Comments: Yours and Ours

"I find the most difficult part to determine the direction of electric field. I need to calculate the magnitude as well as the angle for net magnitude and finally analysis and make the decision."

"i started to get confused at electric dipoles. give the positive and negative charges of same magnitude, how do you know what direction the field points in? also the infinite line stuff made 0 sense"

"So what that point charge is 1/r^2 and infinite line is 1/r? Does that dimensional dependence matter?"

"I am very confused on the dipole charges and the infinite lines of charge. Could you explain more about those?"

"I'd like to spend more time with setting up integrals for continuous distributions."

"The integral of the continuous line charges is a bit abstract for me. I don't quite understand how to add up for the Y component."

"I didn't get the part in superposition of vectors, when he talks about dimensions and how its reflected in the formula."

1. Please respect your fellow students –
2. Quite a bit of homework this week ☹
3. Don't panic!......Don't be intimidated by integrals!

Electricity & Magnetism
Lecture 2

Today's Concepts:

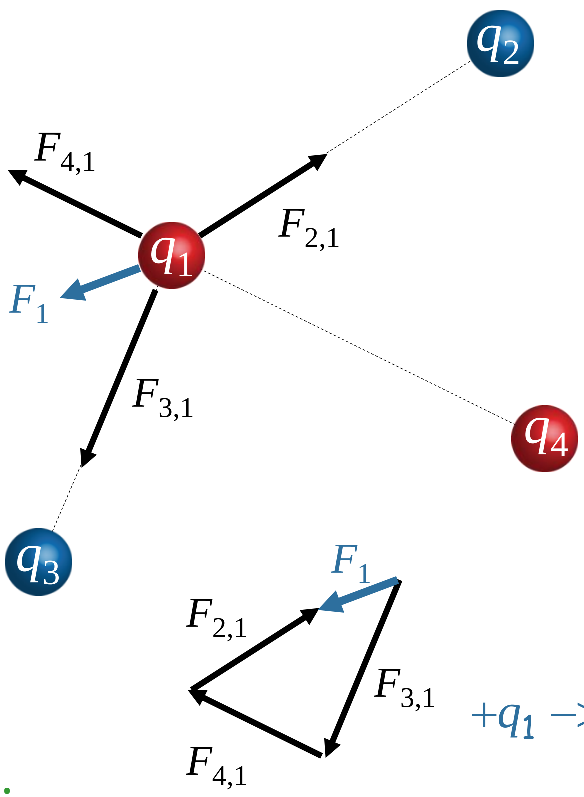
A) The Electric Field

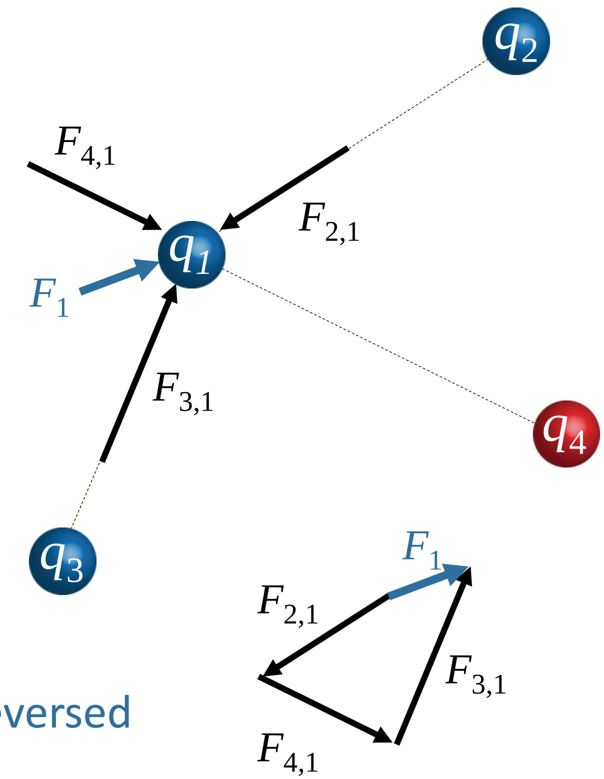
B) Continuous Charge Distributions

Pretty cool stuff man, but like, what does it all meannnnn?

Coulomb's Law (from last time)

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:





$+q_1 \rightarrow -q_1 \Rightarrow$ direction reversed

MATH:

$$\vec{F}_1 = \frac{k q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{k q_1 q_3}{r_{13}^2}\hat{r}_{13} + \frac{k q_1 q_4}{r_{14}^2}\hat{r}_{14} \longrightarrow \vec{E} = \frac{\vec{F}_1}{q_1} = \frac{k q_2}{r_{12}^2}\hat{r}_{12} + \frac{k q_3}{r_{13}^2}\hat{r}_{13} + \frac{k q_4}{r_{14}^2}\hat{r}_{14}$$

Electric Field

"What is the essence of an electric field? "

The electric field E at a point in space is simply the force per unit charge at that point.

