

# Your Comments

“What is the quantity  $E_0$  exactly?” “What does epsilon nought signify? How was it derived from the equations presented in the prelecture?”

IT'S JUST A  
CONSTANT

$$\vec{E} = k \frac{q}{r^2} \hat{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{q}{r^2} \hat{r}$$

$$k \equiv \frac{1}{4\pi\epsilon_o}$$

$$k = 9 \times 10^9 \text{ N m}^2 / \text{C}^2$$

$$\epsilon_o = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

I'm completely confused(or at least I think I am), I hope lecture tomorrow helps

The electric flux is very complex to get the idea in the first reading.

I'm not fully understanding what flux means. I can do calculations but I'm not sure what it signifies in a problem.

This to me was a difficult prelecture. Can we go over electric field lines and using spheres.

flux maxwell equations

# *Electricity & Magnetism*

## *Lecture 3*

Today's Concepts:

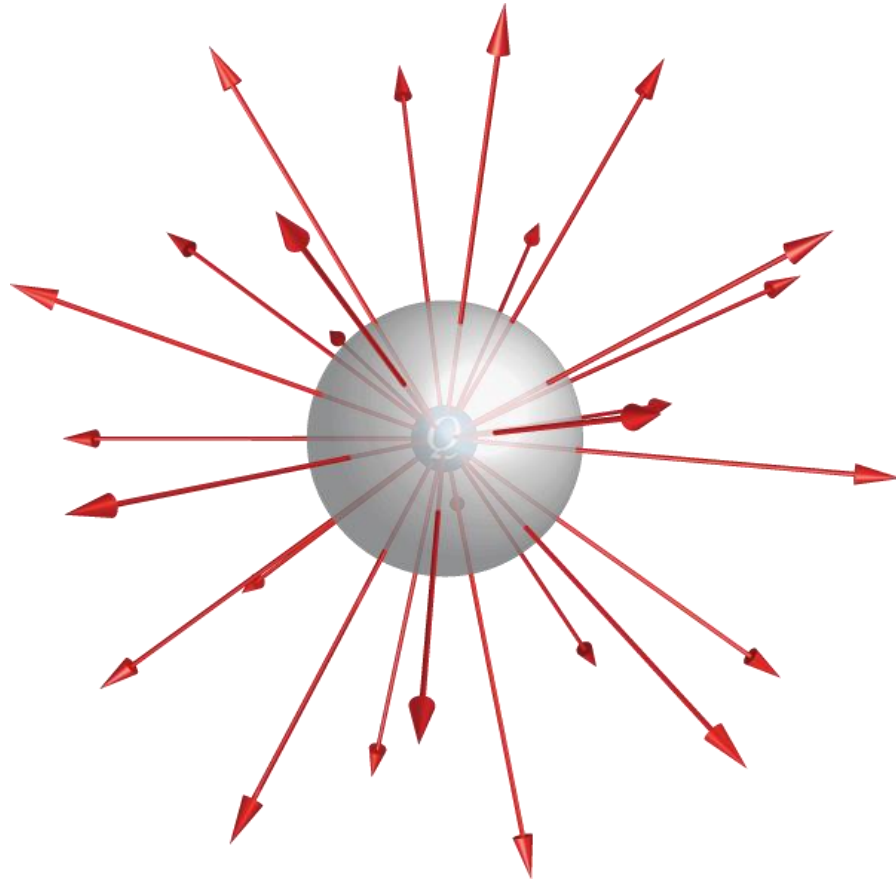
A) Electric Flux

B) Field Lines



Gauss' Law

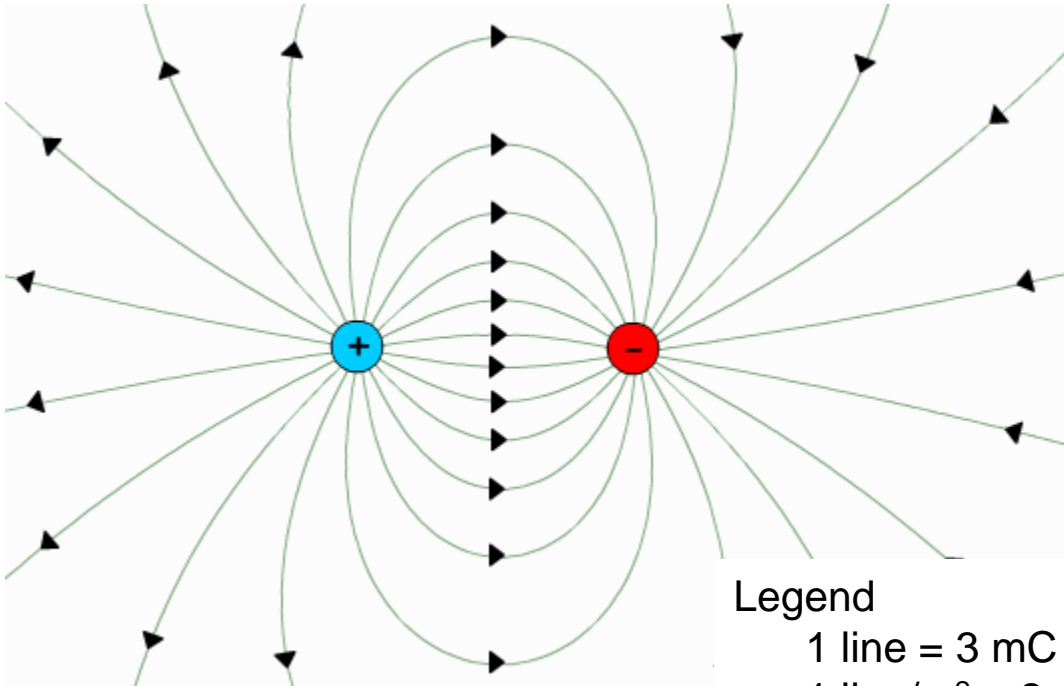
# Electric Field Lines



Direction & Density of Lines  
represent  
Direction & Magnitude of  $E$

Point Charge:  
Direction is radial  
Density  $\propto 1/R^2$

# Electric Field Lines



Legend

1 line = 3 mC

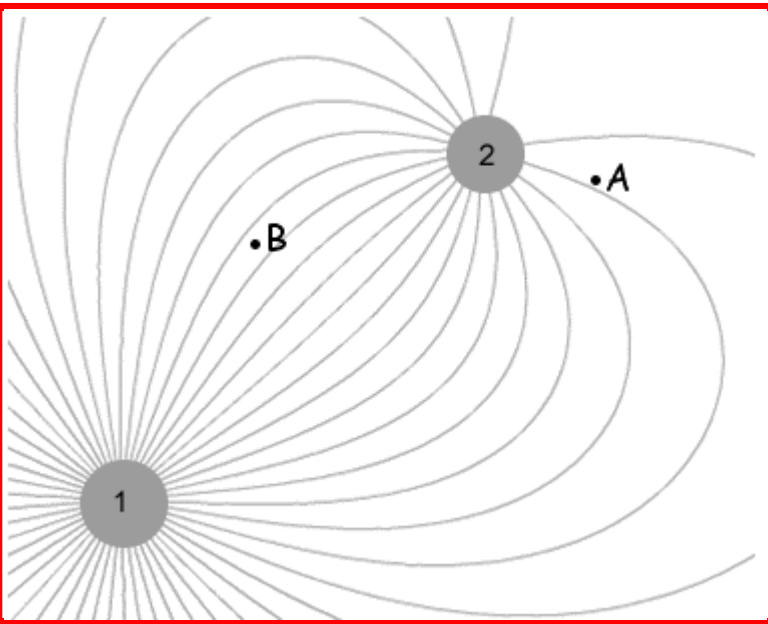
1 line/m<sup>2</sup> =  $2.4 \times 10^{11}$  N/C

Dipole Charge Distribution:  
Direction & Density  
much more interesting.

