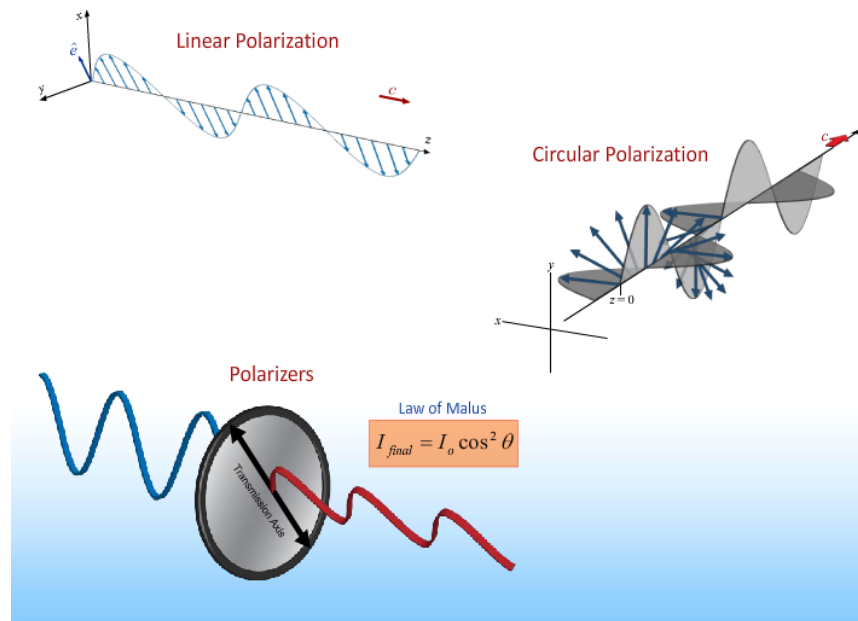


Physics 212

Lecture 24



Your Comments

Why do we want to polarize light? What is polarized light used for?

I feel like after the polarization lecture the Professor laughs and goes tell his friends, "I ran out of things to teach today so I made some stuff up and the students totally bought it."

I really wish you would explain the new right hand rule. I cant make it work in my mind

I can't wait to see what demos are going to happen in class!!! This topic looks like so much fun!!!!

With E related to B by $E=cB$ where $c=(\mu_0\epsilon_0)^{-0.5}$, does the ratio between E and B change when light passes through some material m for which $\epsilon_m \neq \epsilon_0$?

I feel like if specific examples of homework were done for us it would help more, instead of vague general explanations, which of course help with understanding the theory behind the material.

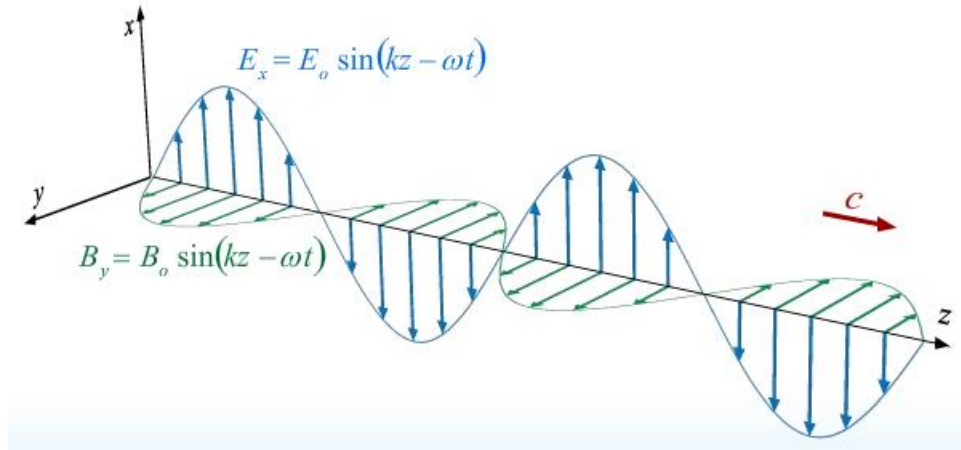
THIS IS SO COOL!

Could you explain what polarization looks like? The lines that are drawn through the polarizers symbolize what? Are they supposed to be slits in which light is let through?

Real talk? The Law of Malus is the most metal name for a scientific concept ever devised. Just say it in a deep, commanding voice, "DESPAIR AT THE LAW OF MALUS." Awesome!

Linearly Polarized Light

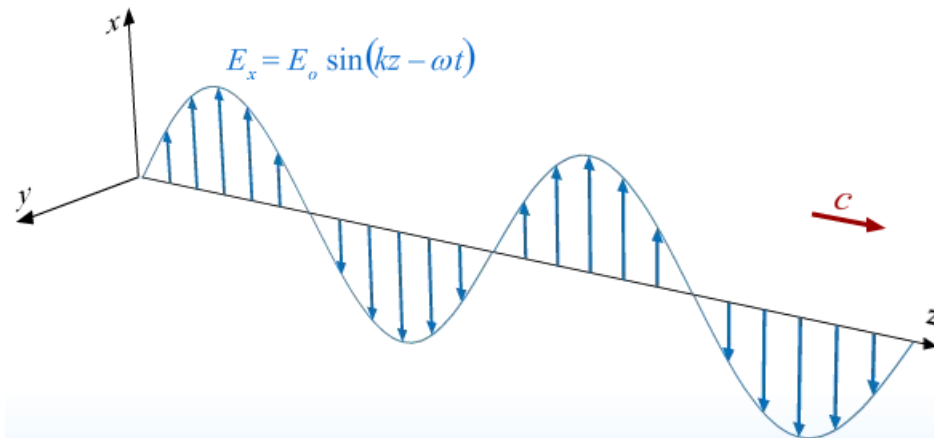
So far we have considered plane waves that look like this:



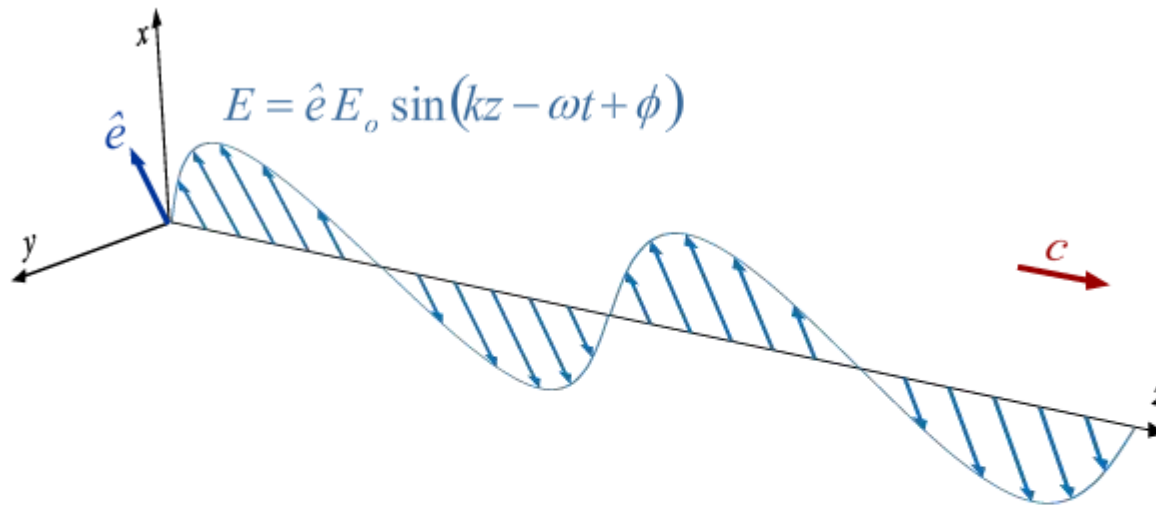
$$\omega = kc$$
$$E_o = cB_o$$
$$c = \frac{1}{\sqrt{\mu_o \epsilon_o}}$$

From now on just draw E and remember that B is still there:

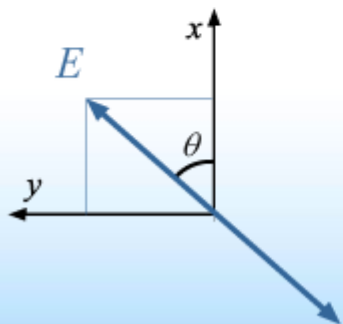
\vec{E} Field determines Polarization



Linear Polarization

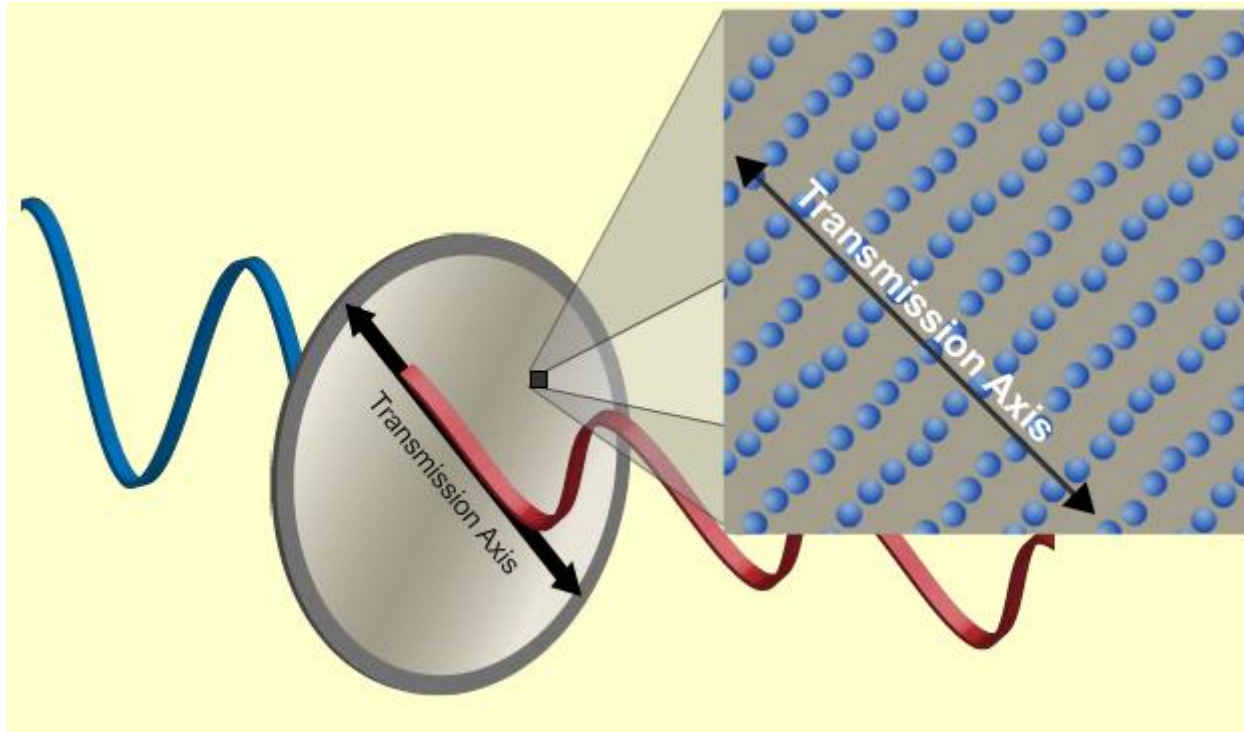


$$\omega = kc$$
$$E_o = cB_o$$
$$c = \frac{1}{\sqrt{\mu_o \epsilon_o}}$$



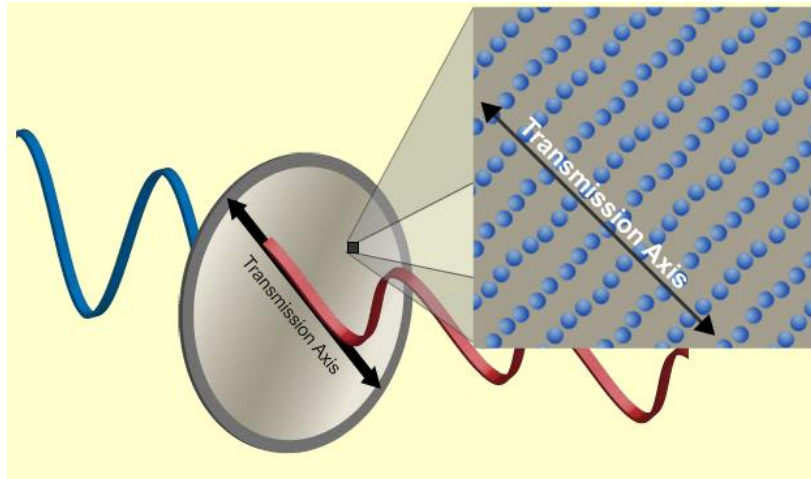
Wave Coming Out of the Screen

Polarizer



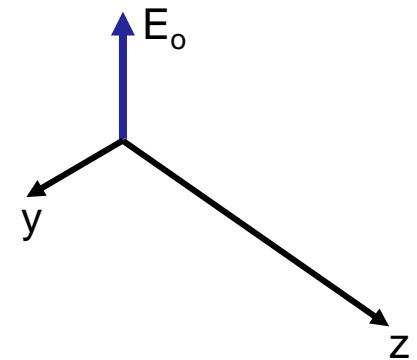
The molecular structure of a polarizer causes the component of the E field perpendicular to the Transmission Axis to be absorbed.

Clicker Question



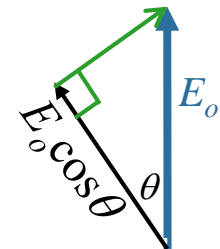
The molecular structure of a polarizer causes the component of the E field perpendicular to the Transmission Axis to be absorbed.

Suppose we have a beam traveling in the $+z$ – direction. At $t = 0$ and $z = 0$, the electric field is aligned along the positive x – axis and has a magnitude equal to E_0



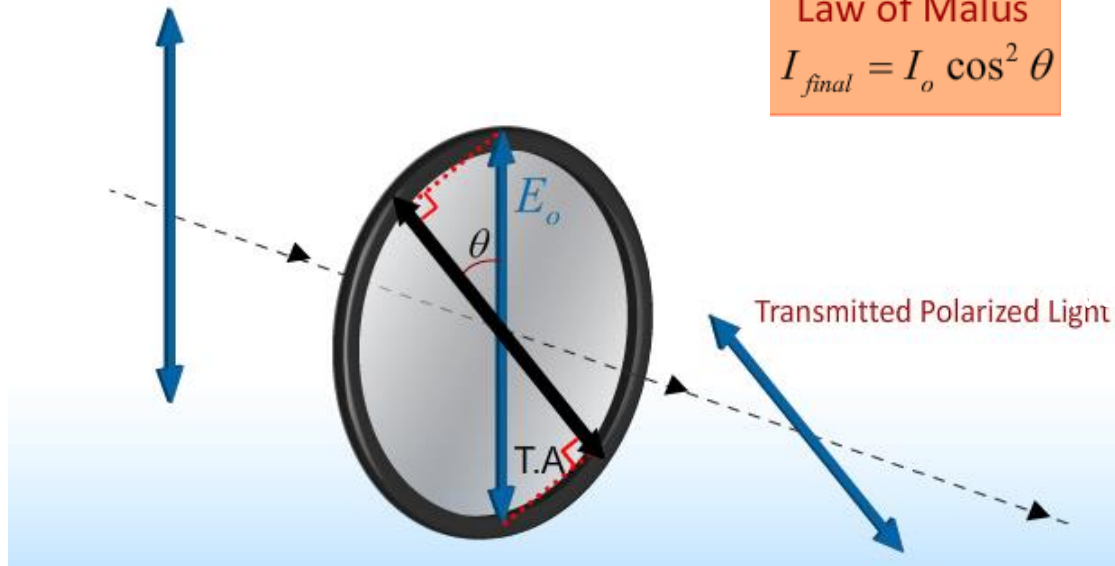
What is the component of E_0 along a direction in the $x - y$ plane that makes an angle of θ with respect to the x – axis?

- A) $E_0 \sin \theta$ B) $E_0 \cos \theta$ C) 0 D) $E_0 / \sin \theta$ E) $E_0 / \cos \theta$



