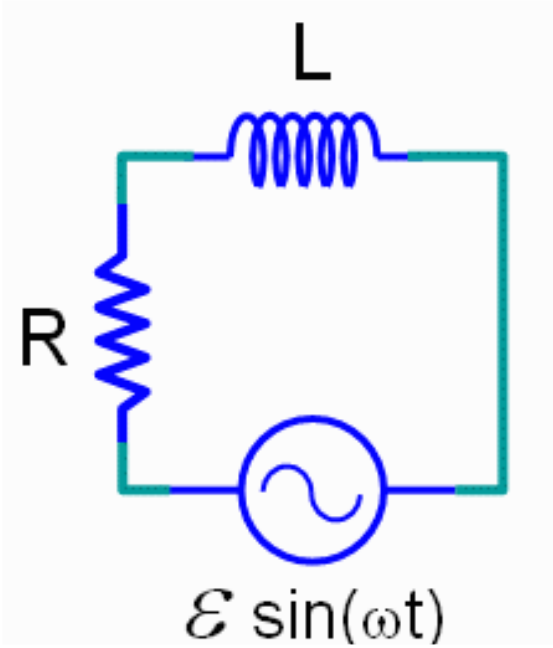
V_L 90° ahead of I

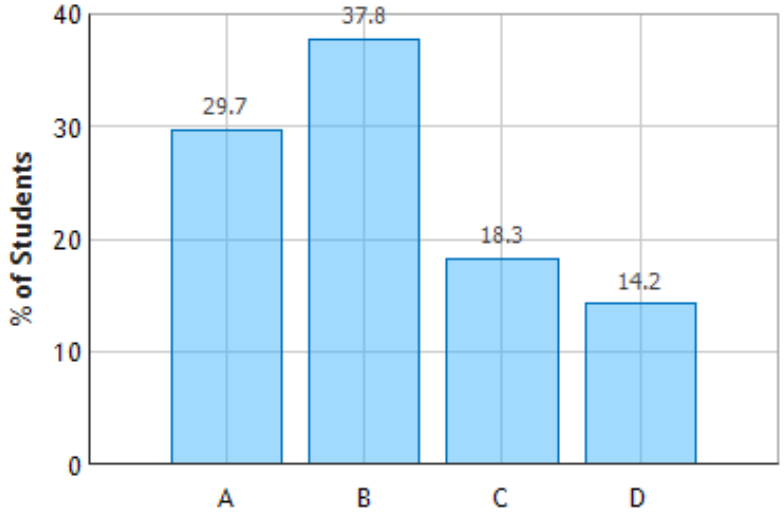
Opposite of capacitor

Like a wire at low ω

CheckPoint 1c

A RL circuit is driven by an AC generator as shown in the figure.





The phase difference between the CURRENT through the resistor and inductor

A) Is always zero

B) Is always 90°

C) Depends on the value of L and R

D) Depends on L, R and the generator voltage

The CURRENT is THE CURRENT

There is only 1 current in this circuit

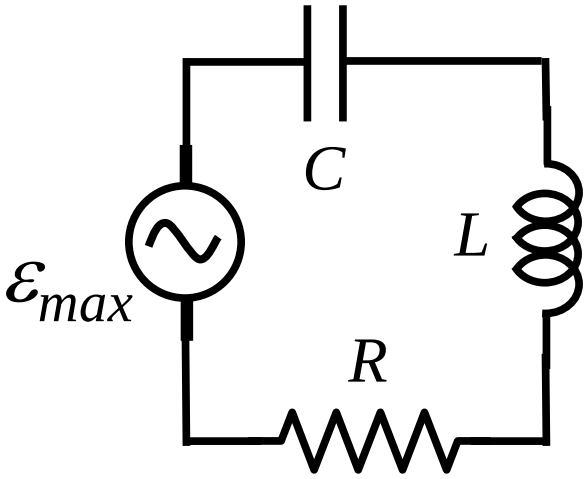
Same everywhere in circuit

Driven RLC Circuit

Makes sense to write everything in terms of I since this is the same everywhere in a one-loop circuit:

$V_{max} = I_{max} X_C$

V 90° behind I



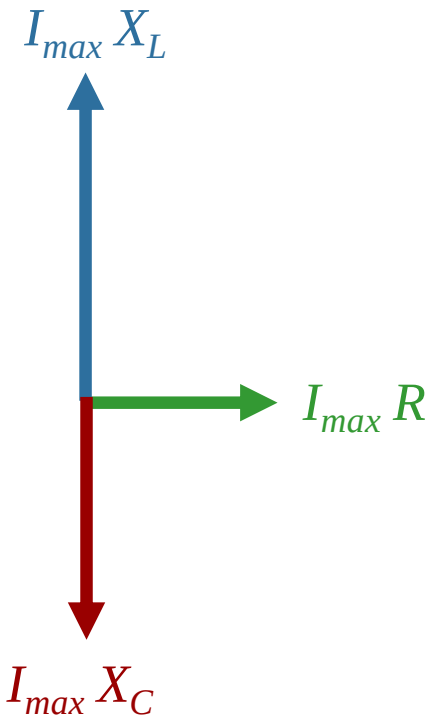
$V_{max} = I_{max} X_L$

V 90° ahead of I

$V_{max} = I_{max} R$

V in phase with I

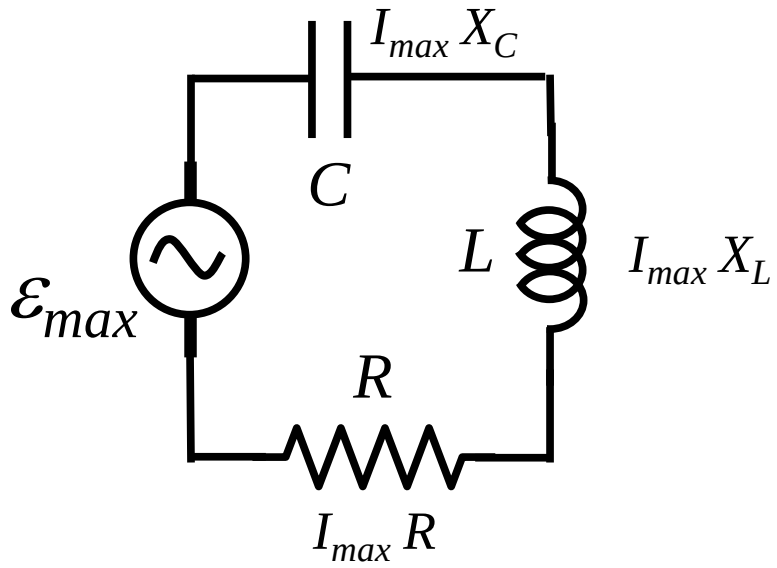
Phasors make this simple to see

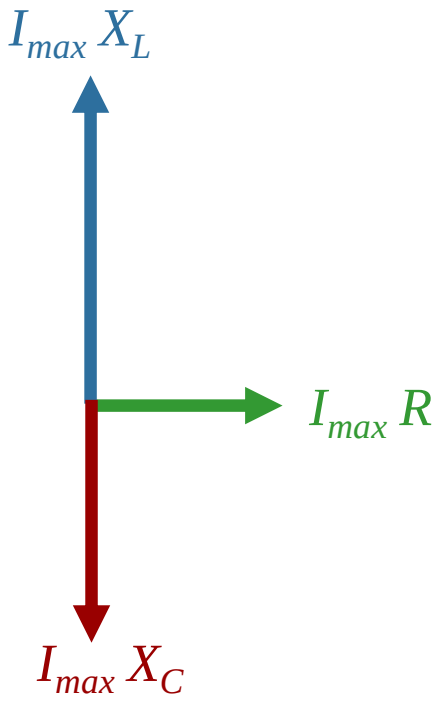


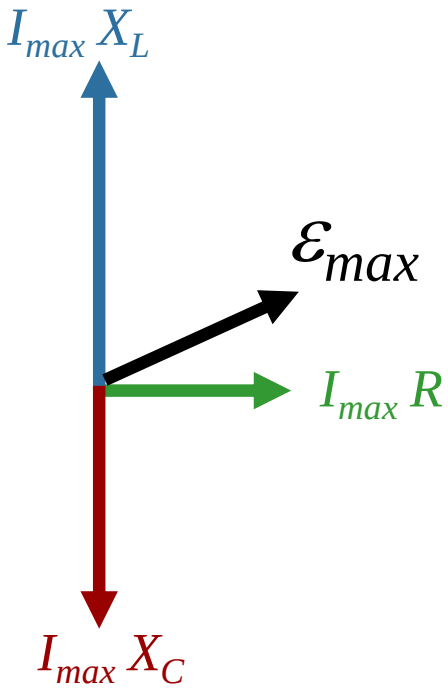
Always looks the same.
Only the lengths will change

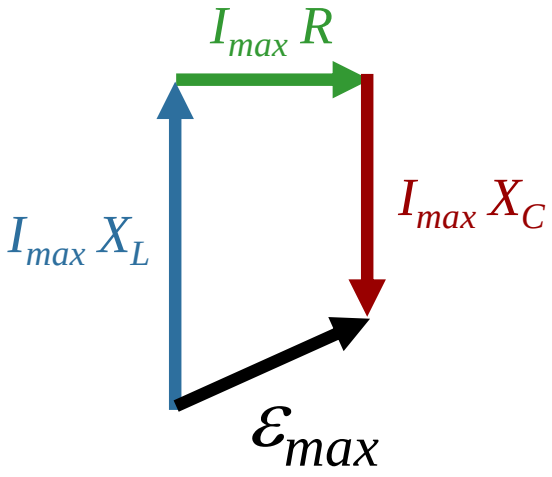
The Voltages still Add Up

But now we are adding vectors:

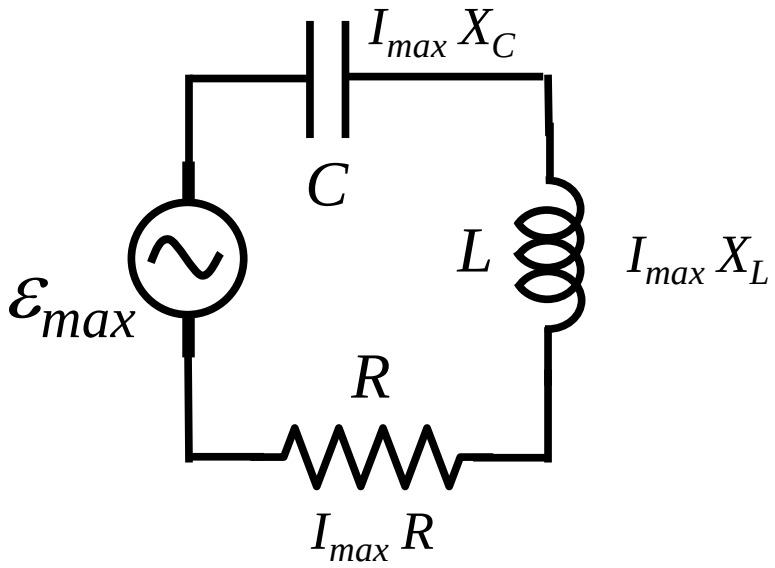


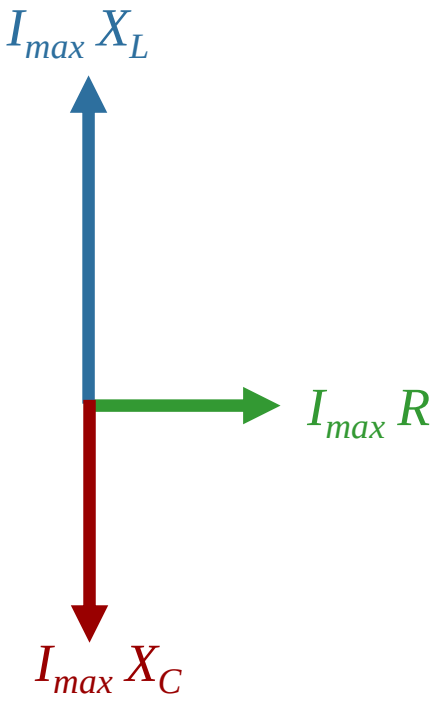


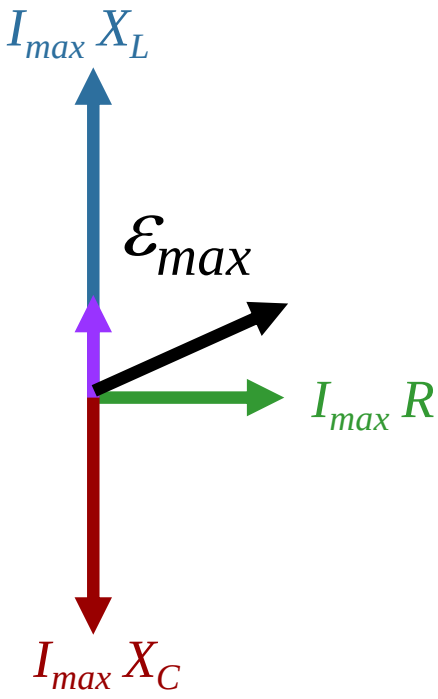




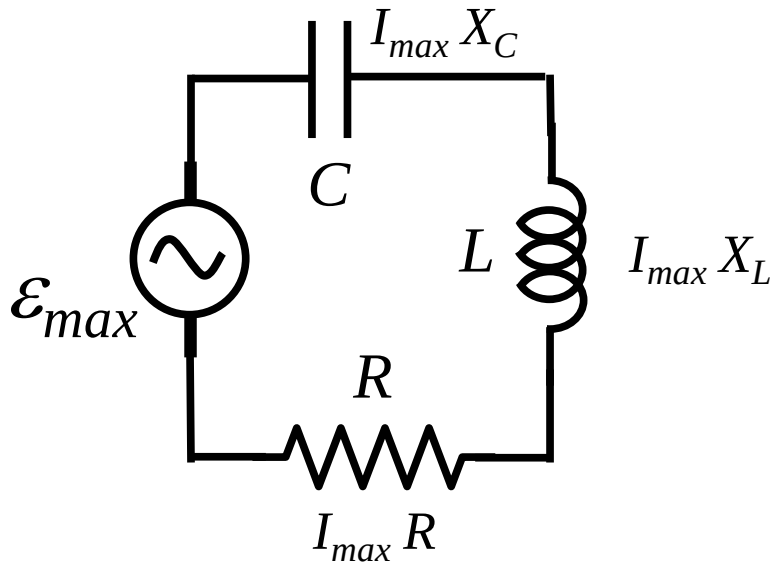
Make this Simpler

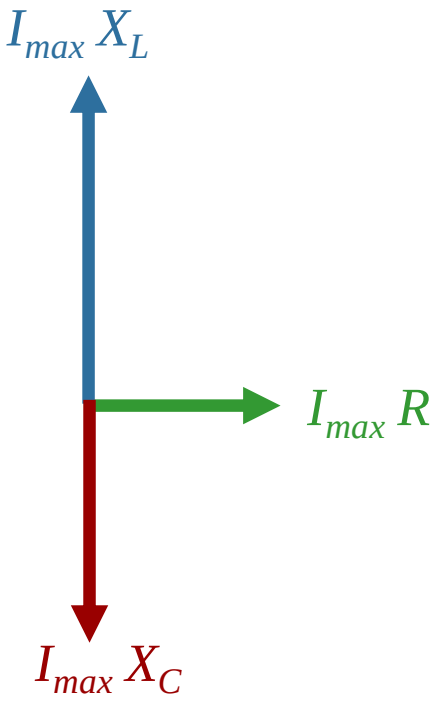




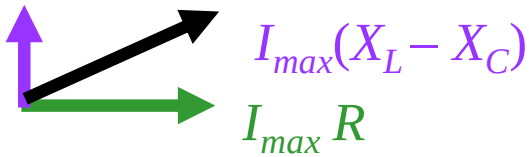


Make this Simpler

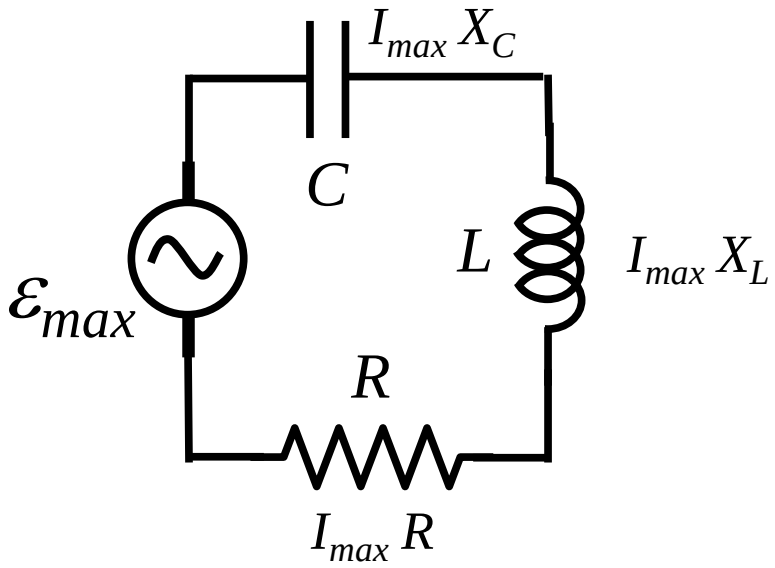




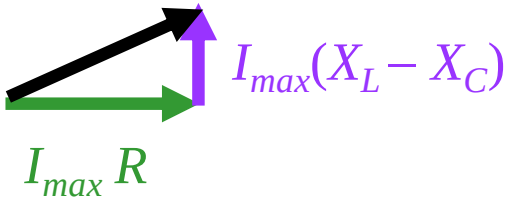
$$\mathcal{E}_{max} = I_{max} Z$$



Make this Simpler

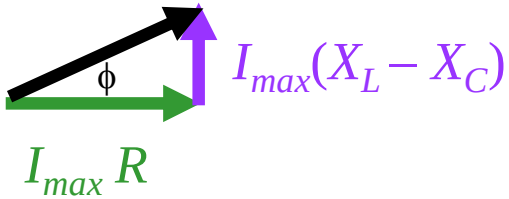


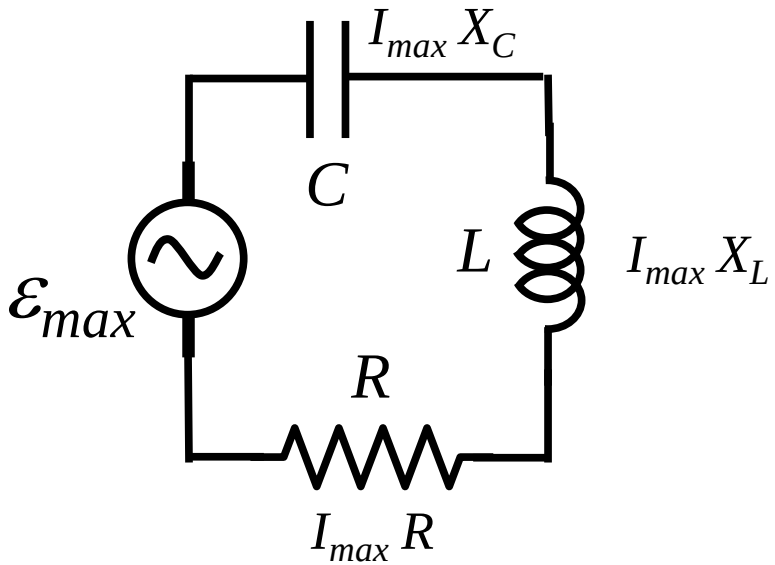
$$\mathcal{E}_{max} = I_{max} Z$$

