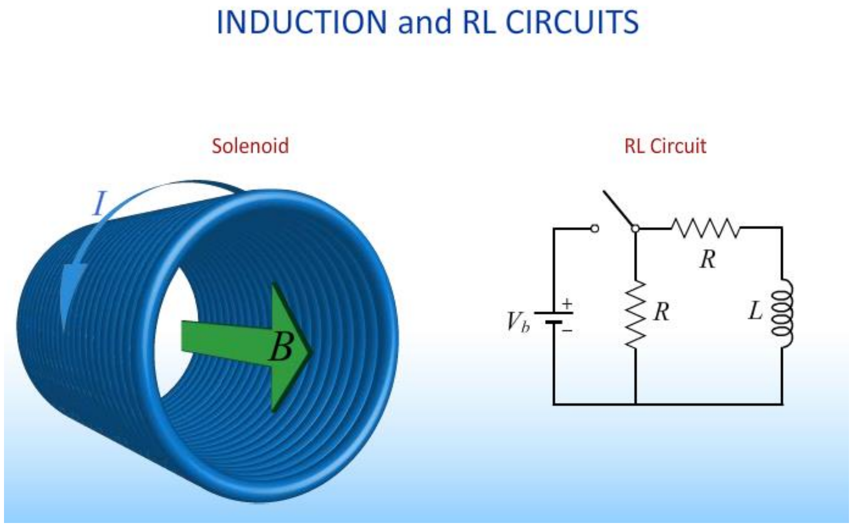
Physics 212
Lecture 18



Today's Concepts:

A) Induction

B) RL Circuits

Hour Exam 2 is April 2nd
Lectures 9-18

Your Comments

I think I understand the basics of how inductors work, but I don't understand the purpose. If they just store energy like a capacitor, what is the advantage of inductors versus capacitors?

I understand that it is difficult to teach a large lecture, but could you please control the amount of talking that goes on in the lecture? I cannot focus on what you are teaching when everyone sitting around me is having their own conversation. Thank You.

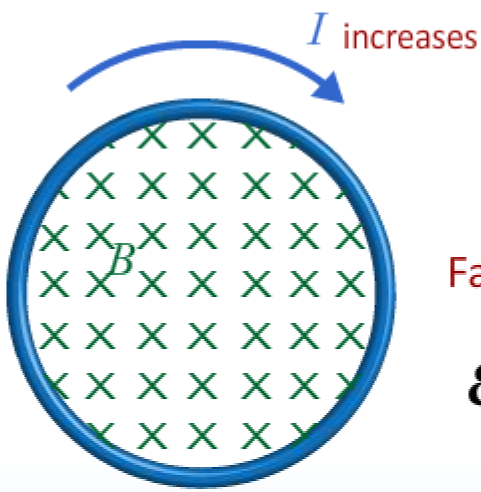
The similarities between these circuits and the RC circuits are easy to understand, but is there another reason for the spike in the voltage across the inductor when the switch is moved besides KVL? For instance, the voltage across the capacitors could be accounted for by the charge on each one of them. Is there a similar explanation for an inductor?

Pretty new stuff pretty hard

This class introduces so many new concepts that I've never heard of. What is self-inductance and why is it important?

With the exam coming up, I didn't concentrate much on this prelecture- we might need some review when we get back from break...

From the Prelecture: Self Inductance



Self-Inductance

$$L \equiv \frac{\Phi_B}{I}$$

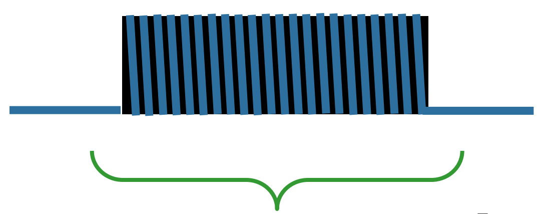
SI Unit

$$H = T\text{-}m^2/A$$

Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

Wrap a wire into a coil to make an “inductor”…



$$\mathcal{E} = -L\frac{dI}{dt}$$

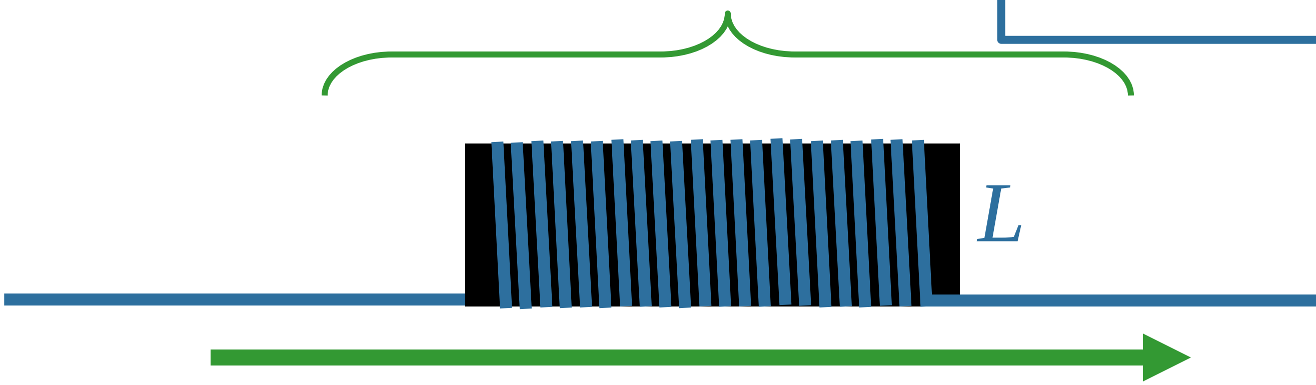
What this really means:

emf induced across L tries to keep I constant.

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

Short Term $I_{before} = I_{after}$

Long Term $V_L = 0$



Inductors prevent discontinuous current changes!

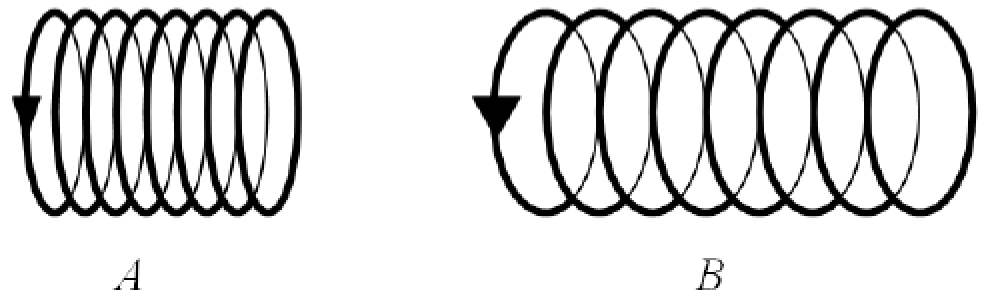
It's like inertia!

