Your Comments

Is it possible to have a spherical 3D case for Ampere's Law like we did for Gauss' law, and if not, what does it tell us about the magnetic field versus the electric field? Is it at a higher level?

I think I understand what is going on, but I won't know for sure until we do some clicker questions.

Why was the magnetic field 0 for the coiled tube, if an integral in the center would have no current? Also, this is easy, hooray!

I feel like I might actually understand this part. Also, thanks for adding the "Sorry prof. but I didn't think about this." answer on the checkpoints. Keep doing it, it's the honest answer sometimes.

I don't see what the purpose of $\int B \cdot dl$ is. Is it strictly a trick used to calculate a certain B? I'm confused.

This prelecture was actually not too bad. Its weird, I left it with a small amount of confidence remaining in my ability to physics correctly.

What does the value of the line integral of the magnetic field over a closed loop mean conceptually?

Physics 212
Lecture 15

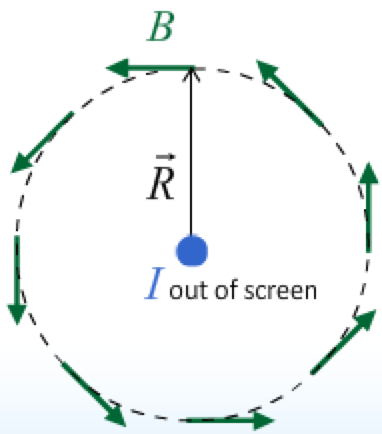
Today's Concept:

Ampere's Law

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enclosed}$$

Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I$$



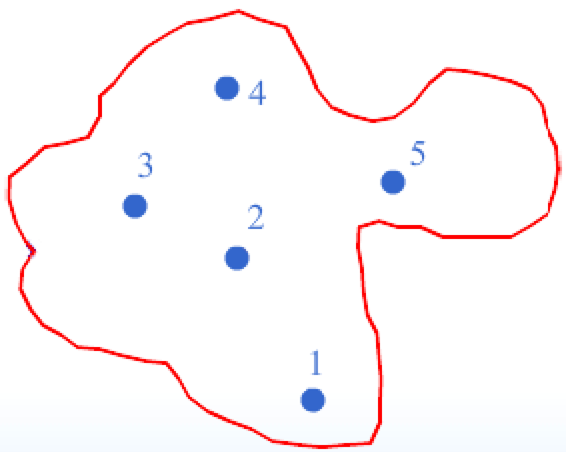
Infinite current-carrying wire

LHS: $\oint \vec{B}\bullet d\vec{\ell} = \oint B d\ell = B\oint d\ell = B\cdot 2\pi R$

RHS: $I_{enclosed} = I$

$$\longrightarrow \quad B = \frac{\mu_o I}{2\pi R}$$

General Case

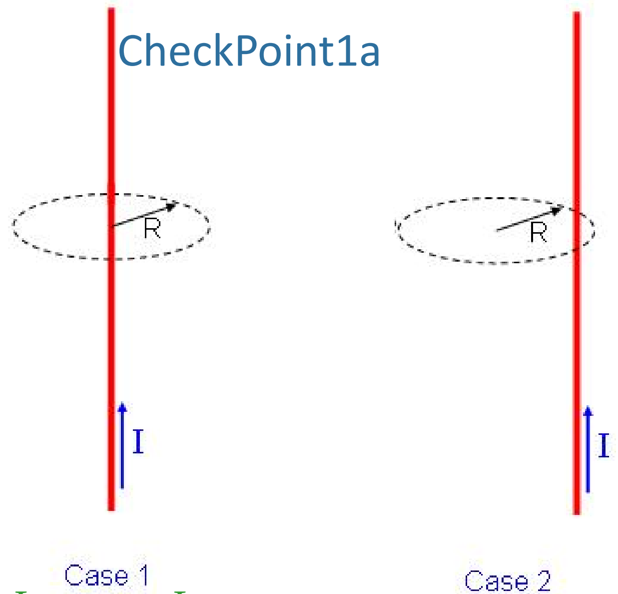


Ampere's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_o I_{enclosed}$$

Practice on Enclosed Currents

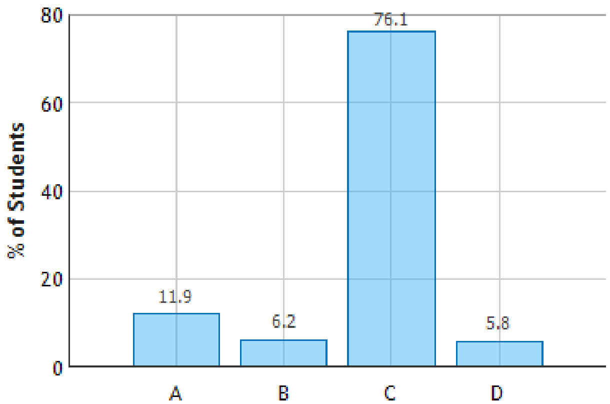
$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

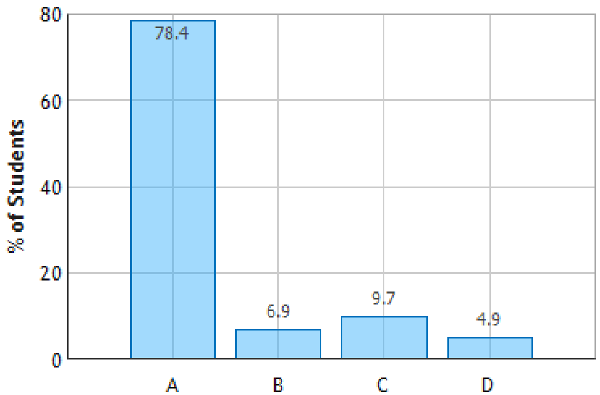


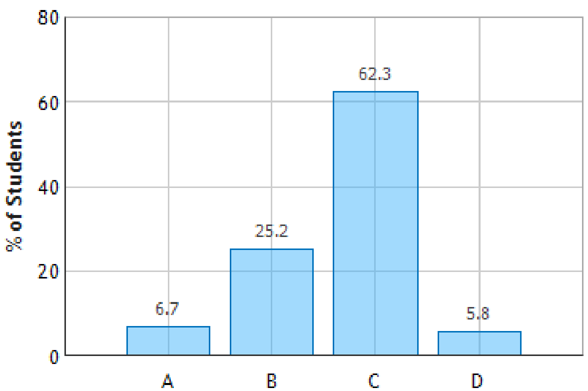
$I_{enclosed} = I$ $I_{enclosed} = I$

For which loop is ∫**B**·d**l** the greatest?

A. Case 1 ***B.*** Case 2 ***C.*** Same

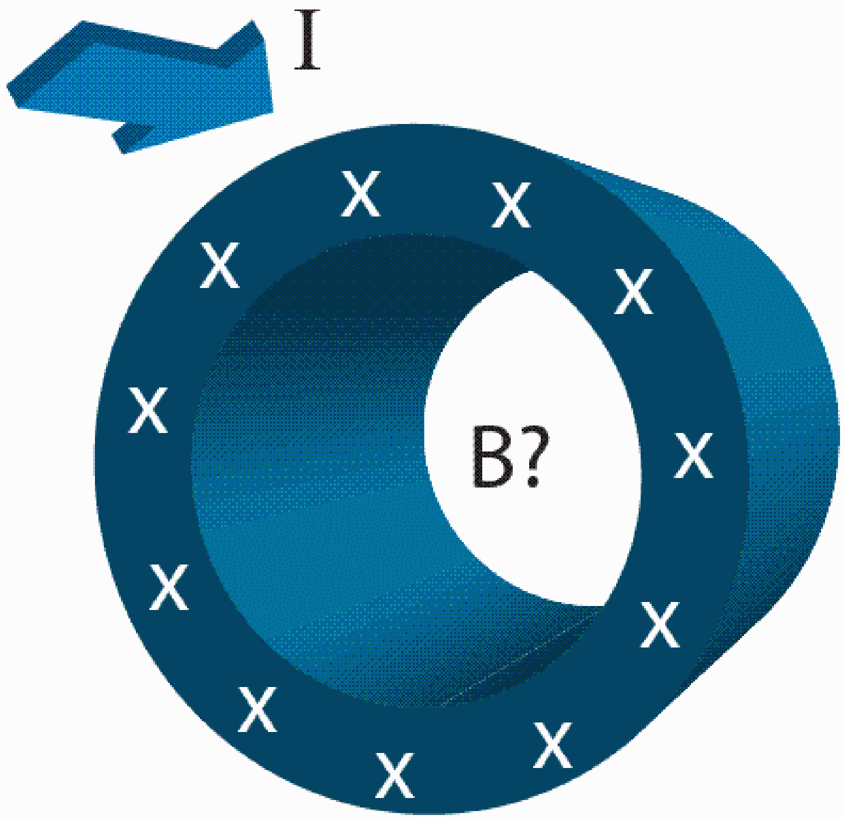






CheckPoint 2a

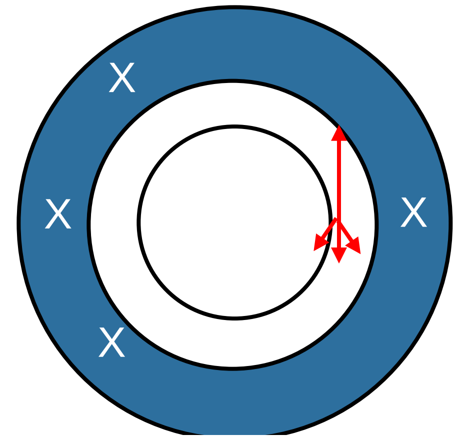
An infinitely long hollow conducting tube carries current I in the direction shown.



What is the direction of the magnetic field inside the tube?