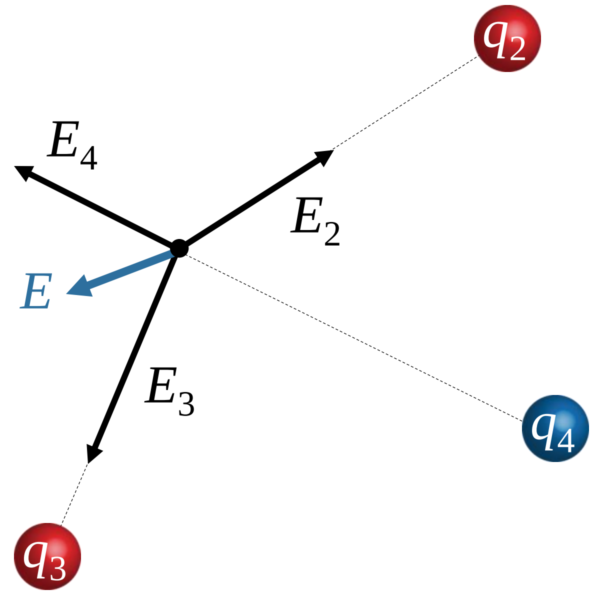
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

Superposition

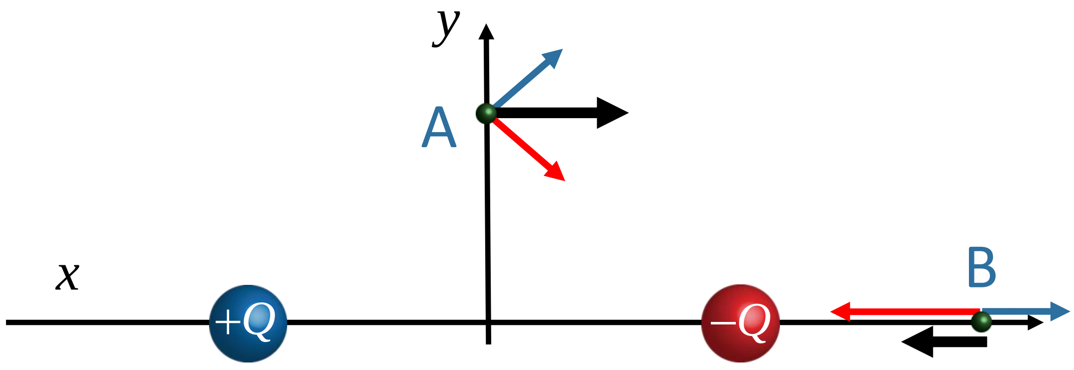
$$\vec{E} = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i$$



Field points toward negative and Away from positive charges.

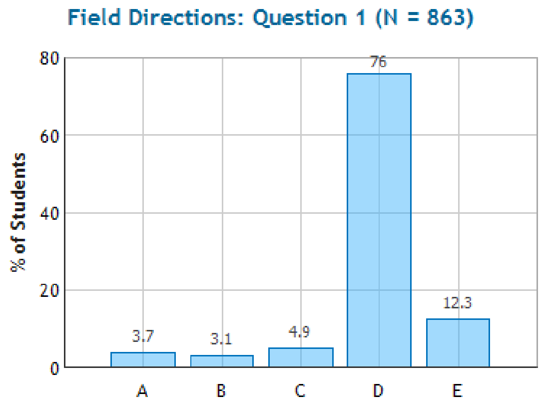
CheckPoint

“I can't figure out the field directions at all.”



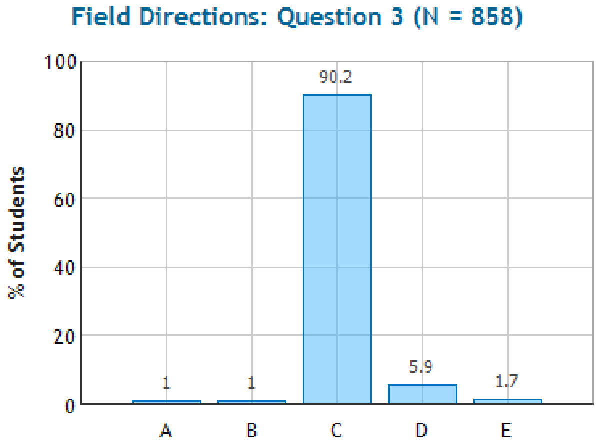
Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is direction at point A