Simulation

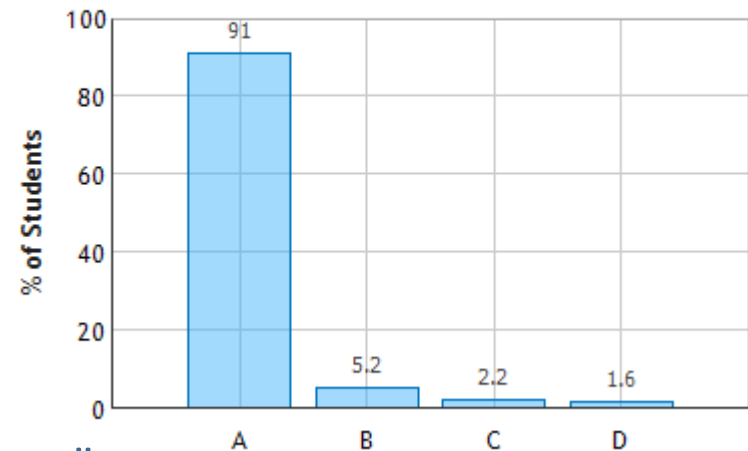
I don't quite understand the concept of charge being proportional to number of electric field lines. Isn't there an electric field at all points surrounding a charge? If that is the case, shouldn't there always be an infinite number of electric field lines surrounding a charge?

# CheckPoint 3.1



- A.  $|Q_1| > |Q_2|$
- B.  $|Q_1| = |Q_2|$
- C.  $|Q_1| < |Q_2|$
- D. Not enough info

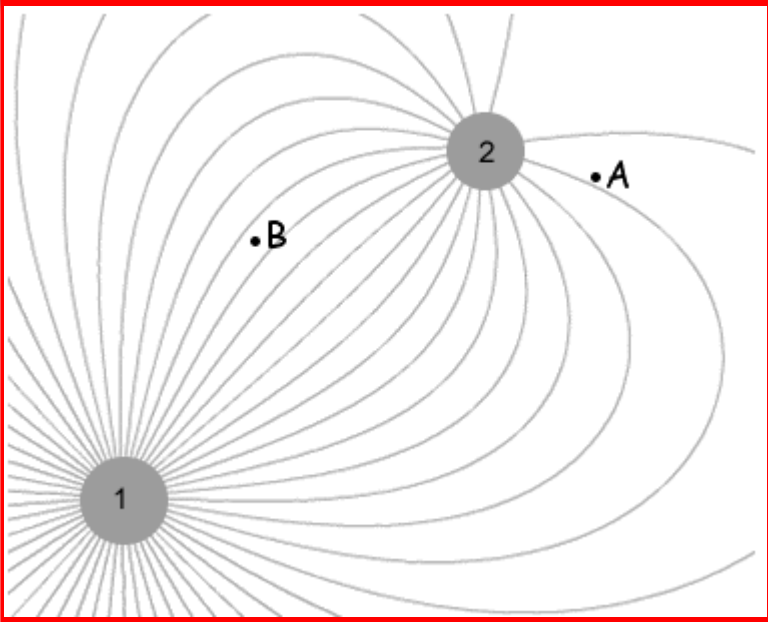
Field Lines from Two Point Charges: Question 1  
(N = 877)



“more lines = higher magnitude of charge”