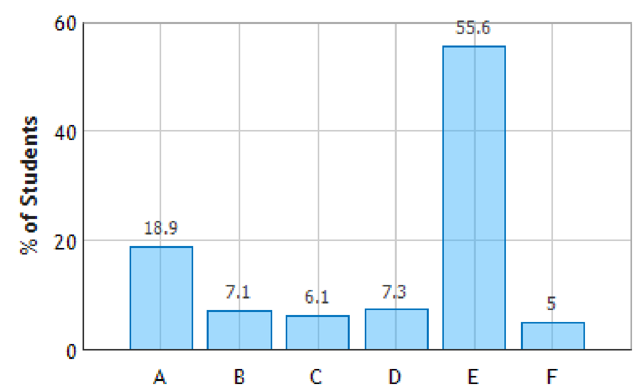
A. clockwise
B. counterclockwise
C. radially inward to the center
D. radially outward from the center
E. the magnetic field is zero

Cylindrical Symmetry



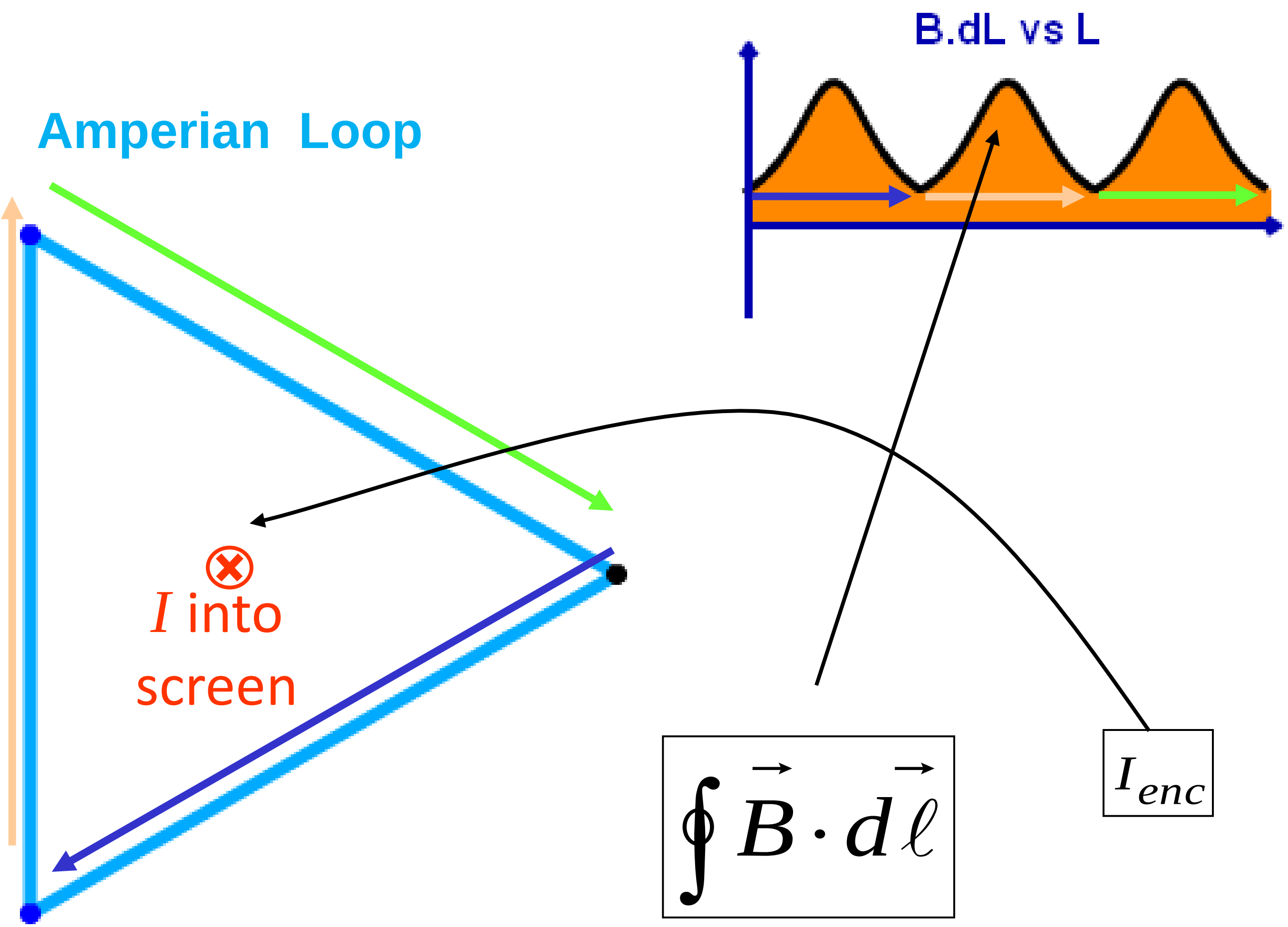
Enclosed Current = 0

Check cancellations



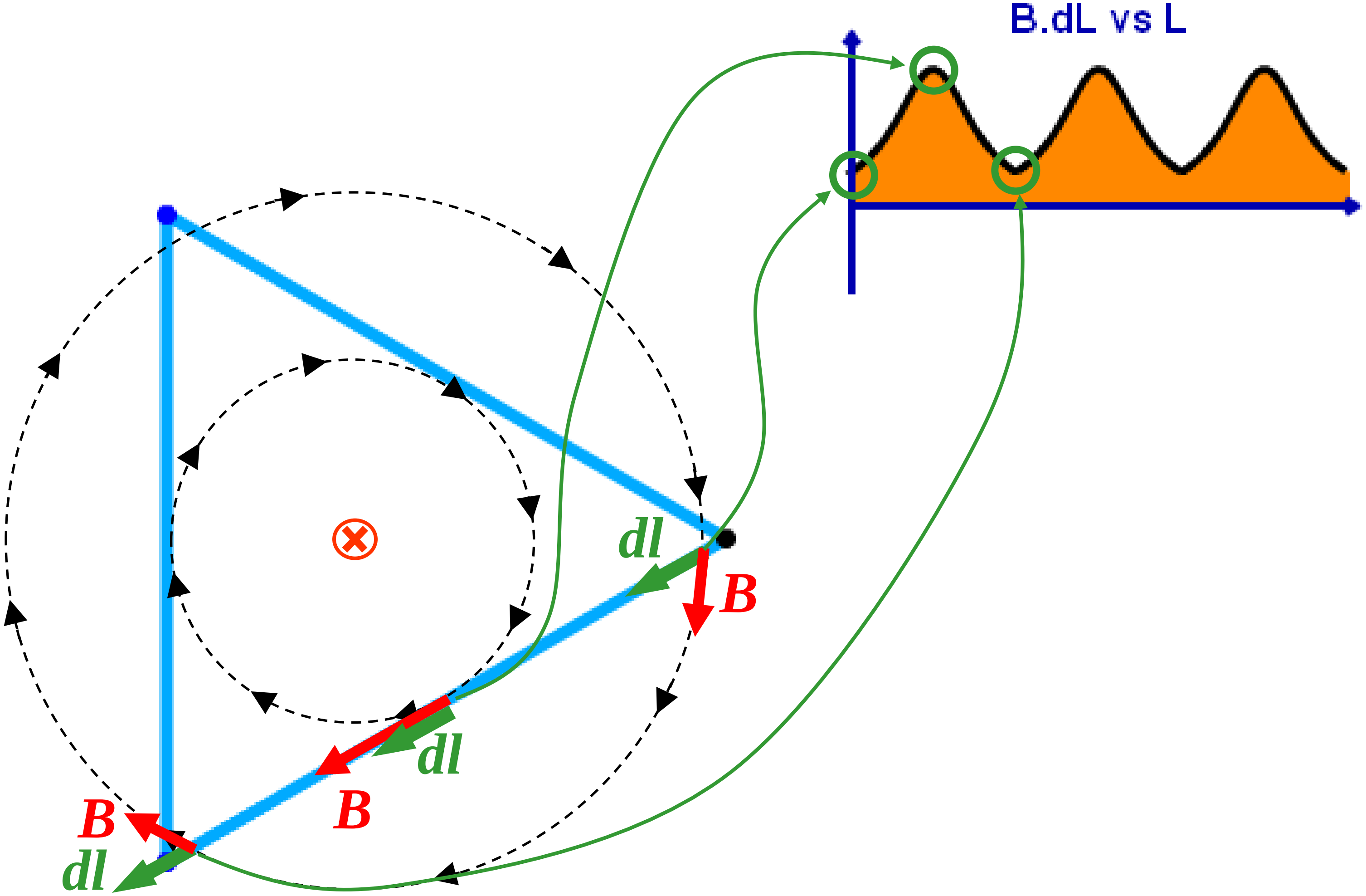
Ampere's Law

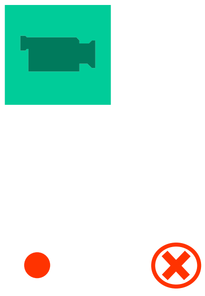
+integrals + magnetic field directinos



Ampere's Law

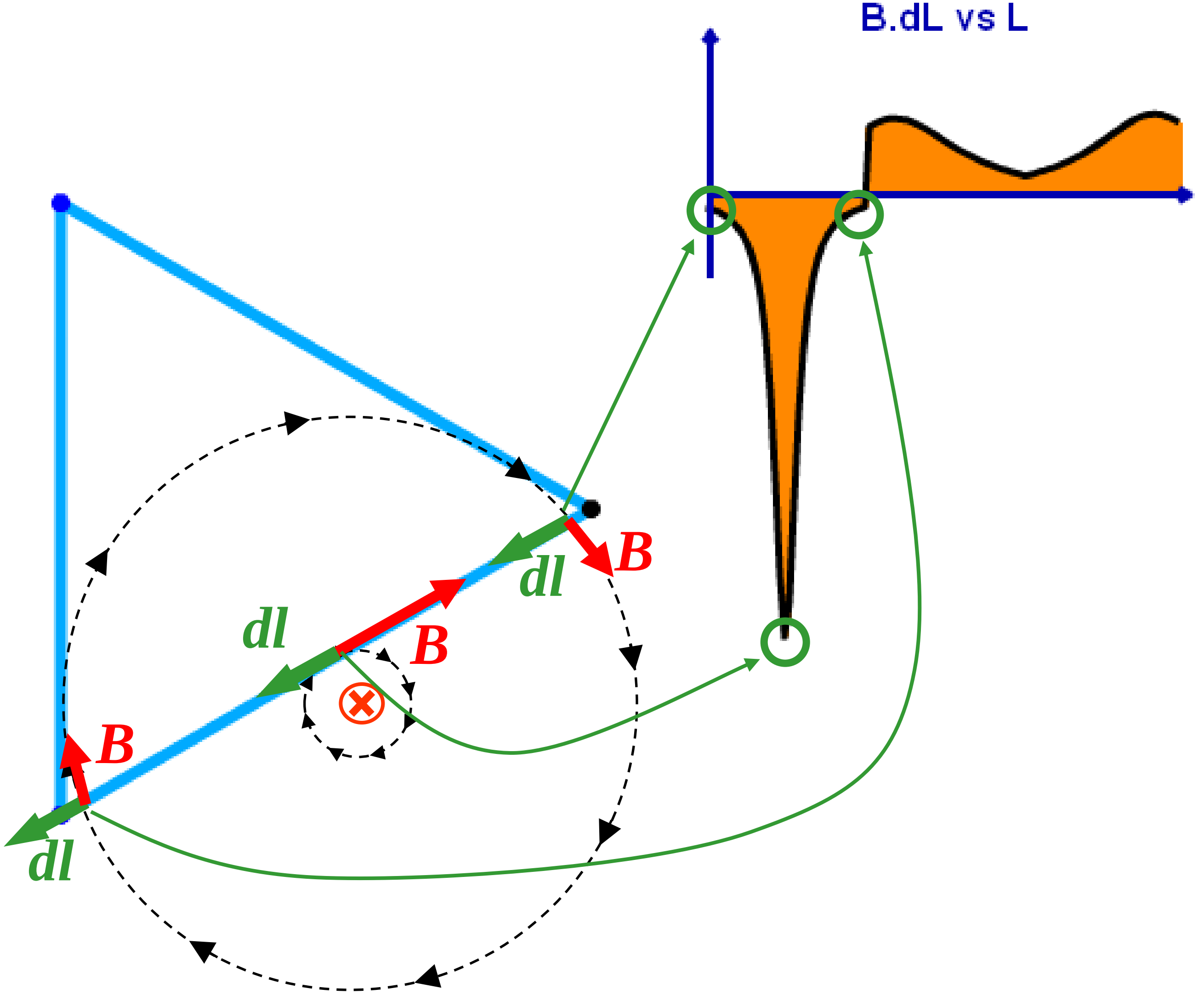
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$