a) Up b) down c) Left d) Right e) zero

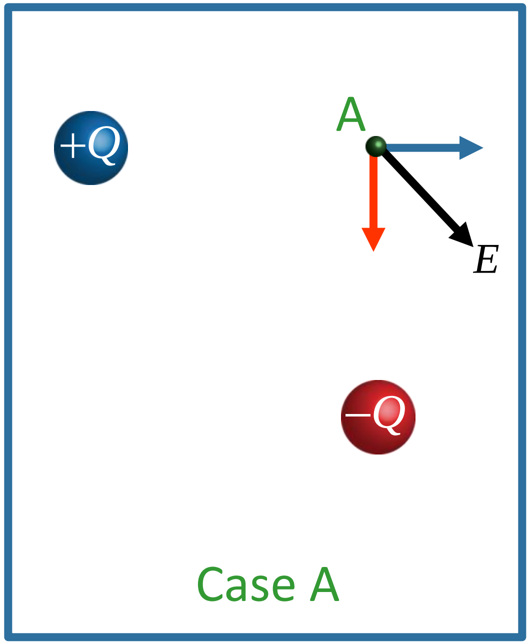
What is direction at point B



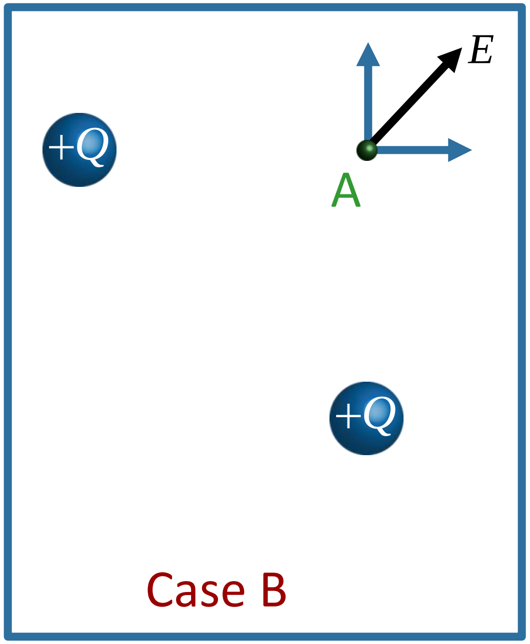
a) Up b) down c) Left d) Right e) zero

CheckPoint

In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest? (Select C if you think they are equal)

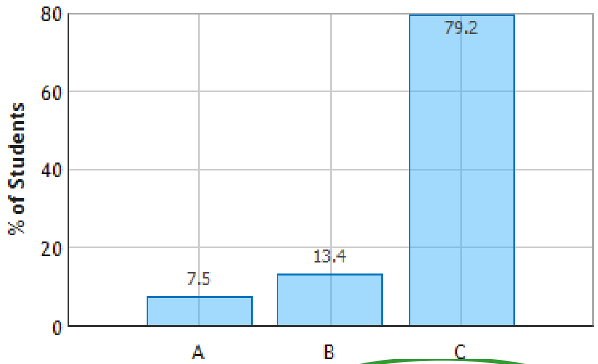


in case B the charges will cancel each other



"The magnitudes combine together instead of canceling out like they do in case A."

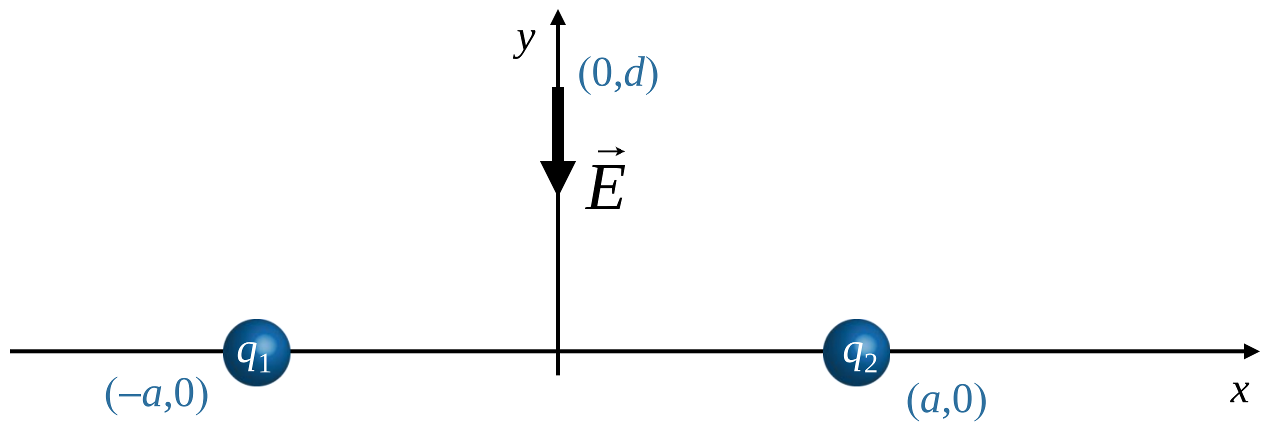
"The charges are equal distance and since they are perpendicular they dont cancel out."



