Llamas, and why it is so difficult to get someone to buy me one as a pet. All I ask is that it breathes fire. I have simple needs people. Seriously. p.s. this lecture was hard.

did not understand any of this

wTF. so over my head

I have no idea what I am doing :(

Wow! That was cool. There is so much that you learn in high school that you just take as factual and move on. But when you can finally see it derived and shown that it is actually true...WOW! Pretty awesome.

This was undoubtedly my favorite prelecture of the semester, its so cool!

I was doing fine until we combined ampere's modified law and faraday's law. Also go over the magnetic and electric fields in empty space. Are they generated from the empty space itself, or do they come from some ambient source like stars?

   its mindblowing that E0 and mu0 are related to the speed of light. Can you calculate the speed of light more accurately through measuring E0 and mu0, or by measuring it directly (with lasers, etc)?

I suspect that people's reactions to electric and magnetic waves that are out of phase will be quite polarized.
Physics 212
Lecture 22

DISPLACEMENT CURRENT and EM WAVES

Displacement Current

\[ I_D = \varepsilon_0 \frac{d\Phi_E}{dt} \]

Modified Ampere’s Law

\[ \int \vec{B} \cdot d\vec{l} = \mu_0 (I + I_D) \]
Maxwell's Equations

Gauss' Law for E Fields
\[ \oint E \cdot dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \]

Gauss' Law for B Fields
\[ \oint B \cdot dA = 0 \]

Faraday's Law
\[ \oint E \cdot dl = -\frac{d}{dt} \oint B \cdot dA \]

Ampere's Law
\[ \oint B \cdot dl = \mu_0 I_{\text{enclosed}} \]
After Prelecture 21: Modify Ampere’s Law

**Ampere's Law**

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}} = \mu_0 (I + I_D) \]

\[ I = \frac{dQ}{dt} \]

\[ I_D = \varepsilon_0 \frac{d\Phi_E}{dt} \]

\[ E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \]

\[ \Phi = EA = \frac{Q}{\varepsilon_0} \]

\[ Q = \varepsilon_0 \Phi \]

\[ \frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi}{dt} \equiv I_D \]
Real Current: Charge $Q$ passes through area $A$ in time $t$:

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area $A$ changes in time

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$
Switch $S$ has been open a long time when at $t = 0$, it is closed. Capacitor $C$ has circular plates of radius $R$. At time $t = t_1$, a current $I_1$ flows in the circuit and the capacitor carries charge $Q_1$.

At time $t_1$, what is the magnetic field $B_1$ at a radius $r$ (point $d$) in between the plates of the capacitor?

**Conceptual and Strategic Analysis**

Charge $Q_1$ creates electric field between the plates of $C$.

Charge $Q_1$ changing in time gives rise to a changing electric flux between the plates.

Changing electric flux gives rise to a displacement current $I_D$ in between the plates.

Apply (modified) Ampere’s law using $I_D$ to determine $B$. 

---

**Calculation**

![Diagram showing current $I_1$ and charge $Q_1$ between the plates of capacitor $C$.]
Switch \( S \) has been open a long time when at \( t = 0 \), it is closed. Capacitor \( C \) has circular plates of radius \( R \). At time \( t = t_1 \), a current \( I_1 \) flows in the circuit and the capacitor carries charge \( Q_1 \).

Capacitor \( C \) has circular plates of radius \( R \). At time \( t = t_1 \), a current \( I_1 \) flows in the circuit and the capacitor carries charge \( Q_1 \).

Compare the magnitudes of the \( B \) fields at points \( c \) and \( d \).

- A) \( B_c < B_d \)
- B) \( B_c = B_d \)
- C) \( B_c > B_d \)

What is the difference?

Apply (modified) Ampere’s Law

**point c:**
\[ I_{(enclosed)} = I_1 \]

**point d:**
\[ I_{D(\text{enclosed})} < I_1 \]
Switch $S$ has been open a long time when at $t = 0$, it is closed. Capacitor $C$ has circular plates of radius $R$. At time $t = t_1$, a current $I_1$ flows in the circuit and the capacitor carries charge $Q_1$.

