

Your Comments

There were some parts of this prelecture I grasped well while other parts like the generator and the loops I still have trouble with.

Can you please clarify how Faraday's and Lenz's law are produced.

This stuff is alright. We all wish it could be Spring Break though, right?

In one of the equations we were solving, we ended up using dI/dt , and it brings up the question again of the relationship between the electric and magnetic fields. Will we have the chance to take a comprehensive look at Maxwell's equations?

I got so confused with Faraday's Law and Lenz's Law... AREN'T THESE TWO LAWS SUPPOSED TO MEAN THE SAME THING???

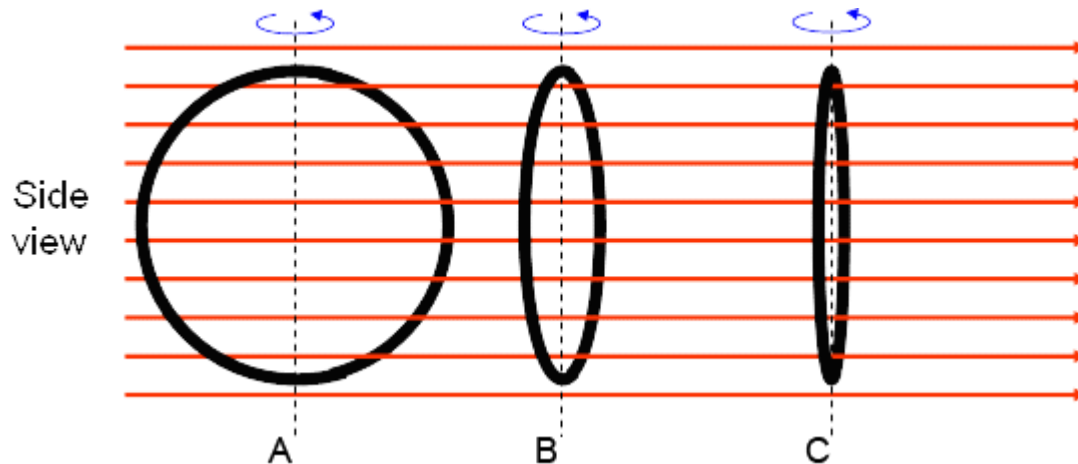
I'm still a little fuzzy on directions of EMF and flux. (Flux is same direction of magnetic field and EMF opposite?). I'm also having a tough time understanding the graphical representation of the EMF, flux and current.

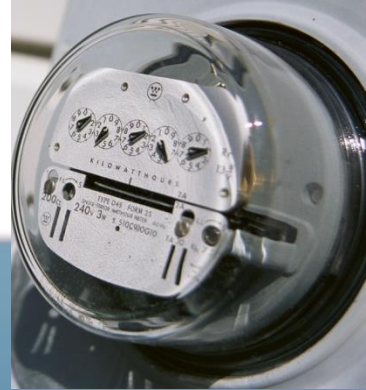
For Question 2 of the Prelecture: "Even though the flux through the loop is zero at this point, the rate of change of flux through the loop, $d\Phi/dt$, is greatest, resulting the greatest emf." What. Explain this, please. I see nothing in the prelecture that could have possibly given me the tools in order to figure out which orientation of theta has the greatest induced emf. Wouldn't it be natural to think that the induced emf would be smallest at this point, since $B \cdot A$ is equal to zero? Since the magnetic field and angular velocity are the same in all cases, wouldn't the only variable that changes with time have to be the cross-sectional area? This prelecture seems to be just a bunch of equations, but no concepts.

Prelecture: Faraday's Law

A circular wire loop is placed in a uniform magnetic field pointing to the right. The loop is rotated with *constant angular velocity* around a vertical axis (dashed line).

At which of the three times shown is the induced *emf* greatest?



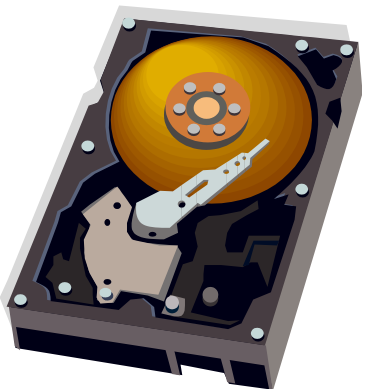


Physics 212

Lecture 17

Today's Concept:

Faraday's Law



$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$



Faraday's Law

Faraday's Law:

$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

Looks scary but it's not – its amazing and beautiful!



A changing magnetic flux produces an electric field.

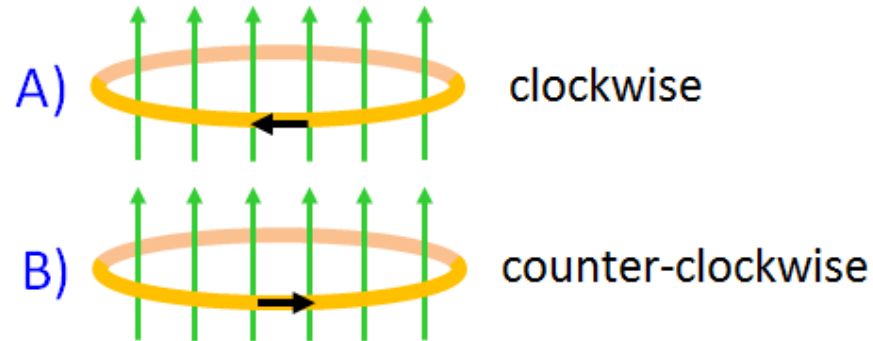


Electricity and magnetism are deeply connected.

Checkpoint 1



Suppose a current flows in a horizontal conducting loop in such a way that the magnetic flux produced by this current points upward.



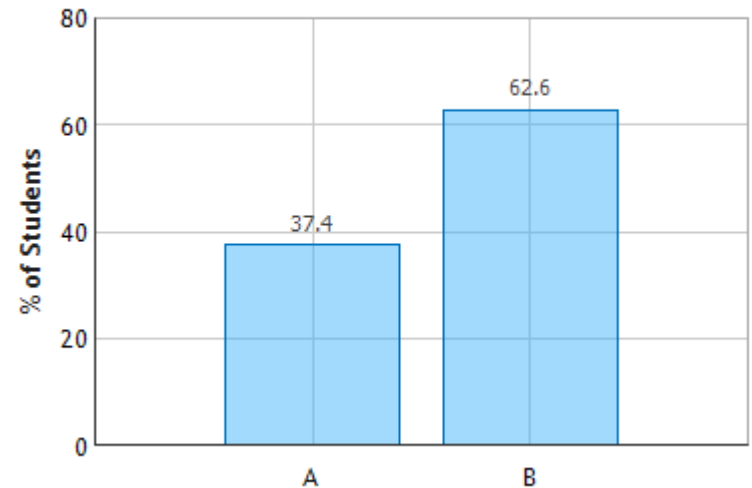
As viewed from above, in which direction is this current flowing?

A. clockwise

B. counterclockwise

Right hand rule. B field created by current points upwards.