A **B** **Equal**

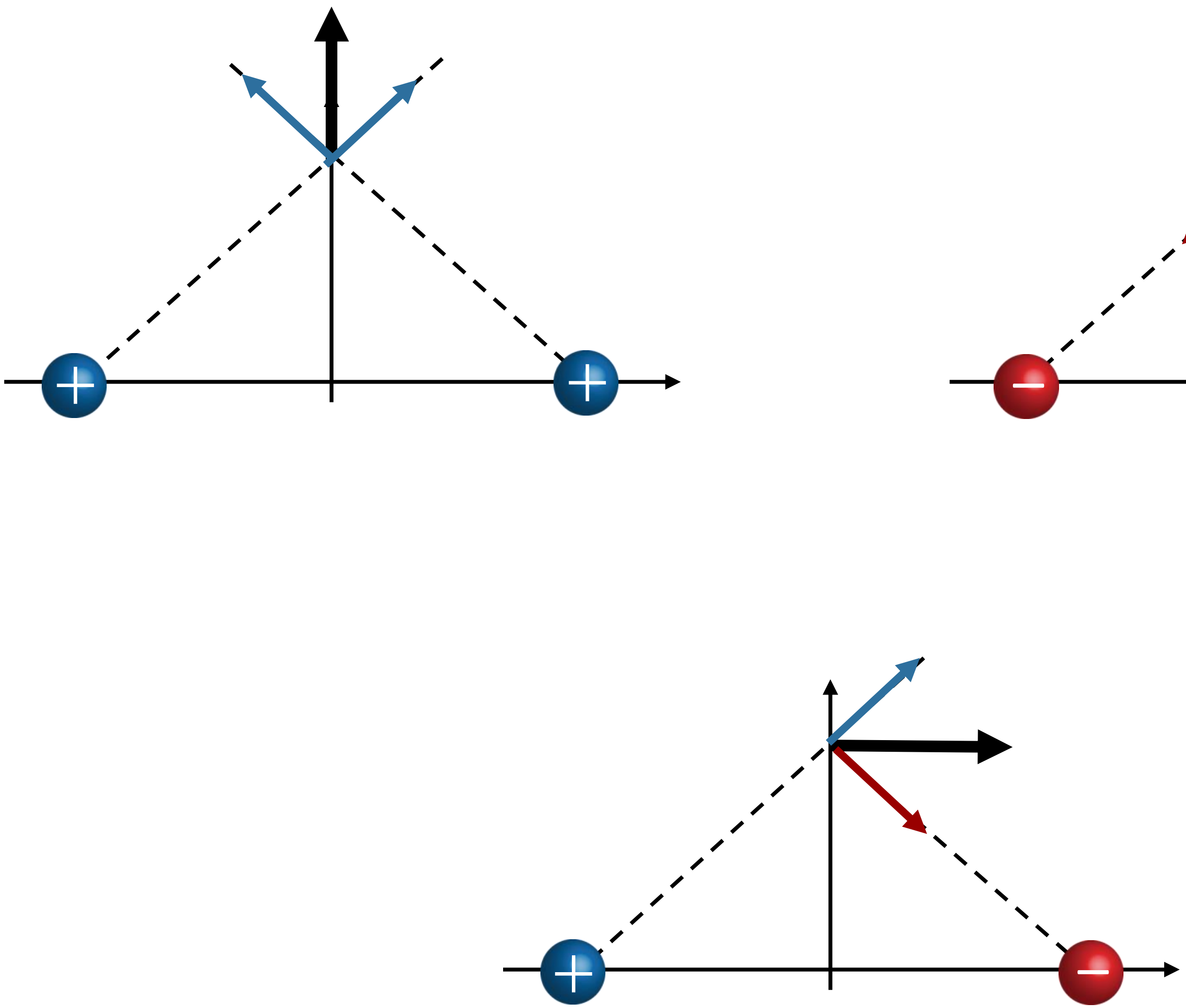
Two Charges

Two charges q_1 and q_2 are fixed at points $(-a,0)$ and $(a,0)$ as shown. Together they produce an electric field at point $(0,d)$ which is directed along the negative *y*-axis.



Which of the following statements is true:

A) Both charges are negative
B) Both charges are positive
C) The charges are opposite
D) There is not enough information to tell how the charges are related



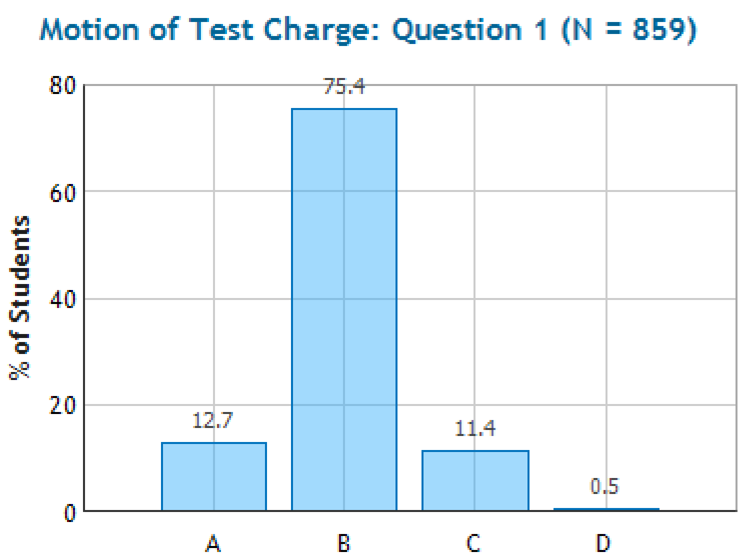
CheckPoint

A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q. How will the test charge move immediately after being released?

8) **How will the test charge move immediately after being released?**

○ to the left ○ to the right ○ stay still ○ other

A B C D



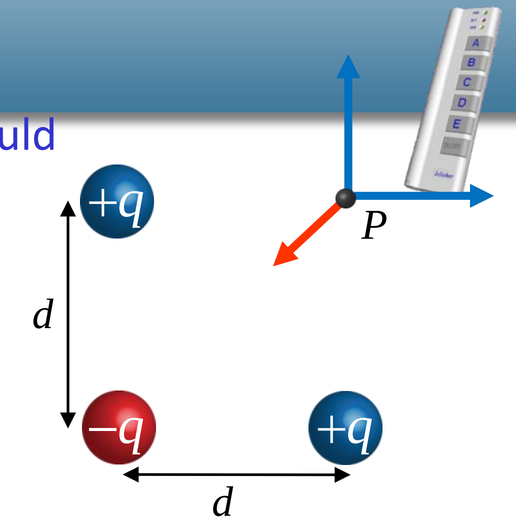
"(A LEFT) Since the radius is squared, the distance between the 2Q charge makes its pull smaller than that of the 1Q charge."