What is the magnitude of the electric field between the plates?

A) $E = \frac{Q_1}{\pi R^2 \varepsilon_0}$  
B) $E = \frac{Q_1 \pi R^2}{\varepsilon_0}$
C) $E = \frac{Q_1}{\varepsilon_0}$  
D) $E = \frac{Q_1}{r}$
Switch $S$ has been open a long time when at $t = 0$, it is closed. Capacitor $C$ has circular plates of radius $R$. At time $t = t_1$, a current $I_1$ flows in the circuit and the capacitor carries charge $Q_1$.

What is the electric flux through a circle of radius $r$ in between the plates?

A) $\Phi_E = \frac{Q_1}{\varepsilon_0} \pi r^2$  
B) $\Phi_E = \frac{Q_1}{\varepsilon_0} \pi R^2$  
C) $\Phi_E = \frac{Q_1 r^2}{\varepsilon_0 R^2}$  
D) $\Phi_E = \frac{Q_1 \pi r^2}{\varepsilon_0 R^2}$
Switch $S$ has been open a long time when at $t = 0$, it is closed. Capacitor $C$ has circular plates of radius $R$. At time $t = t_1$, a current $I_1$ flows in the circuit and the capacitor carries charge $Q_1$. What is the displacement current enclosed by circle of radius $r$?

A) $I_D = I_1 \frac{R^2}{r^2}$  
B) $I_D = I_1 \frac{r}{R}$  
C) $I_D = I_1 \frac{r^2}{R^2}$  
D) $I_D = I_1 \frac{R}{r}$

$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ_1}{dt} \frac{r^2}{R^2} = I_1 \frac{r^2}{R^2}$
Switch $S$ has been open a long time when at $t = 0$, it is closed. Capacitor $C$ has circular plates of radius $R$. At time $t = t_1$, a current $I_1$ flows in the circuit and the capacitor carries charge $Q_1$.

What is the magnitude of the $B$ field at radius $r$?

A) $B = \frac{\mu_0 I_1}{2\pi R}$

B) $B = \frac{\mu_0 I_1}{2\pi r}$

C) $B = \frac{\mu_0 I_1 R}{2\pi r^2}$

D) $B = \frac{\mu_0 I_1 r}{2\pi R^2}$

Ampere’s Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_D)$

$\rightarrow B \cdot 2\pi r = \mu_0 \left(0 + I_1 \frac{r^2}{R^2}\right)$

$\rightarrow B = \frac{\mu_0 I_1}{2\pi \frac{r}{R^2}}$
At time t=0 the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; **A is at the center** and B is toward the outer edge.

Compare the magnitudes of the magnetic fields at points A and B just after the switch is closed:

A. B_A < B_B  
B. B_A = B_B  
C. B_A > B_B

From the calculation we just did:

\[ B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2} \]
At time $t=0$ the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; A is at the center and B is toward the outer edge.

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

$B$ is proportional to $I$

but

At $A$, $B = 0$!!
Switch $S$ has been open a long time when at $t = 0$, it is closed. Capacitor $C$ has circular plates of radius $R$. At time $t = t_1$, a current $I_1$ flows in the circuit and the capacitor carries charge $Q_1$.

What is the time dependence of the magnetic field $B$ at a radius $r$ between the plates of the capacitor?

$B$ at fixed $r$ is proportional to the current $I$.

Close switch: $V_C = 0 \Rightarrow I = V/R_a$ (maximum)

$I$ exponentially decays with time constant $\tau = R_a C$
Suppose you were able to charge a capacitor with constant current (does not change in time).

Does a $B$ field exist in between the plates of the capacitor?