CheckPoint 2

Two solenoids are made with the same cross sectional area and total number of turns. Inductor *B* is twice as long as inductor *A*

$$L_B = \mu_0 n^2 \pi r^2 z$$

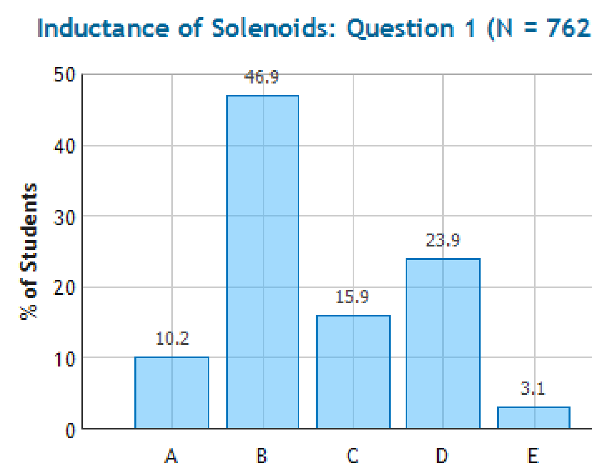
$(1/2)^2$ 2



$$\longrightarrow \quad L_B = \tfrac{1}{2} L_A$$

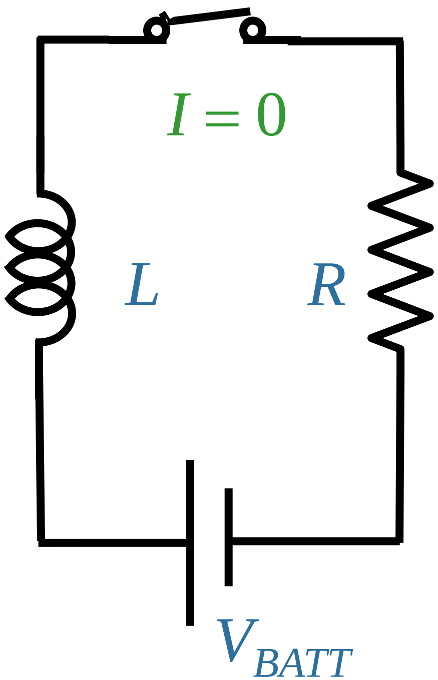
Compare the inductance of the two solenoids

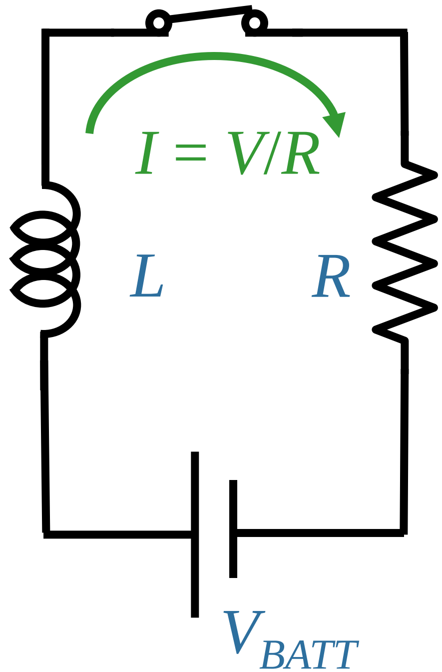
A) $L_A = 4\ L_B$
B) $L_A = 2\ L_B$
C) $L_A = L_B$
D) $L_A = (1/2)\ L_B$
E) $L_A = (1/4)\ L_B$

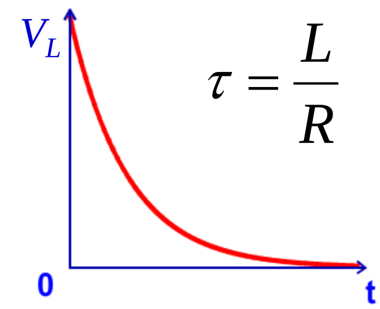


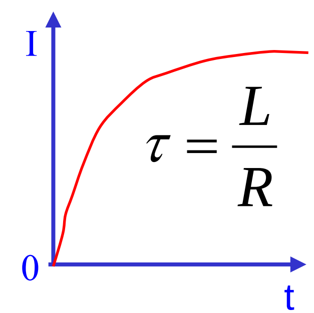
How to think about RL circuits Episode 1:

When no current is flowing initially:









At $t = 0$: I_L unchanged

$I_L = 0$
$V_R = 0$
$V_L = V_{BATT}$
(L is like an open circuit)

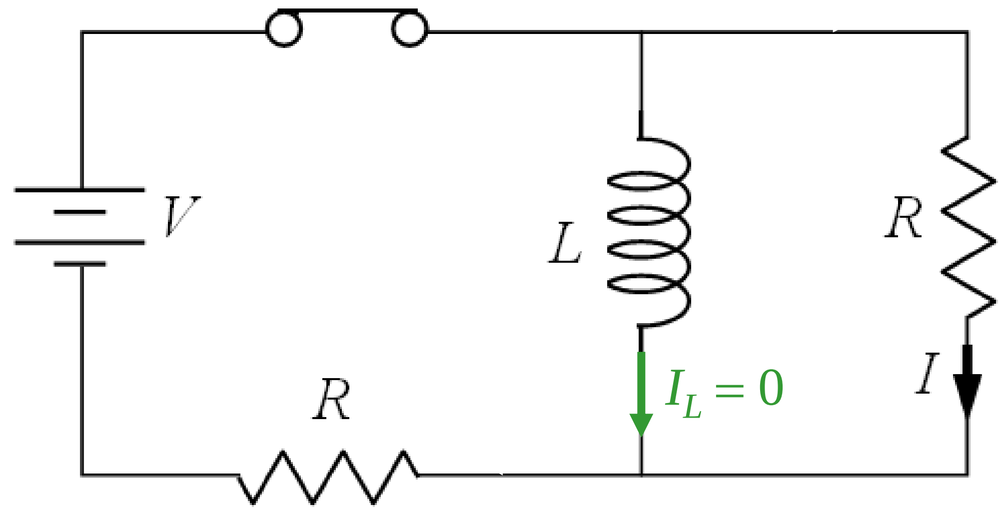
At $t >> L/R$: $V_L = 0$

$V_L = 0$
$V_R = V_{BATT}$
$I = V_{BATT}/R$
(L is like a wire)

CheckPoint 2a

In the circuit, the switch has been open for a long time, and the current is zero everywhere.

At time $t = 0$ the switch is closed.



What is the current I through the vertical resistor immediately after the switch is closed?