"(B) Force from Q charge is greater than 2Q because kQq/r^2>2kQq/4r^2 => 1>1/2 and test charge is repelled"

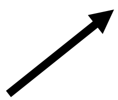
"(C Still) the two electric field are equal in magnitude but opposite in direction"

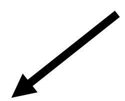
Example

"More examples of electric fields of different charge distributions would be nice (for example, charges arranged in a circle or square).!"



What is the direction of the electric field at point P, the unoccupied corner of the square?

A) 

B) 

C) $E = 0$

D) Need to know d

E) Need to know d & q

Calculate E at point P.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

$$E_x = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\cos\frac{\pi}{4}\right)$$

$$E_y = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\sin\frac{\pi}{4}\right)$$

Continuous Charge Distributions

"I would like to go over the process of integration with the infinite line of charge example again in class if possible.."

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

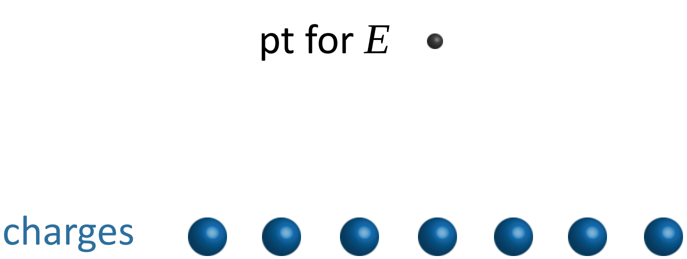
WHAT DOES THIS MEAN ?

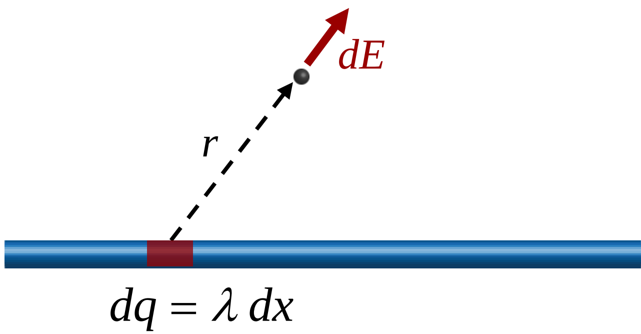
Integrate over all charges (dq)

r is vector from dq to the point at which E is defined

Linear Example:

$$\lambda = Q/L$$





Charge Density

"What exactly is Lambda? Q/L ?? Charge density??."

Linear $(\lambda = Q/L)$ Coulombs/meter

Surface $(\sigma = Q/A)$ Coulombs/meter^2

Volume $(\rho = Q/V)$ Coulombs/meter^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?.

A) A sphere w/ radius 4 meters and volume charge density ρ = 2 C/m^3

B) A sphere w/ radius 4 meters and surface charge density σ = 2 C/m^2

C) Both A) and B) have the same net charge.

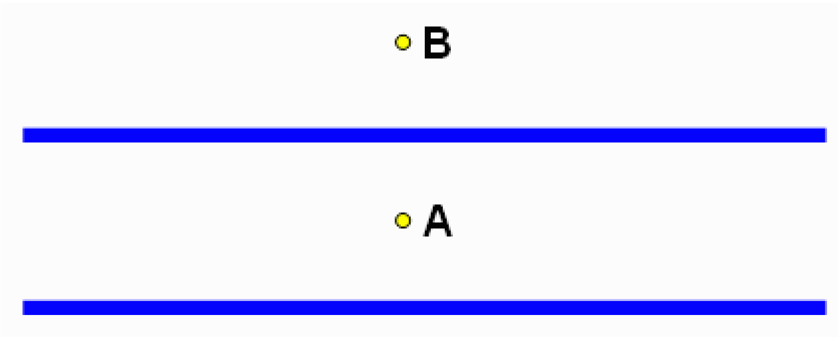
$$Q_A = \rho V = \rho \tfrac{4}{3}\pi R^3$$

$$Q_B = \sigma A = \sigma 4\pi R^2$$

$$\frac{Q_A}{Q_B} = \frac{\rho \frac{4}{3}\pi R^3}{\sigma 4\pi R^2} = \frac{1}{3}\frac{\rho}{\sigma}R = \frac{1}{3}\frac{2}{2}4 = \frac{4}{3}$$