Linear Polarizers

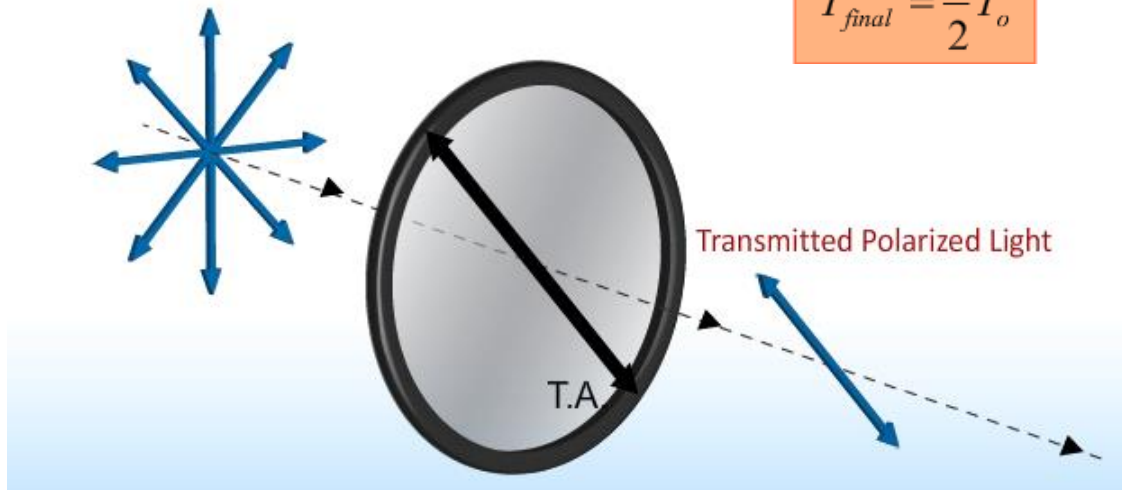
Incident Polarized Light



Law of Malus

$$I_{final} = I_o \cos^2 \theta$$

Incident Unpolarized Light

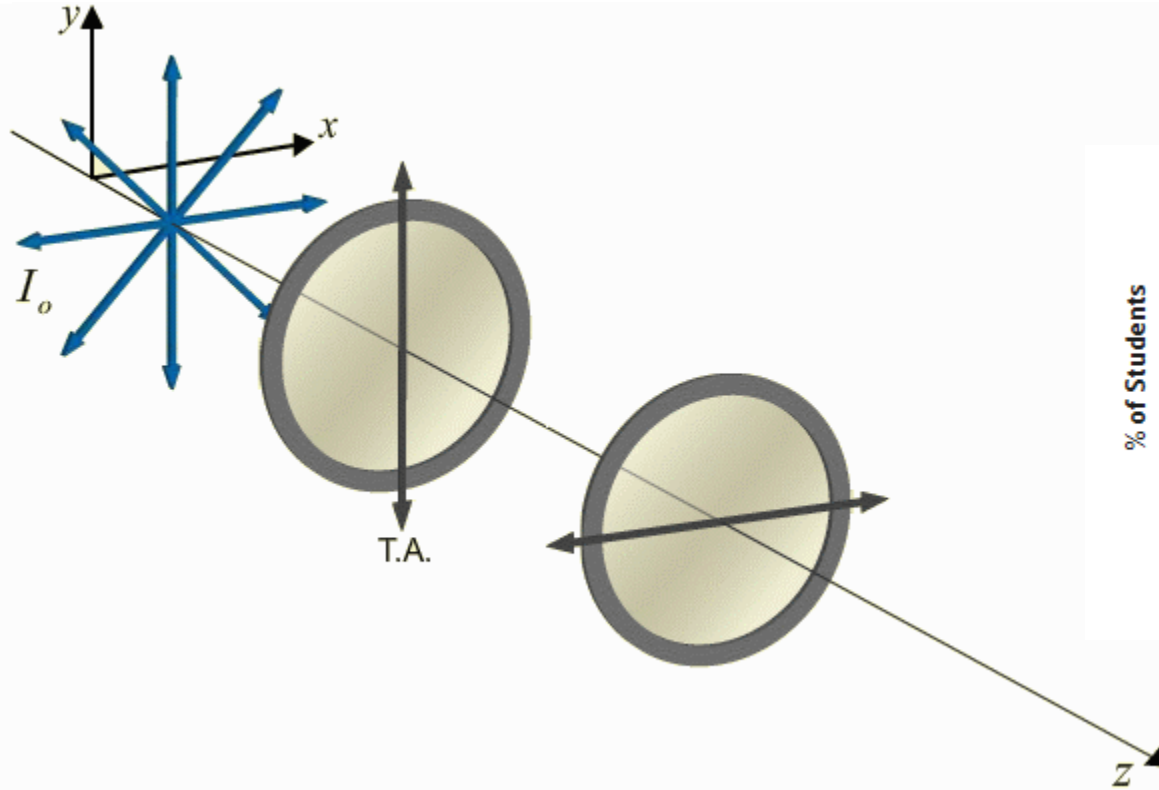


$$I_{final} = \frac{1}{2} I_o$$

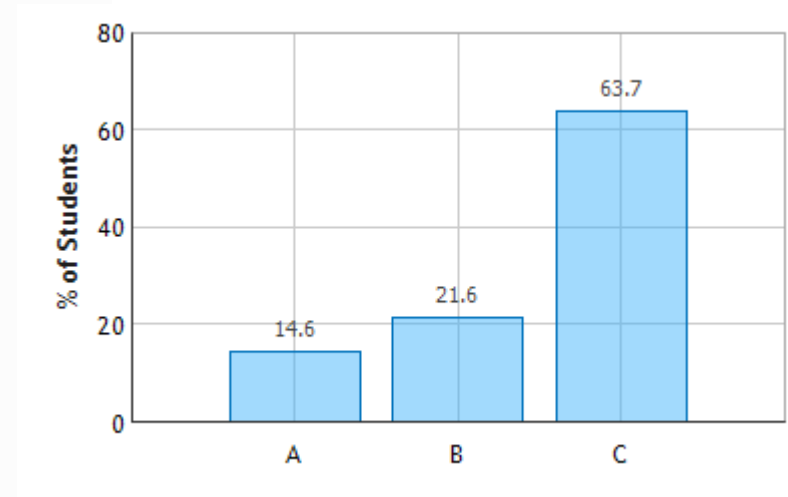
CheckPoint 1a



An unpolarized EM wave is incident on two orthogonal polarizers.



Two Polarizers



What percentage of the intensity gets through both polarizers?

A. 50%

B. 25%

C. 0%

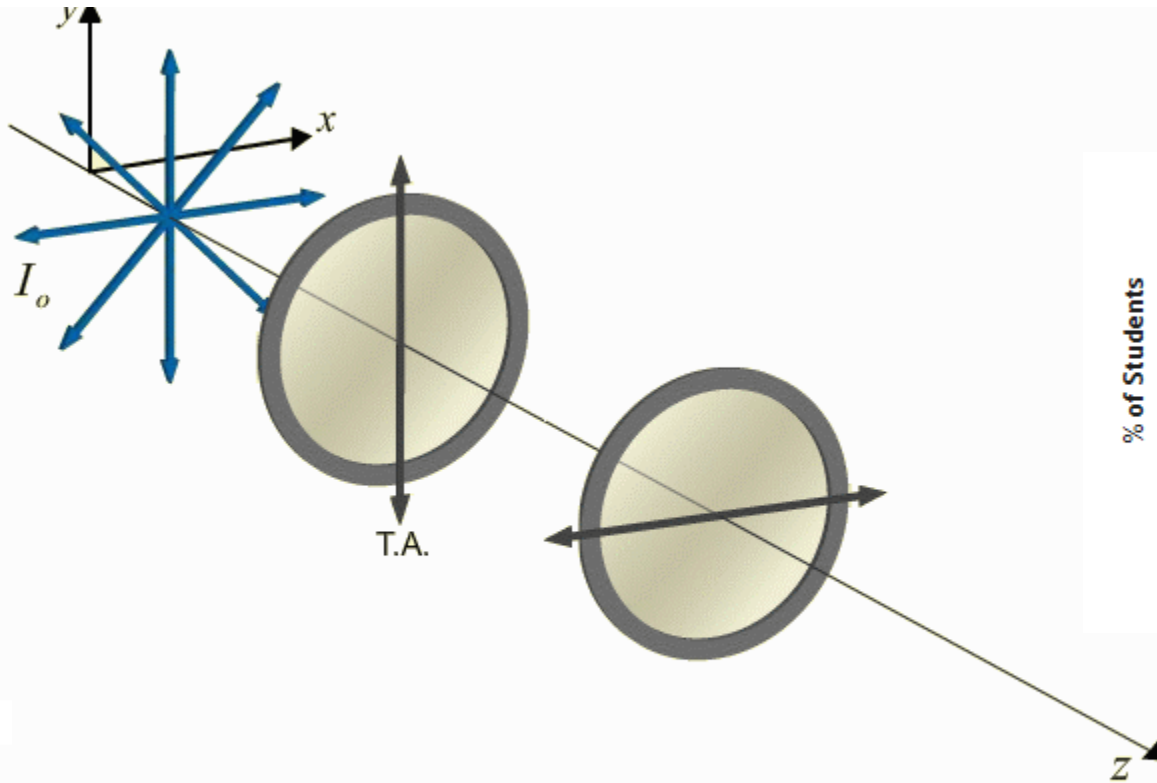


No light will come through. $\cos(90^\circ) = 0$

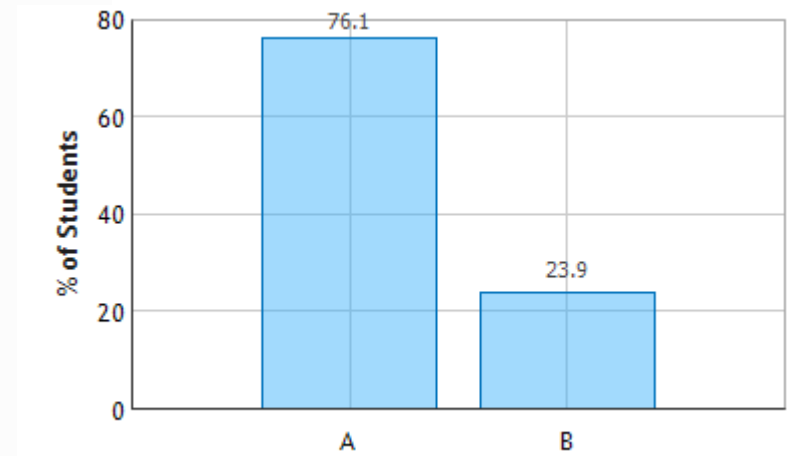
CheckPoint 1b



An unpolarized EM wave is incident on two orthogonal polarizers.



Two Polarizers



Is it possible to increase this percentage by inserting another Polarizer between the original two?

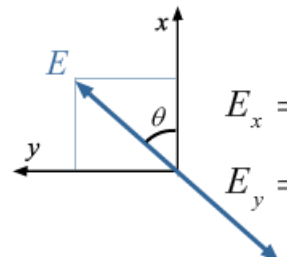
A. Yes

B. No

DEMO

Circularly Polarized Light

There is no reason that ϕ has to be the same for E_x and E_y :



Wave Coming Out of the Screen

$$\left. \begin{aligned} E_x &= E_o \cos \theta \sin(kz - \omega t + \phi_x) \\ E_y &= E_o \sin \theta \sin(kz - \omega t + \phi_y) \end{aligned} \right\} \begin{array}{l} \text{Satisfies} \\ \text{Wave Equation} \end{array} \left. \begin{aligned} \frac{\partial^2 E_x}{\partial z^2} &= \mu_o \epsilon_o \frac{\partial^2 E_x}{\partial t^2} \\ \frac{\partial^2 E_y}{\partial z^2} &= \mu_o \epsilon_o \frac{\partial^2 E_y}{\partial t^2} \end{aligned} \right.$$

Making ϕ_x different from ϕ_y causes circular or elliptical polarization:

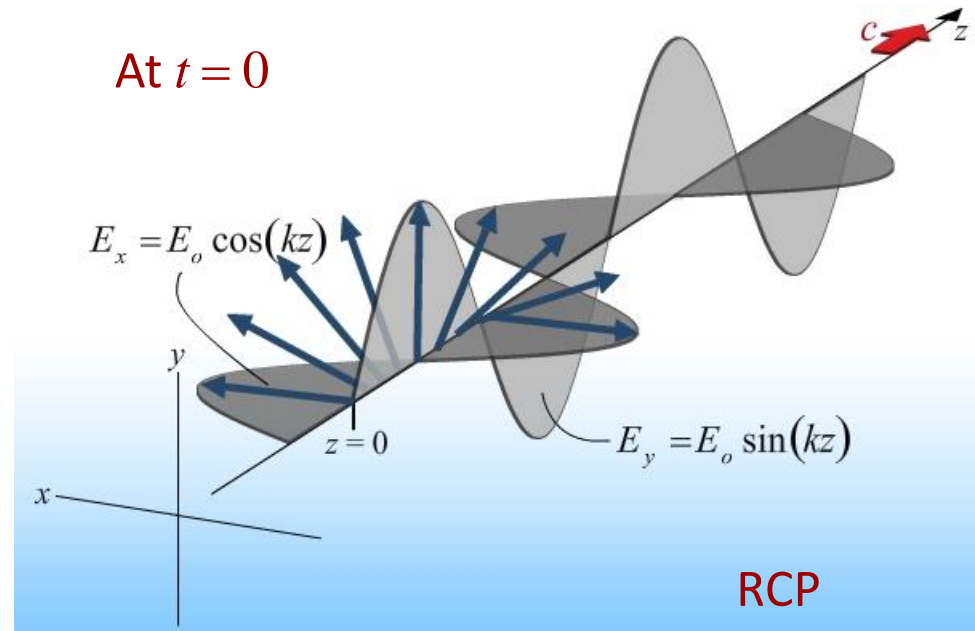
Example:

$$\phi_x - \phi_y = 90^\circ = \frac{\pi}{2}$$

$$\theta = 45^\circ = \pi/4$$

$$E_x = \frac{E_0}{\sqrt{2}} \cos(kz - \omega t)$$

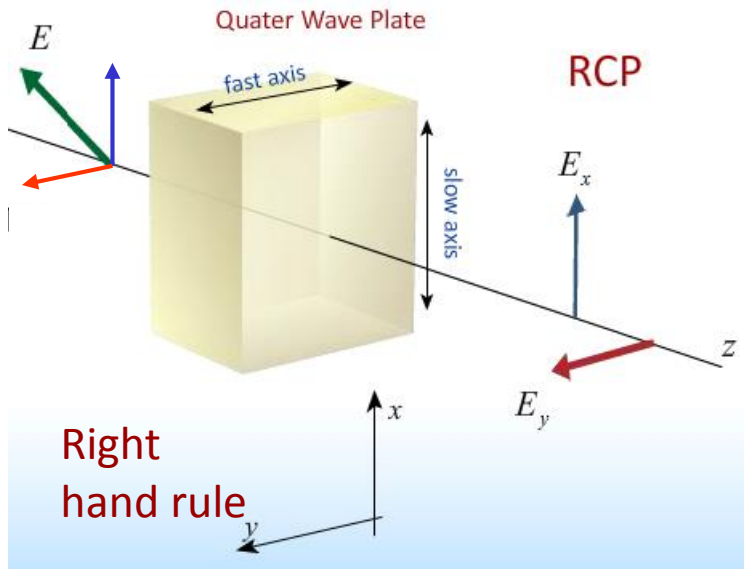
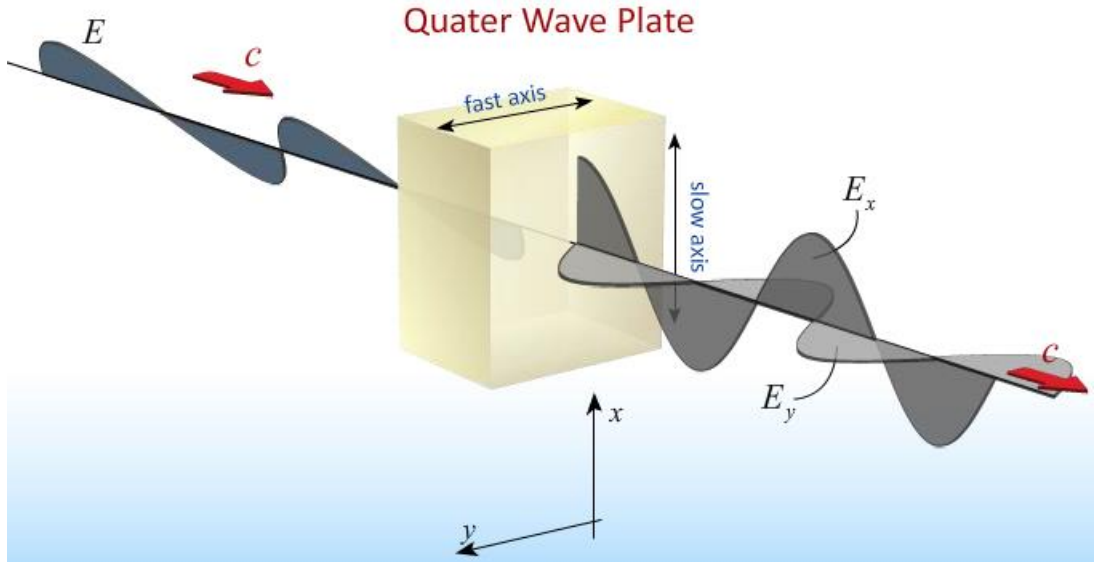
$$E_y = \frac{E_0}{\sqrt{2}} \sin(kz - \omega t)$$



Quarter Waveplates

Q: How do we change the relative phase between E_x and E_y ?

A: Birefringence



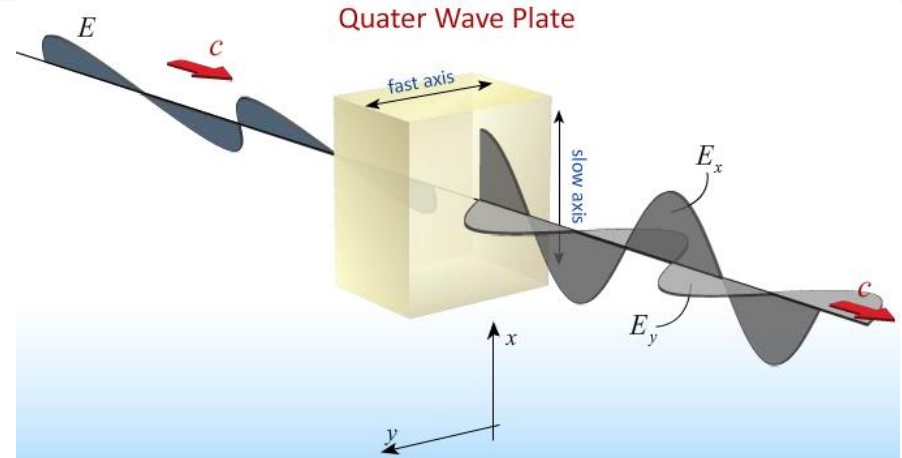
By picking the right thickness we can change the relative phase by exactly 90° .

This changes linear to circular polarization and is called a *quarter wave plate*

Intensity does not change!

“talk something about intensity”

NOTE: No Intensity is lost passing through the QWP !



BEFORE QWP:

$$E = E_o \sin(kz - \omega t) \left[\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right] \quad \longrightarrow \quad I = c\epsilon_o \langle E^2 \rangle = c\epsilon_o \langle E_x^2 + E_y^2 \rangle$$

$$= c\epsilon_o \left(\frac{E_o^2}{2} + \frac{E_o^2}{2} \right) \langle \sin^2(kz - \omega t) \rangle = c\epsilon_o E_o^2 \frac{1}{2}$$

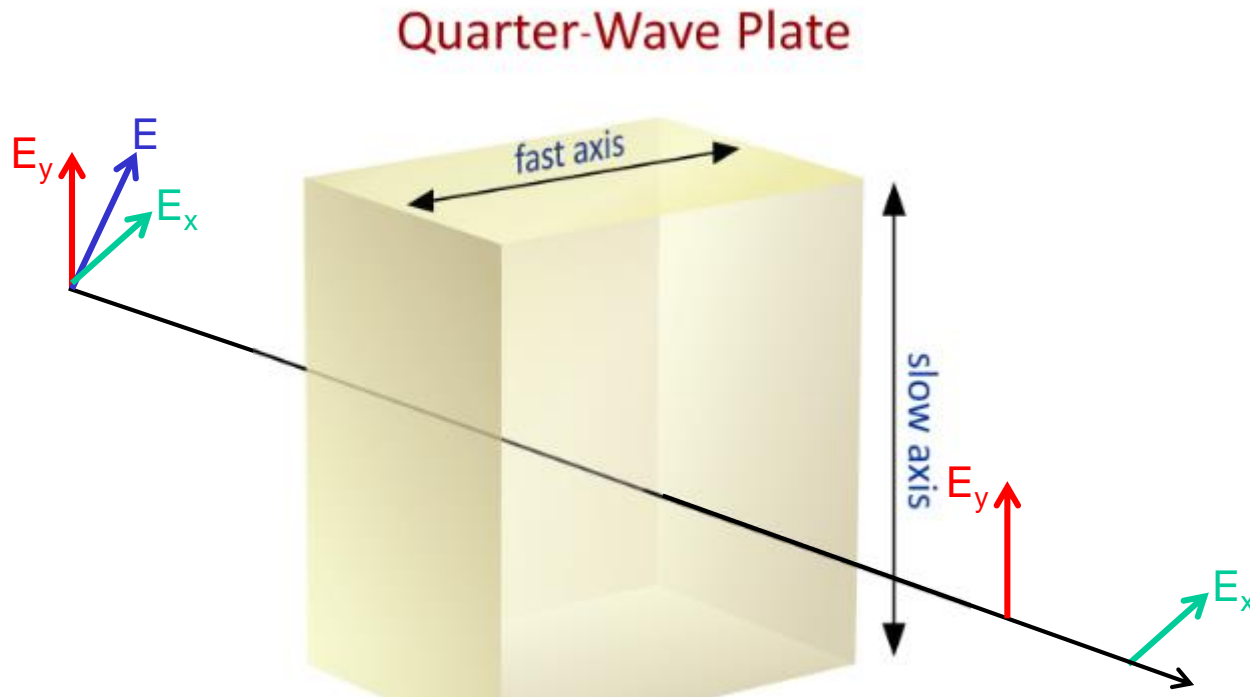
AFTER QWP:

$$E = \frac{E_o}{\sqrt{2}} \left[\hat{i} \cos(kz - \omega t) + \hat{j} \sin(kz - \omega t) \right] \quad \longrightarrow \quad I = c\epsilon_o \langle E^2 \rangle = c\epsilon_o \langle E_x^2 + E_y^2 \rangle$$

$$= c\epsilon_o \frac{E_o^2}{2} \langle \cos^2(kz - \omega t) + \sin^2(kz - \omega t) \rangle$$

$$= c\epsilon_o \frac{E_o^2}{2} \langle 1 \rangle = c\epsilon_o \frac{E_o^2}{2} \quad \text{— THE SAME!}$$

Right or Left Circularly Polarized



A linearly polarized EM wave is incident on a quarter-wave plate as shown above. The resulting wave is

- A) Right Circularly Polarized
- B) Left Circularly Polarized**
- C) Linearly Polarized

Curve fingers from slow to fast, thumb must be direction of propagation.