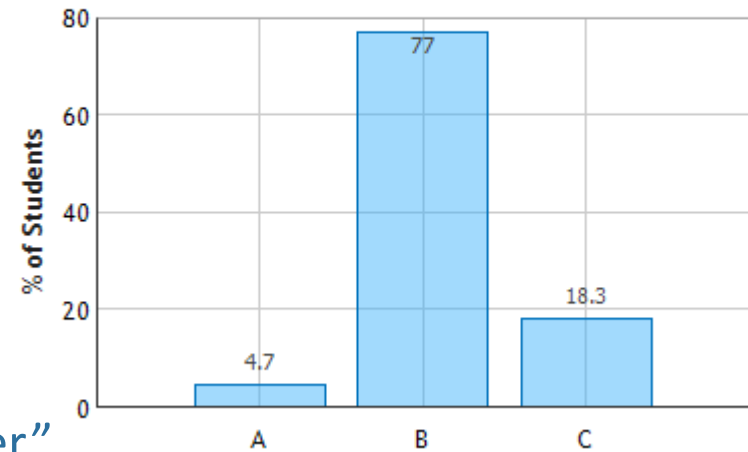
Simulation

## CheckPoint 3.3



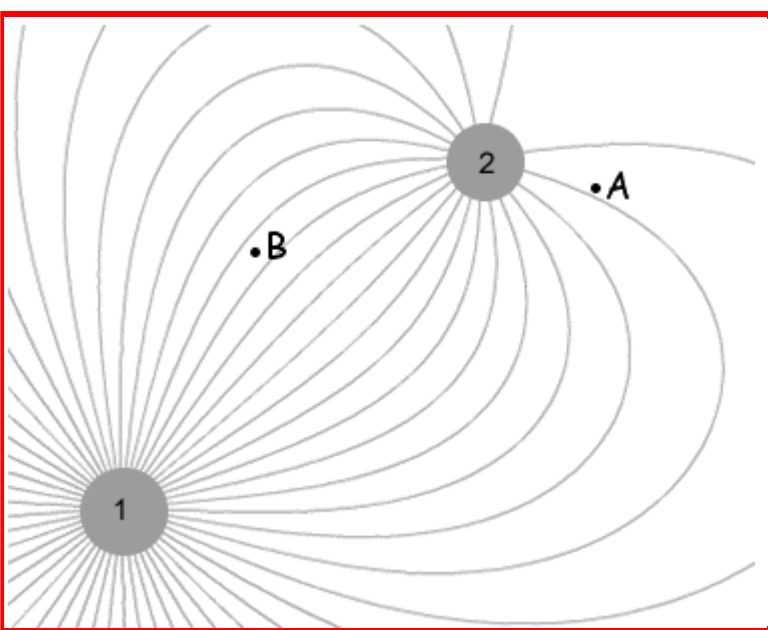
- A.  $Q_1$  and  $Q_2$  have the same sign
- B.  $Q_1$  and  $Q_2$  have opposite signs
- C. C. Not enough info

Field Lines from Two Point Charges: Question 3  
(N = 875)



“Because they go from one to the other.”