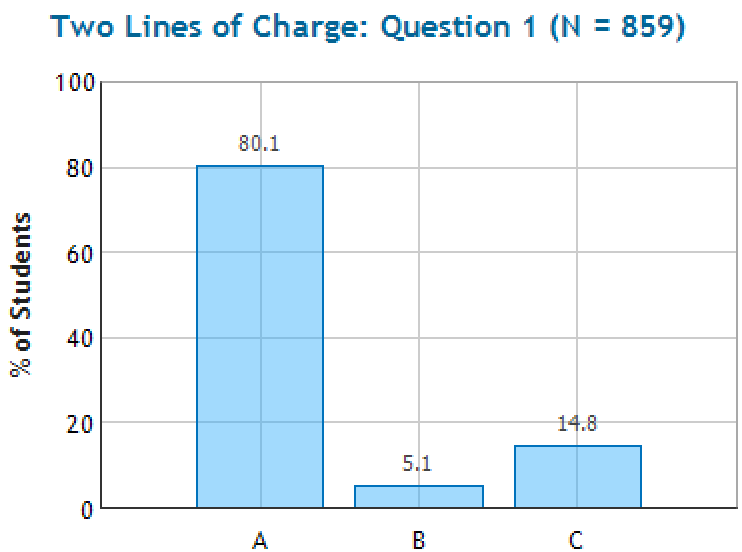
CheckPoint

Two infinite lines of charge are shown below.



Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown. How does E_A, the magnitude of the electric field at point A, compare to E_B, the magnitude of the electric field at point B?

- ○ $E_A < E_B$
- ○ $E_A = E_B$
- ○ $E_A > E_B$



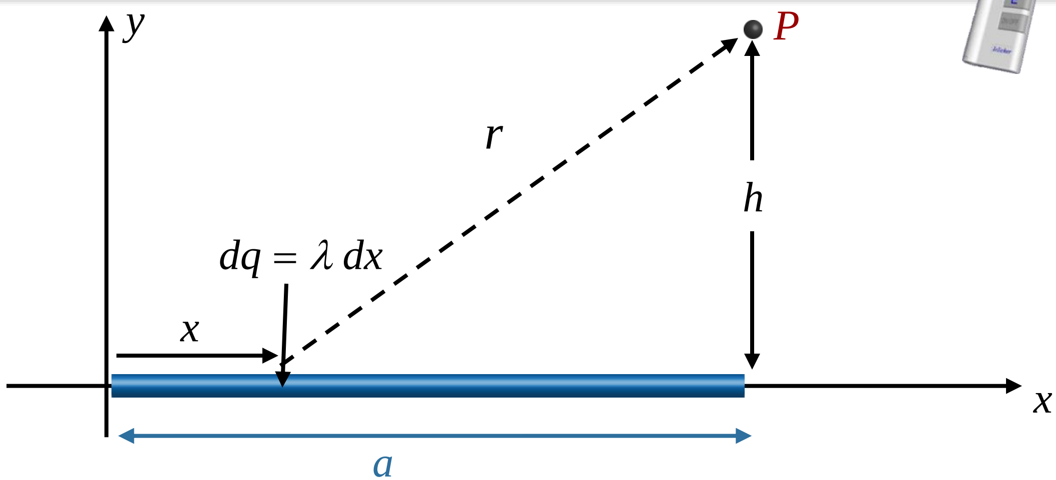
A) ($E_A<E_B$) “As the electric field by both lines have equal magnitude on A but opposite directions they get cancelled out hence electric field at A=0 whereas that is not the case at B and electric field at B>0”

C) ($E_A>E_B$) “E_A will be grater becuase it has two lines of charge acting on it from equl distance were E_B would be less because it is fruther from the lower line of charge.”

Calculation

"How is the integration of dE over L worked out, step by step?"

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What is $\dfrac{dq}{r^2}$?

A) $\dfrac{dx}{x^2}$

B) $\dfrac{dx}{a^2 + h^2}$

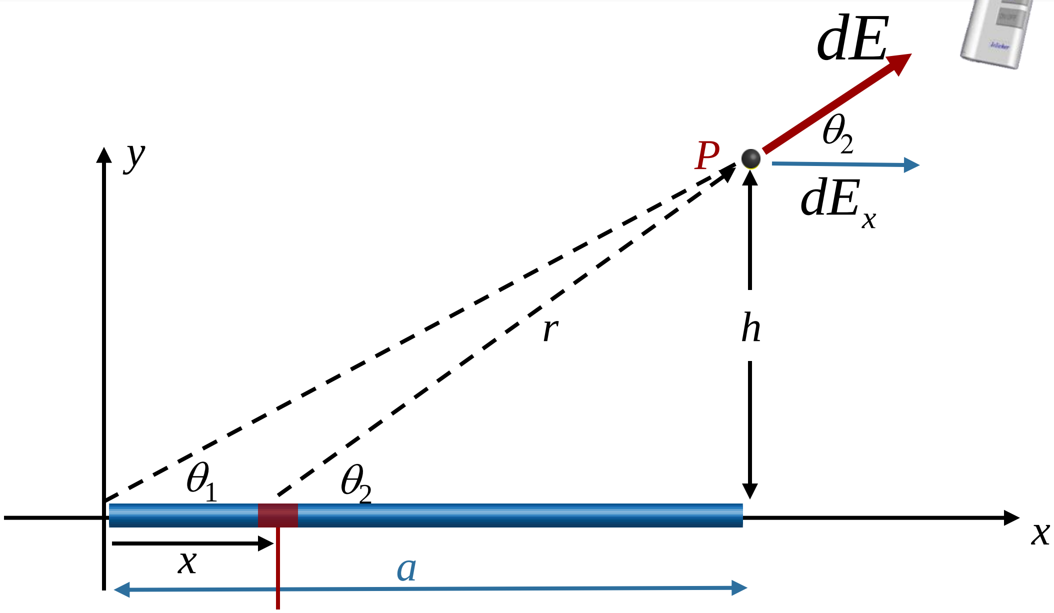
C) $\dfrac{\lambda dx}{a^2 + h^2}$

D) $\dfrac{\lambda dx}{(a - x)^2 + h^2}$

E) $\dfrac{\lambda dx}{x^2}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

We want:

$$E_x = \int dE_x$$

What is dE_x?