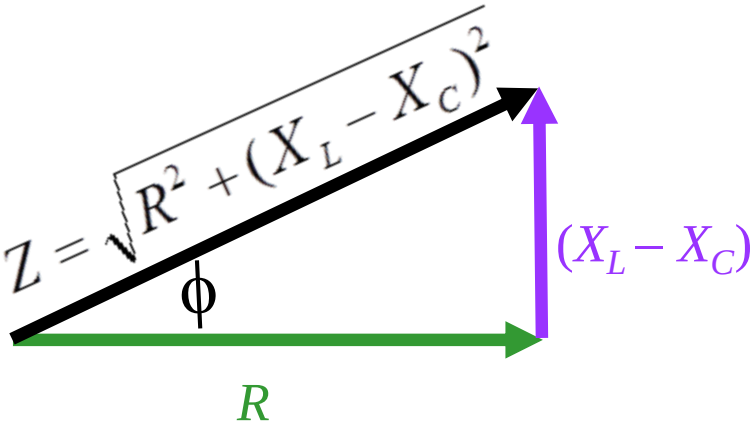


Make this Simpler

$\mathcal{E}_{max} = I_{max} Z$





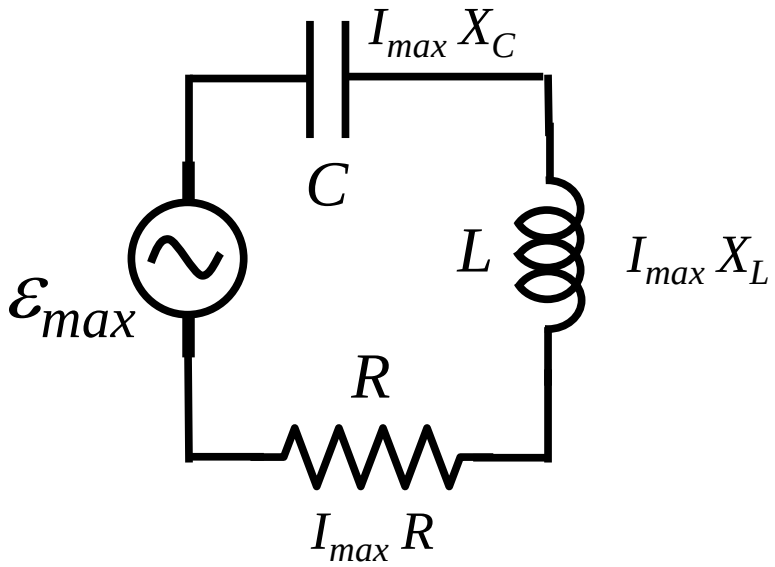


Impedance Triangle

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

Summary

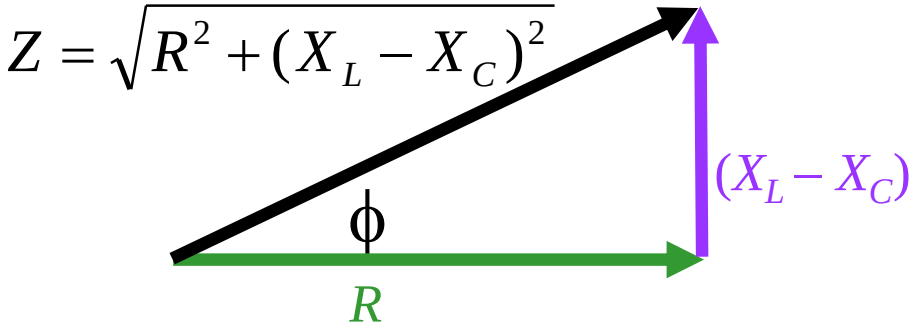
$$V_{Cmax} = I_{max} X_C$$
$$V_{Lmax} = I_{max} X_L$$
$$V_{Rmax} = I_{max} R$$
$$\mathcal{E}_{max} = I_{max} Z$$



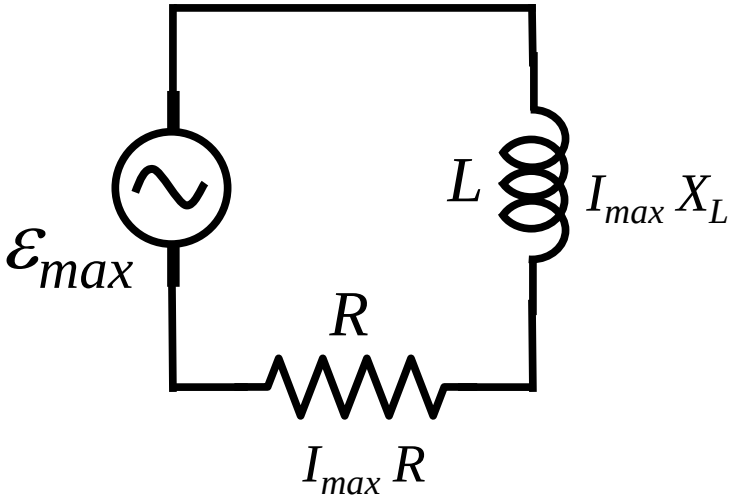
$$I_{max} = \mathcal{E}_{max} / Z$$

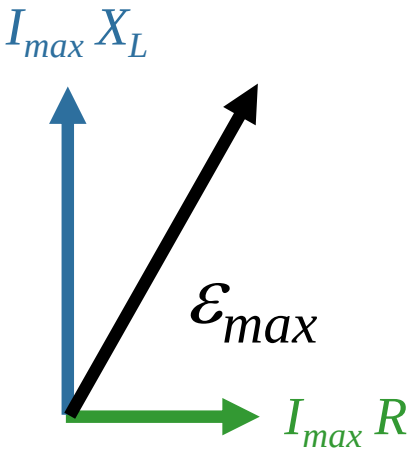
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan(\phi) = \frac{X_L - X_C}{R}$$