Ampere's Law

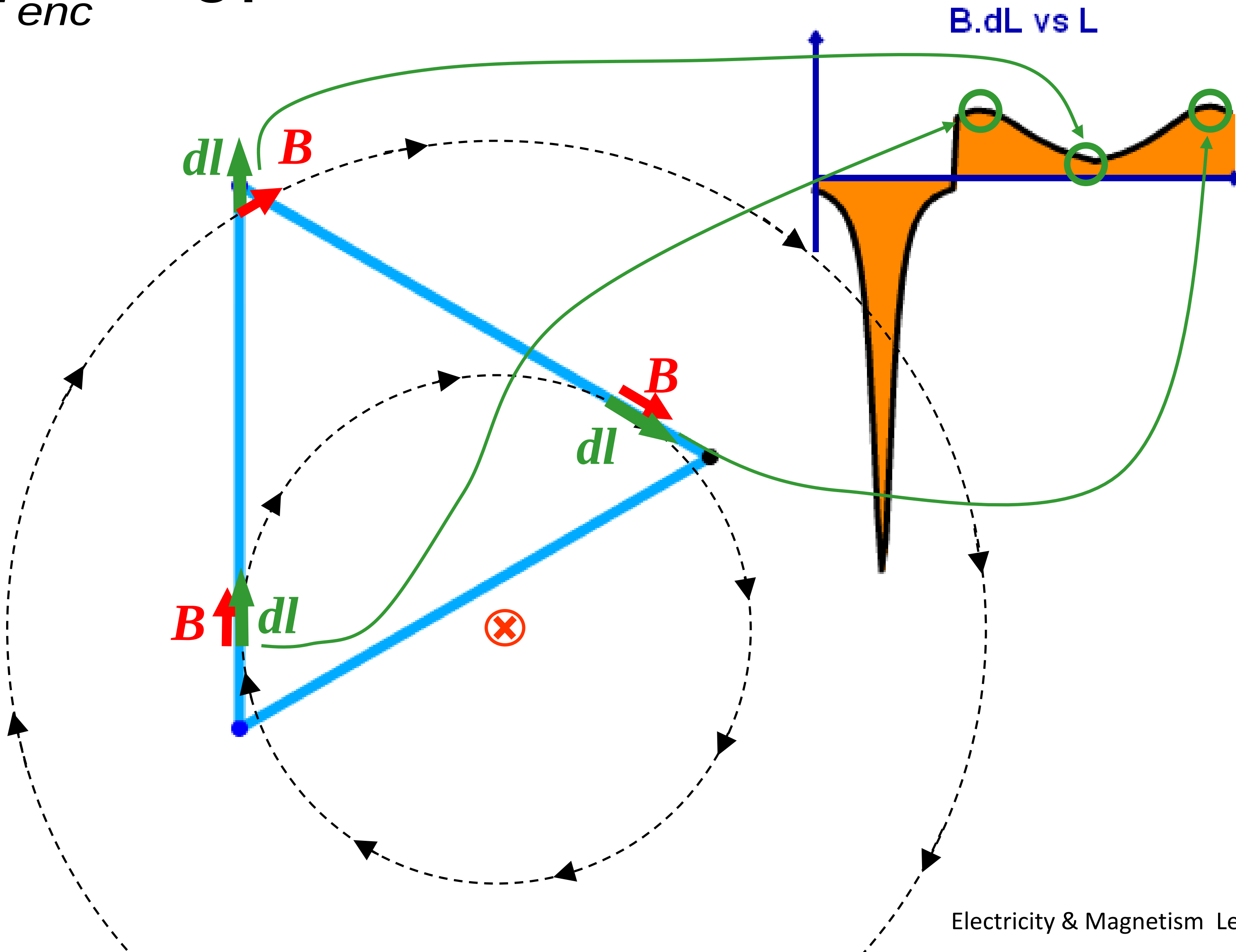
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



Ampere's Law

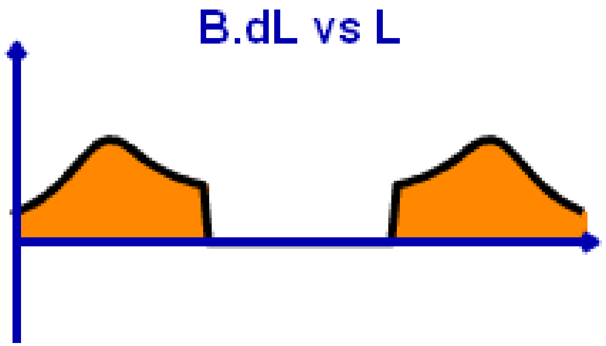
$I_{enc} = 0!$

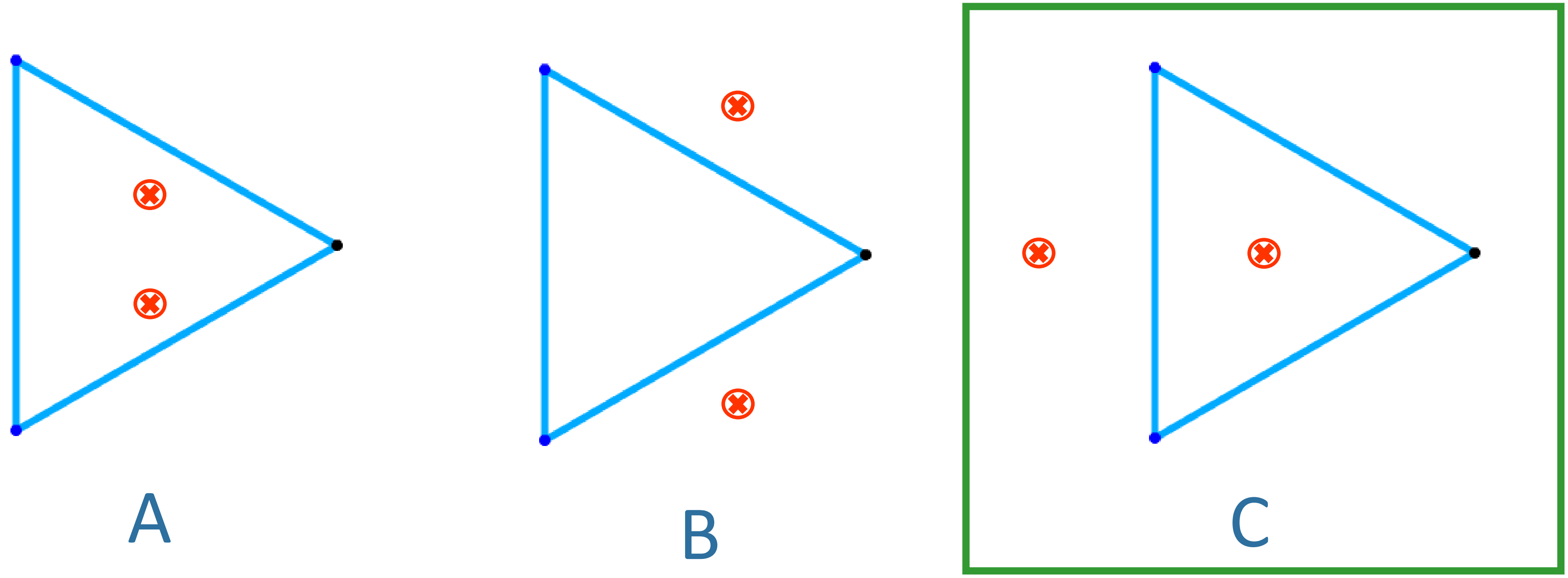
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



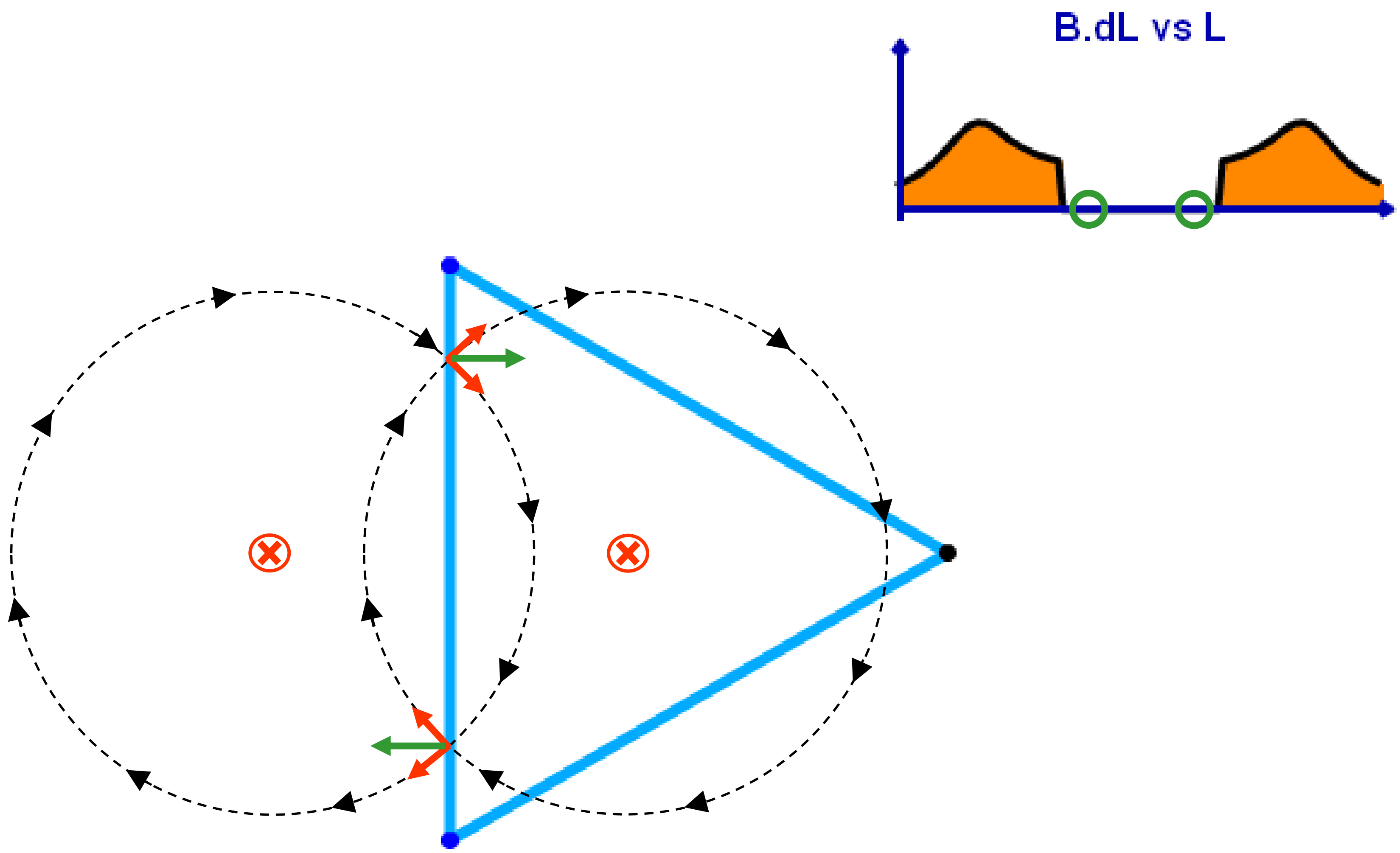
Ampere's Law Clicker Question

Which of the following current distributions would give rise to the $B \cdot dL$ distribution at the right?