Circular Light on Linear Polarizer



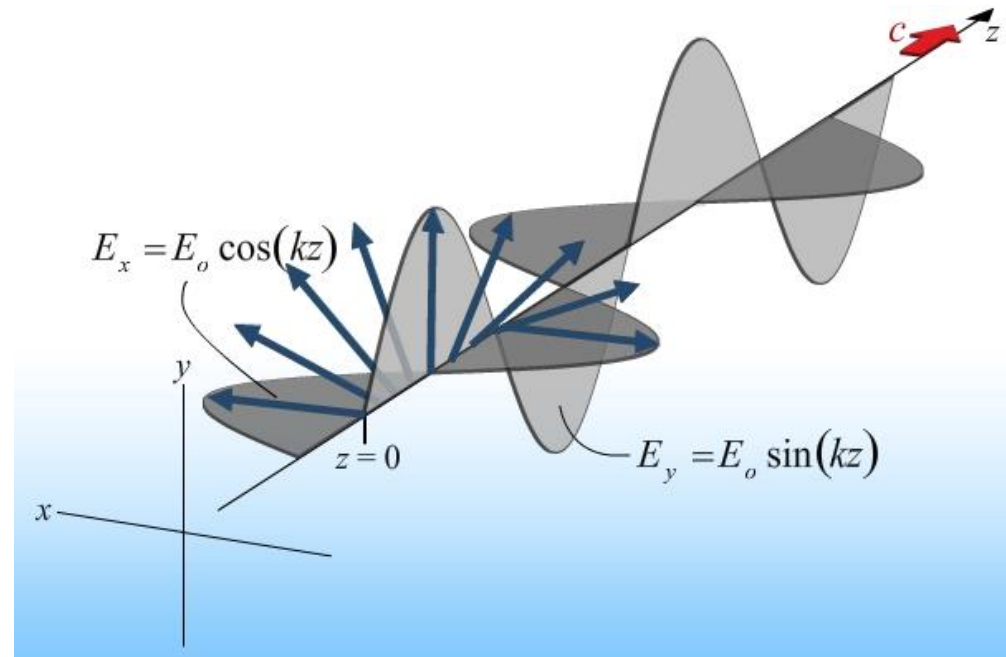
Q: What happens when circularly polarized light is put through a polarizer along the y (or x) axis ?

A) $I = 0$

B) $I = \frac{1}{2} I_0$

C) $I = I_0$

$$\begin{aligned} I &= \varepsilon_0 c \langle E^2 \rangle \\ &= \varepsilon_0 c \langle E_x^2 + \cancel{E_y^2} \rangle \\ &= \varepsilon_0 c \frac{E_0^2}{2} \underbrace{\langle \cos^2(kz - \omega t) \rangle}_{1/2} \end{aligned}$$



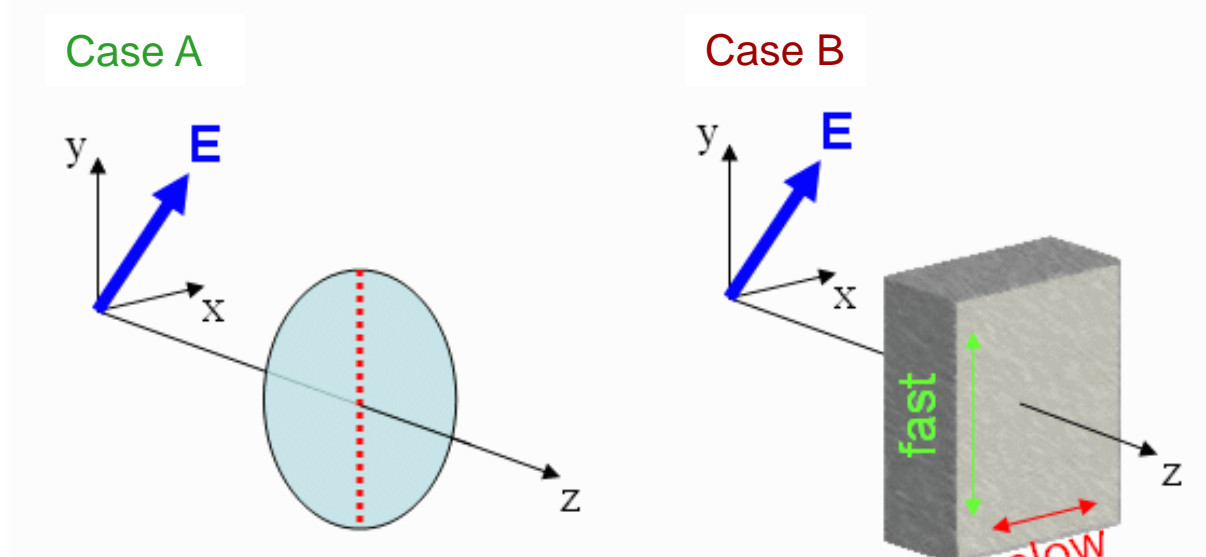
$$= \frac{1}{2} \cdot \frac{1}{2} \varepsilon_0 c E_0^2$$

Half of before

CheckPoint 2a



Identical linearly polarized light at 45° from the y-axis and propagating along the z axis is incident on two different objects. In Case A the light intercepts a linear polarizer with polarization along the y-axis. In Case B, the light intercepts a quarter wave plate with fast axis along the y-axis.



Compare the intensities of the light waves after transmission.

A. $I_A < I_B$

B. $I_A = I_B$

C. $I_A > I_B$

Case A:

E_x is absorbed

$$I_A = I_0 \cos^2(45^\circ)$$

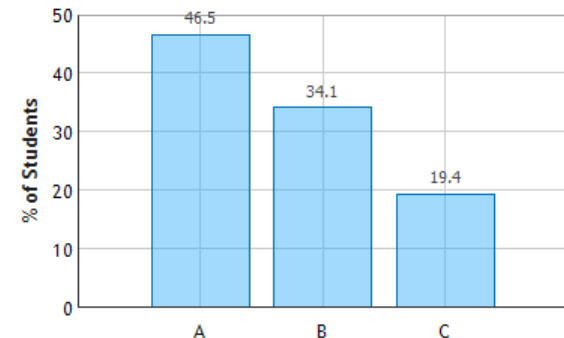
$$I_A = \frac{1}{2} I_0$$

Case B:

(E_x, E_y) phase changed

$$I_B = I_0$$

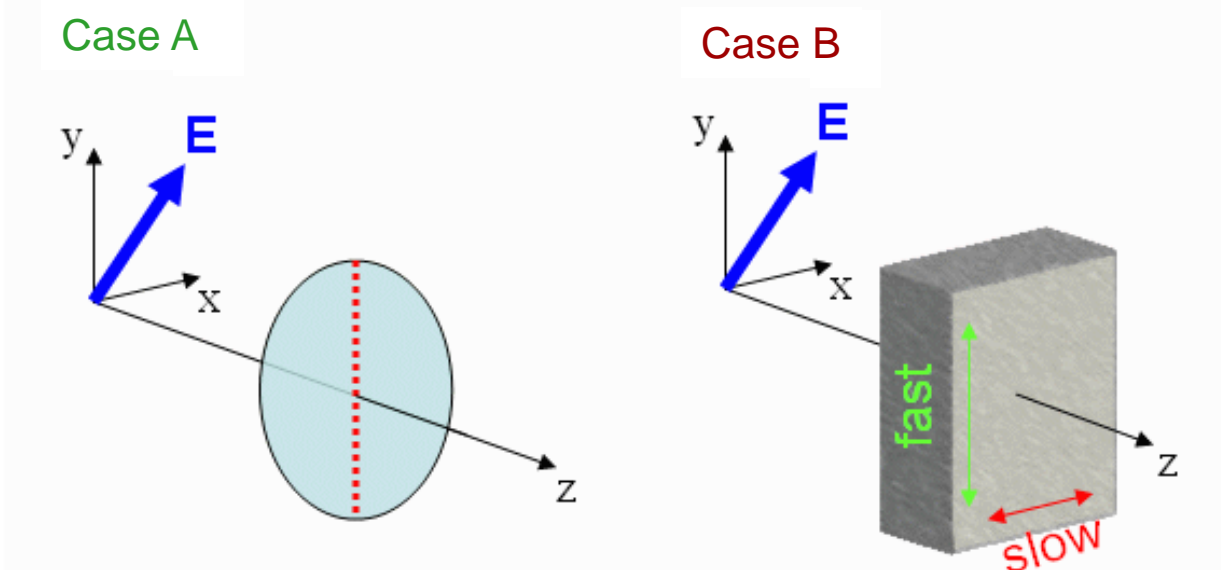
A Polarizer and a Quarter-Wave Plate: Question 1 (N = 725)





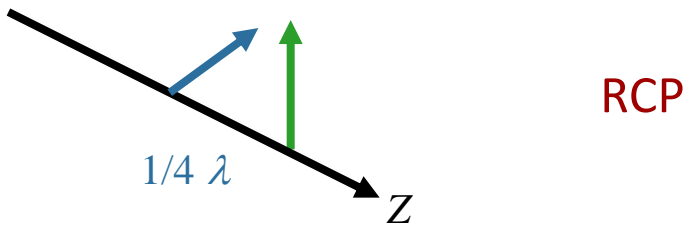
Identical linearly polarized light at 45° from the y -axis and propagating along the z axis is incident on two different objects. In Case A the light intercepts a linear polarizer with polarization along the y -axis. In Case B, the light intercepts a quarter wave plate with vast axis along the y -axis.

the light intercepts a quarter wave plate with fast axis along the y -axis

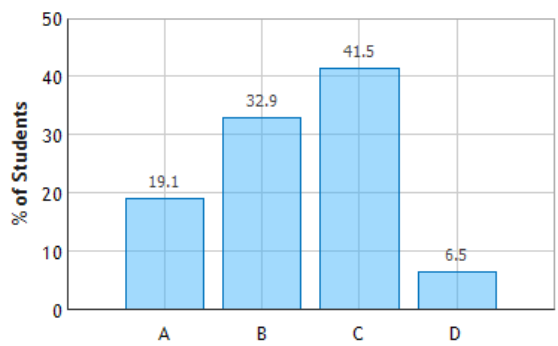


What is the polarization of the light wave in Case B after it passes through the quarter wave plate?.

