We have a midterm and we're learning new material? Ain't nobody got time fo dat.

Feels good to be an ECE major right about now. I finally know what the MechE's felt like during 211.

Couple logistical things: 1. No lab today? 2. Where can we find the video of you solving the old midterms?

I am stressing out for this exam! What concepts should I focus most of my time on?

I just wanted to thank you for making the second deadline 100% because I have four midterms this week and I am on the brink of quitting school and working as a roadkillpickerupper.

I'm really nervous about the exam. In terms of difficulty level how hard is it? Also one of my professors told me that if the lecture before an exam isn't review then the professor must give the next lecture off. Apparently it's university policy. As much as i love this class, how come this isn't true for physics 212?

What does EMF stand for? ElectroMotiveForce

I couldn't help but notice that the constant/equation sheet that we are supplied with in our discussion book is pretty weak. Is that completely identical to the one we will Wednesday for our midterm? The 211 sheet was pretty solid, this one just feels it's lacking.

I accept that wires with a larger cross sectional area have less resistance, but why isn't surface area taken into account instead considering charges only move 'on the surfaces' of conductors? Does this have to do with scattering from electron-electron interactions?

If we all get A's on the exam can we have a pizza party instead of a lecture this Thursday?
Today’s Concept:

Ohm’s Law, Resistors in circuits
A Big Idea Review

Coulomb’s Law
Force law between point charges
\[ \vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2} \hat{r}_{1,2} \]

Electric Field
Force per unit charge
\[ \vec{E} \equiv \frac{\vec{F}}{q} \]

Gauss’ Law
Flux through closed surface is always proportional to charge enclosed
\[ \int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \]

Electric Potential
Potential energy per unit charge
\[ \Delta V_{a \to b} \equiv \frac{\Delta U_{a \to b}}{q} = -\int_a^b \vec{E} \cdot d\vec{l} \]

Capacitance
Relates charge and potential for two conductor system
\[ C \equiv \frac{Q}{V} \]

Electric Potential
Scalar Function that can be used to determine \( \vec{E} \)
\[ \vec{E} = -\nabla V \]
Applications of Big Ideas

Conductors
Charges free to move

What Determines
How They Move?

They move until
\( E = 0 \)!

Spheres
Cylinders
Infinite Planes

Gauss’ Law

\( E = 0 \) in conductor determines charge densities on surfaces

Field Lines &
Equipotentials

Work Done By E Field

\[
W_{a \to b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{a}^{b} q \vec{E} \cdot d\vec{l}
\]

Change in Potential Energy

\[
\Delta U_{a \to b} = -W_{a \to b} = -\int_{a}^{b} q \vec{E} \cdot d\vec{l}
\]

Capacitor Networks

Series:

\[
\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3}
\]

Parallel

\[
C_{123} = C_1 + C_{23}
\]
Current and Resistance

Key Concepts:

1) How resistance depends on $A, L, \sigma, r$

2) How to combine resistors in series and parallel

3) Understanding resistors in circuits

Today’s Plan:

1) Review of resistance & preflights

2) Work out a circuit problem in detail
Observables:

\[ V = EL \]
\[ I = JA \]

Conductivity – high for good conductors.

Ohm's Law:

\[ \sigma E = \frac{V}{L} \]

Note: “Conventional current flow”, \( I \), is opposite to direction electrons flow.

Resistance

\[ R = \frac{1}{\sigma} \]

\[ I = \frac{V}{R} \]

\[ R = \frac{L}{\sigma A} \]
The SAME amount of current I passes through three different resistors. $R_2$ has twice the cross-sectional area and the same length as $R_1$, and $R_3$ is three times as long as $R_1$ but has the same cross-sectional area as $R_1$.

In which case is the CURRENT DENSITY through the resistor the smallest?

A. Case 1  
B. Case 2  
C. Case 3

$$J \equiv \frac{I}{A}$$  

$J_1 = J_3 = 2J_2$ 

Same Current  

$J \propto \frac{1}{A}$
$I$ is like flow rate of water
$V$ is like pressure
$R$ is how hard it is for water to flow in a pipe

\[ R = \frac{L}{\sigma A} \]

To make $R$ big, make $L$ long or $A$ small
To make $R$ small, make $L$ short or $A$ big

Electricity & Magnetism Lecture 9, Slide 8
Same current through both resistors

Compare voltages across resistors

\[ R \propto \frac{L}{A} \]

\[ V = IR \propto \frac{L}{A} \]

Two Resistors: Question 1 (N = 785)