Loop of Current: Question 1 (N = 821)



Faraday's Law:

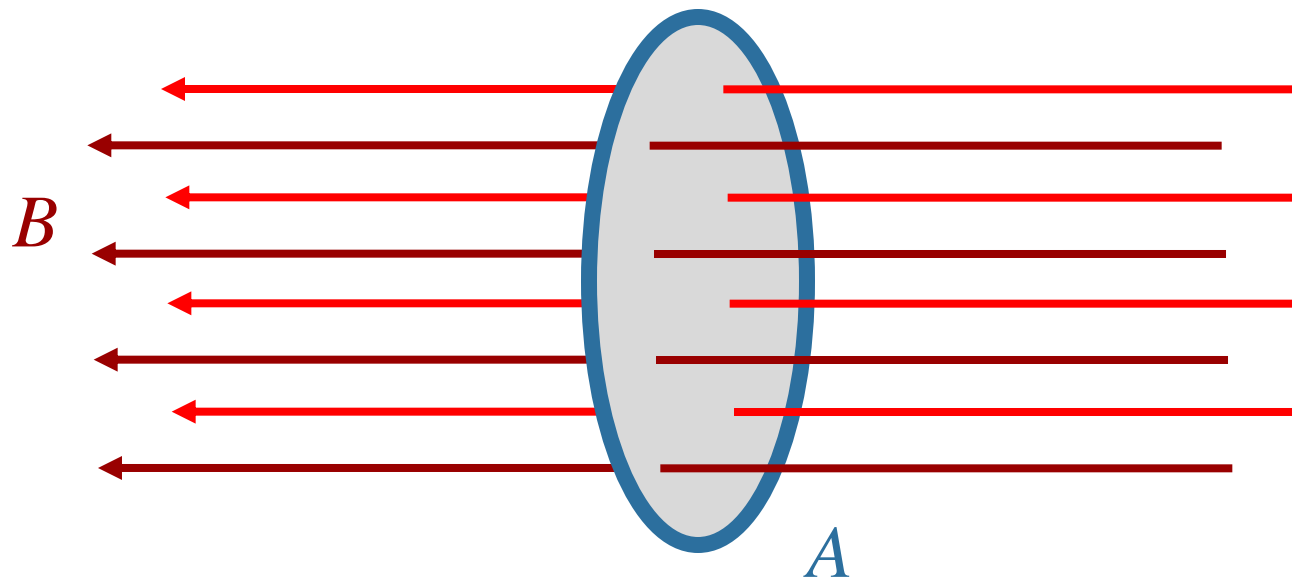
$$emf = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

where

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In Practical Words:

1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.



Think of Φ_B as the number of field lines passing through the surface

There are many ways to change this...

Faraday's Law:

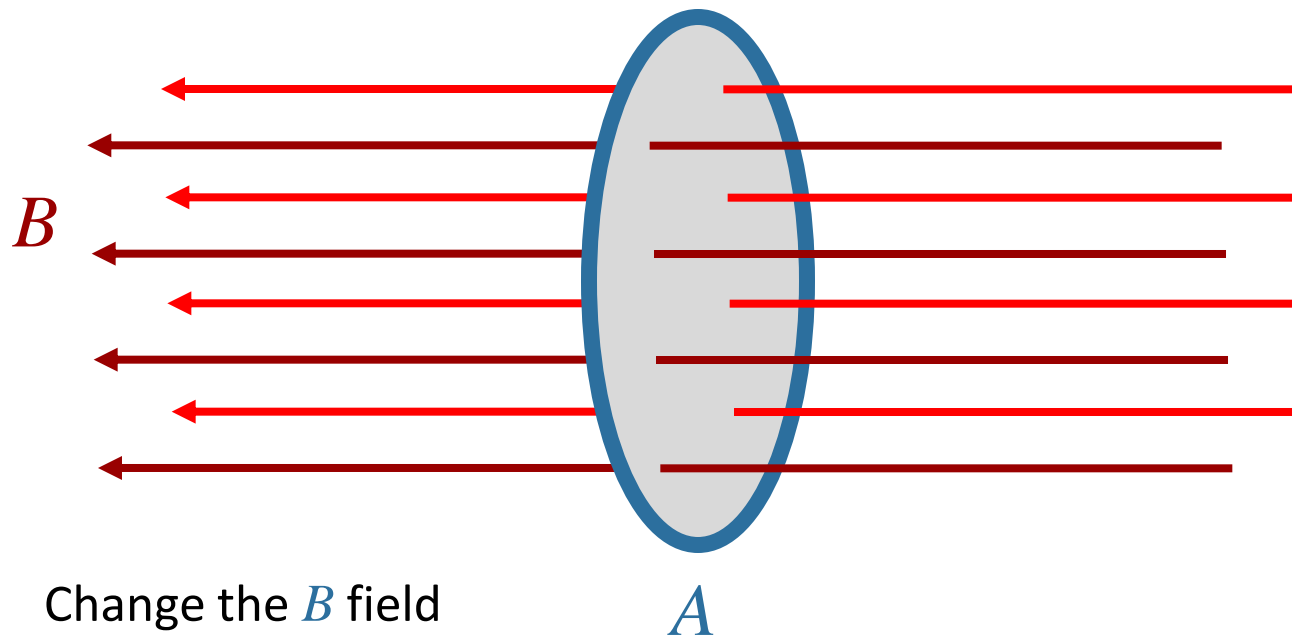
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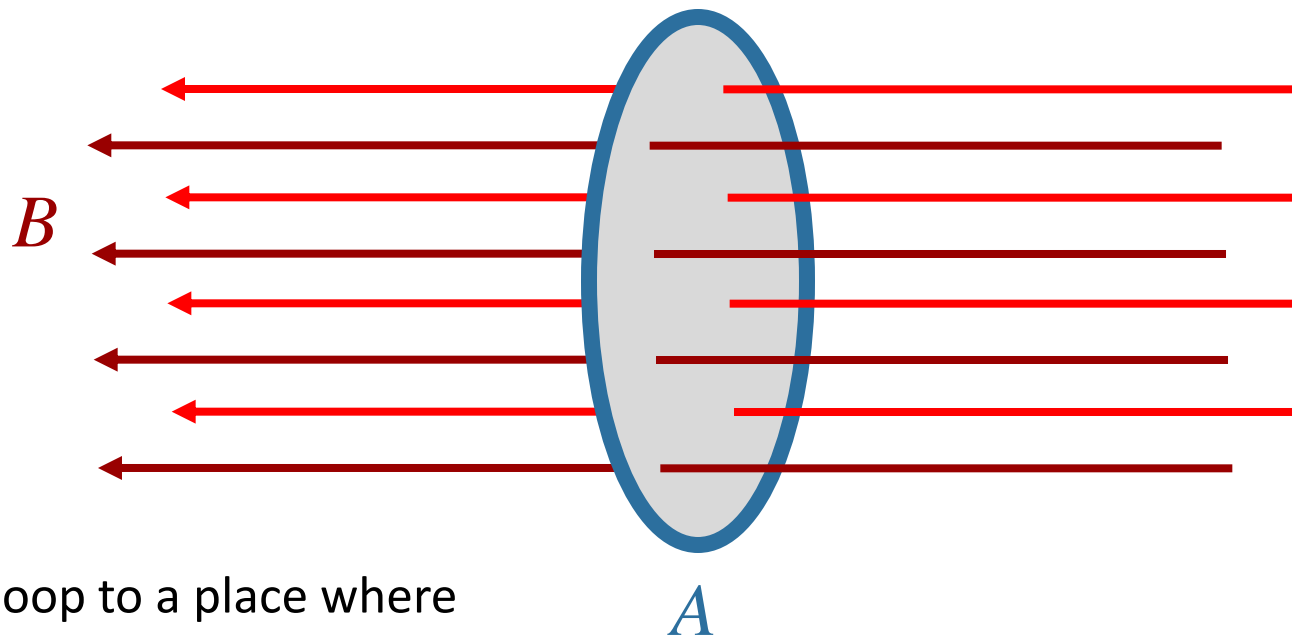
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Move loop to a place where the B field is different