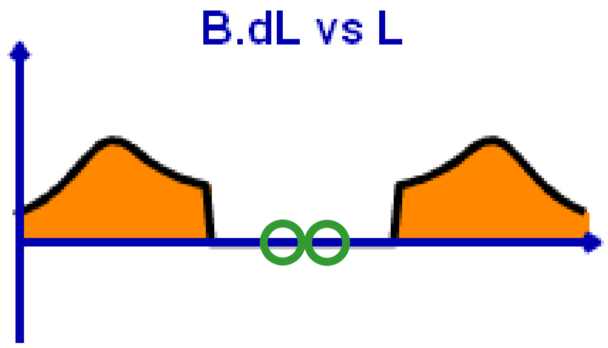


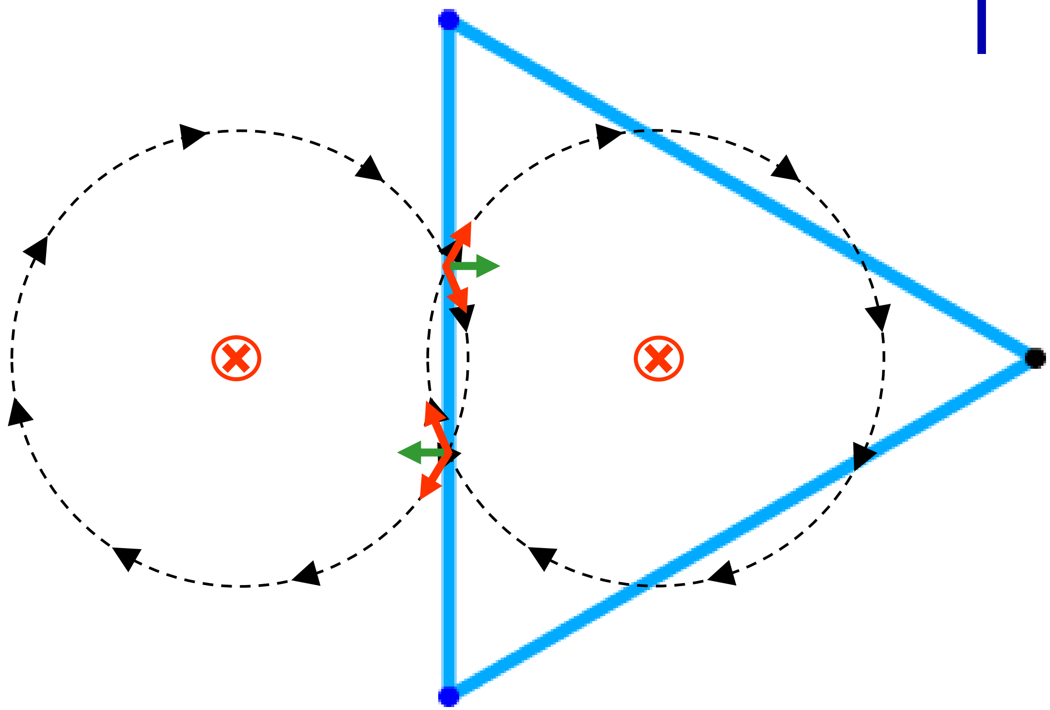


Ampere's Law Clicker Question

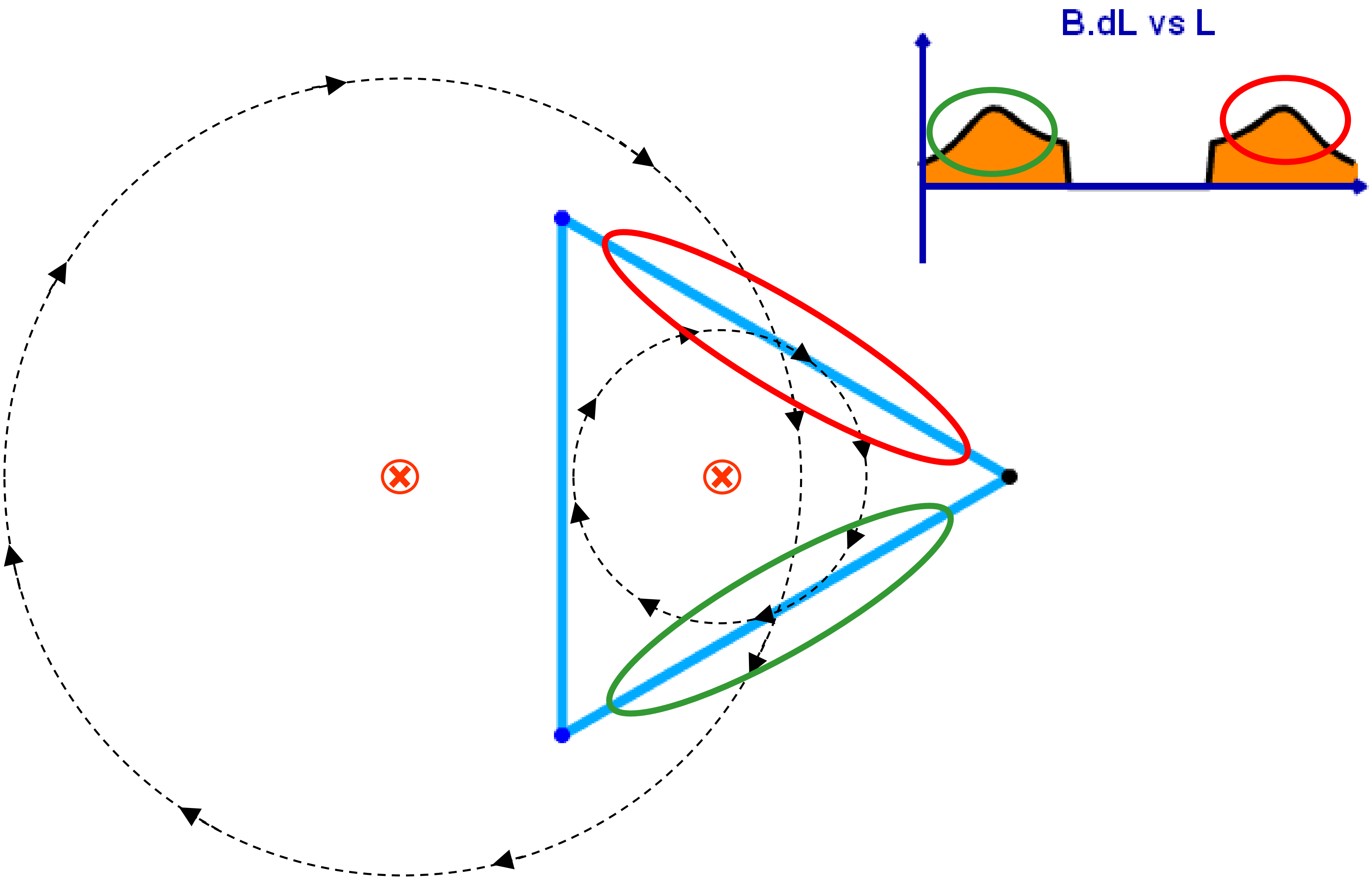


Ampere's Law Clicker Question

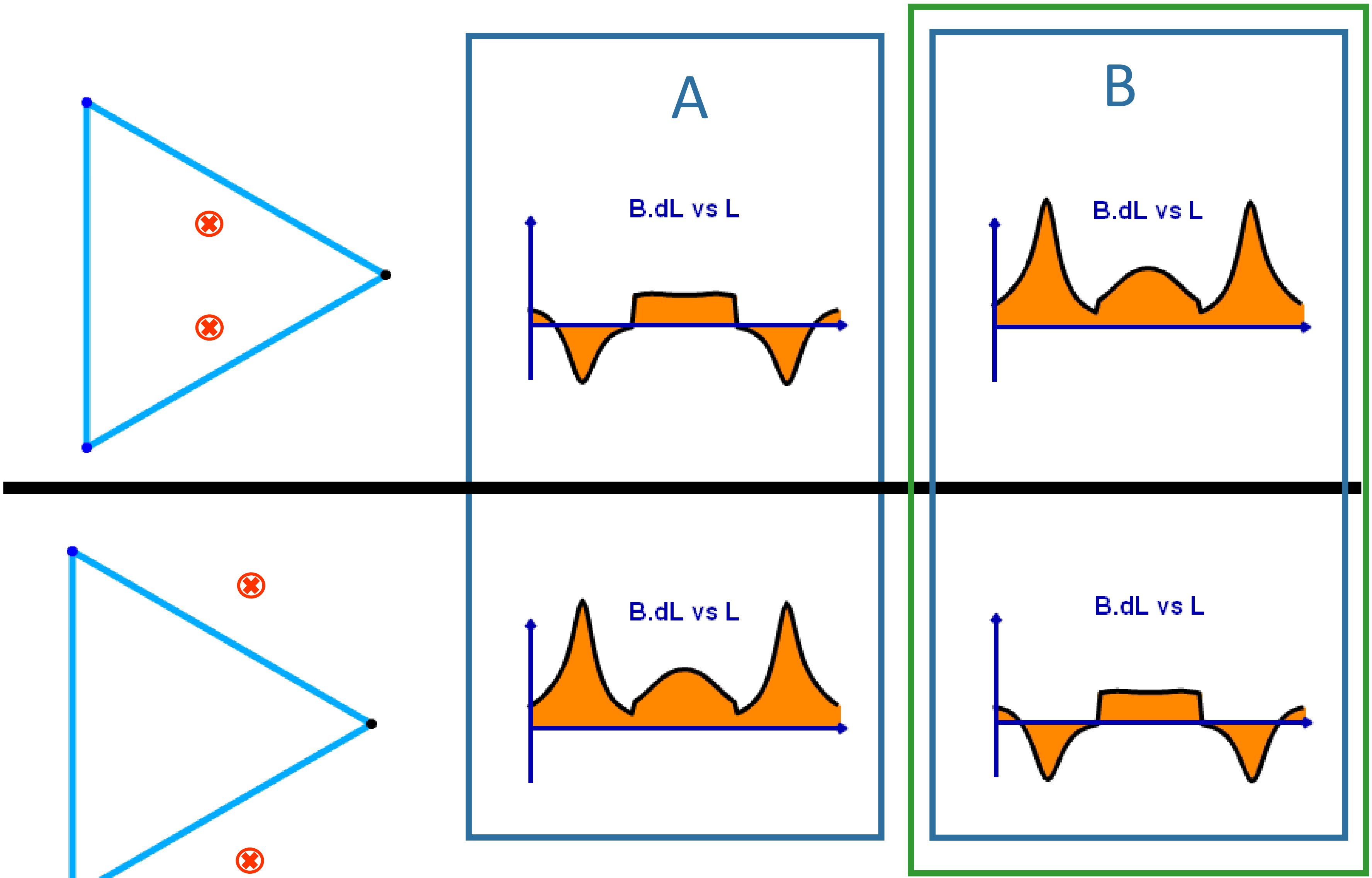




Ampere's Law Clicker Question

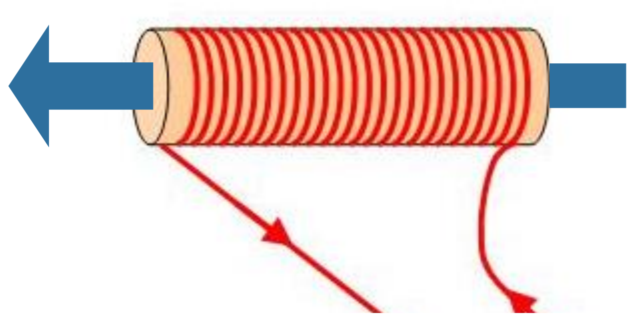


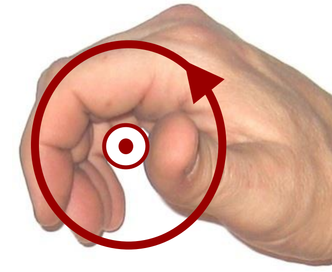
Match the other two:



CheckPoint 2b

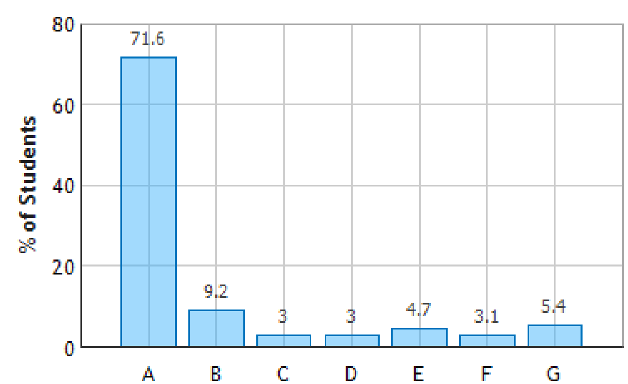
A current carrying wire is wrapped around cardboard tube as shown below.





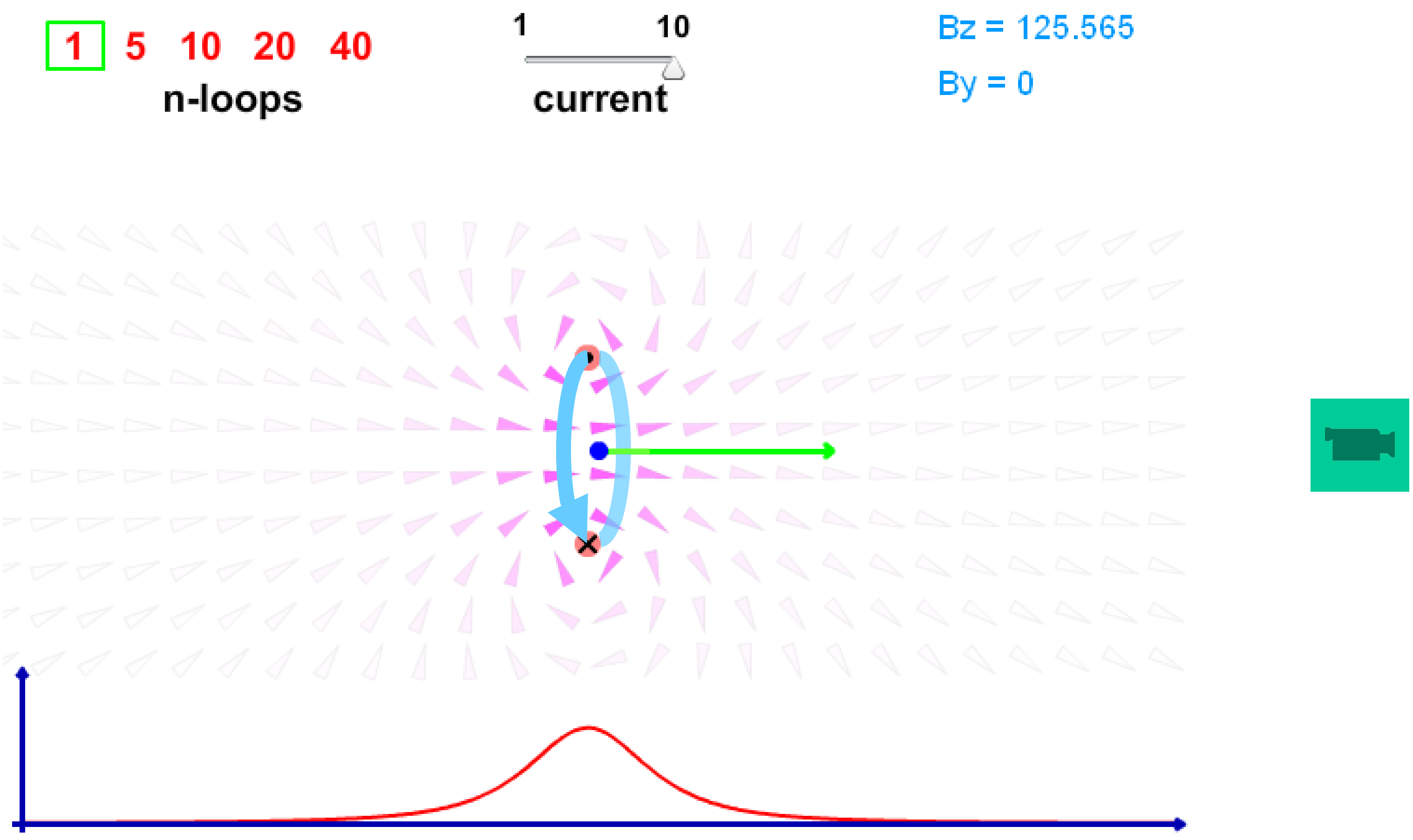
In which direction does the magnetic field point inside the tube?

A. Left   ***B.*** Right   ***C.*** Up   ***D.*** Down   ***E.*** Out of screen



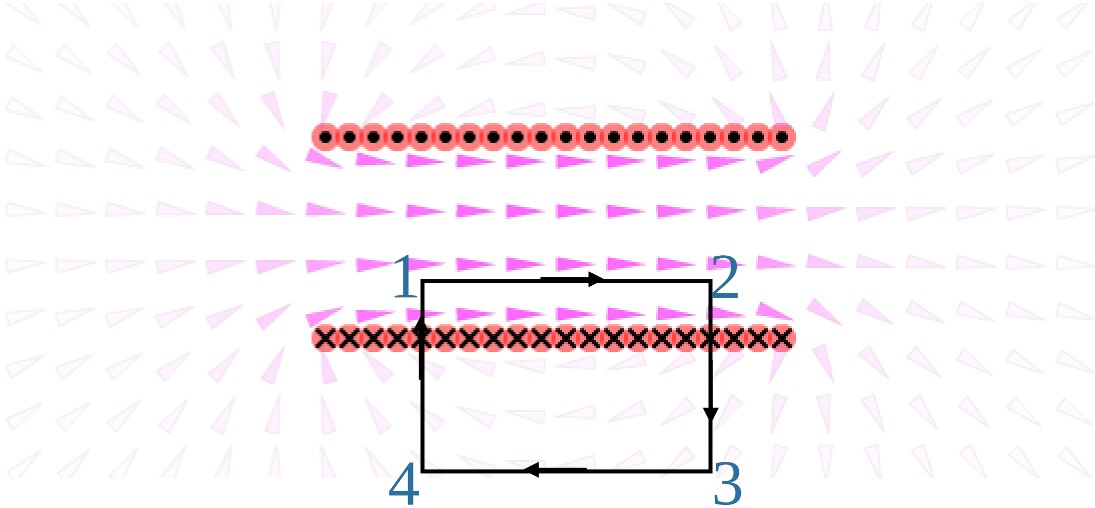
Use the right hand rule and curl your fingers along the direction of the current.

Simulation



Solenoid

Several loops packed tightly together form a uniform magnetic field inside, and nearly zero magnetic field outside.



From this simulation, we can assume a constant field inside the solenoid and zero field outside the solenoid, and apply Ampere's law to find the magnitude of the constant field inside the solenoid!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \longrightarrow \int_1^2 \vec{B} \bullet d\vec{\ell} + \int_2^3 \vec{B} \bullet d\vec{\ell} + \int_3^4 \vec{B} \bullet d\vec{\ell} + \int_4^1 \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

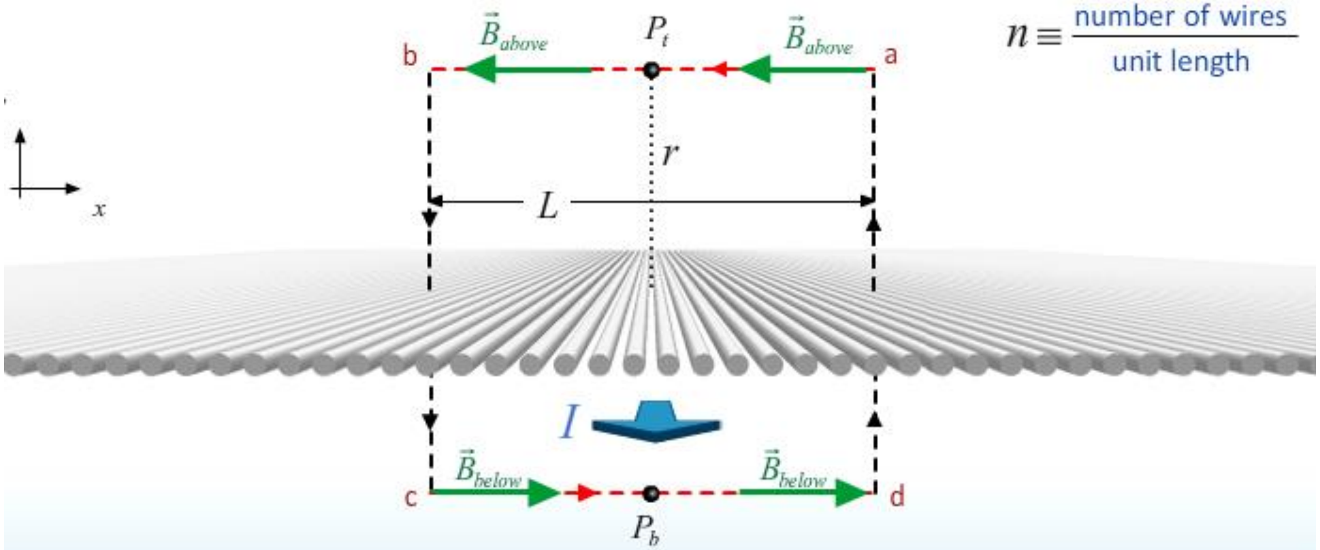
$$BL + 0 + 0 + 0 = \mu_o I_{enc} \longrightarrow BL = \mu_o nLI \longrightarrow B = \mu_o nI$$