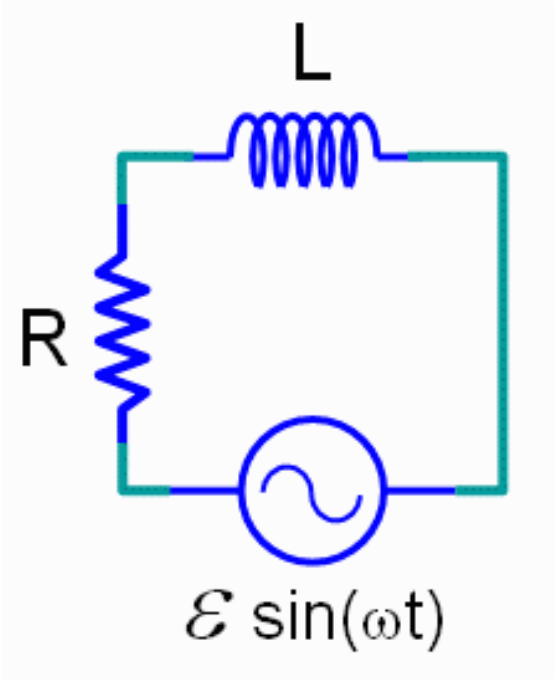
Example: RL Circuit $X_c = 0$



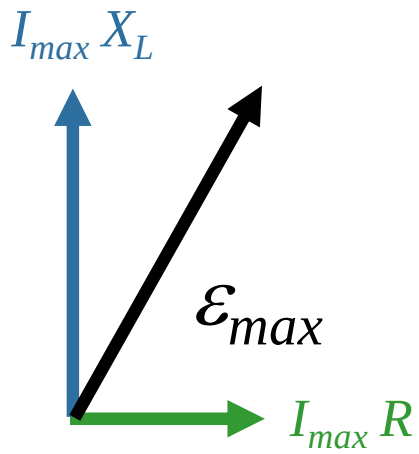


CheckPoint 1a

2) A RL circuit is driven by an AC generator as shown in the figure.



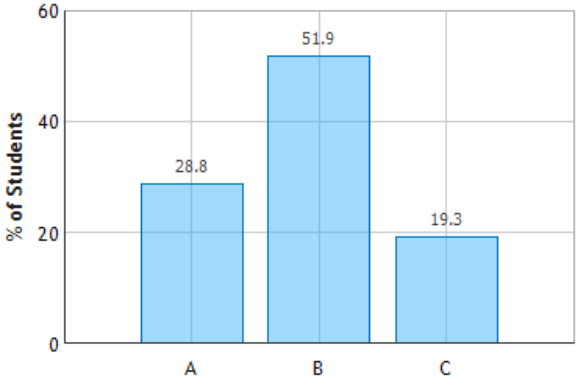
Draw Voltage Phasors



The voltages across the resistor and generator are

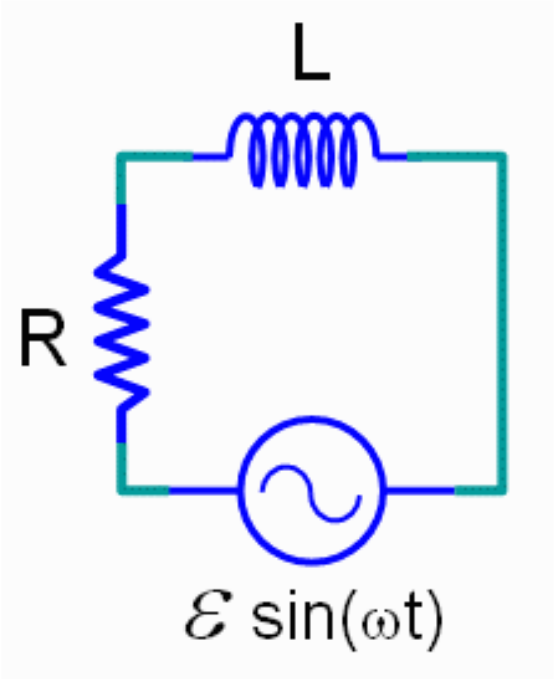
A) Always out of phase ***B)*** Always in phase

C) Sometimes in and sometimes out of phase

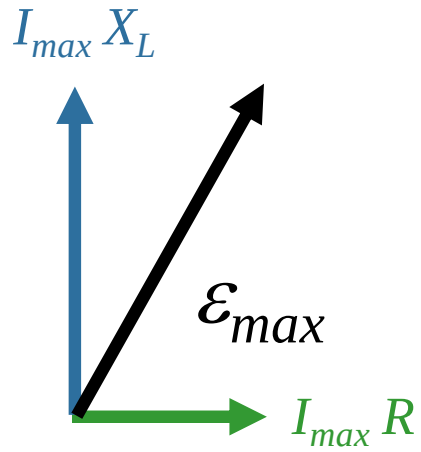


CheckPoint 1b

A RL circuit is driven by an AC generator as shown in the figure.