Faraday's Law:

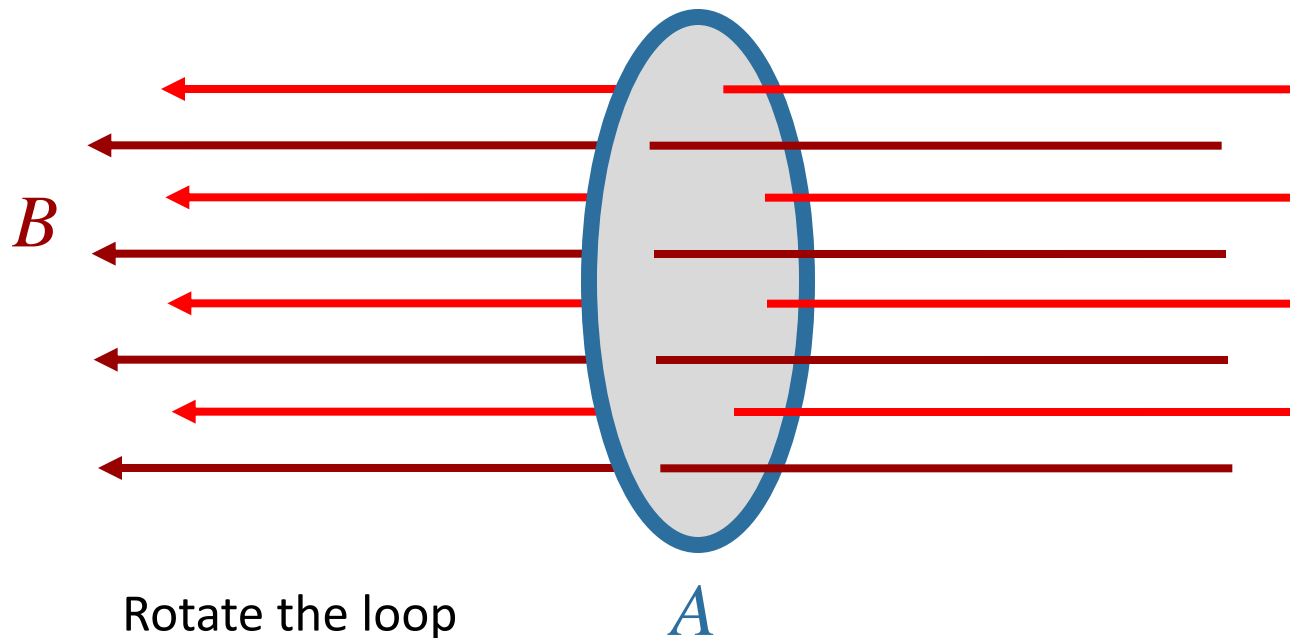
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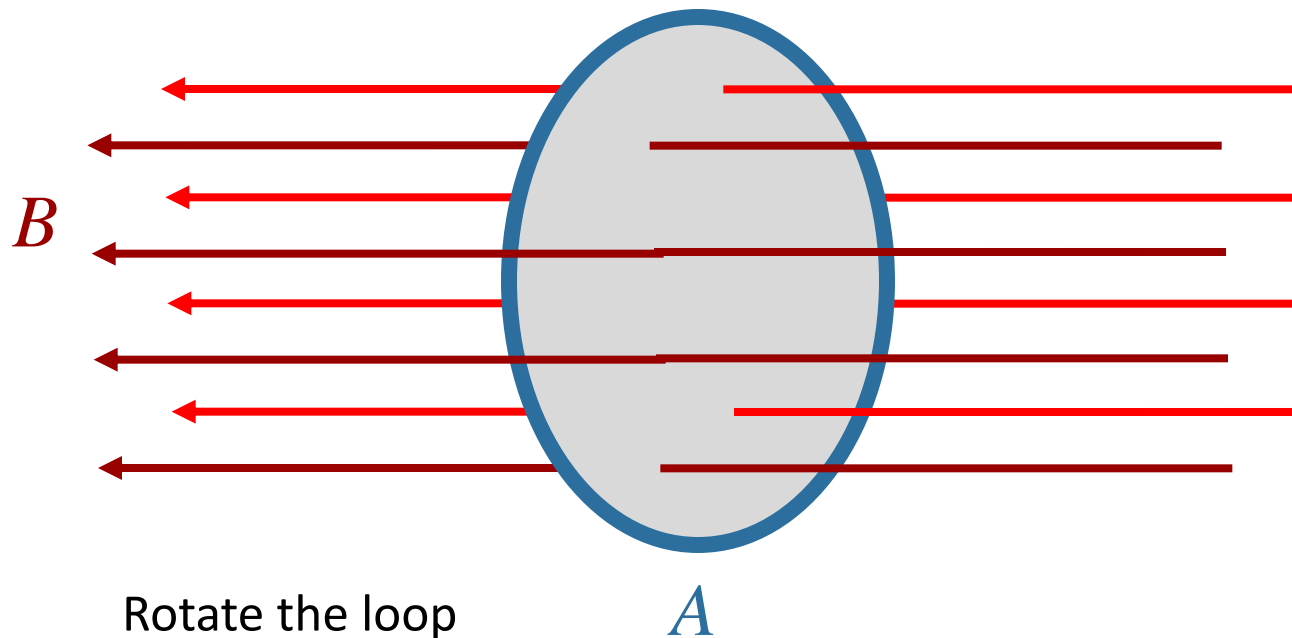
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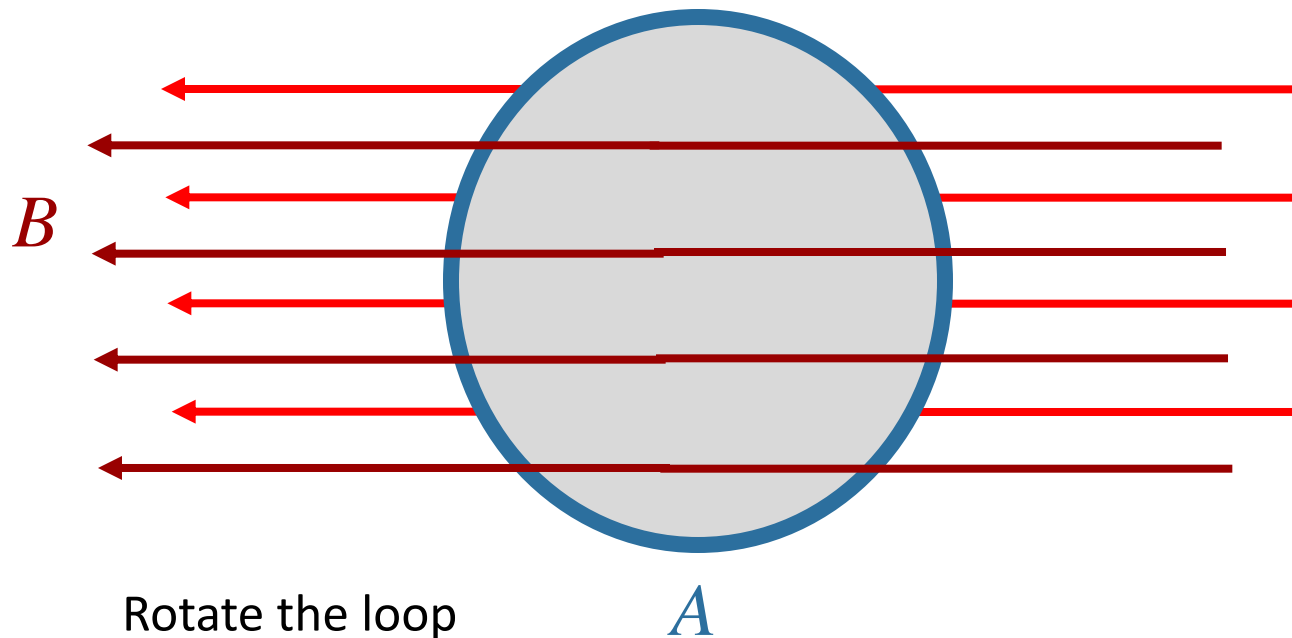
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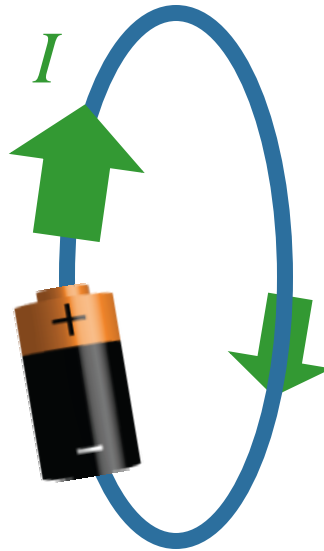
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In Practical Words:

- 1) When the flux Φ_B through a loop changes, an *emf* is induced in the loop.
- 2) The *emf* will make a current flow if it can (like a battery).



Demo
Coil and magnet

Faraday's Law:

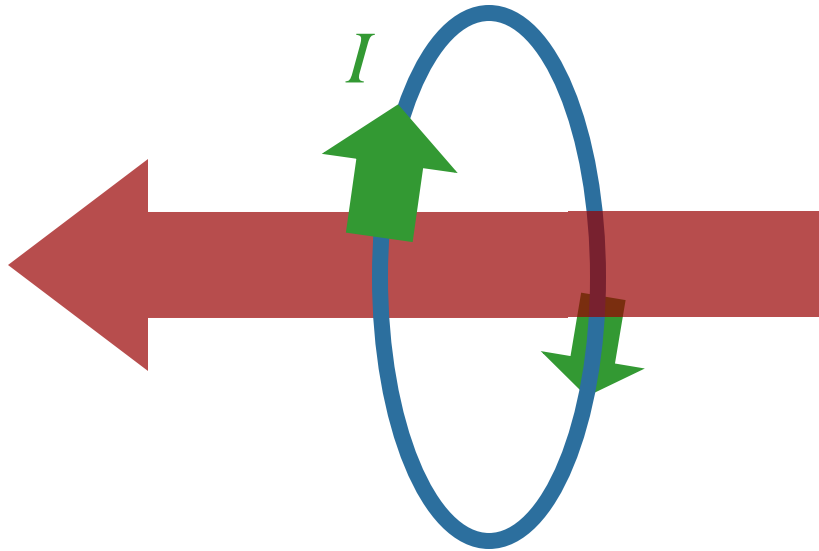
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- 3) The current that flows induces a new magnetic field.



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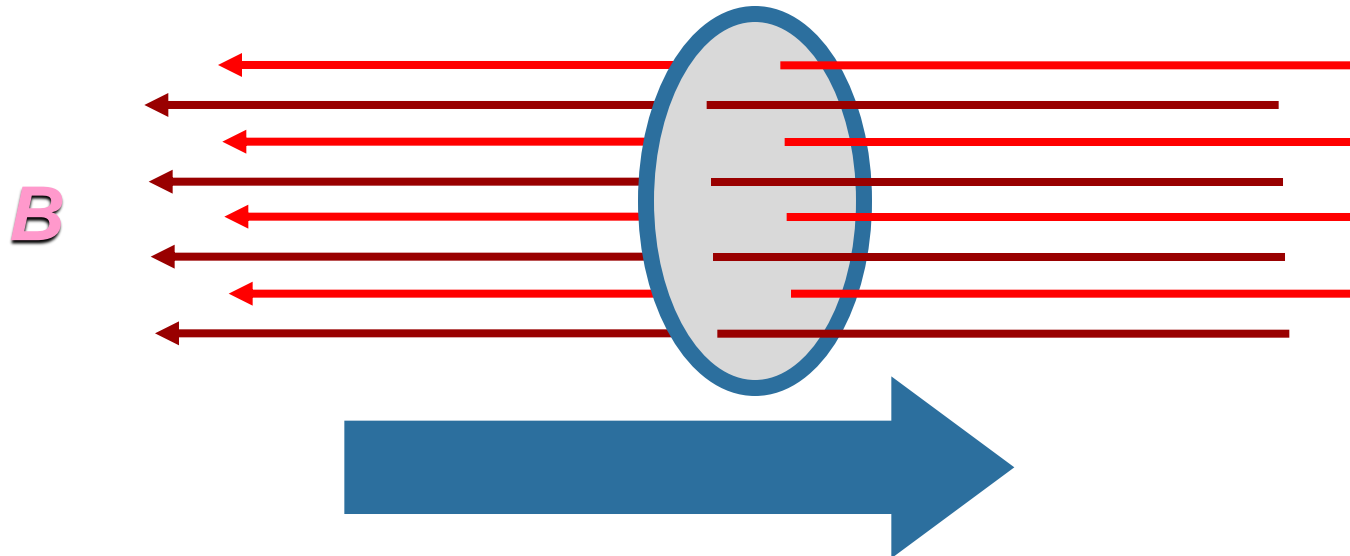
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- 4) The new magnetic field opposes the change in the original magnetic field that created it. (Lenz' Law)



Faraday's Law:

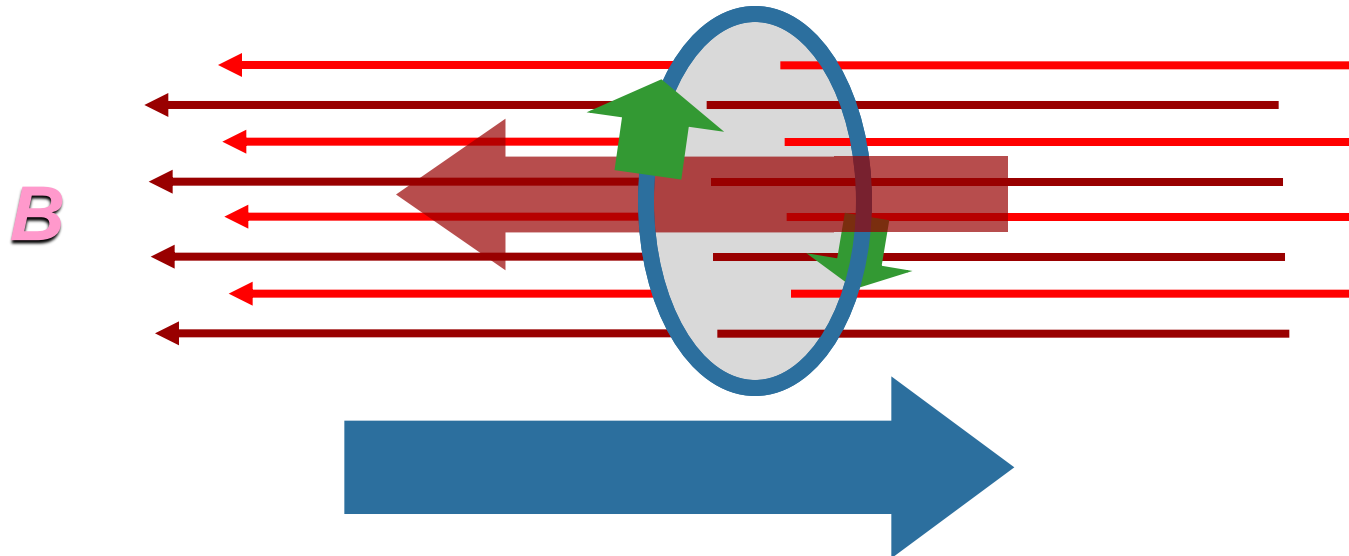
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Demo

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Executive Summary:

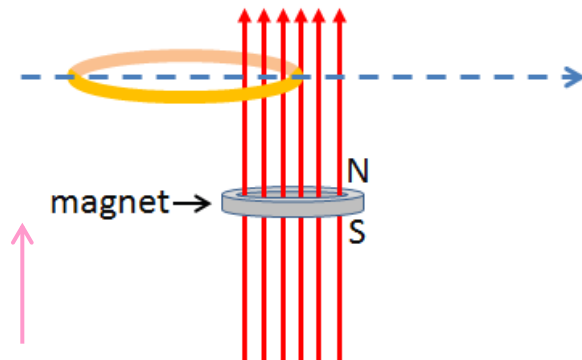


$emf \rightarrow$ current \rightarrow field a) induced **only** when **flux is changing**
b) **opposes the change**

Checkpoint 2

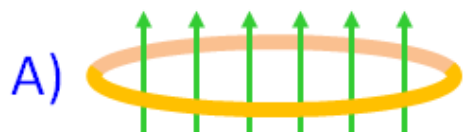


A magnet makes the vertical magnetic field shown by the red arrows. A horizontal conducting loop is entering the field as shown.



Direction of positive flux

At the instant shown above, what is the direction of the additional flux produced by the current induced in the loop?



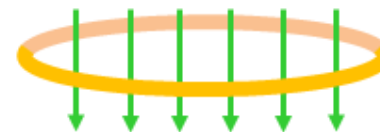
up

B)

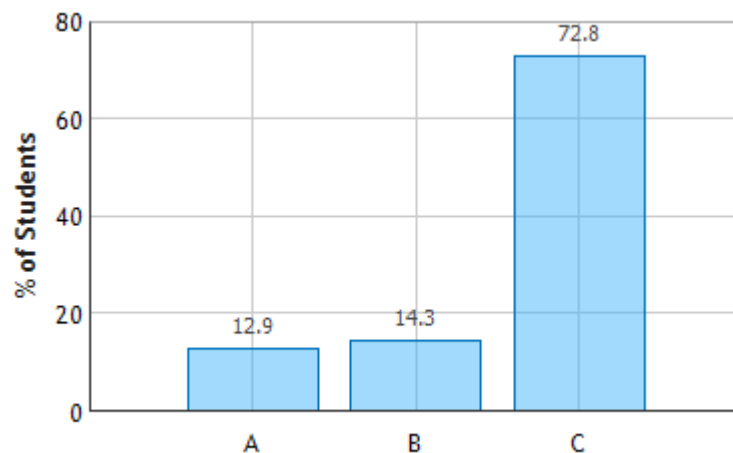


zero

C)



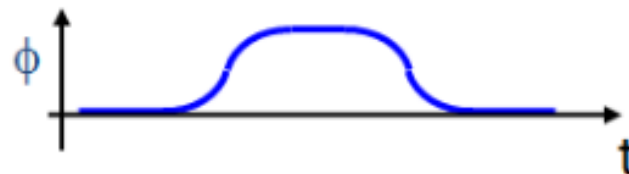
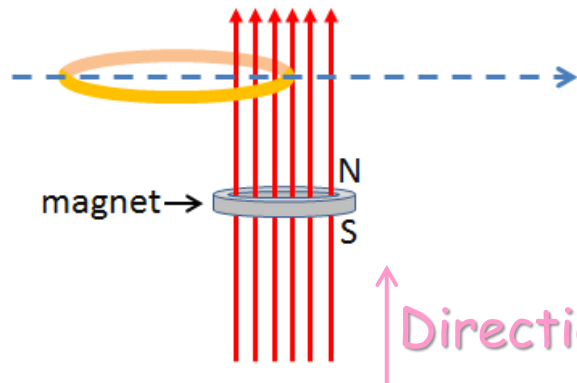
down



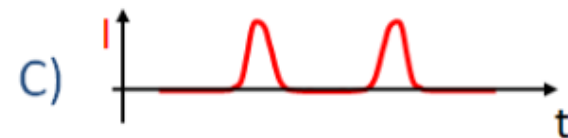
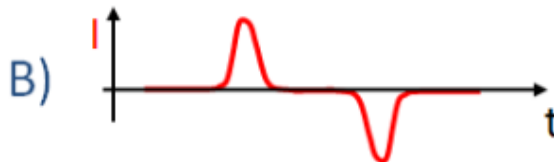
Checkpoint 3



A magnet makes the vertical magnetic field shown by the red arrows. A horizontal conducting loop is entering the field as shown.