(+ is in the direction of the arrow)

A) $I = V/R$

B) $I = V/2R$

C) $I = 0$

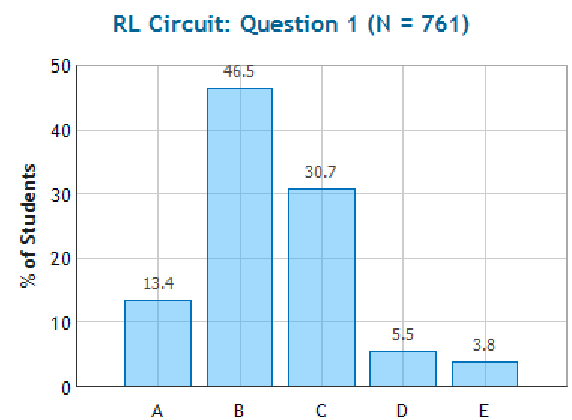
D) $I = -V/2R$

E) $I = -V/R$

Before: $I_L = 0$

After: $I_L = 0$

→ $I = +\,V/2R$

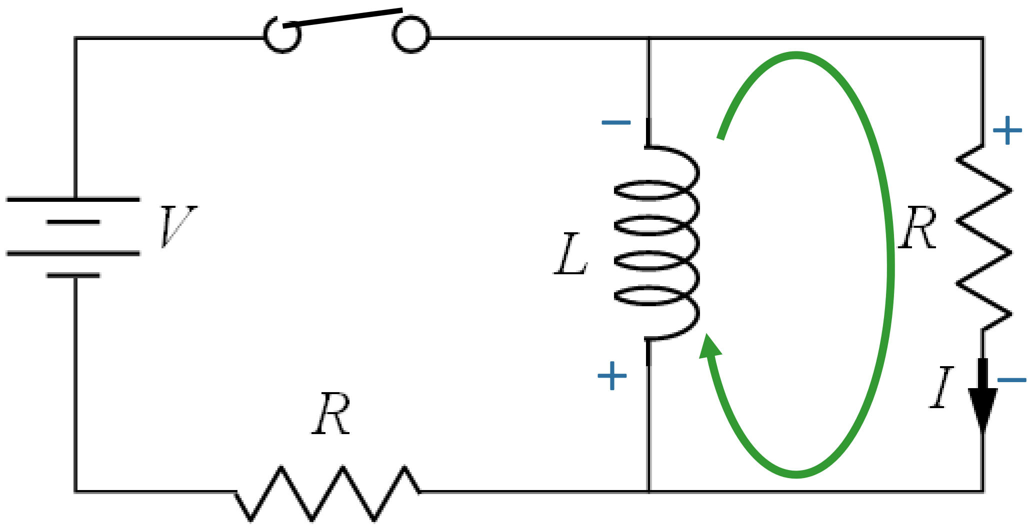


RL Circuit (Long Time)

What is the current I through the vertical resistor after the switch has been closed for a long time?

(+ is in the direction of the arrow)

A) $I = V/R$
B) $I = V/2R$
C) $I = 0$
D) $I = -V/2R$
E) $I = -V/R$

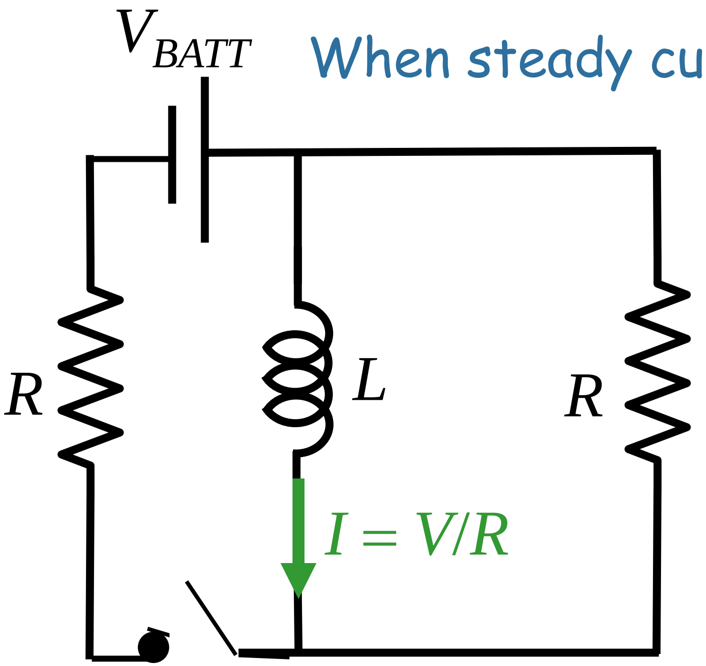


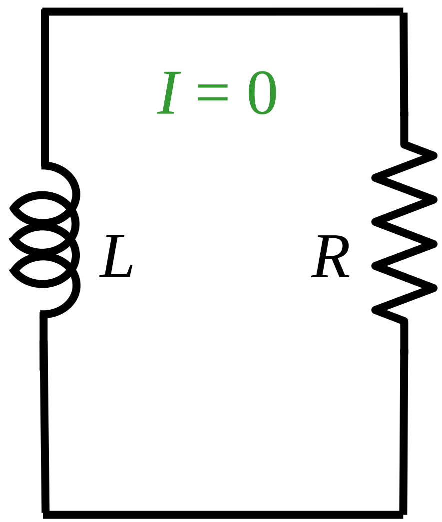
After a long time in any static circuit: $V_L = 0$

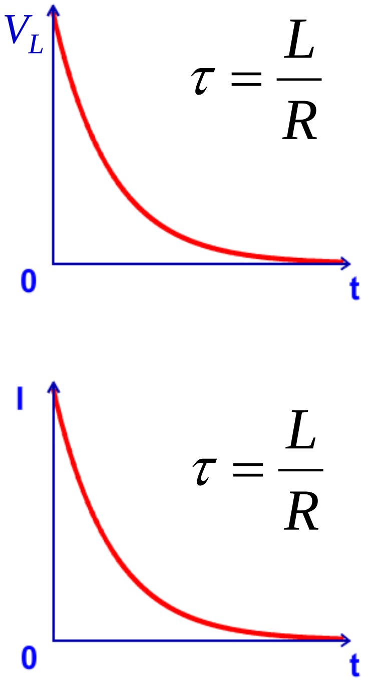
KVR:
$$V_L + IR = 0$$

How to Think about RL Circuits Episode 2:

When steady current is flowing initially: then switch is opened







At $t = 0$: I_L unchanged

$I = V_{BATT}/R$

$V_R = IR$

$V_L = V_R$

At $t >> L/R$: $V_L = 0$

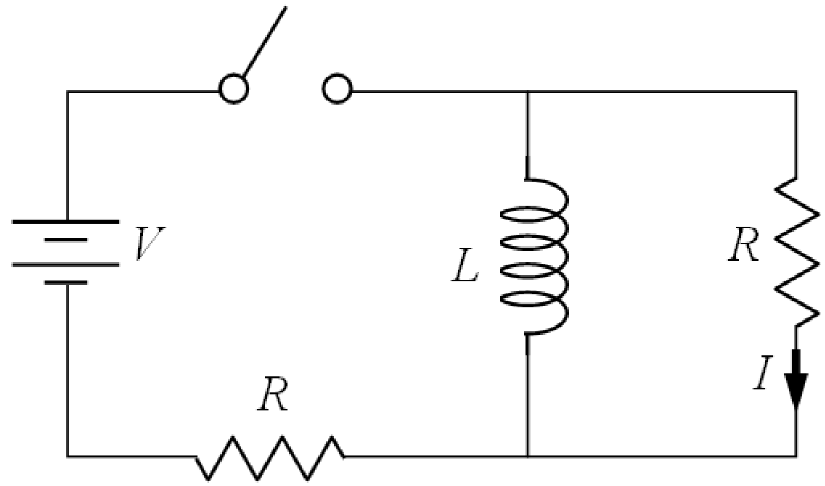
$V_L = 0$

$V_R = 0$

$I = 0$

CheckPoint 2b

After a long time, the switch is opened, abruptly disconnecting the battery from the circuit.

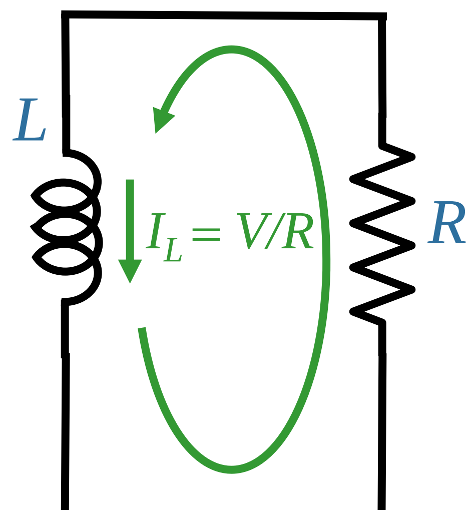


What is the current I through the vertical resistor immediately after the switch is opened?

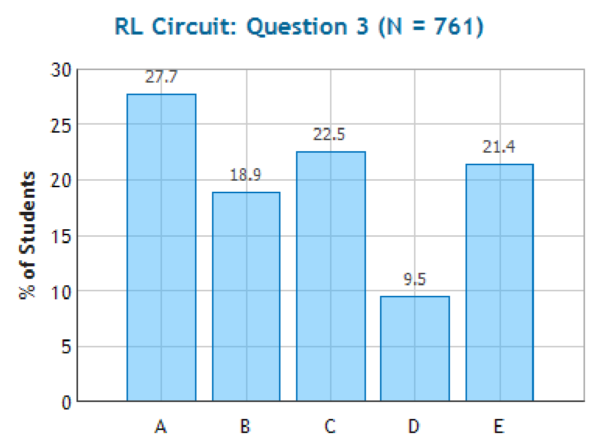
(+ is in the direction of the arrow)

A) $I = V/R$
B) $I = V/2R$
C) $I = 0$
D) $I = -V/2R$
E) $I = -V/R$

circuit when switch opened

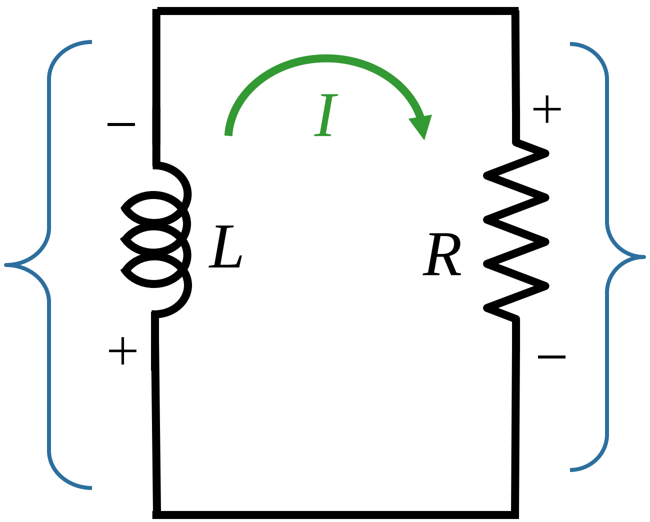


Current through inductor cannot change DISCONTINUOUSLY

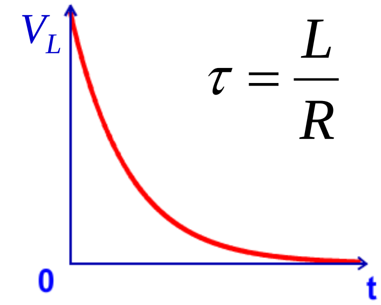


Why is there Exponential Behavior?