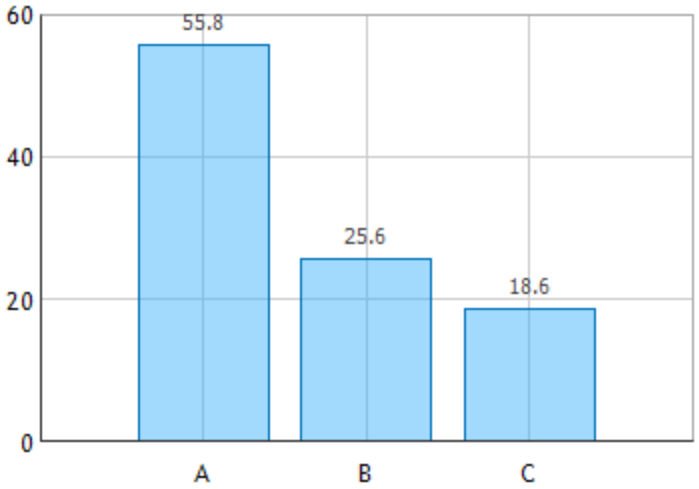
Draw Voltage Phasors



The voltages across the resistor and inductor are

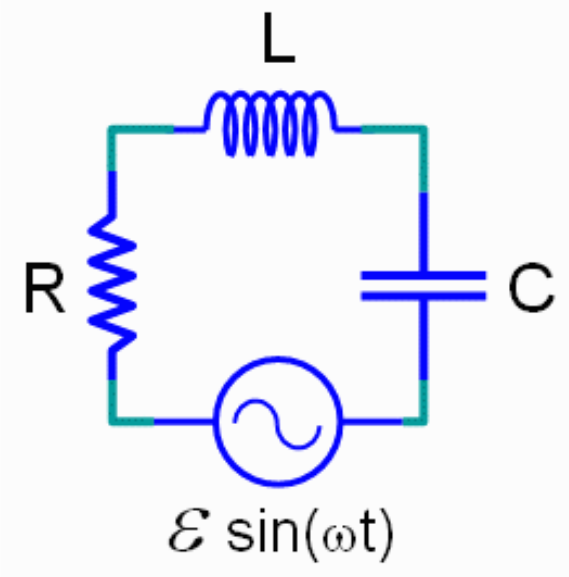
A) Always out of phase ***B)*** Always in phase

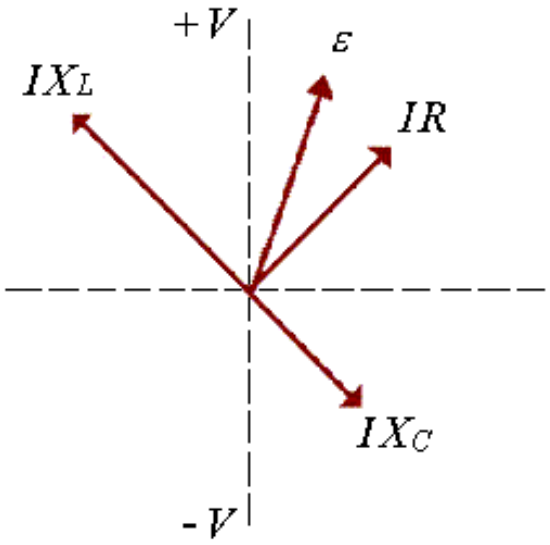
C) Sometimes in and sometimes out of phase

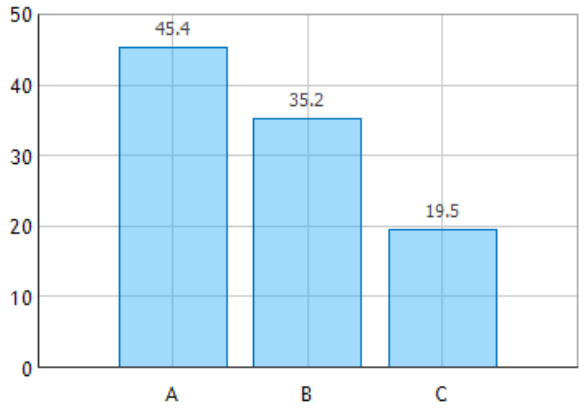


CheckPoint 2a

A driven RLC circuit is represented by the phasor diagram below.



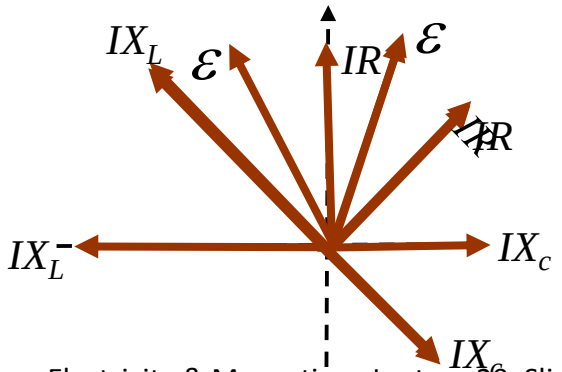




The vertical axis of the phasor diagram represents voltage. When the current through the circuit is maximum, what is the potential difference across the inductor?

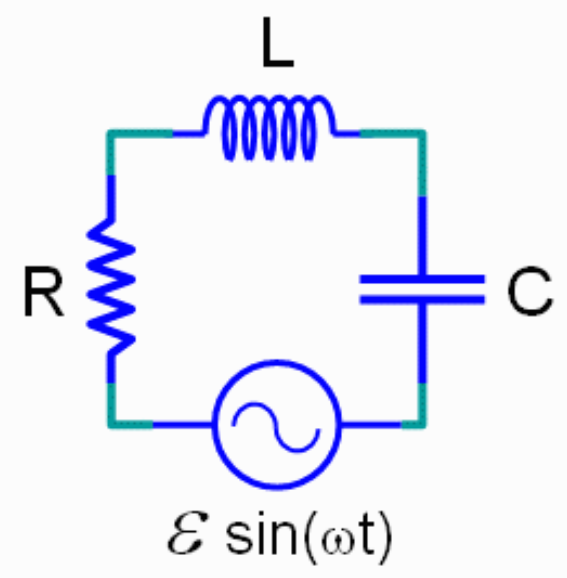
A) $V_L = 0$ ***B)*** $V_L = V_{L,max}/2$ ***C)*** $V_L = V_{L,max}$

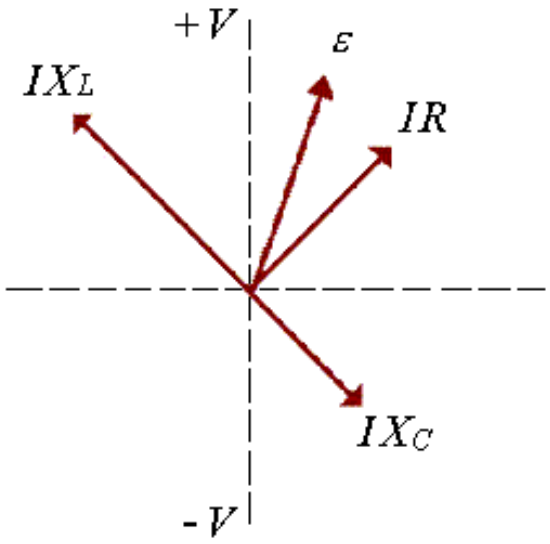
What does the voltage phasor diagram look like when the current is a maximum?