The upward flux through the loop as a function of time is shown by the blue trace. Which of the red traces below it best represents the current induced in the loop as a function of time as it passes over the magnet? (Positive means counter-clockwise as viewed from above):

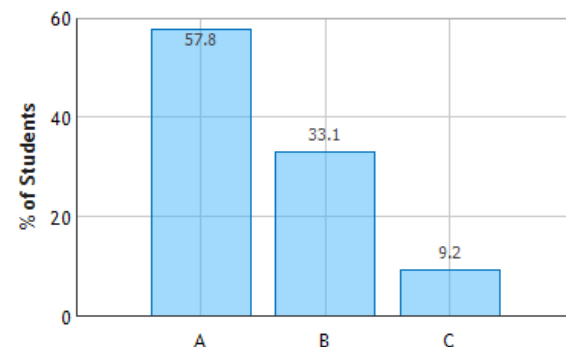


Flux is changing!

Induced flux is initially negative (opposing increasing positive flux – last checkpoint)

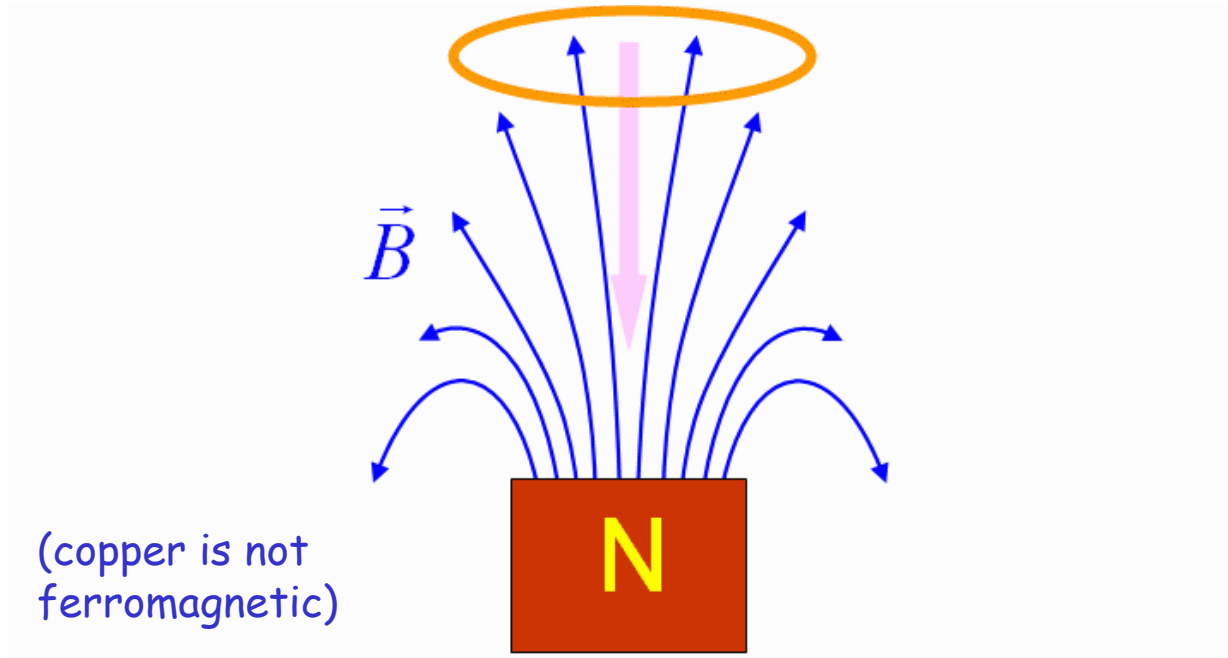
THEREFORE, initial *induced current* must be CW as viewed from above

- Current direction from right-hand rule 😊



Cool Example

A horizontal copper ring is dropped from rest directly above the north pole of a permanent magnet



As the ring falls, in which direction will the induced field point?

- A.** up **B. down** **C.** No induced field

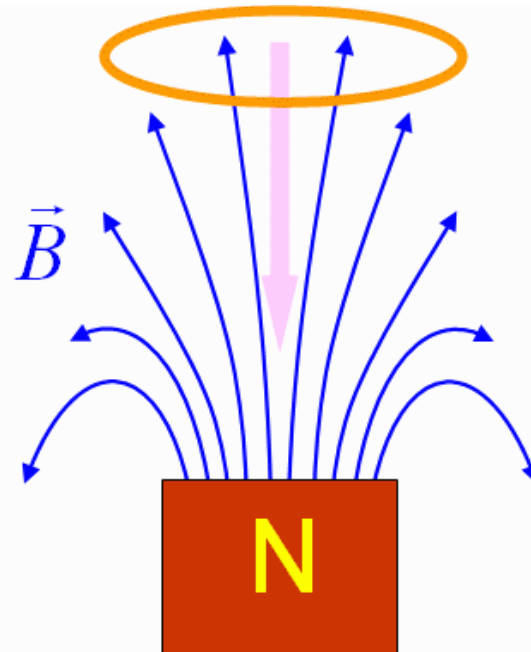
Choose direction for positive flux to be in same direction as external field (easiest), then

Increase in flux from external field is opposed by induced flux with opposite sign

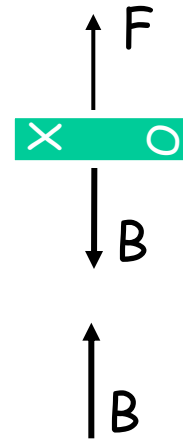
Cool Example



A horizontal copper ring is dropped from rest directly above the north pole of a permanent magnet



(copper is not
ferromagnetic)



Like poles repel

→ $F_{\text{total}} < mg$

→ $a < g$

Will the acceleration a of the falling ring in the presence of the magnet be any different than it would have been under the influence of just gravity (i.e. g)?

A. $a > g$

B. $a = g$

C. $a < g$

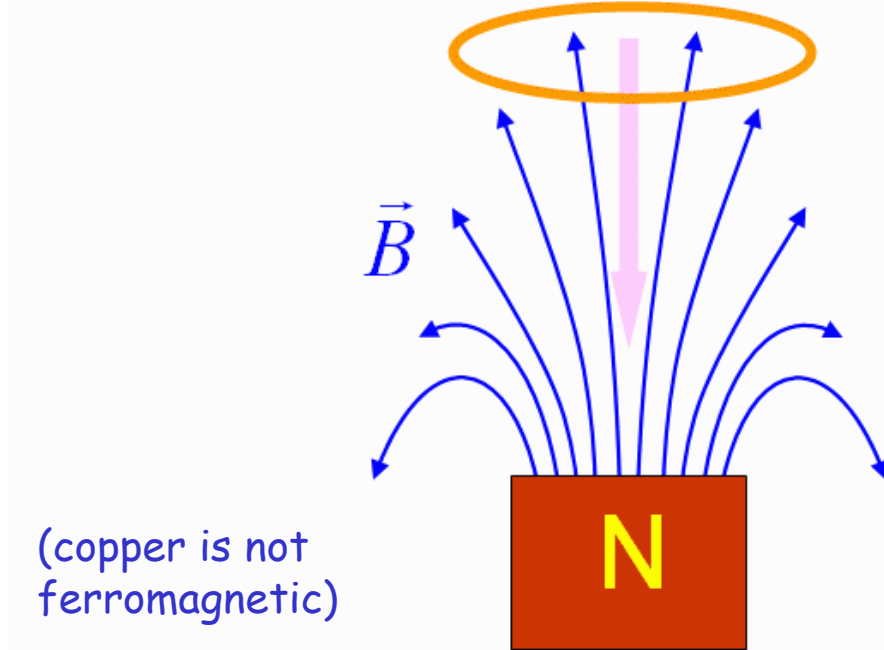
This one is hard !