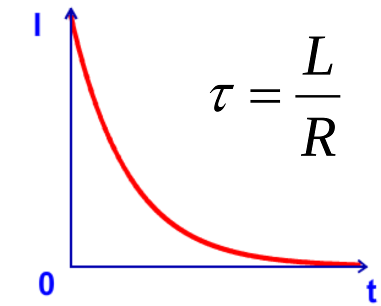
$$V = L\frac{dI}{dt}$$



$$V = IR$$



$$\tau = \frac{L}{R}$$

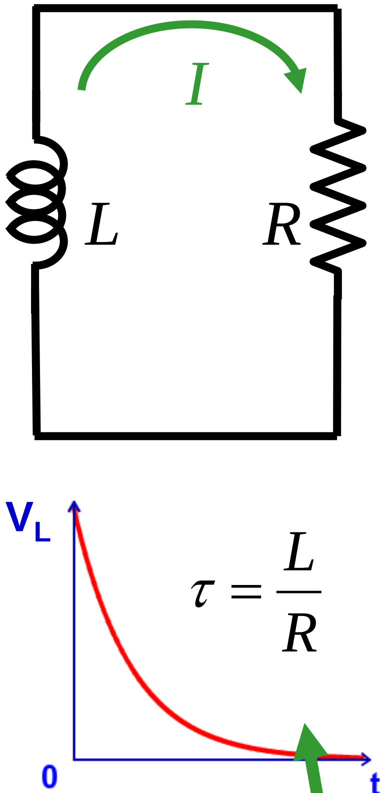


$$\tau = \frac{L}{R}$$

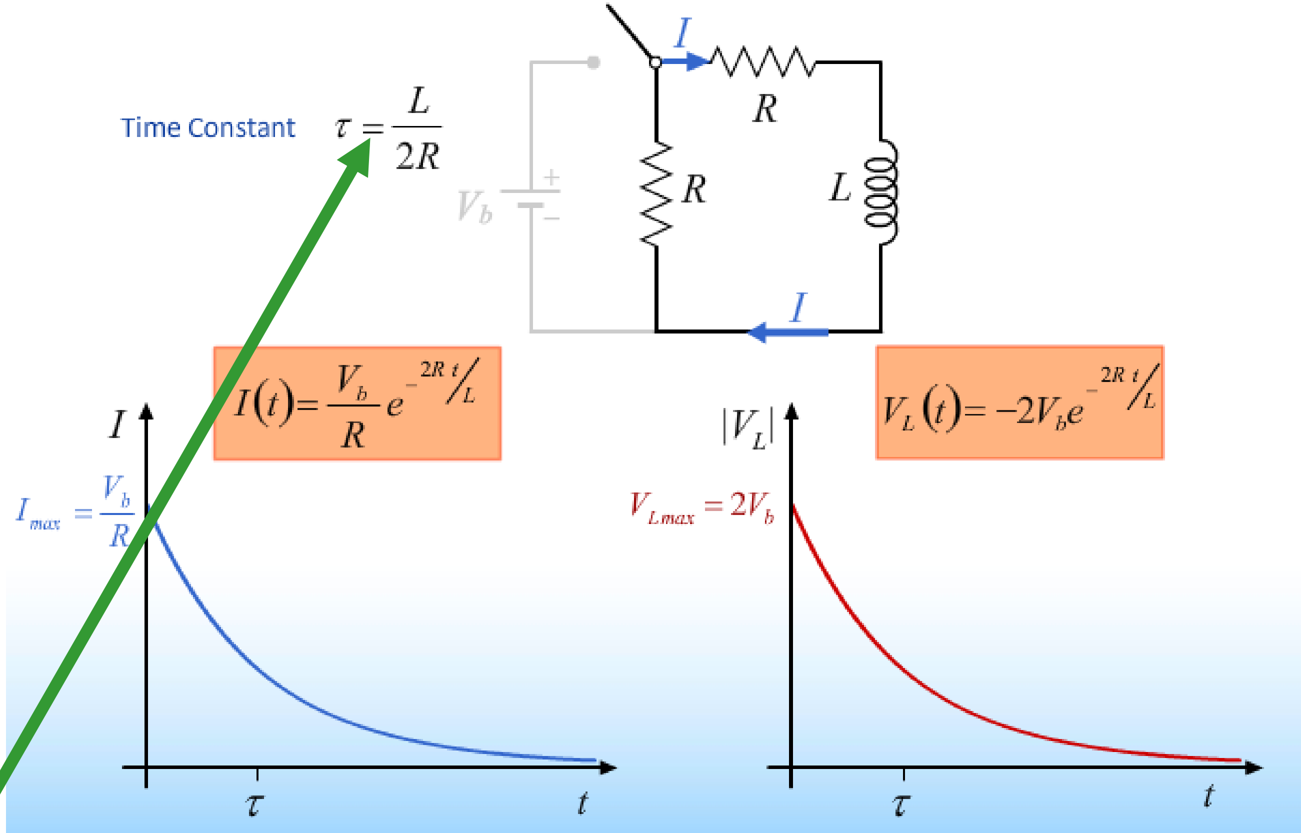
$$L\frac{dI}{dt} + IR = 0$$

$$I(t) = I_0 e^{-tR/L} = I_0 e^{-t/\tau}$$

where $\tau = \frac{L}{R}$



Lecture:



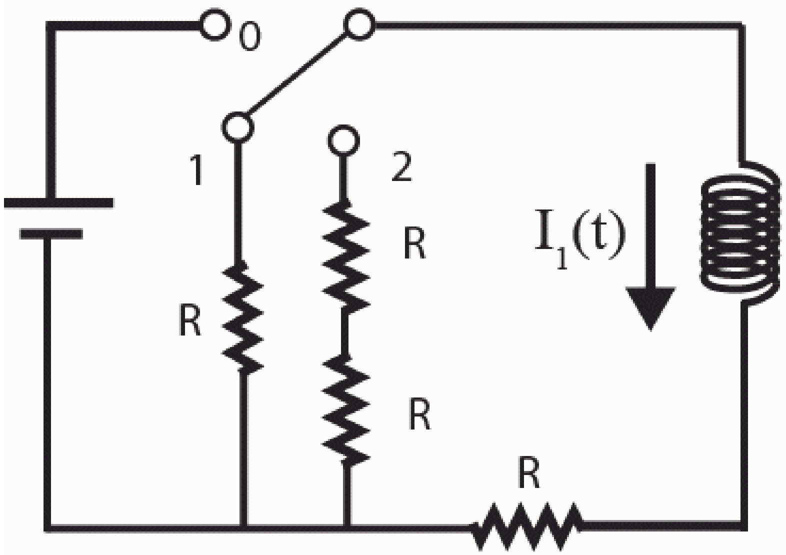
Prelecture:

Did we mess up?

No: The resistance is simply twice as big in one case.

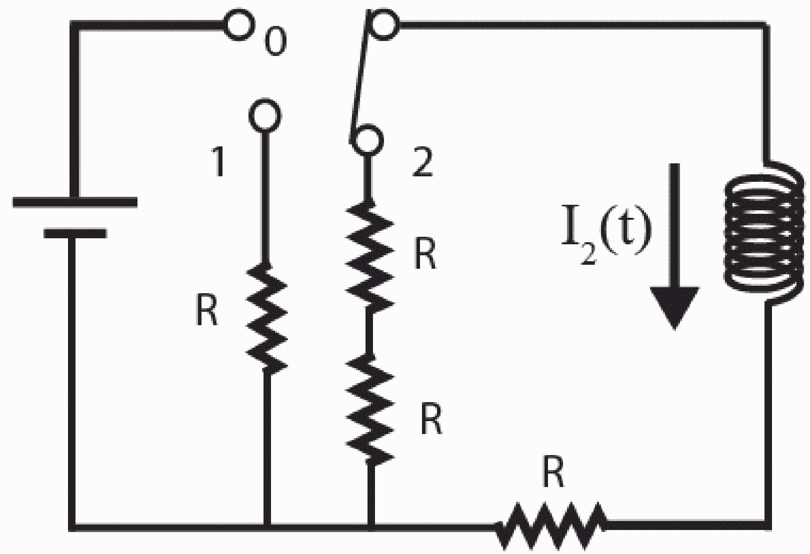
CheckPoint 3a

After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2

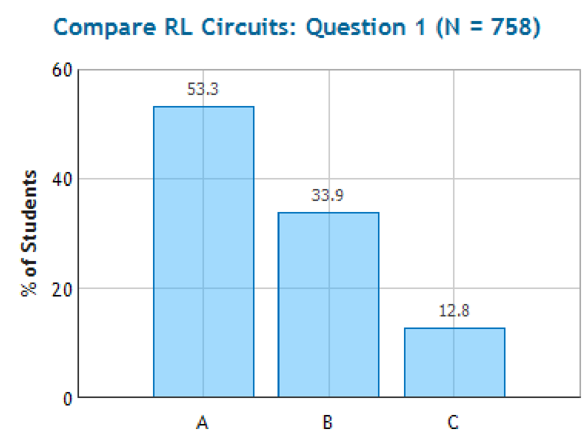


Case 2

After switch moved, which case has larger time constant?