A) $dE \cos\theta_1$

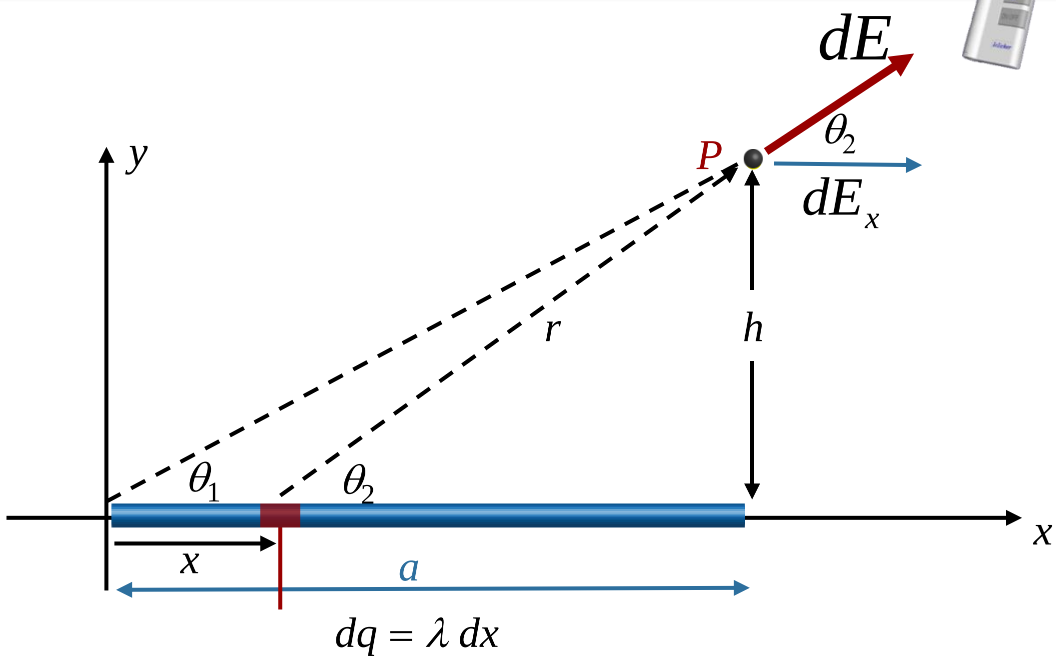
B) $dE \cos\theta_2$

C) $dE \sin\theta_1$

D) $dE \sin\theta_2$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE \cos\theta_2$$

What is E_x ?

A) $\displaystyle\int_0^a \frac{k\lambda \cos\theta_2 dx}{(a-x)^2 + h^2}$

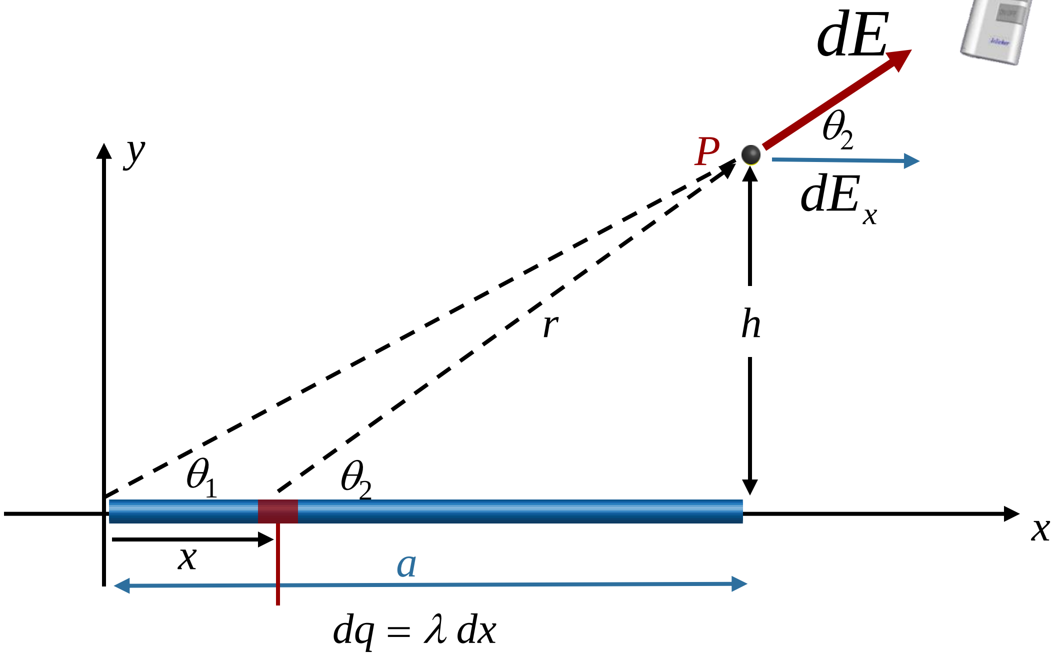
B) $\displaystyle\lambda k \cos\theta_2 \int_0^a \frac{dx}{h^2 + (x-a)^2}$

C) A and B are both OK

$\cos\theta_2$ DEPENDS ON x!