- A. linearly polarized
- C. right circularly polarized**
- B. left circularly polarized
- D. undefined



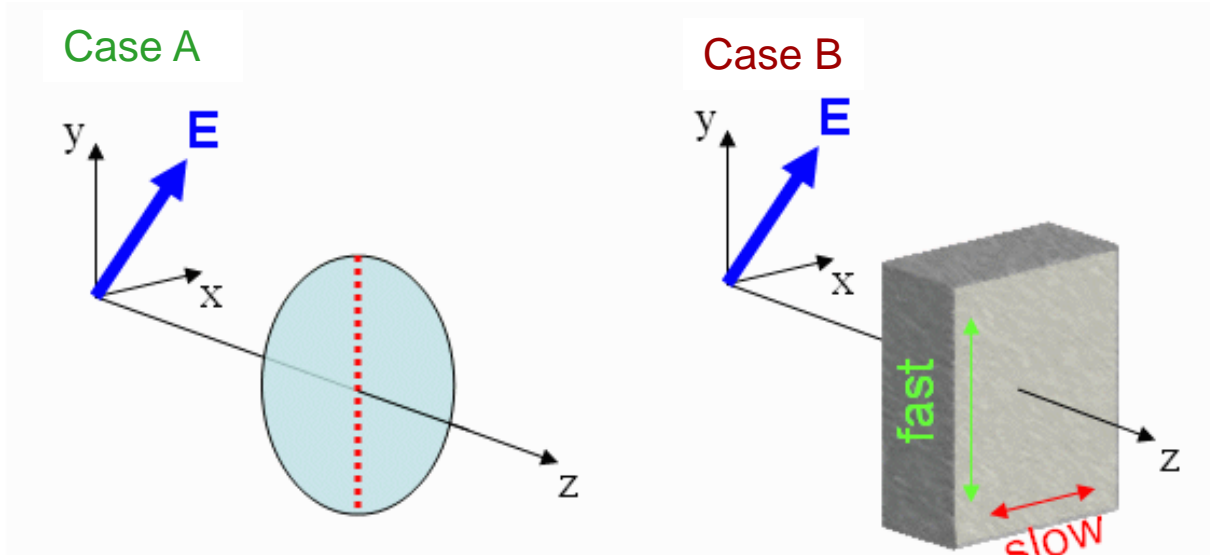
A Polarizer and a Quarter-Wave Plate: Question 3 (N = 726)



Checkpoint 2c

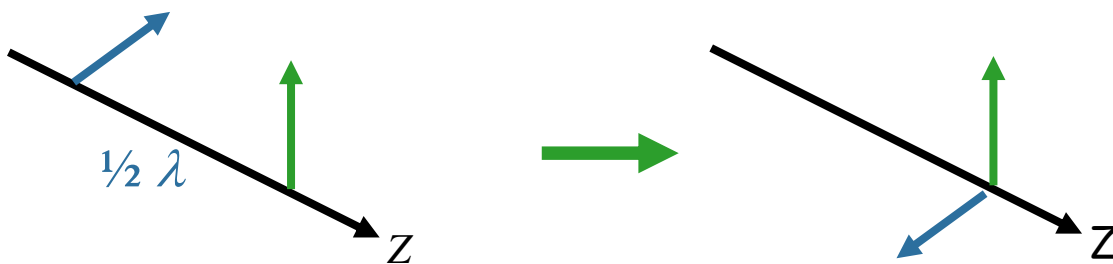


Identical linearly polarized light at 45° from the y-axis and propagating along the z axis is incident on two different objects. In Case A the light intercepts a linear polarizer with polarization along the y-axis. In Case B, the light intercepts a quarter wave plate with fast axis along the y-axis.

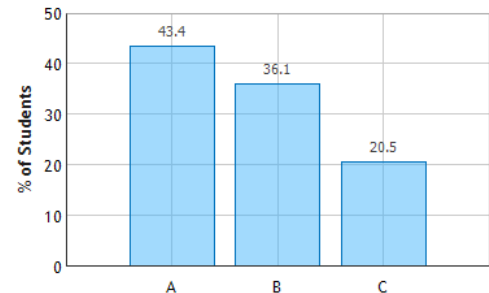


If the thickness of the quarter-wave plate in Case B is doubled, what is the polarization of the wave after passing through the wave plate?

- A.** linearly polarized
- B.** circularly polarized
- C.** undefined



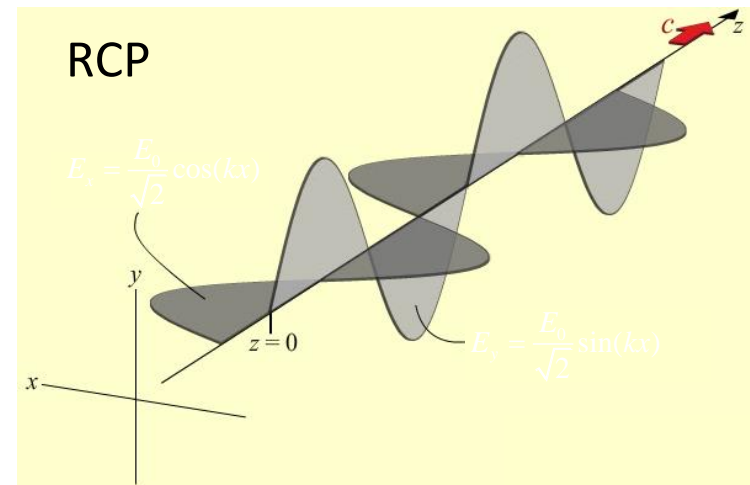
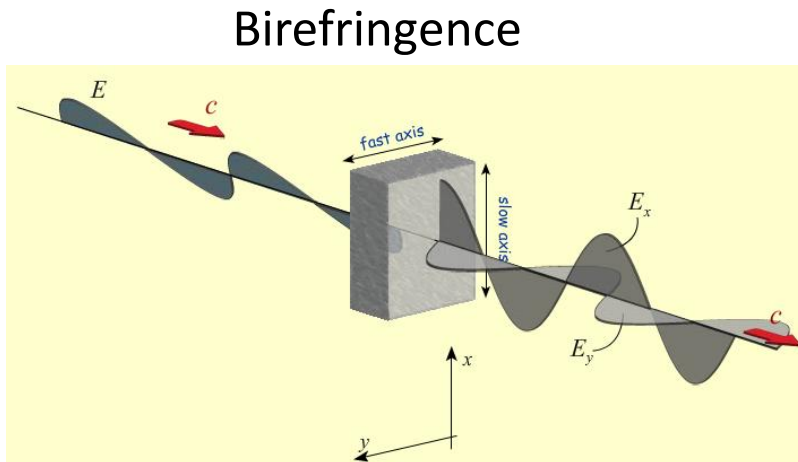
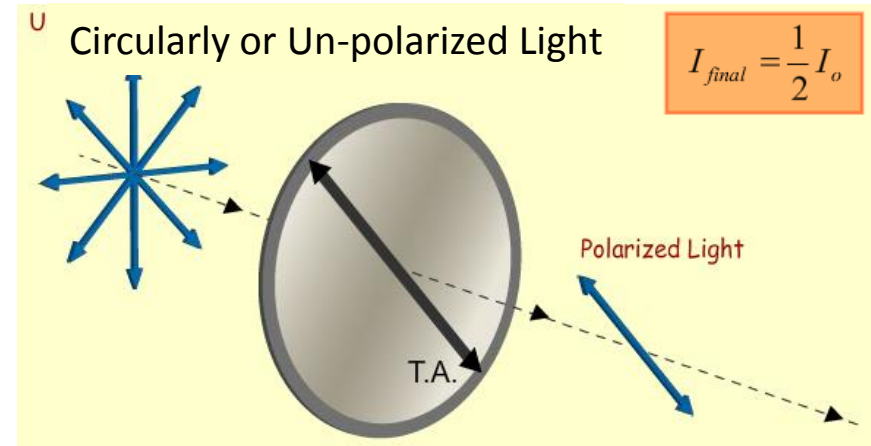
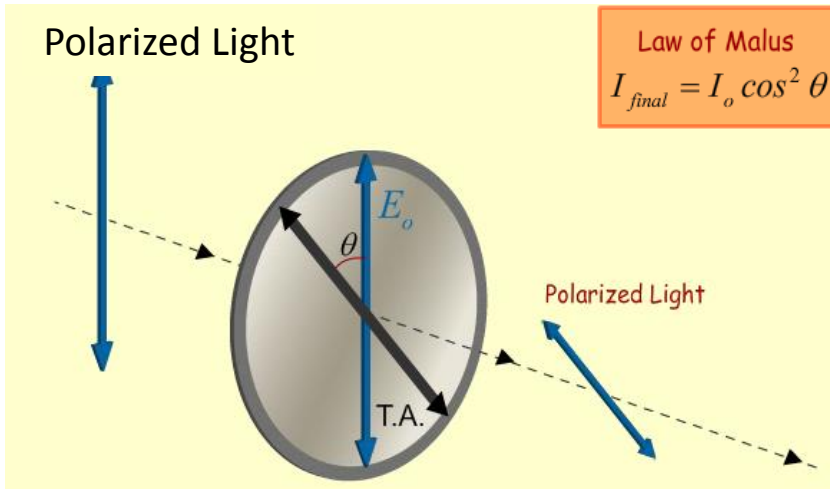
A Polarizer and a Quarter-Wave Plate: Question 5 (N = 723)



Elect

Executive Summary:

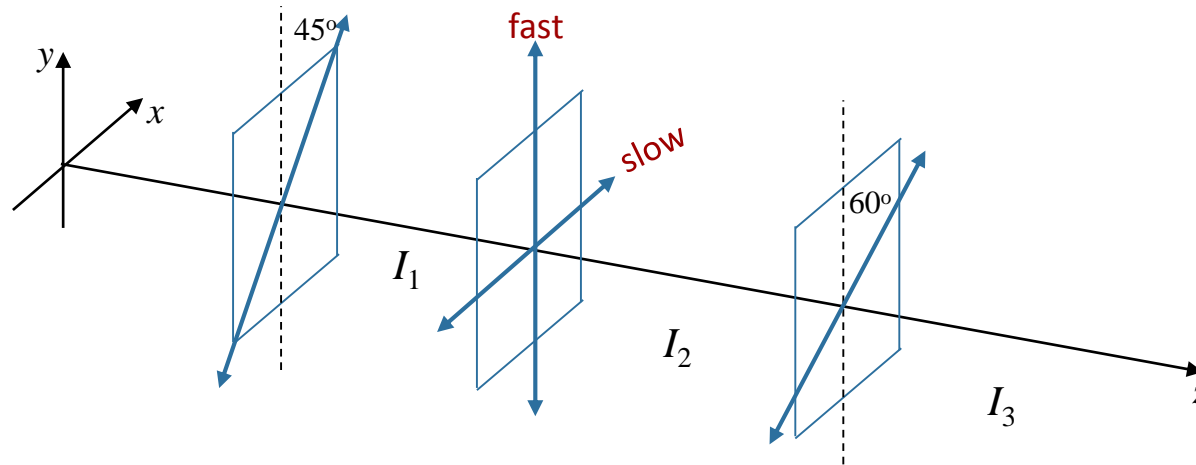
Polarizers & QW Plates:



Calculation

Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.