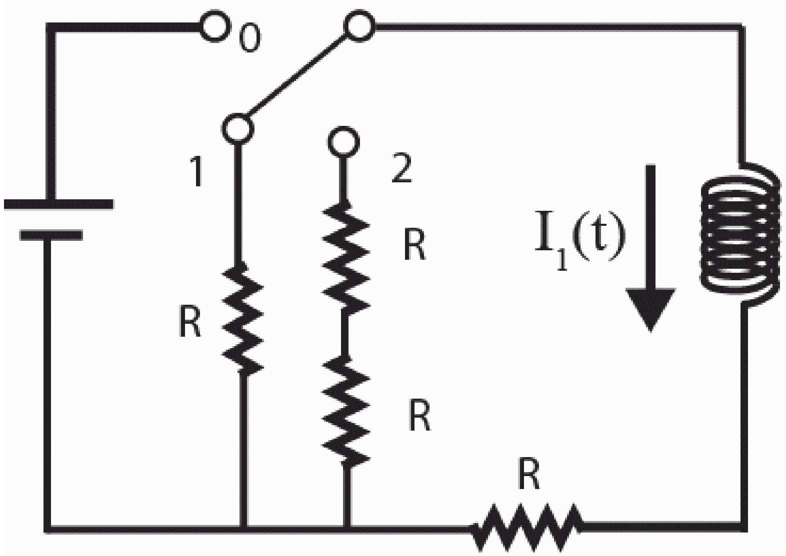
A) Case 1
B) Case 2
C) The same

$$\tau_1 = \frac{L}{2R} \qquad \tau_2 = \frac{L}{3R}$$



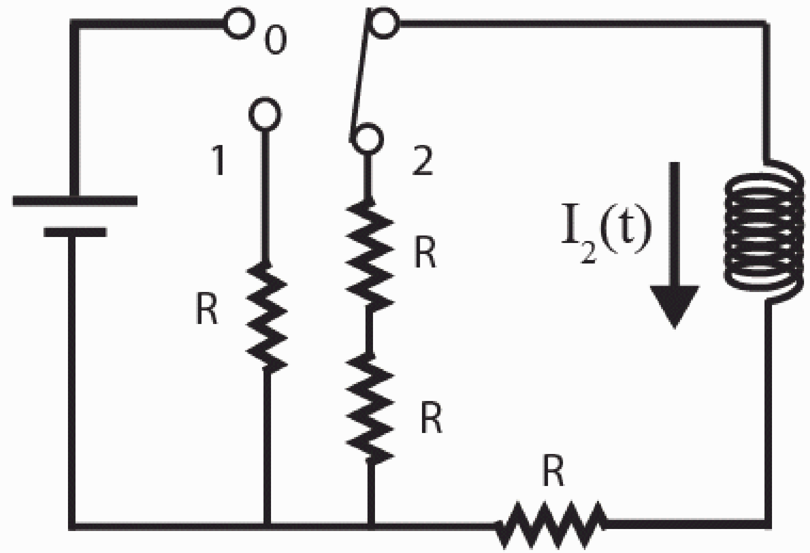
CheckPoint 3b

After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

Immediately after switch moved, in which case is the voltage across the inductor larger?

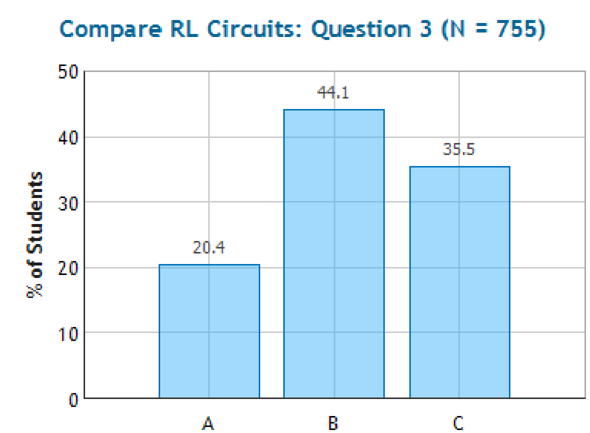
A) Case 1
B) Case 2
C) The same

After switch moved:

$$V_{L1} = \frac{V}{R}2R$$

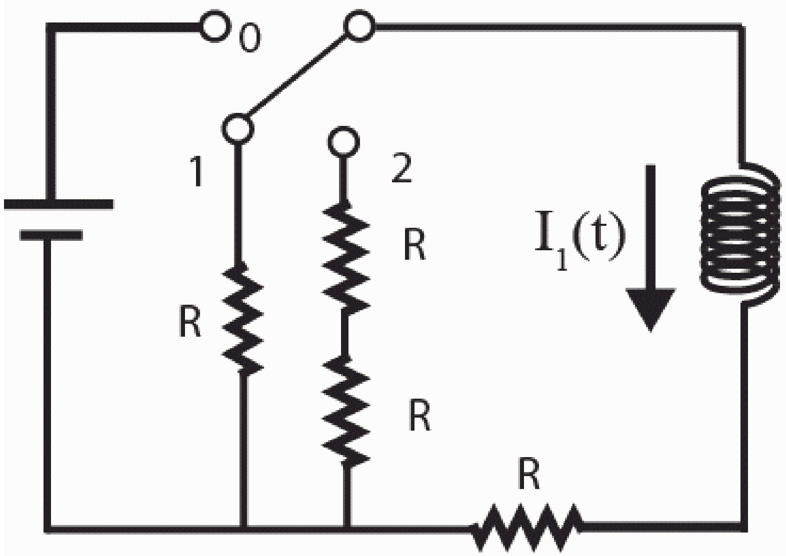
Before switch moved: $I = \frac{V}{R}$

$$V_{L2} = \frac{V}{R}3R$$



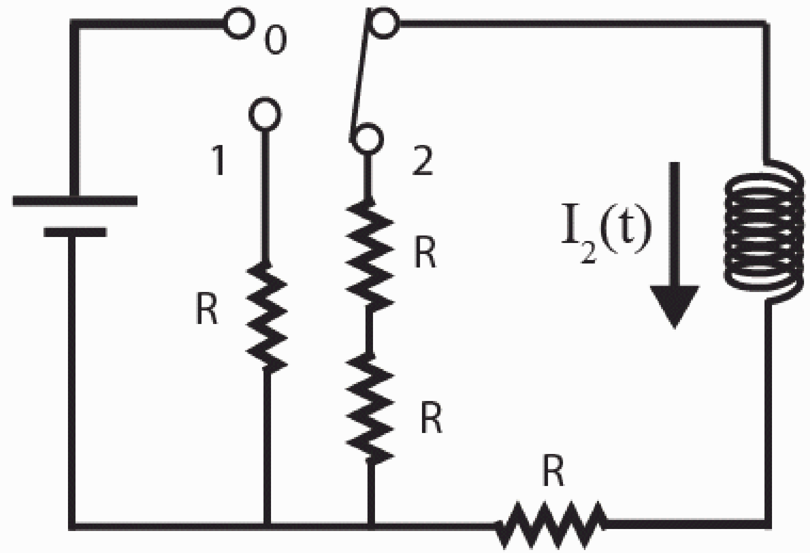
CheckPoint 3c

After long time at 0, moved to 1



Case 1

After long time at 0, moved to 2



Case 2

After switch moved for finite time, in which case is the current through the inductor larger?

A) Case 1
B) Case 2
C) The same

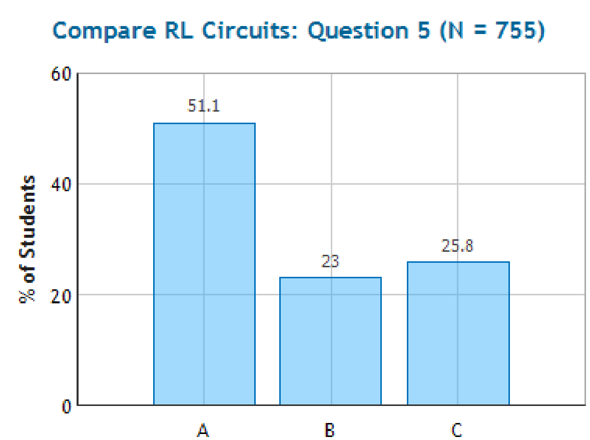
Immediately after: $I_1 = I_2$

After awhile

$$I_1 = Ie^{-t/\tau_1}$$

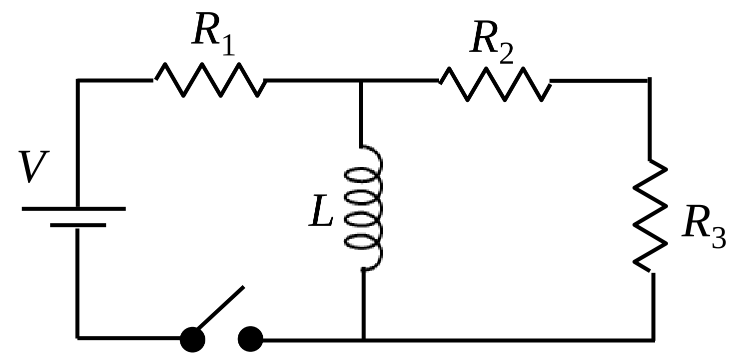
$$I_2 = Ie^{-t/\tau_2}$$

$$\tau_1 > \tau_2$$



Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is dI_L/dt, the time rate of change of the current through the inductor immediately after switch is closed

Conceptual Analysis

Once switch is closed, currents will flow through this 2-loop circuit.

KVR and KCR can be used to determine currents as a function of time.

Strategic Analysis

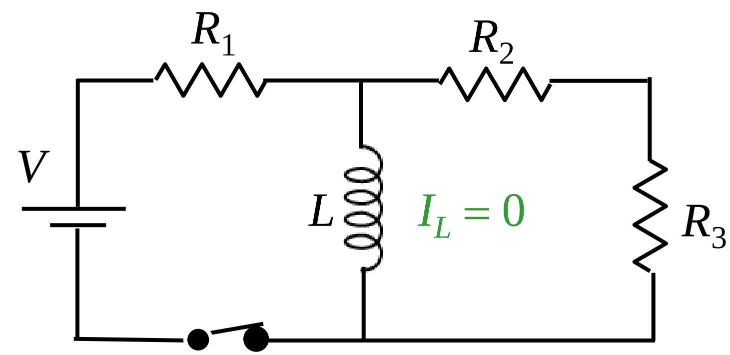
Determine currents immediately after switch is closed.

Determine voltage across inductor immediately after switch is closed.

Determine dI_L/dt immediately after switch is closed.

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



What is I_L, the current in the inductor, immediately after the switch is closed?

A) $I_L = V/R_1$ up

B) $I_L = V/R_1$ down

C) $I_L = 0$

INDUCTORS: Current cannot change discontinuously !

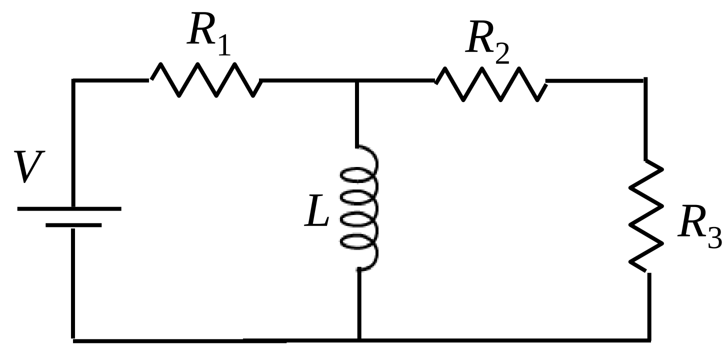
↓

Current through inductor immediately after switch is closed
is the same as
the current through inductor immediately before switch is closed

Immediately before switch is closed: $I_L = 0$ since no battery in loop

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$$I_L(t = 0+) = 0$$

What is the magnitude of I_2, the current in R_2, immediately after the switch is closed?