n = # turns/length

Similar to the Current Sheet

Wire Density

$$n \equiv \frac{\text{number of wires}}{\text{unit length}}$$



$$\oint_{loop} \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l}$$

$$= \quad BL \quad + \quad BL$$

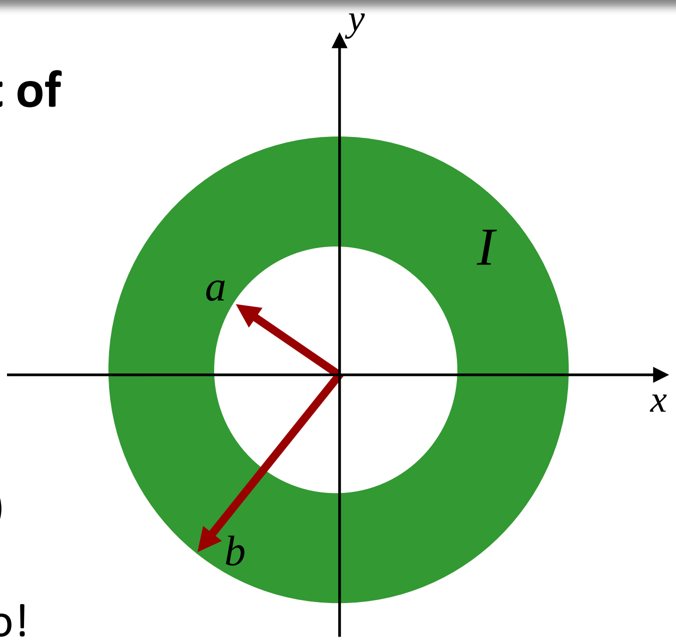
Total integral around the loop

$$\oint_{loop} \vec{B}\cdot d\vec{\ell} = 2BL = \mu_0 I_{enclosed}$$

$$\therefore B = \frac{\mu_0 N I}{2L} = \frac{\mu_0 n I}{2}$$

Example Problem

An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current I **out of the screen.**



Sketch $|B|$ as a function of r.

Conceptual Analysis

Complete cylindrical symmetry (can only depend on r)

$\Rightarrow$ can use Ampere's law to calculate B

B field can only be clockwise, counterclockwise or zero!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$B\oint d\ell = \mu_o I_{enc}$$ For circular path concentric with shell.

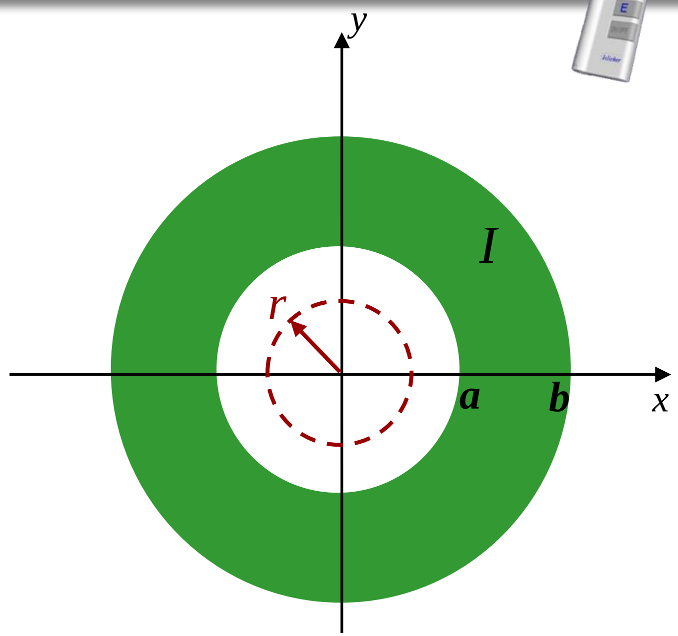
Strategic Analysis

Calculate B for the three regions separately:

1) $r < a$
2) $a < r < b$
3) $r > b$

Example Problem

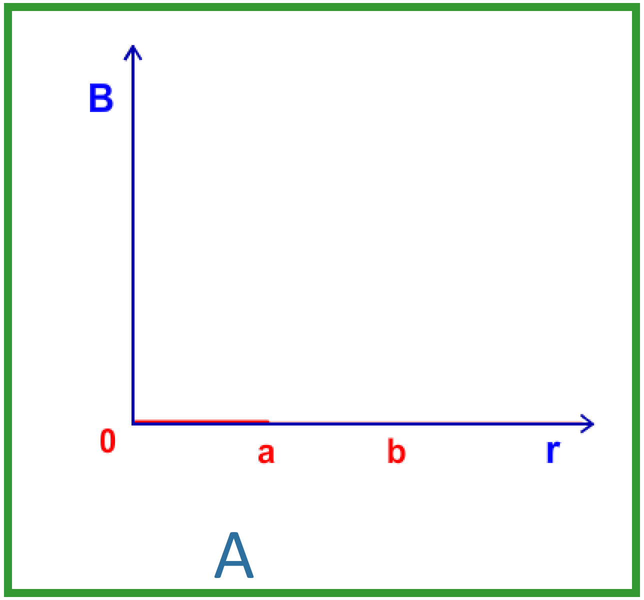
What does $|B|$ look like for $r < a$?

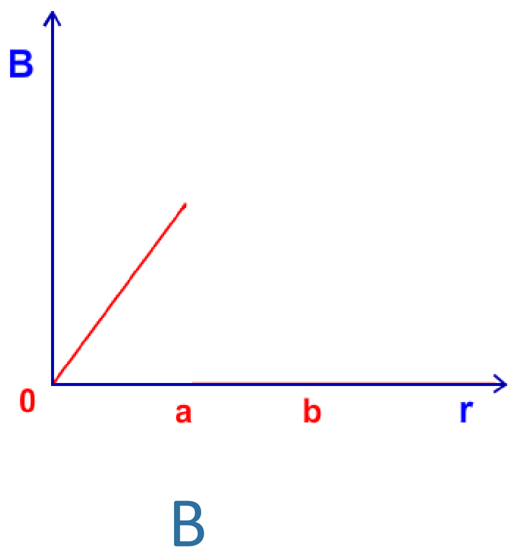


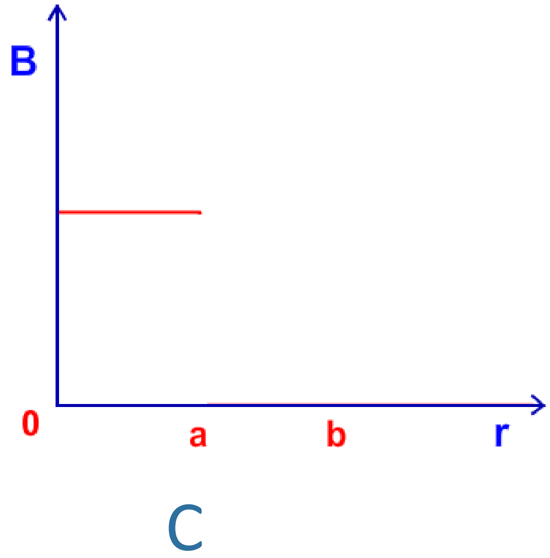
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{inc}$$

0

so $\vec{B} = 0$

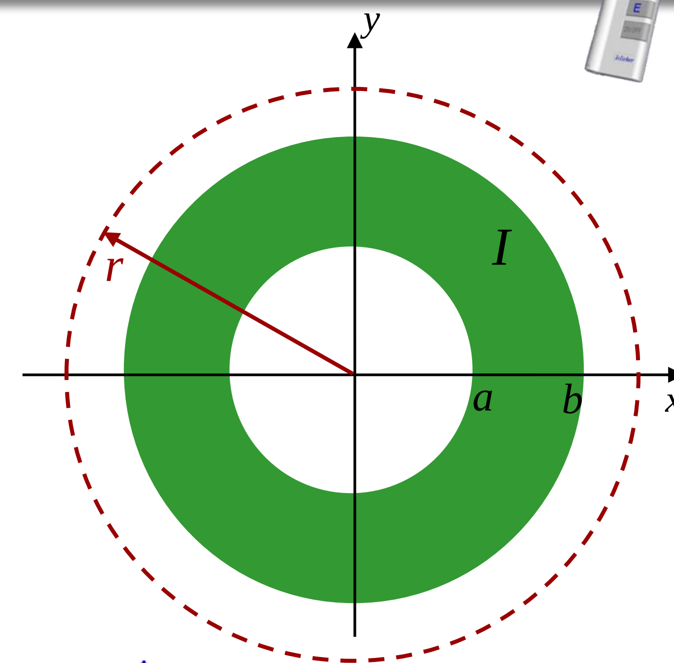






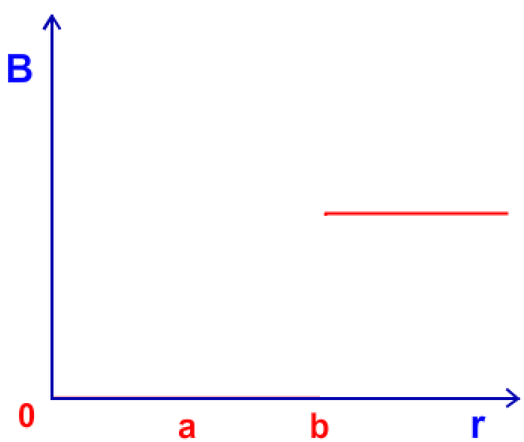
Example Problem

What does $|B|$ look like for $r > b$?