What is the intensity I_3 in terms of I_1 ?



Conceptual Analysis

Linear Polarizers: absorbs E field component perpendicular to TA

Quarter Wave Plates: Shifts phase of E field components in fast-slow directions

Strategic Analysis

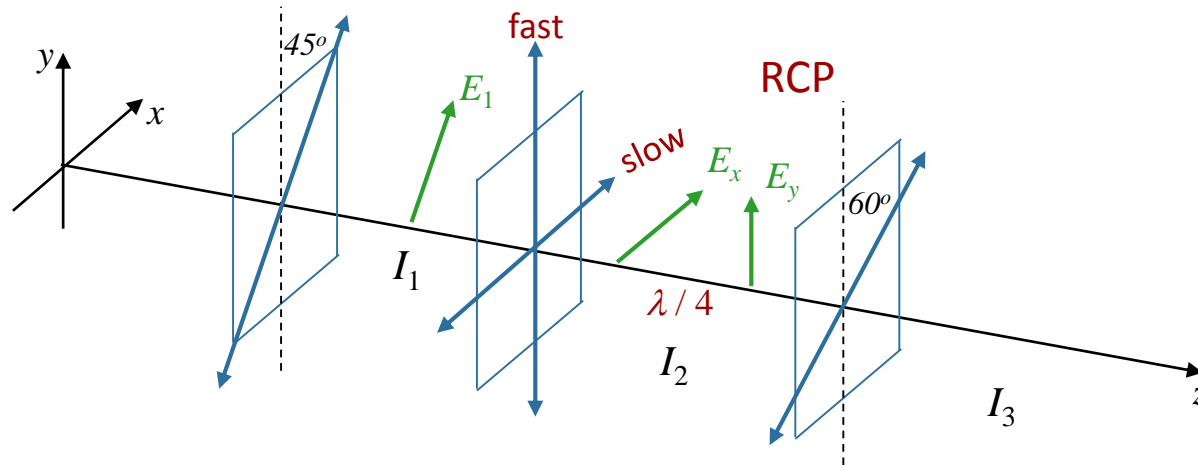
Determine state of polarization and intensity reduction after each object

Multiply individual intensity reductions to get final reduction.

Calculation



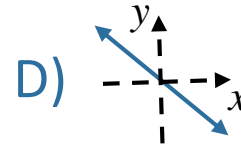
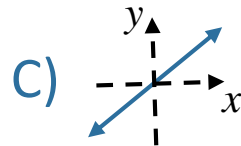
Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



What is the polarization of the light after the QWP?

A) LCP

B) RCP



E) un-polarized

Light incident on QWP is linearly polarized at 45° to fast axis



Light will be circularly polarized after QWP

LCP or RCP? Easiest way:
Right Hand Rule:

Curl fingers of RH back to front
Thumb points in dir of propagation
if right hand polarized.

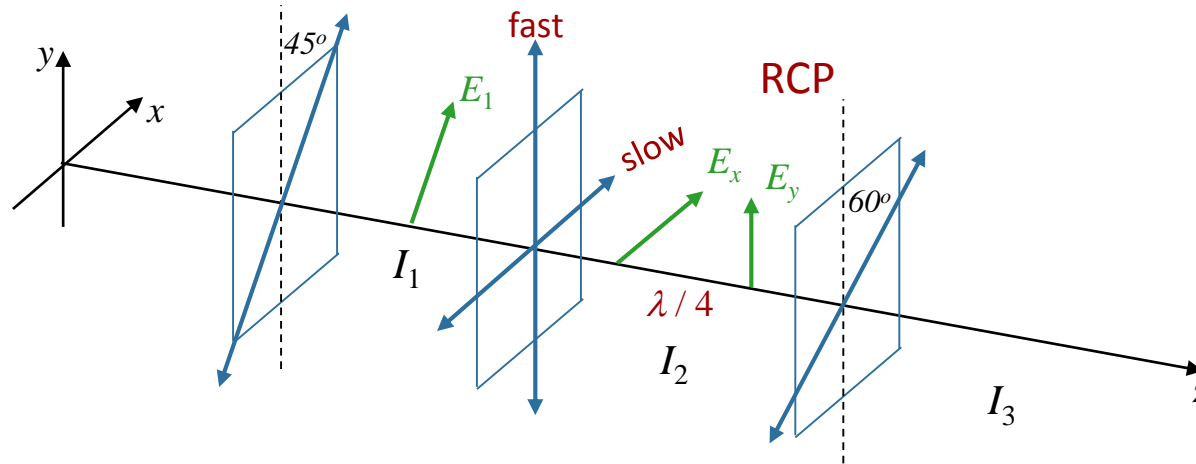


RCP

Calculation



Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



What is the intensity I_2 of the light after the QWP?

A) $I_2 = I_1$

B) $I_2 = \frac{1}{2} I_1$

C) $I_2 = \frac{1}{4} I_1$

Before:

$$E_x = \frac{E_1}{\sqrt{2}} \sin(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}} \sin(kz - \omega t)$$

No absorption: Just a phase change!

$$I = \epsilon_0 c \left[\langle E_x^2 \rangle + \langle E_y^2 \rangle \right]$$

Same before & after!

After:

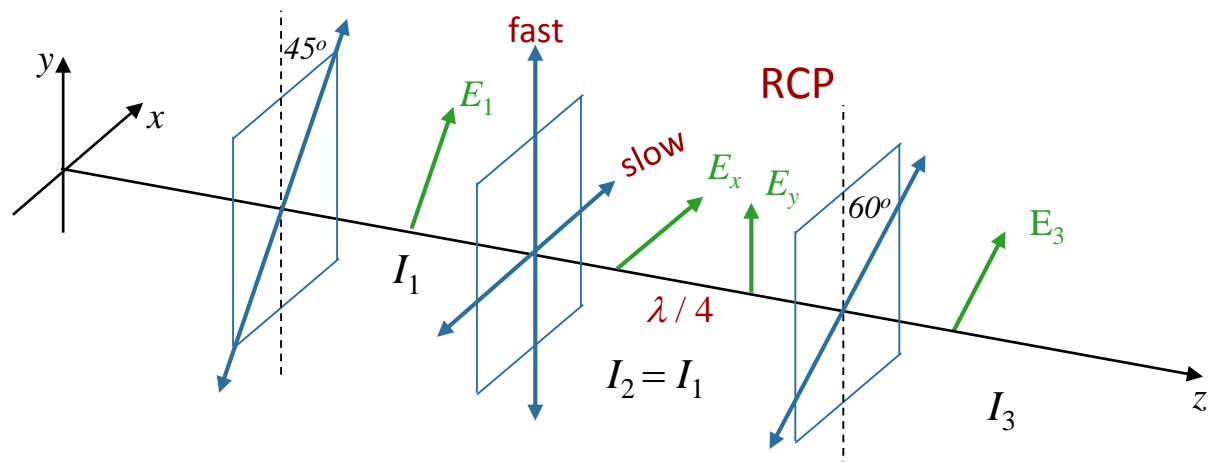
$$E_x = \frac{E_1}{\sqrt{2}} \cos(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}} \cos(kz - \omega t)$$

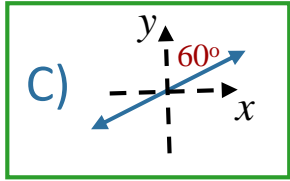
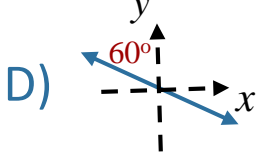
Calculation



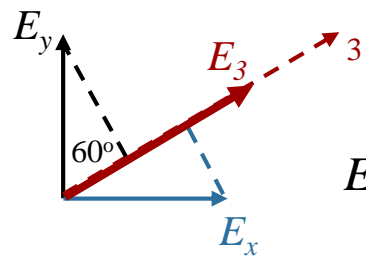
Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



What is the polarization of the light after the 60° polarizer?