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

We know:

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE \cos\theta_2$$

What is $\cos\theta_2$?

A) $\dfrac{x}{\sqrt{a^2+h^2}}$

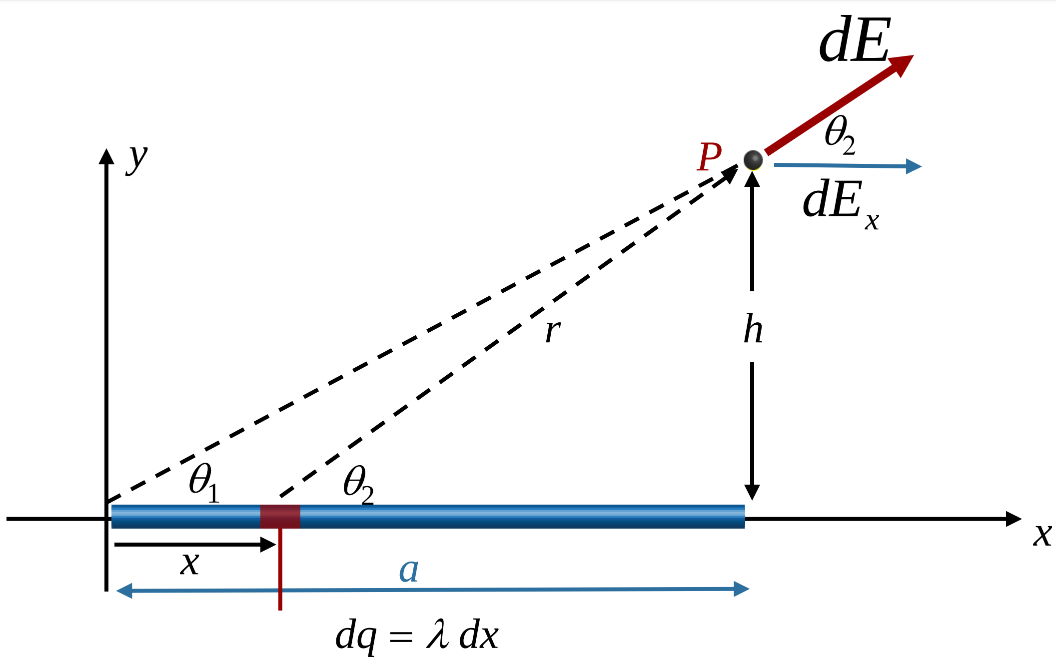
B) $\dfrac{a-x}{\sqrt{(a-x)^2+h^2}}$

C) $\dfrac{a}{\sqrt{a^2+h^2}}$

D) $\dfrac{a}{\sqrt{(a-x)^2+h^2}}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know: $$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2} \qquad E_x = \int dE\cos\theta_2 \qquad \cos\theta_2 = \frac{a-x}{\sqrt{(a-x)^2 + h^2}}$$

Putting it all together

$$E_x(P) = \lambda k \int_0^a dx \frac{a-x}{\left((a-x)^2 + h^2\right)^{3/2}} \longrightarrow E_x(P) = \frac{\lambda k}{h}\left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right)$$

Homework Problem

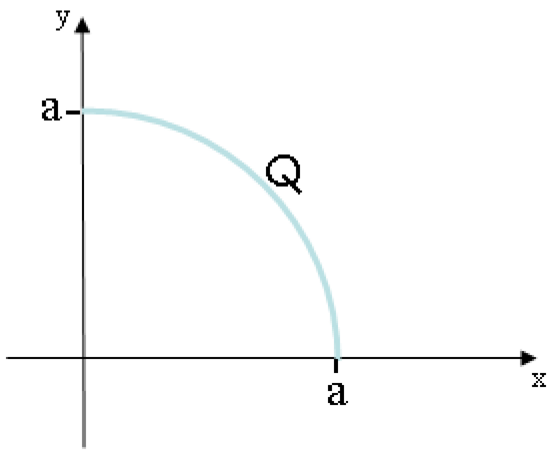
Homework: Coulomb's Law

Deadline: 100% until Tuesday, January 22 at 8:00 AM

Electric Field from Arc of Charge

1 2 3 4 5

A total charge Q = -4.2 μC is distributed uniformly over a quarter circle arc of radius a = 7.7 cm as shown.



Please try to solve this problem on your own. Only use help if you get stuck, and have thought hard about it for at least 10 minutes!

1) What is λ the linear charge density along the arc?

C/m Submit

2) What is E_x, the value of the x-component of the electric field at the origin (x,y) = (0,0) ?

N/C Submit Help