Upward B field increases as loop falls

Clockwise current (viewed from top) is induced

Cool Example

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B field increases upward as loop falls

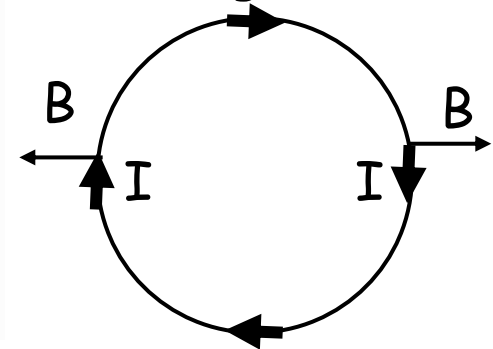
Clockwise current (viewed from top) is induced

Main Field produces horizontal forces

"Fringe" Field produces vertical force

HOW
IT
WORKS

Looking down



$\vec{I} \times \vec{B}$ points UP

$\vec{F}_{\text{total}} < mg$

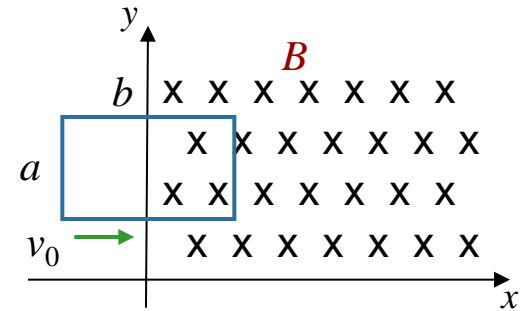
$a < g$

Demo !

dropping magnets
e-m cannon

Calculation

A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the direction and the magnitude of the force on the loop when half of it is in the field?

Conceptual Analysis

Once loop enters B field region, flux will be changing in time
Faraday's Law then says emf will be induced

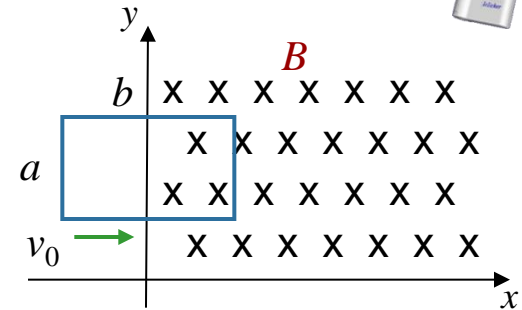
Strategic Analysis

- Find the emf
- Find the current in the loop
- Find the force on the current

Calculation

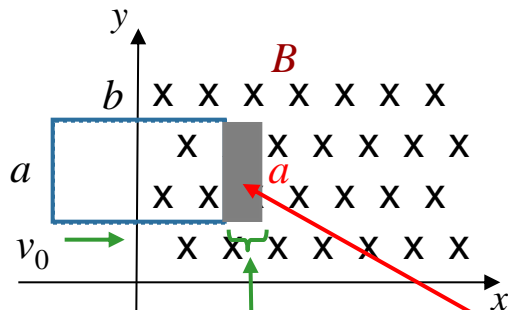


A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



$$emf = -\frac{d\Phi_B}{dt}$$

- A) $\varepsilon = Babv_0^2$ B) $\varepsilon = \frac{1}{2} Bav_0$ C) $\varepsilon = \frac{1}{2} Bbv_0$ **D) $\varepsilon = Bav_0$** E) $\varepsilon = Bbv_0$



In a time dt
it moves by $v_0 dt$

The area in field
changes by $dA = v_0 dt a$

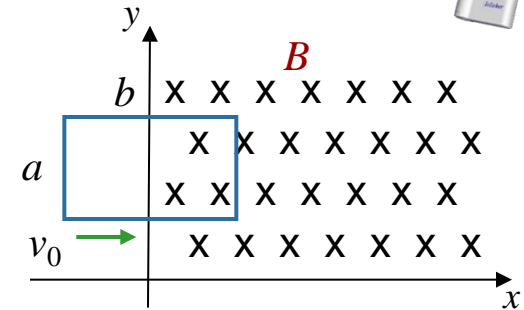
→ Change in Flux = $d\Phi_B = BdA = Bav_0 dt$

→
$$\frac{d\Phi_B}{dt} = Bav_0$$

Calculation



A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the direction of the current induced in the loop just after it enters the field?

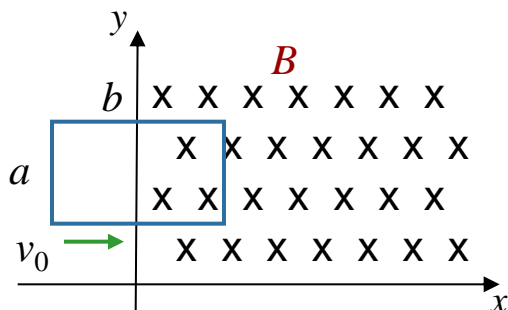
$$\text{emf} = -\frac{d\Phi_B}{dt}$$

A) clockwise

B) counterclockwise

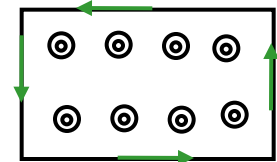
C) no current is induced

emf is induced in direction to oppose the change in flux that produced it



Flux is increasing into the screen

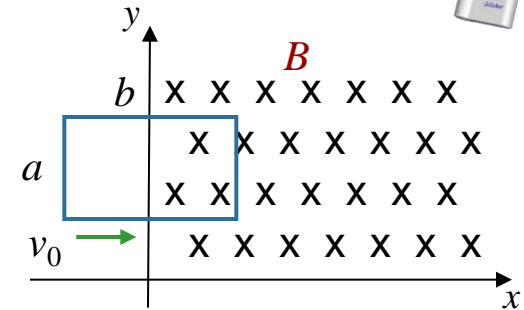
Induced emf produces flux out of screen



Calculation



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$$emf = -\frac{d\Phi_B}{dt}$$

What is the direction of the net force on the loop just after it enters the field?