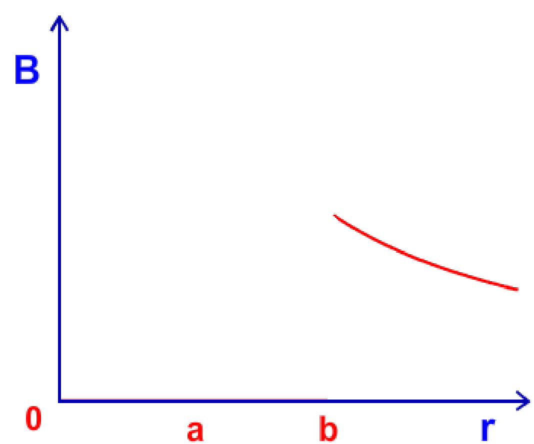


$$\oint \vec{B}\bullet d\vec{\ell} = \mu_o I_{inc}$$

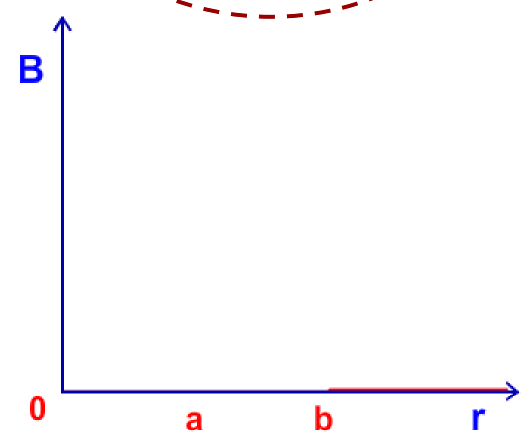
I



A



B



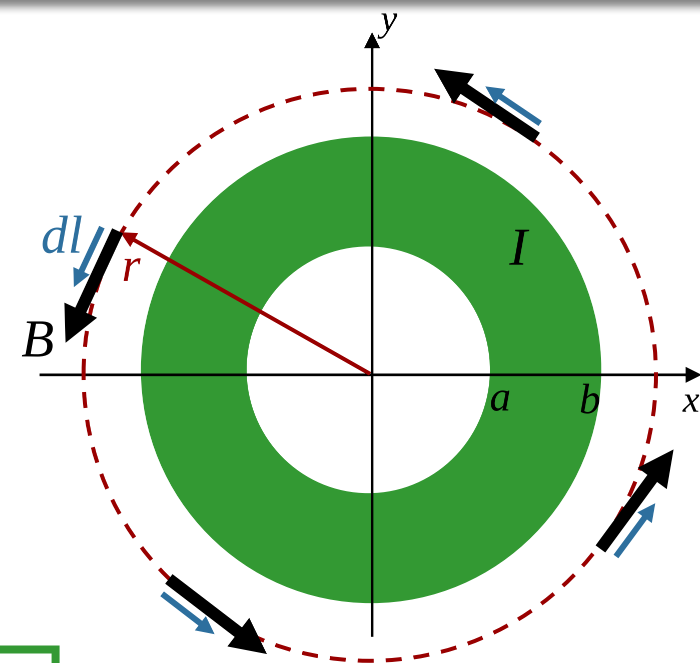
C

Example Problem

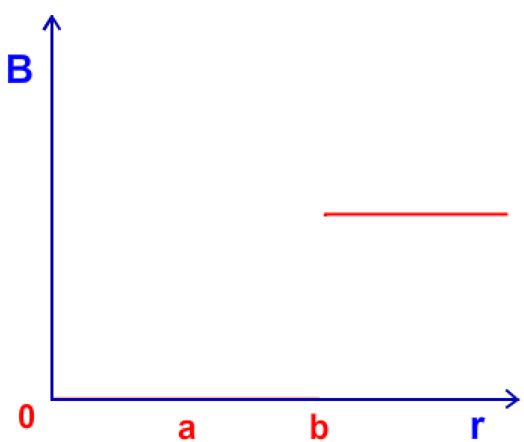
What does $|B|$ look like for $r > b$?

LHS: $\oint \vec{B} \bullet d\vec{\ell} = \oint B d\ell = B \oint d\ell = B \cdot 2\pi r$

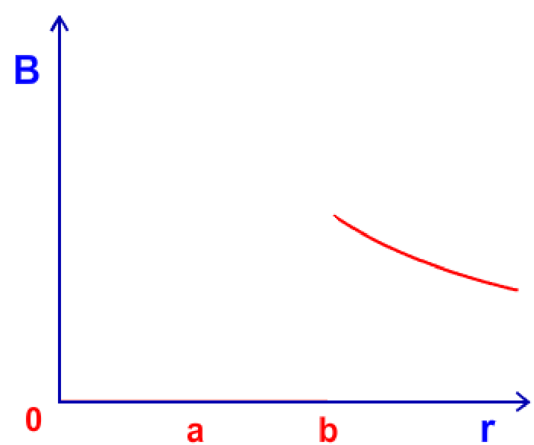
RHS: $I_{enclosed} = I$



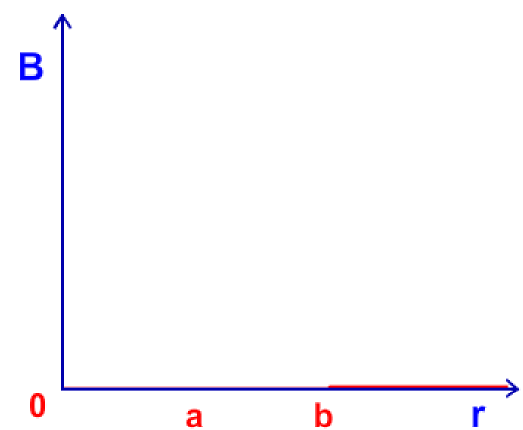
$$B = \frac{\mu_o I}{2\pi r}$$



A



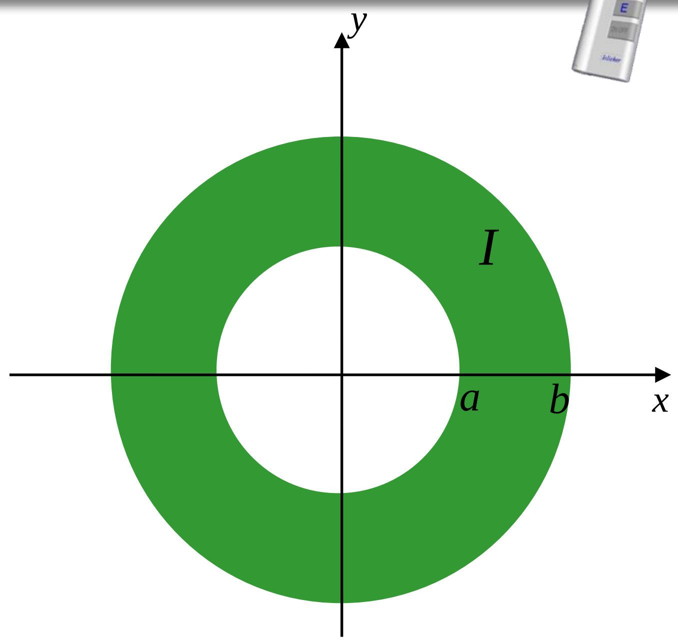
B



C

Example Problem

What is the current density j (Amp/m^2) in the conductor?



A) $j = \frac{I}{\pi b^2}$

B) $j = \frac{I}{\pi b^2 + \pi a^2}$

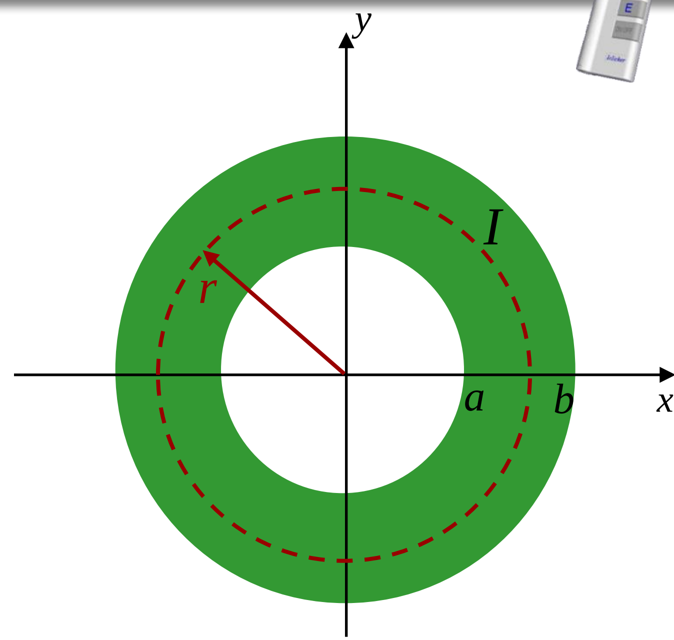
C) $j = \frac{I}{\pi b^2 - \pi a^2}$

$j = I / area$

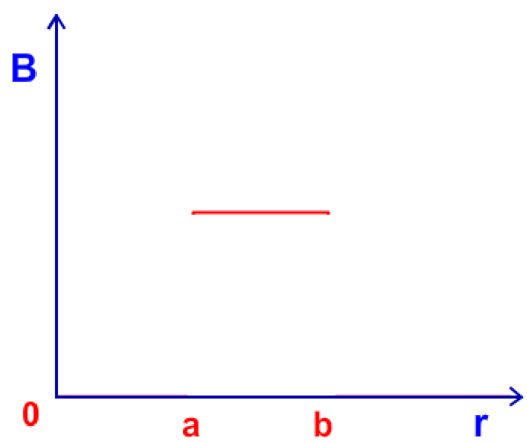
$area = \pi b^2 - \pi a^2$

$$j = \frac{I}{\pi b^2 - \pi a^2}$$

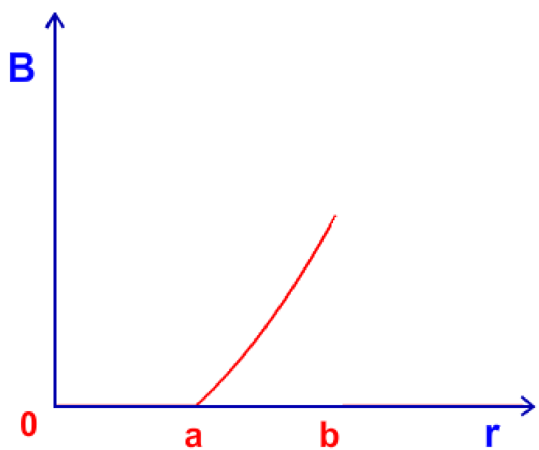
Example Problem



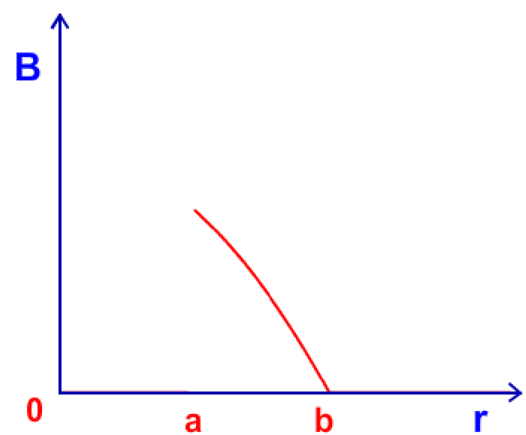
What does $|B|$ look like for $a < r < b$?



A



B

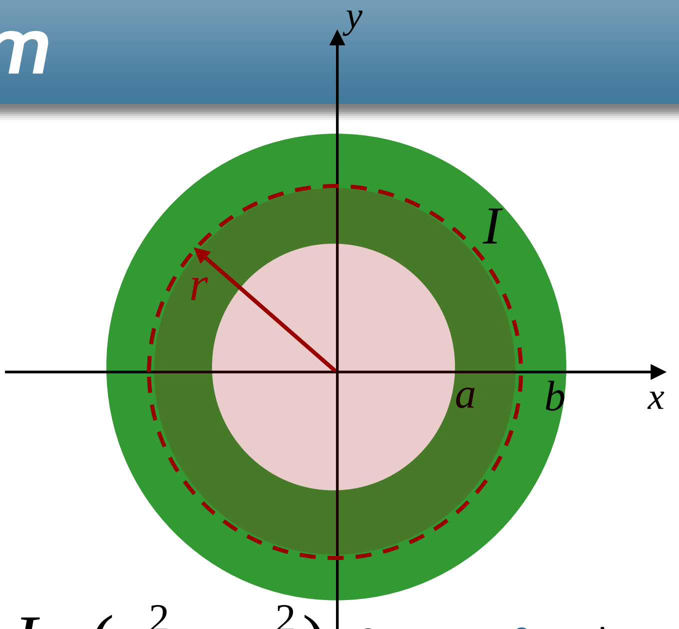


C

Example Problem

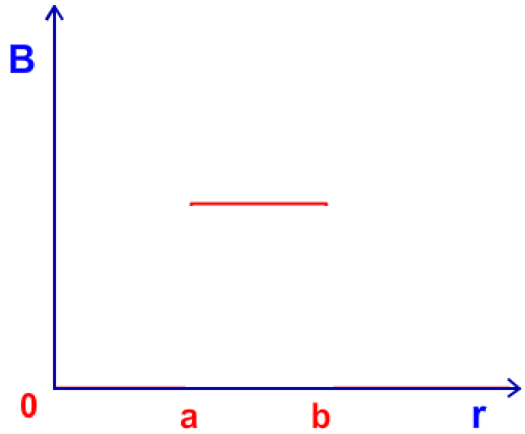
What does $|B|$ look like for $a < r < b$?