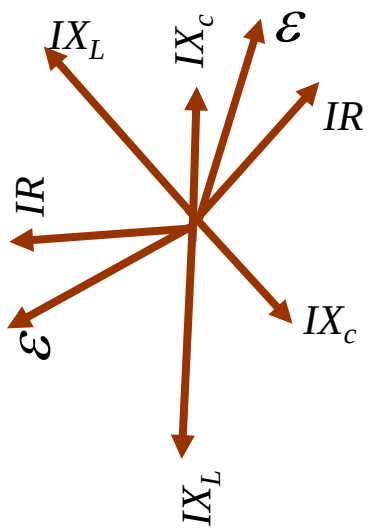


CheckPoint 2b

A driven RLC circuit is represented by the phasor diagram below.



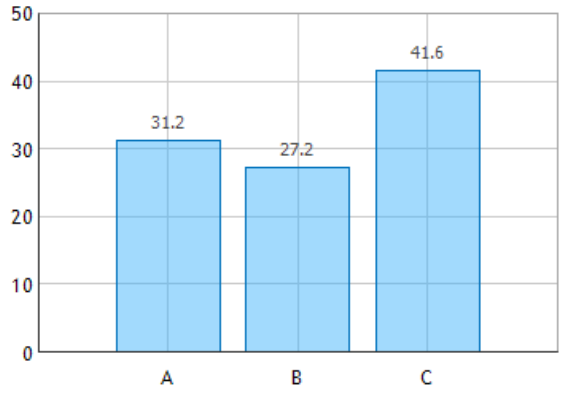




When the capacitor is fully charged, what is the magnitude of the voltage across the inductor?

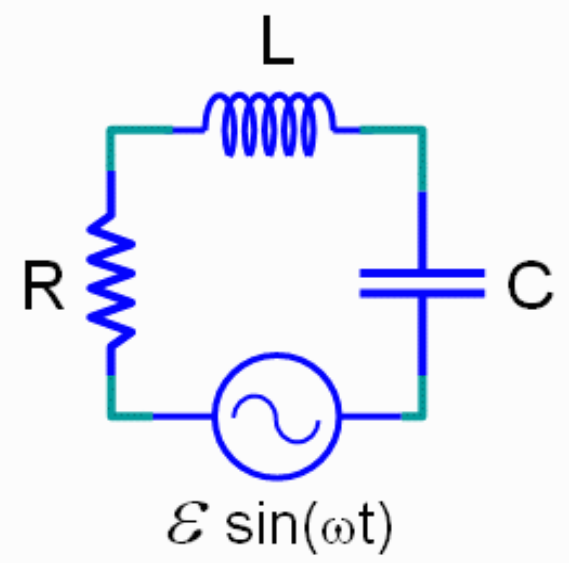
A) $V_L = 0$ ***B)*** $V_L = V_{L,max}/2$ ***C)*** $V_L = V_{L,max}$

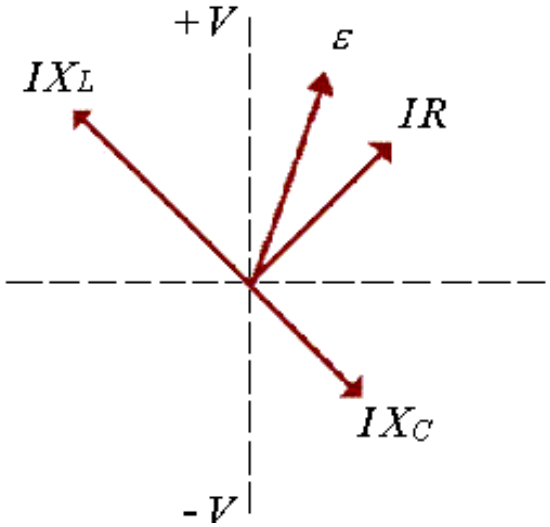
What does the voltage phasor diagram look like when the capacitor is fully charged?

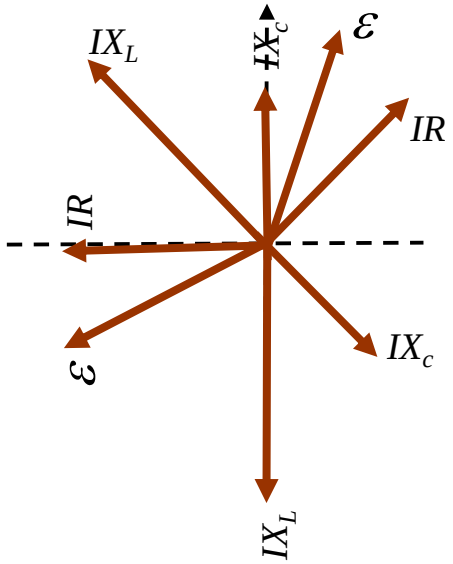


CheckPoint 2C

A driven RLC circuit is represented by the phasor diagram below.

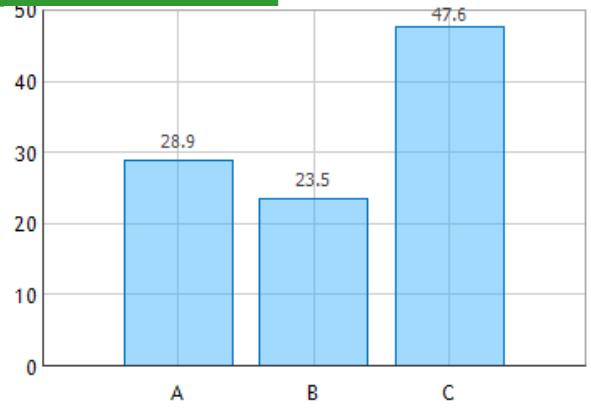






When the voltage across the capacitor is at its positive maximum, $V_C = +V_{C,max}$, what is the voltage across the inductor ?