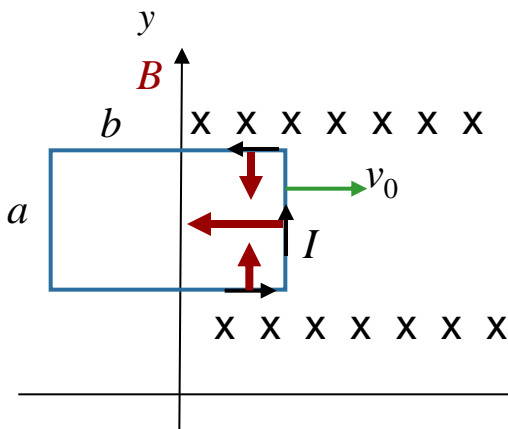
A) $+y$

B) $-y$

C) $+x$

D) $-x$

Force on a current in a magnetic field: $\vec{F} = I\vec{L} \times \vec{B}$



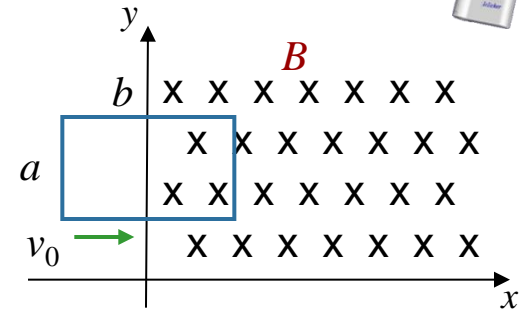
Force on top and bottom segments cancel (red arrows)

Force on right segment is directed in $-x$ direction.

Calculation



A rectangular loop (height = a , length = b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



What is the magnitude of the net force on the loop just after it enters the field?

$$\vec{F} = I\vec{L} \times \vec{B} \quad \varepsilon = Bav_0 \quad \text{emf} = -\frac{d\Phi_B}{dt}$$

A) $F = 4aBv_0R$

B) $F = a^2Bv_0R$

C) $F = a^2B^2v_0^2 / R$

D) $F = a^2B^2v_0 / R$

$\vec{F} = I\vec{L} \times \vec{B} \quad \rightarrow \quad F = ILB \quad \text{since} \quad \vec{L} \perp \vec{B}$

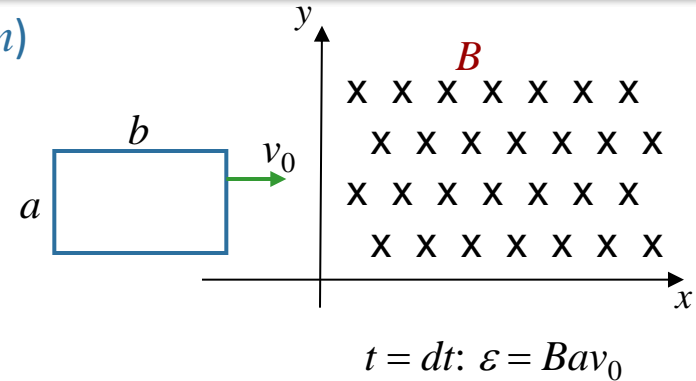
$I = \frac{\varepsilon}{R} = \frac{Bav_0}{R} \quad \rightarrow \quad F = \left(\frac{Bav_0}{R} \right) aB = \frac{B^2a^2v_0}{R}$

$\uparrow \uparrow$
 ILB

Follow Up

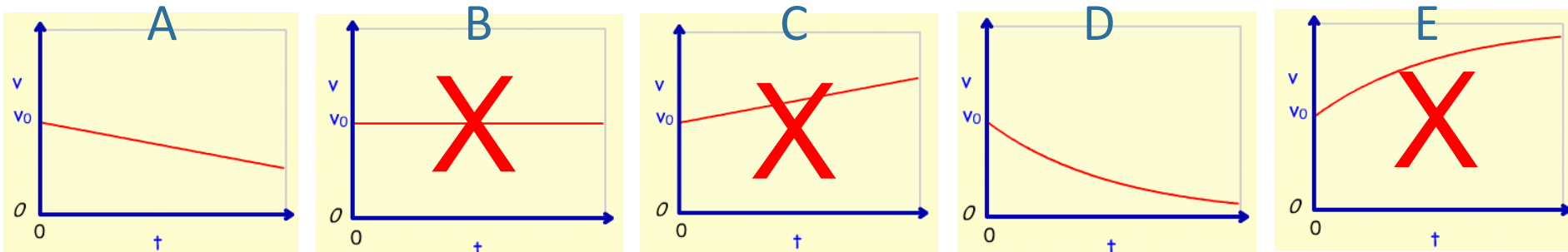


A rectangular loop (sides = a, b , resistance = R , mass = m) coasts with a constant velocity v_0 in $+x$ direction as shown. At $t = 0$, the loop enters a region of constant magnetic field B directed in the $-z$ direction.



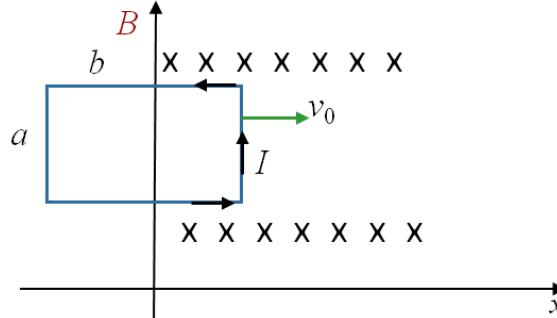
What is the velocity of the loop when half of it is in the field?

Which of these plots best represents the velocity as a function of time as the loop moves from entering the field to halfway through?



This is not obvious, but we know v must decrease

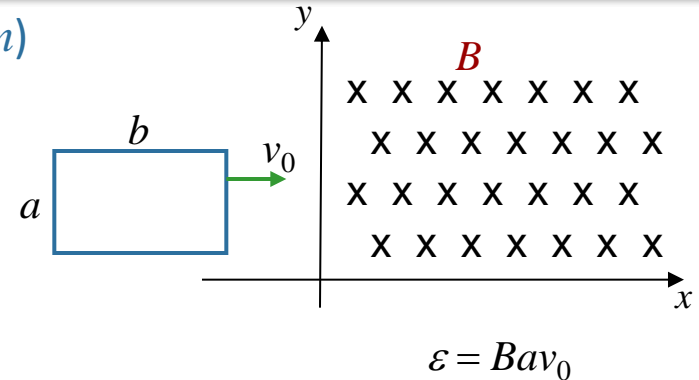
Why?



F_{right} points to left
Acceleration negative
Speed must decrease

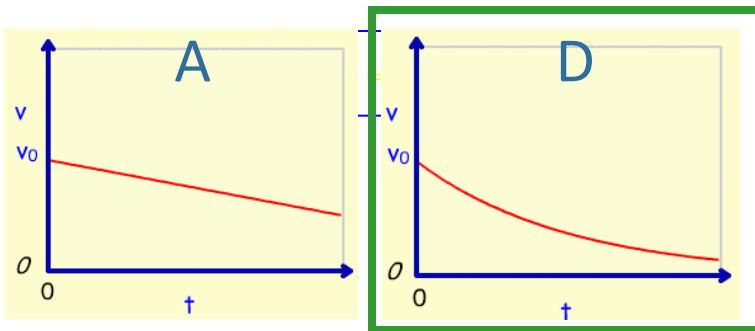
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Why D, not A?

F is not constant, depends on v

$$F = -\frac{a^2 B^2 v}{R} = m \frac{dv}{dt} \quad \longrightarrow \quad v = v_0 e^{-\alpha t}$$

where $\alpha = \frac{a^2 B^2}{mR}$

Challenge: Look at energy



Claim: The decrease in kinetic energy of loop is equal to the energy dissipated as heat in the resistor. Can you verify?