- A) LCP
- B) RCP
- C) 
- D) 
- E) un-polarized

Absorption: only passes components of E parallel to TA ($\theta = 60^\circ$)



$$E_3 = E_x \sin \theta + E_y \cos \theta$$

$$E_3 = \frac{E_1}{\sqrt{2}} (\cos(kz - \omega t) \sin \theta + \sin(kz - \omega t) \cos \theta)$$

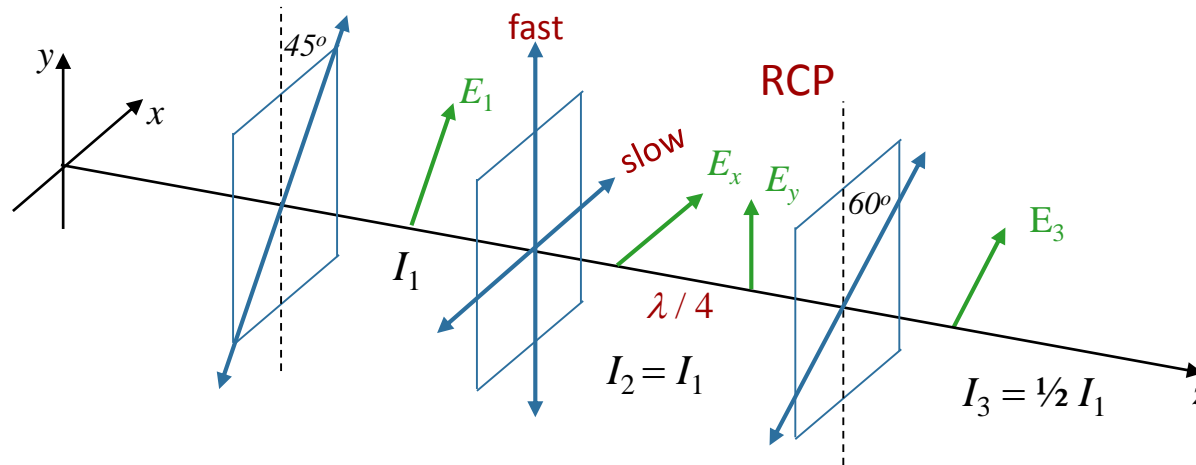
$$E_3 = \frac{E_1}{\sqrt{2}} (\sin(kz - \omega t + \theta))$$



Calculation



Light is incident on two linear polarizers and a quarter wave plate (QWP) as shown.



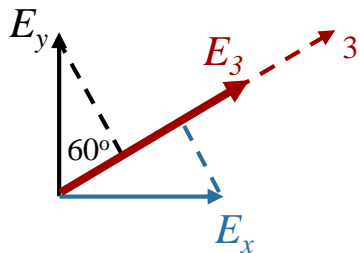
What is the intensity I_3 of the light after the 60° polarizer?

A) $I_3 = I_1$

B) $I_3 = \frac{1}{2} I_1$

C) $I_3 = \frac{1}{4} I_1$

$$E_3 = \frac{E_1}{\sqrt{2}} \quad I \propto E^2 \quad \rightarrow \quad I_3 = \frac{1}{2} I_1$$

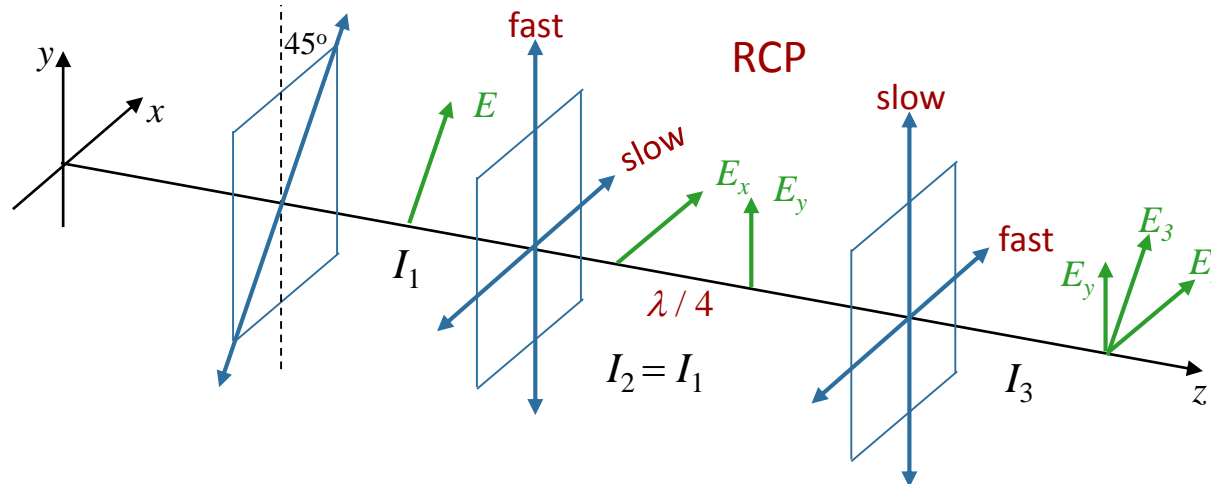


NOTE: This does not depend on θ !

Follow-Up 1



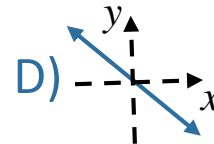
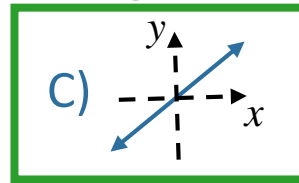
Replace the 60° polarizer with another QWP as shown.



What is the polarization of the light after the last QWP?

A) LCP

B) RCP



E) un-polarized

Easiest way:

E_{fast} is $\lambda / 4$ ahead of E_{slow}

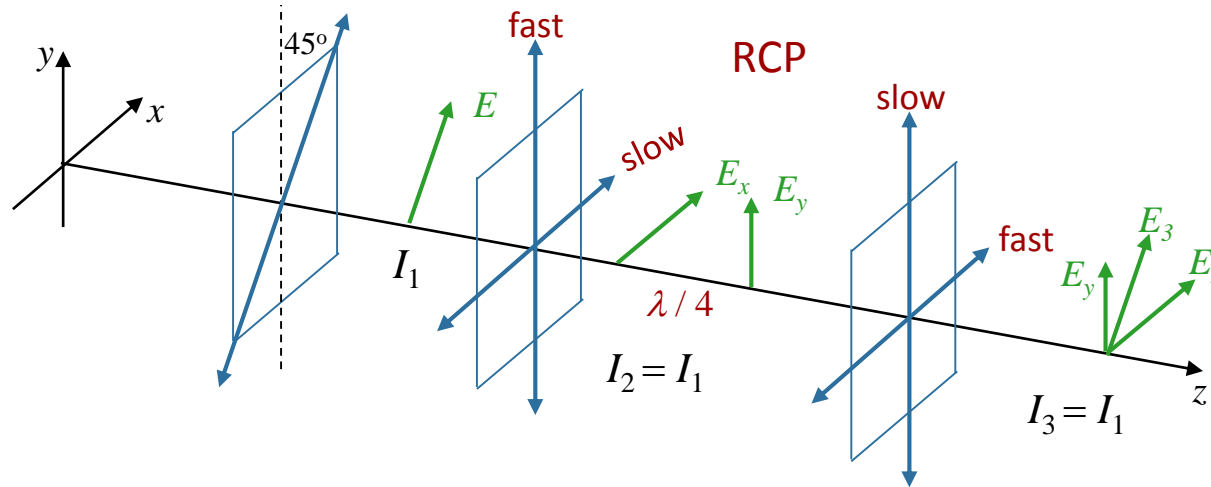


Brings E_x and E_y back in phase!

Follow-Up 2



Replace the 60° polarizer with another QWP as shown.



What is the intensity I_3 of the light after the last QWP?

A) I_1

B) $\frac{1}{2} I_1$

C) $\frac{1}{4} I_1$

Before:

$$E_x = \frac{E_1}{\sqrt{2}} \cos(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}} \sin(kz - \omega t)$$

No absorption: Just a phase change!

Intensity = $\langle E^2 \rangle$

$$I_{before} = \frac{E_1^2}{2}$$



$$I_{after} = \frac{E_1^2}{2}$$

After:

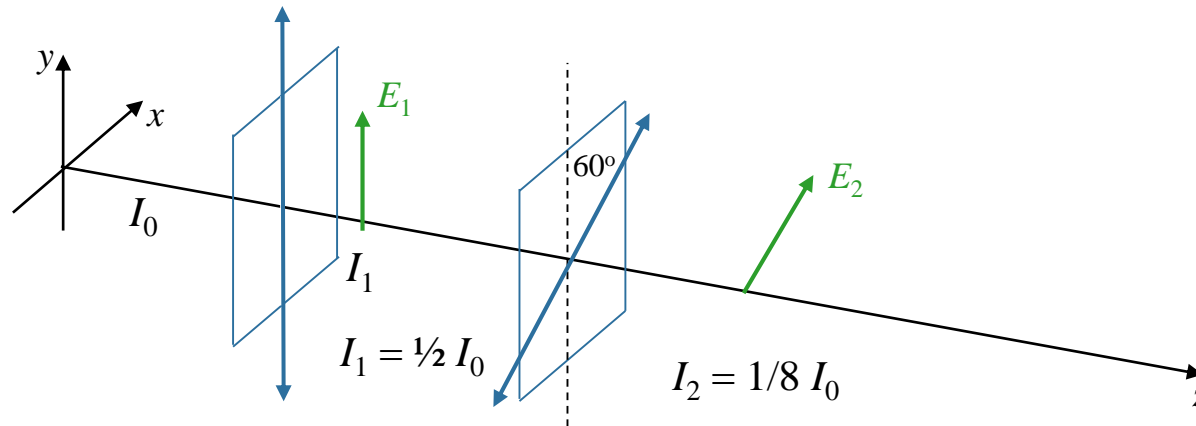
$$E_x = \frac{E_1}{\sqrt{2}} \cos(kz - \omega t)$$

$$E_y = \frac{E_1}{\sqrt{2}} \cos(kz - \omega t)$$

Follow-Up 3

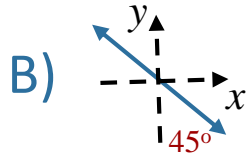


Consider light incident on two linear polarizers as shown. Suppose $I_2 = 1/8 I_0$



What is the possible polarization of the input light?

A) LCP



B)

C) un-polarized

D) all of above

E) none of above

After first polarizer: LP along y-axis with intensity I_1

After second polarizer: LP at 60° wrt y-axis

Intensity: $I_2 = I_1 \cos^2(60^\circ) = 1/4 I_1$

$I_2 = 1/8 I_0 \Rightarrow I_1 = 1/2 I_0$

Question is: What kind of light loses $1/2$ of its intensity after passing through vertical polarizer?

Answer: Everything except LP at θ other than 45°