## CheckPoint 3.5



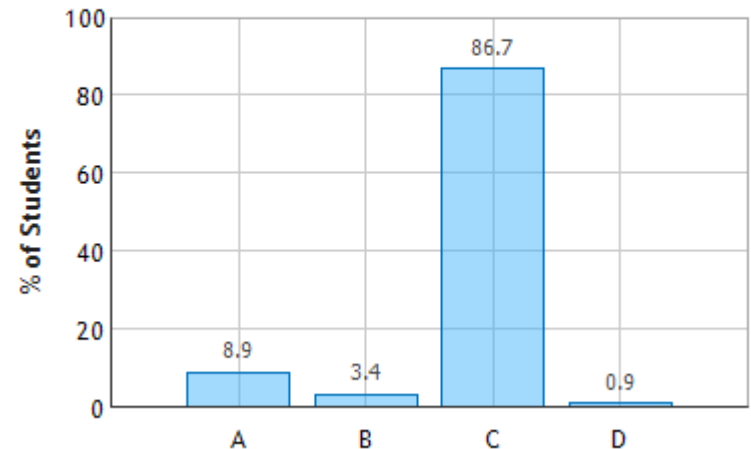
A.  $|E_A| > |E_B|$

B.  $|E_A| = |E_B|$

C.  $|E_A| < |E_B|$

D. Not enough info

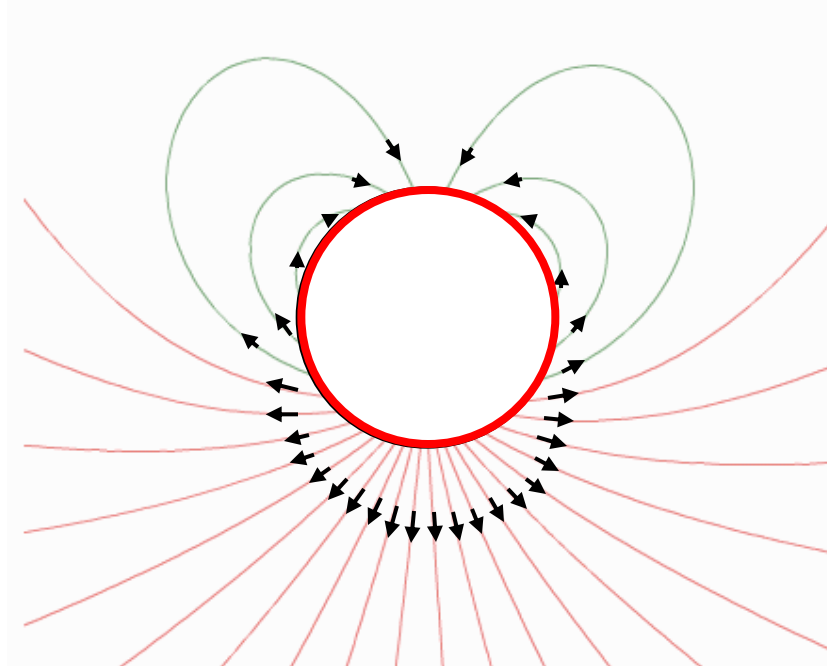
Field Lines from Two Point Charges: Question 5  
(N = 875)



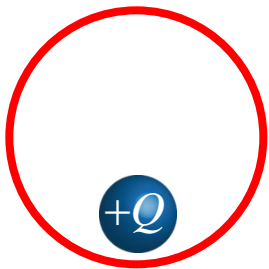
“the lines are closer together at B, meaning electric field is stronger”

# Point Charges

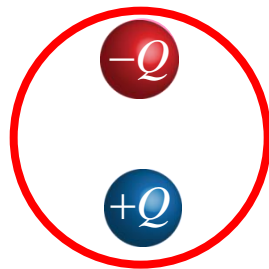
“Telling the difference between positive and negative charges while looking at field lines. Does field line density from a certain charge give information about the sign of the charge?”



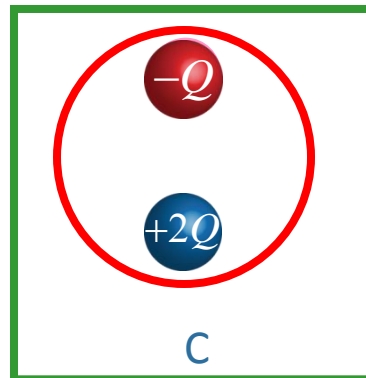
What charges are inside the red circle?