A) YES  
B) NO

Constant current $\Rightarrow Q$ increases linearly with time

Therefore $E$ increases linearly with time ($E = Q/(A \varepsilon_0)$)

d$E$/dt is not zero $\Rightarrow$ Displacement current is not zero
$\Rightarrow B$ is not zero!
We learned about waves in Physics 211

1-D Wave Equation
\[ \frac{d^2 h}{dx^2} = \frac{1}{v^2} \frac{d^2 h}{dt^2} \]

Solution
\[ h(x,t) = h_1(x - vt) + h_2(x + vt) \]

Common Example: Harmonic Plane Wave

\[ h(x,t) = A \cos(kx - \omega t) \]

Variable Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude:</td>
<td>( A )</td>
</tr>
<tr>
<td>Wave Number:</td>
<td>( k = \frac{2\pi}{\lambda} )</td>
</tr>
<tr>
<td>Wavelength:</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Angular Frequency:</td>
<td>( \omega = \frac{2\pi}{T} )</td>
</tr>
<tr>
<td>Period:</td>
<td>( T )</td>
</tr>
<tr>
<td>Frequency:</td>
<td>( f = \frac{1}{T} )</td>
</tr>
<tr>
<td>Velocity:</td>
<td>( v = \lambda f = \frac{\omega}{k} )</td>
</tr>
</tbody>
</table>
The Electromagnetic Spectrum

Penetrates Earth Atmosphere?
Y  N  Y  N

Wavelength (meters)

- **Radio**: $10^3$
- **Microwave**: $10^{-2}$
- **Infrared**: $10^{-5}$
- **Visible**: $0.5 \times 10^{-6}$
- **Ultraviolet**: $10^{-8}$
- **X-ray**: $10^{-10}$
- **Gamma Ray**: $10^{-12}$

About the size of...

- Buildings
- Humans
- Honey Bee
- Pinpoint
- Protozoans
- Molecules
- Atoms
- Atomic Nuclei

Frequency (Hz)

- $10^4$
- $10^8$
- $10^{12}$
- $10^{15}$
- $10^{16}$
- $10^{18}$
- $10^{20}$

Temperature of bodies emitting the wavelength (K)

- **1 K**
- **100 K**
- **10,000 K**
- **10 Million K**
Faraday's Law
\[ \int E \cdot dl = -\frac{d}{dt} \int B \cdot dA \]

Plane Wave Solution
\[ E \rightarrow E(z,t) \quad B \rightarrow B(z,t) \]

Modified Ampere's Law
\[ \oint B \cdot dl = \mu_0 \varepsilon_0 \frac{d}{dt} \int E \cdot dA \]

Special Relativity (1905)
Speed of Light is Constant

How can light move at the same velocity in any inertial frame of reference? That's really trippy."

see PHYS 225
Example: A Harmonic Solution

\[
\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}
\]

\[
\frac{\partial^2 B_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_y}{\partial t^2}
\]

\[
E_x = E_0 \cos(kz - \omega t)
\]

\[
\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}
\]

\[
B_y = \frac{k}{\omega} E_0 \cos(kz - \omega t)
\]

Two Important Features

1. \(B_y\) is in phase with \(E_x\)

2. \(B_0 = \frac{E_0}{c}\)
An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same z coordinate.

\[ E_x = E_0 \sin(kz - \omega t) \]

Compare the magnitudes of the electric fields at points A and B:

A. \( E_A < E_B \)

B. \( E_A = E_B \)  

C. \( E_A > E_B \)

\[ E = E_0 \sin (kz - \omega t) \]:

\( E \) depends only on z coordinate for constant \( t \).

z coordinate is same for A, B, C.
An electromagnetic plane-wave is traveling in the $+z$ direction. The illustration below shows this wave at some instant in time. Points A, B and C have the same $z$ coordinate.

\[ E_x = E_0 \sin(\kappa z - \omega t) \]

Electricity & Magnetism Lecture 22, Slide 21

Compare the magnitudes of the electric fields at points A and C

A. $E_A < E_C$

B. $E_A = E_C$

C. $E_A > E_C$

\[ E = E_0 \sin (\kappa z - \omega t): \]

$E$ depends only on $z$ coordinate for constant $t$.

$z$ coordinate is same for A, B, C.
Consider a point \((x, y, z)\) at time \(t\) when \(E_x\) is negative and has its maximum value.

At \((x, y, z)\) at time \(t\), what is \(B_y\)?

A) \(B_y\) is positive and has its maximum value
B) \(B_y\) is negative and has its maximum value
C) \(B_y\) is zero
D) We do not have enough information