Two Resistors: Question 3 (N = 783)
### Resistor Summary

<table>
<thead>
<tr>
<th><strong>Series</strong></th>
<th><strong>Parallel</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wiring</strong></td>
<td>Each resistor on a different wire.</td>
</tr>
<tr>
<td><strong>Voltage</strong></td>
<td>Different for each resistor.</td>
</tr>
<tr>
<td>( V_{total} = V_1 + V_2 )</td>
<td>( V_{total} = V_1 = V_2 )</td>
</tr>
<tr>
<td><strong>Current</strong></td>
<td>Same for each resistor</td>
</tr>
<tr>
<td>( I_{total} = I_1 = I_2 )</td>
<td>( I_{total} = I_1 + I_2 )</td>
</tr>
<tr>
<td><strong>Resistance</strong></td>
<td>Increases</td>
</tr>
<tr>
<td>( R_{eq} = R_1 + R_2 )</td>
<td>( 1/R_{eq} = 1/R_1 + 1/R_2 )</td>
</tr>
</tbody>
</table>
Three resistors are connected to a battery with emf $V$ as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.

\[ V = I_1 + I_2 + I_3 \]

Current through $R_2$ and $R_3$ is the same

\[ I_{23} = \frac{V}{R_2 + R_3} \]

Compare the current through $R_2$ with the current through $R_3$:

A. $I_2 > I_3$

B. $I_2 = I_3$

C. $I_2 < I_3$
CheckPoint 2b

Compare the current through $R_1$ with the current through $R_2$

- A $I_1/I_2 = 1/2$
- B $I_1/I_2 = 1/3$
- C $I_1/I_2 = 1$
- D $I_1/I_2 = 2$
- E $I_1/I_2 = 3$

We know:

$$I_{23} = \frac{V}{R_2 + R_3}$$

Similarly:

$$I_1 = I_{23} \frac{R_2 + R_3}{R_1}$$

$$\frac{I_1}{I_{23}} = \frac{R_2 + R_3}{R_1} = 2$$
CheckPoint 2c

Compare the voltage across $R_2$ with the voltage across $R_3$

- $V_2 > V_3$
- $V_2 = V_3 = V$
- $V_2 = V_3 < V$
- $V_2 < V_3$

$V_2 = I_2 R_2$

$V_3 = I_3 R_3$

$I_2 = I_3$ (Series)

$R_2 = R_3$ (Problem statement)

$V_{23} = V$

$V_{23} = V_2 + V_3$

$V_2 = V_3 = \frac{V}{2}$
**CheckPoint 2d**

Compare the voltage across $R_1$ with the voltage across $R_2$

- **A** $V_1 = V_2 = V$
- **B** $V_1 = \frac{1}{2} V_2 = V$
- **C** $V_1 = 2V_2 = V$
- **D** $V_1 = \frac{1}{2} V_2 = \frac{1}{5} V$
- **E** $V_1 = \frac{1}{2} V_2 = \frac{1}{2} V$

$R_1$ in parallel with series combination of $R_2$ and $R_3$

$$V_1 = V_{23}$$

$$R_2 = R_3 \implies V_2 = V_3$$

$$V_{23} = V_2 + V_3 = 2V_2$$

$$\implies V_1 = 2V_2 = V$$

Electricity & Magnetism Lecture 9, Slide 15
In the circuit shown: 

\[ V = 18V, \]
\[ R_1 = 1\Omega, \quad R_2 = 2\Omega, \quad R_3 = 3\Omega, \quad \text{and} \quad R_4 = 4\Omega. \]

What is \( V_2 \), the voltage across \( R_2 \)?

**Conceptual Analysis:**

Ohm’s Law: when current \( I \) flows through resistance \( R \), the potential drop \( V \) is given by:

\[ V = IR. \]

Resistances are combined in series and parallel combinations

\[
R_{\text{series}} = R_a + R_b
\]

\[
\left(1/R_{\text{parallel}}\right) = \left(1/R_a\right) + \left(1/R_b\right)
\]

**Strategic Analysis:**

Combine resistances to form equivalent resistances

Evaluate voltages or currents from Ohm’s Law

Expand circuit back using knowledge of voltages and currents
In the circuit shown: \( V = 18 \text{V}, \ R_1 = 1 \Omega, \ R_2 = 2 \Omega, \ R_3 = 3 \Omega, \) and \( R_4 = 4 \Omega. \)

What is \( V_2, \) the voltage across \( R_2? \)

Combine Resistances:

- \( R_1 \) and \( R_2 \) are connected:
  - A) in series
  - B) in parallel
  - C) neither in series nor in parallel

**Parallel Combination**

- Can make a loop that contains only those two resistors

**Series Combination**

- Series: Every loop with resistor 1 also has resistor 2.
We first will combine resistances $R_2$, $R_3$, $R_4$:
Which of the following is true?