A) $I_2 = \dfrac{V}{R_1}$

B) $I_2 = \dfrac{V}{R_2 + R_3}$

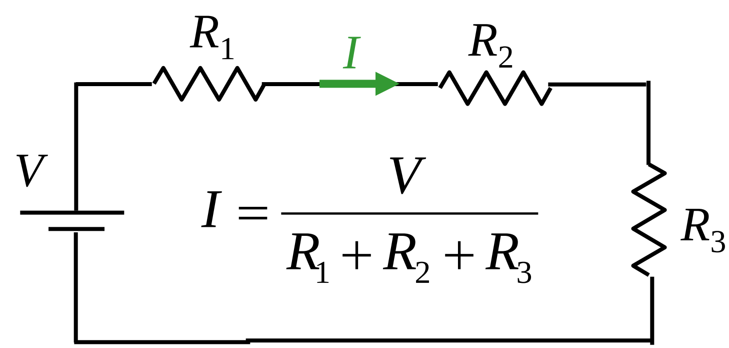
C) $I_2 = \dfrac{V}{R_1 + R_2 + R_3}$

D) $I_2 = \dfrac{VR_2R_3}{R_2 + R_3}$

We know $I_L = 0$ immediately after switch is closed

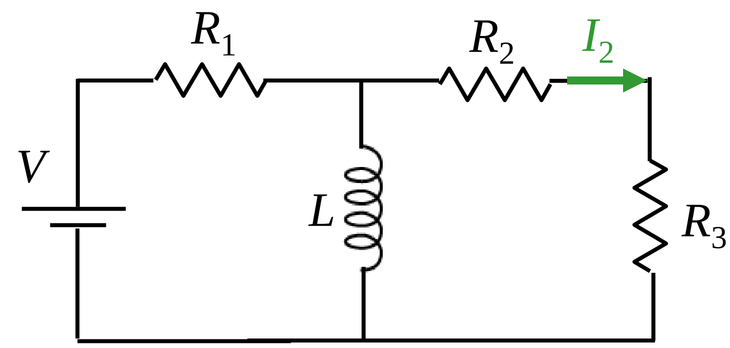
↓

Immediately after switch is closed, circuit looks like:



Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.



$I_L(t = 0+) = 0 \quad I_2(t = 0+) = V/(R_1 + R_2 + R_3)$

What is the magnitude of V_L, the voltage across the inductor, immediately after the switch is closed?

A) $V_L = V \dfrac{R_2 R_3}{R_1}$ B) $V_L = V$ C) $V_L = 0$ D) $V_L = V \dfrac{R_2 R_3}{R_1(R_2 + R_3)}$ E) $V_L = V \dfrac{R_2 + R_3}{R_1 + R_2 + R_3}$

Kirchhoff's Voltage Law,

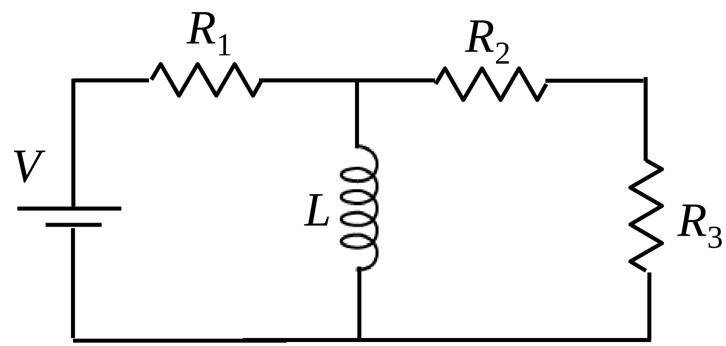
$V_L - I_2 R_2 - I_2 R_3 = 0 \quad V_L = I_2 (R_2 + R_3)$

$$V_L = \frac{V}{R_1 + R_2 + R_3}(R_2 + R_3)$$

Calculation

The switch in the circuit shown has been open for a long time. At $t = 0$, the switch is closed.

What is dI_L/dt, the time rate of change of the current through the inductor immediately after switch is closed



$V_L(t = 0+) = V(R_2 + R_3)/(R_1 + R_2 + R_3)$

A) $\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1}$

B) $\frac{dI_L}{dt} = 0$

C) $\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1 + R_2 + R_3}$

D) $\frac{dI_L}{dt} = \frac{V}{L}$

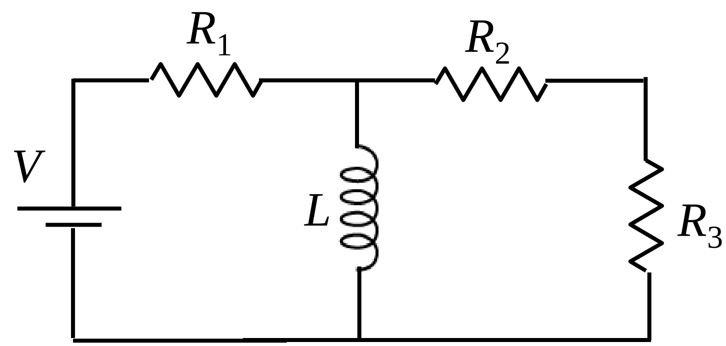
The time rate of change of current through the inductor $(dI_L/dt) = V_L/L$

$$\frac{dI_L}{dt} = \frac{V}{L}\frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

Follow Up

The switch in the circuit shown has been closed for a long time.

What is I_2, the current through R_2 ?
(Positive values indicate current flows to the right)

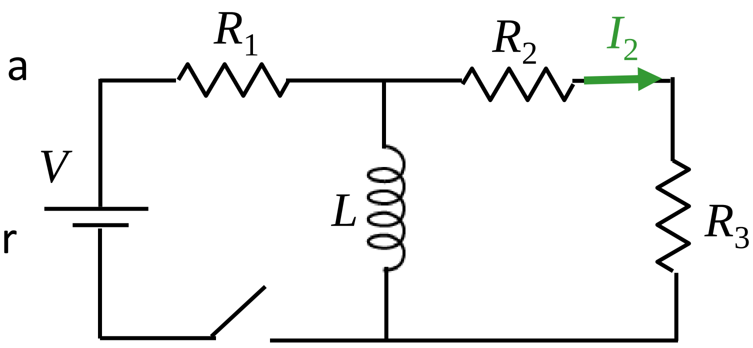


A) $I_2 = +\frac{V}{R_2 + R_3}$ B) $I_2 = +\frac{V(R_2 R_3)}{R_1 + R_2 + R_3}$ C) $I_2 = 0$ D) $I_2 = -\frac{V}{R_2 + R_3}$

After a long time, $dI/dt = 0$
Therefore, the voltage across $L = 0$
Therefore the voltage across $R_2 + R_3 = 0$
Therefore the current through $R_2 + R_3$ must be zero!

Follow Up 2

The switch in the circuit shown has been closed for a long time at which point, the switch is opened.



What is I_2, the current through R_2 immediately after switch is opened ?

(Positive values indicate current flows to the right)

A) $I_2 = +\frac{V}{R_1 + R_2 + R_3}$

B) $I_2 = +\frac{V}{R_1}$

C) $I_2 = 0$

D) $I_2 = -\frac{V}{R_1}$

E) $I_2 = -\frac{V}{R_1 + R_2 + R_3}$

Current through inductor immediately after switch is opened

is the same as

the current through inductor immediately before switch is opened

Immediately before switch is opened: $I_L = V/R_1$

Immediately after switch is opened: I_L flows in right loop

Therefore, $I_L = -V/R_1$