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \quad \rightarrow \quad B \cdot 2\pi r = \mu_o \cdot jA_{enc}$$

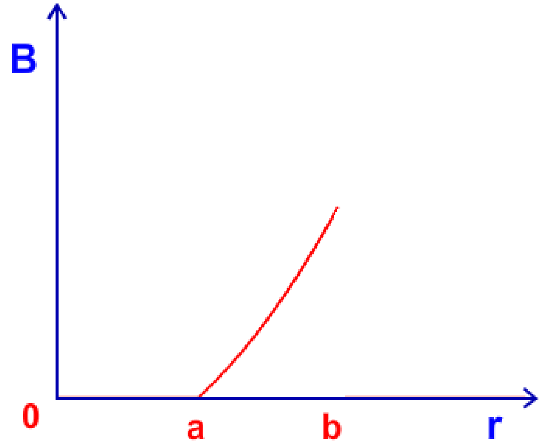


$$B \cdot 2\pi r = \mu_o \cdot \frac{I}{\pi(b^2 - a^2)} \cdot \pi(r^2 - a^2) \quad \rightarrow \quad B = \frac{\mu_o I}{2\pi r} \cdot \frac{(r^2 - a^2)}{(b^2 - a^2)}$$

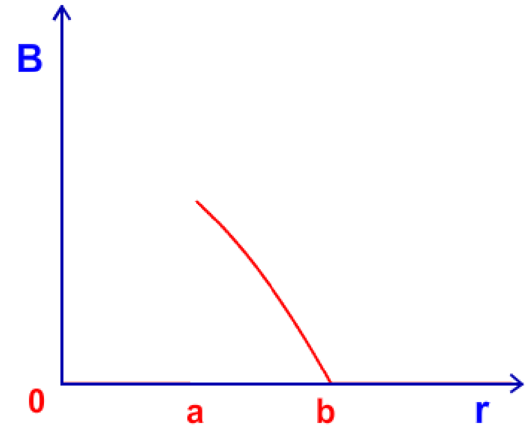
Starts at 0 and increases almost linearly



A



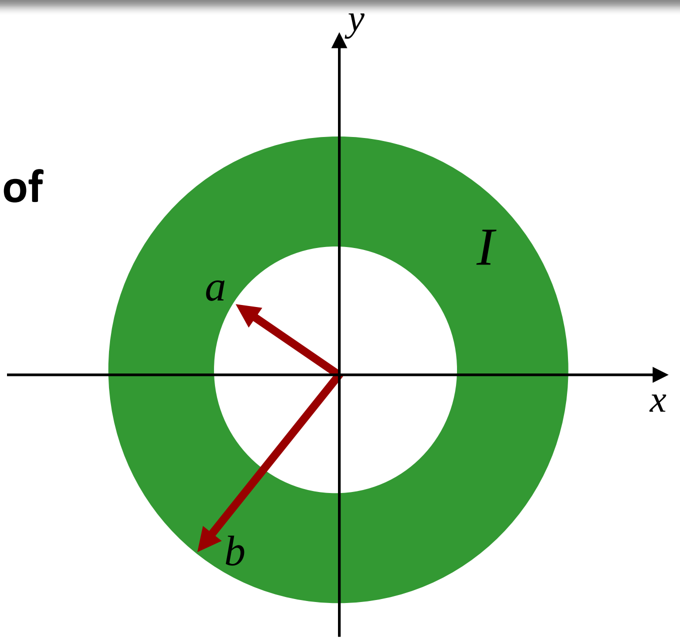
B



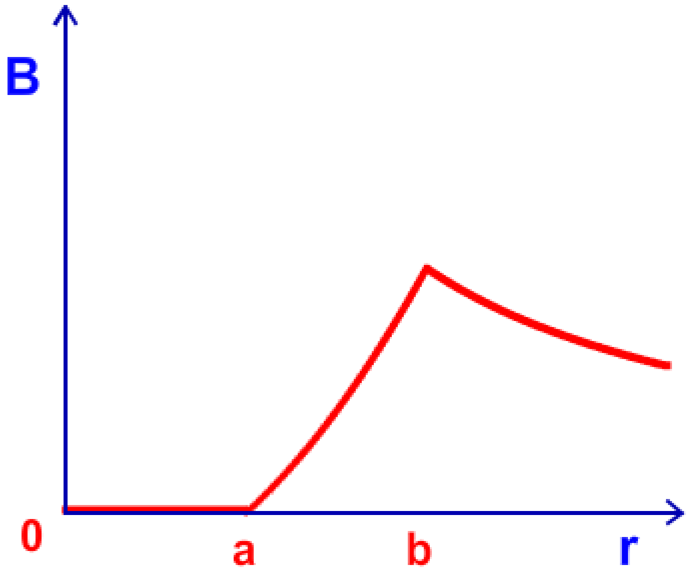
C

Example Problem

An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current **I out of the screen.**

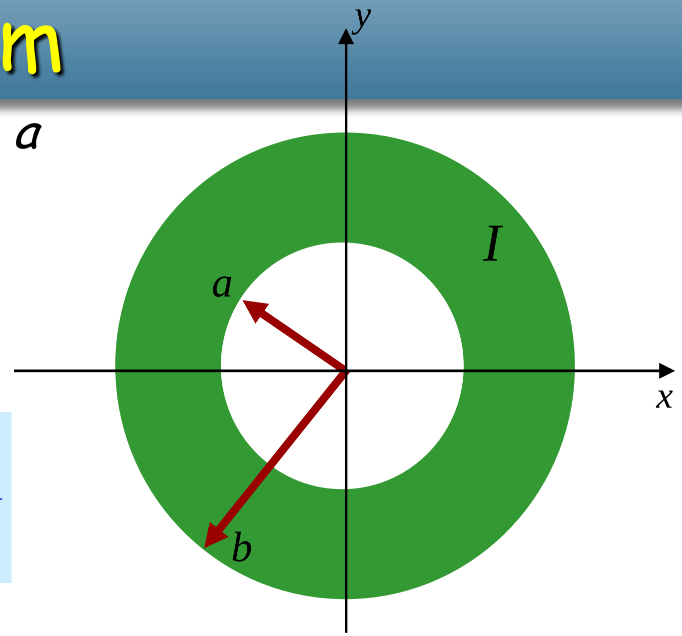


Sketch $|B|$ as a function of r.



Example Problem

An infinitely long cylindrical shell with inner radius *a* and outer radius *b* carries a uniformly distributed current I ***out of the screen***.

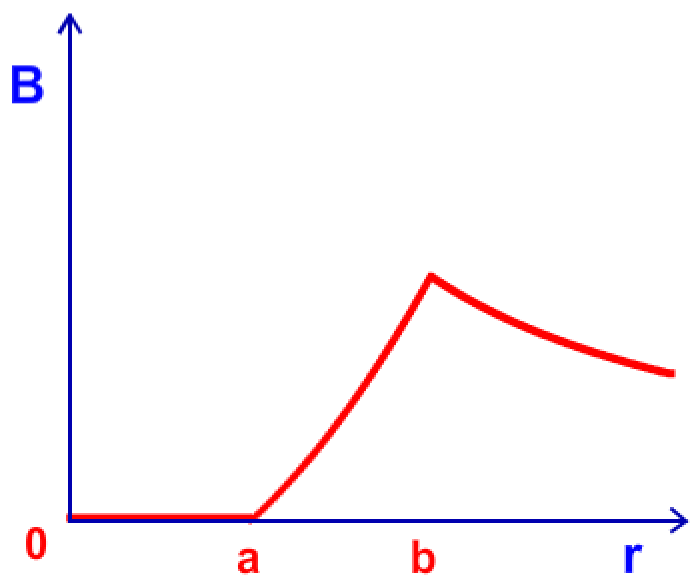


Sketch |B| as a function of *r*.

How big is B at *r* = *b*?

$$B = \frac{\mu_o I}{2\pi r} \cdot \frac{(r^2 - a^2)}{(b^2 - a^2)}$$

Let I = 10 A, b = 1 mm



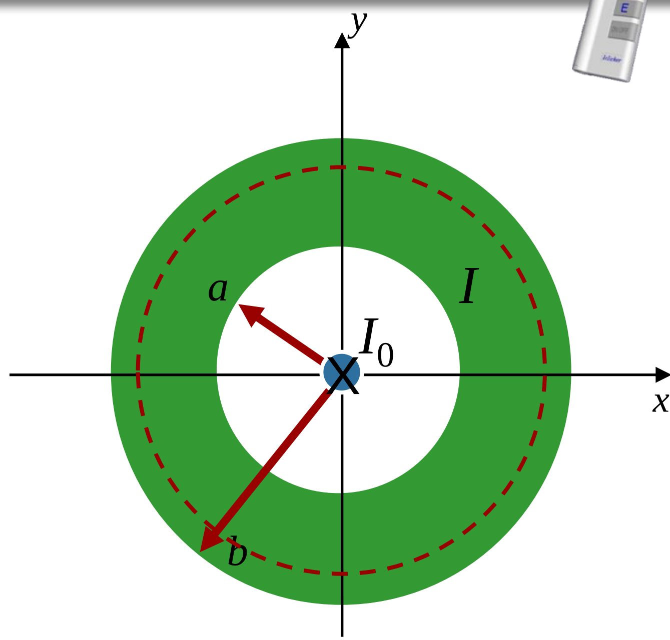
$$B(b) = \frac{\mu_o I}{2\pi b}$$

$$= \frac{4\pi x 10^{-7}\ \mathrm{Tm/A} \cdot 10\mathrm{A}}{2\pi \cdot 0.001\,\mathrm{m}}$$

$$= 2x10^{-3}\ \mathrm{T}$$

Follow-Up

Add an infinite wire along the z axis carrying current I_0.



What must be true about I_0 such that there is some value of r, $a < r < b$, such that $B(r) = 0$?

A) $|I_0| > |I|$ AND I_0 into screen

B) $|I_0| > |I|$ AND I_0 out of screen

C) $|I_0| < |I|$ AND I_0 into screen

D) $|I_0| < |I|$ AND I_0 out of screen

E) There is no current I_0 that can produce $B = 0$ there

B will be zero if total current enclosed $= 0$