A) $R_2$, $R_3$ and $R_4$ are connected in series

B) $R_2$, $R_3$, and $R_4$ are connected in parallel

C) $R_3$ and $R_4$ are connected in series ($R_{34}$) which is connected in parallel with $R_2$

D) $R_2$ and $R_4$ are connected in series ($R_{24}$) which is connected in parallel with $R_3$

E) $R_2$ and $R_4$ are connected in parallel ($R_{24}$) which is connected in parallel with $R_3$

In the circuit shown: $V = 18V$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is $V_2$, the voltage across $R_2$?
In the circuit shown:  \( V = 18 \text{V}, \)
\( R_1 = 1 \Omega, \)  \( R_2 = 2 \Omega, \)  \( R_3 = 3 \Omega, \)  and  \( R_4 = 4 \Omega. \)

What is  \( V_2, \) the voltage across  \( R_2? \)

\( R_2 \)  and  \( R_4 \)  are connected in series \( (R_{24}) \) which is connected in parallel with  \( R_3. \)

Redraw the circuit using the equivalent resistor  \( R_{24} = \) series combination of  \( R_2 \)  and  \( R_4. \)
In the circuit shown: $V = 18V$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is $V_2$, the voltage across $R_2$?

Combine Resistances:

- $R_2$ and $R_4$ are connected in series $= R_{24}$
- $R_3$ and $R_{24}$ are connected in parallel $= R_{234}$

What is the value of $R_{234}$?

- A) $R_{234} = 1\ \Omega$
- B) $R_{234} = 2\ \Omega$
- C) $R_{234} = 4\ \Omega$
- D) $R_{234} = 6\ \Omega$

$R_2$ and $R_4$ in series

$$R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$$

$$\frac{1}{R_{\text{parallel}}} = \left(\frac{1}{R_a}\right) + \left(\frac{1}{R_b}\right)$$

$$1/R_{234} = (1/3) + (1/6) = (3/6)\ \Omega^{-1}$$

$$R_{234} = 2\ \Omega$$
In the circuit shown: \( V = 18\, \text{V} \),
\( R_1 = 1\, \Omega \), \( R_2 = 2\, \Omega \), \( R_3 = 3\, \Omega \), and \( R_4 = 4\, \Omega \).
\( R_{24} = 6\, \Omega \quad R_{234} = 2\, \Omega \)
What is \( V_2 \), the voltage across \( R_2 \)?

\( R_1 \) and \( R_{234} \) are in series. \( R_{1234} = 1 + 2 = 3 \, \Omega \)

Our next task is to calculate the total current in the circuit

Ohm’s Law tells us: \( I_{1234} = \frac{V}{R_{1234}} \)

\[ = \frac{18}{3} \]

\[ = 6 \, \text{Amps} \]
In the circuit shown: $V = 18\, V$, 
$R_1 = 1\, \Omega$, $R_2 = 2\, \Omega$, $R_3 = 3\, \Omega$, and $R_4 = 4\, \Omega$. 
$R_{24} = 6\, \Omega$ \hspace{1cm} $R_{234} = 2\, \Omega$ \hspace{1cm} $I_{1234} = 6\, A$

What is $V_2$, the voltage across $R_2$?

$I_{234} = I_{1234}$ \hspace{1cm} Since $R_1$ in series with $R_{234}$

$V_{234} = I_{234} \cdot R_{234}$

$= 6 \times 2$

$= 12\, \text{Volts}$

What is $V_{ab}$, the voltage across $R_{234}$?

A) $V_{ab} = 1\, V$ \hspace{0.5cm} B) $V_{ab} = 2\, V$ \hspace{0.5cm} C) $V_{ab} = 9\, V$ \hspace{0.5cm} D) $V_{ab} = 12\, V$ \hspace{0.5cm} E) $V_{ab} = 16\, V$
Which of the following are true?

A) $V_{234} = V_{24}$  
B) $I_{234} = I_{24}$  
C) Both A+B  
D) None

$R_3$ and $R_{24}$ were combined in parallel to get $R_{234}$  

Voltages are same!

Ohm’s Law

$I_{24} = \frac{V_{24}}{R_{24}}$

$= \frac{12}{6}$

$= 2$ Amps
Which of the following are true?

A) $V_{24} = V_2$  
B) $I_{24} = I_2$  
C) Both A+B  
D) None

$R_2$ and $R_4$ where combined in series to get $R_{24}$  
Currents are same!

The Problem Can Now Be Solved!

**Ohm’s Law**

\[
V_2 = I_2 \times R_2
= 2 \times 2
= 4 \text{ Volts}!
\]
What is $I_3$?

A) $I_3 = 2 \text{ A}$  
B) $I_3 = 3 \text{ A}$  
C) $I_3 = 4 \text{ A}$

\[ V_3 = V_{234} = 12 \text{ V} \quad \rightarrow \quad I_3 = \frac{V_3}{R_3} = \frac{12 \text{ V}}{3 \Omega} = 4 \text{ A} \]

What is $I_1$?

We know $I_1 = I_{1234} = 6 \text{ A}$

NOTE: $I_2 = \frac{V_2}{R_2} = \frac{4}{2} = 2 \text{ A}$  
$\rightarrow \quad I_1 = I_2 + I_3$  
Make Sense?