A) $V_L = 0$ ***B)*** $V_L = V_{L,max}$ ***C)*** $V_L = -V_{L,max}$



What does the voltage phasor diagram look like when the voltage across capacitor is at its positive maximum?

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

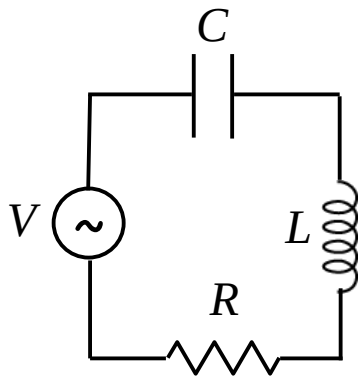
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



Conceptual Analysis

The maximum voltage for each component is related to its reactance and to the maximum current.

The impedance triangle determines the relationship between the maximum voltages for the components

Strategic Analysis

Use V_{max} and I_{max} to determine Z

Use impedance triangle to determine R

Use V_{Cmax} and impedance triangle to determine X_L

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

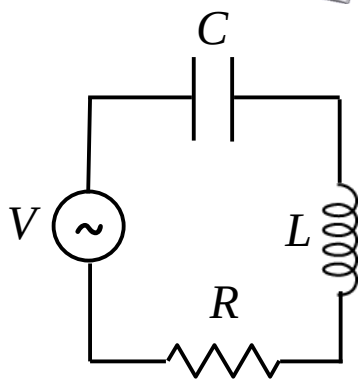
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



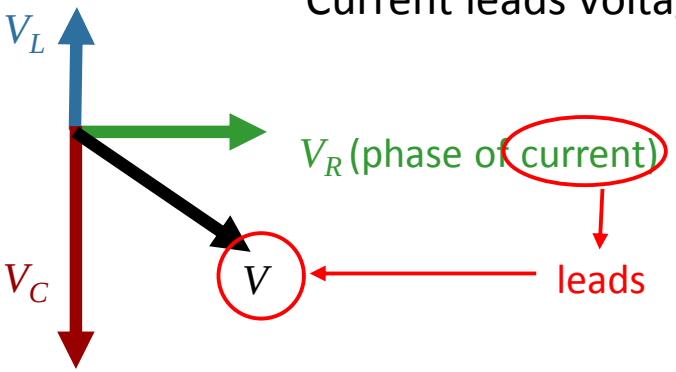
Compare X_L and X_C at this frequency:

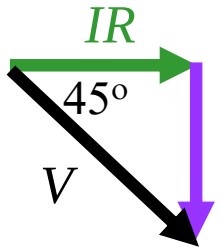
A) $X_L < X_C$ B) $X_L = X_C$ C) $X_L > X_C$ D) Not enough information

This information is determined from the phase

Current leads voltage

$$V_L = I_{max} X_L$$
$$V_C = I_{max} X_C$$





Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

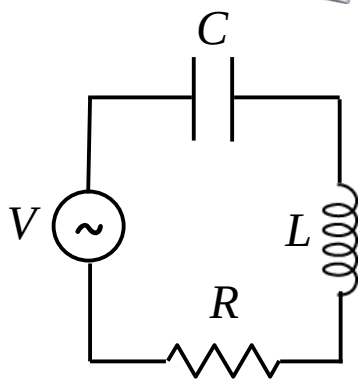
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



What is Z, the total impedance of the circuit?

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

$$Z = \frac{V_{\max}}{I_{\max}} = \frac{100V}{2mA} = 50k\Omega$$

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

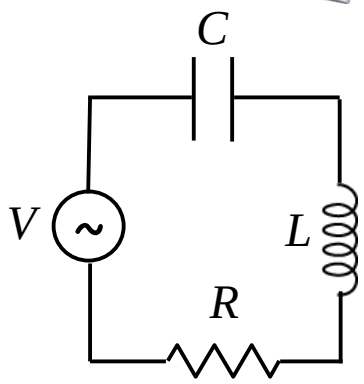
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?



$Z = 50k\Omega$

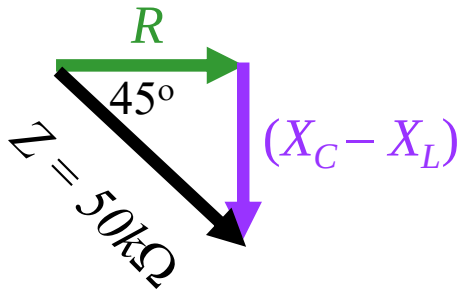
$\sin(45) = .707$

$\cos(45) = .707$

What is R?

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ D) $21.1\ k\Omega$

Determined from impedance triangle



$$\cos(45) = \frac{R}{Z} \rightarrow R = Z\cos(45^o)$$

$$= 50\ k\Omega \text{ x } 0.707$$

$$= 35.4\ k\Omega$$

Calculation

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\ V$

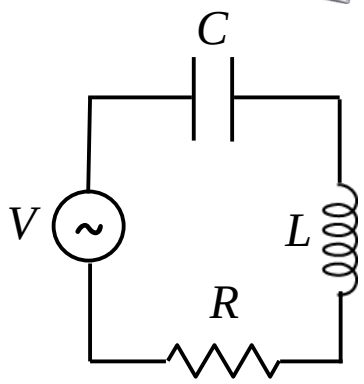
$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$

The current leads generator voltage by 45^o

L and R are unknown.

What is X_L, the reactance of the inductor, at this frequency?

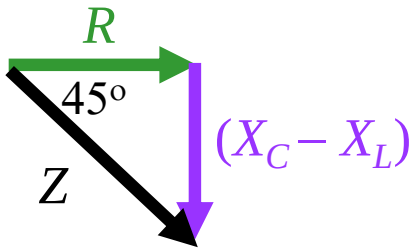


$Z = 50k\Omega$

$R = 35.4k\Omega$

A) $70.7\ k\Omega$ B) $50\ k\Omega$ C) $35.4\ k\Omega$ **D) $21.1\ k\Omega$**

We start with the impedance triangle:



$$\frac{X_C - X_L}{R} = \tan 45^\circ = 1 \rightarrow X_L = X_C - R$$

What is X_C?

$$V_{Cmax} = I_{max} X_C$$

$$X_C = \frac{113}{2} = 56.5k\Omega$$

$$X_L = 56.5\ k\Omega - 35.4\ k\Omega$$