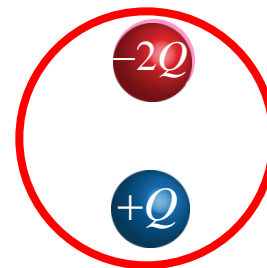
A



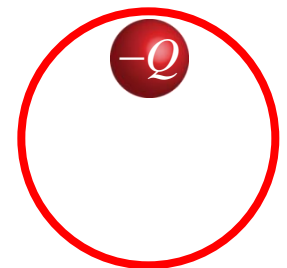
B



C



D

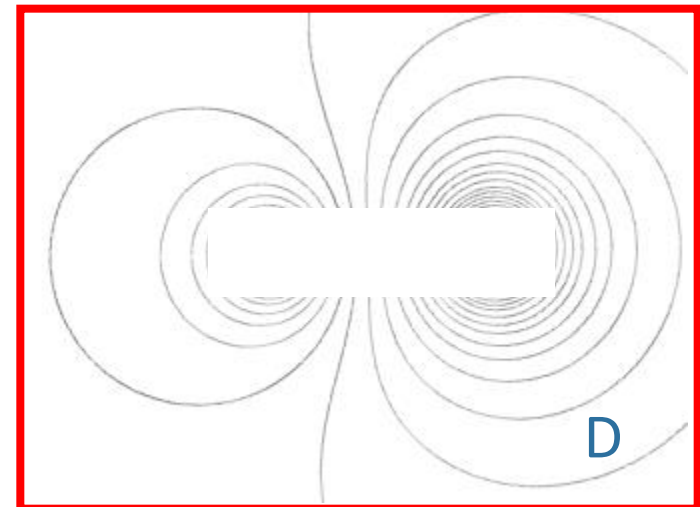
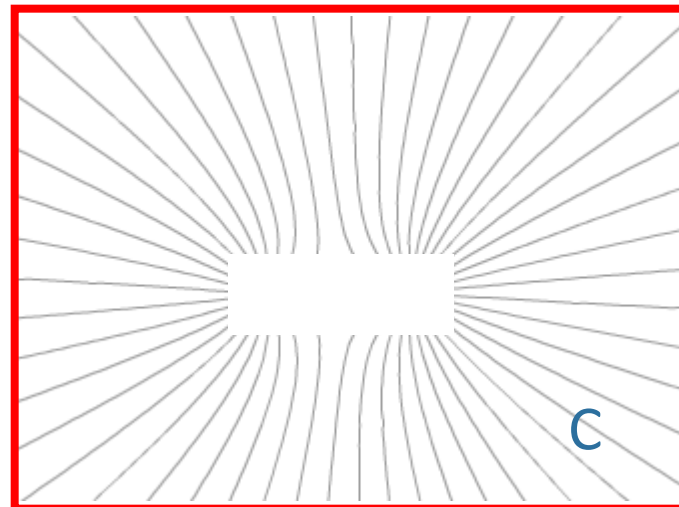
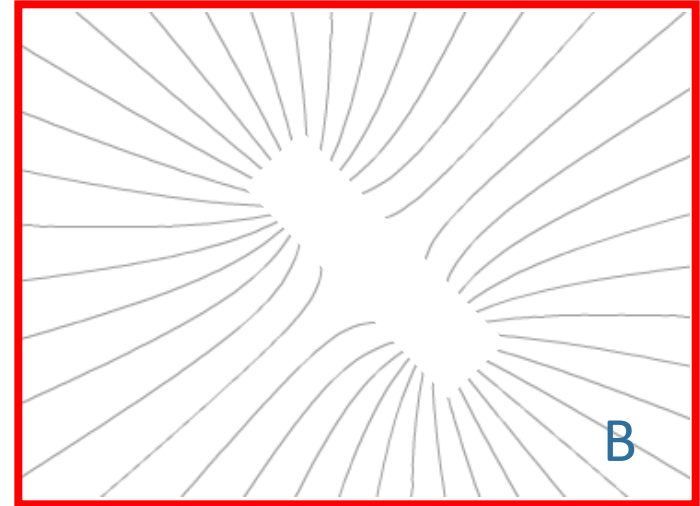
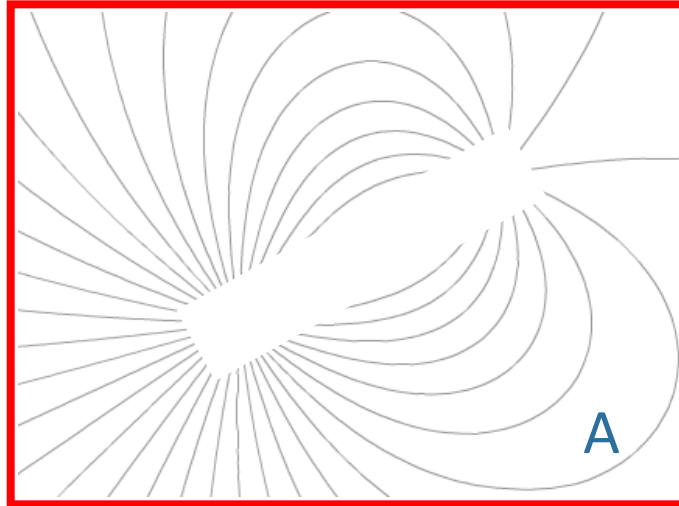


E



# Electric Field lines

Which of the following field line pictures best represents the electric field from two charges that have the **same** sign but different magnitudes?



Simulation

# Electric Flux “Counts Field Lines”

I had two quick questions that I thought might be good food for thought for other students as well: Why is it that the net flux equals zero when the charge is placed outside the object (obviously canceling the contributions of the field lines to the flux), but not equal to 0 when the charge is inside the object? Is it because all the field lines start inside the object and end outside the object rather than starting and ending outside the object? What are the units of Phi, and how is this "flux" actually measured in real life?

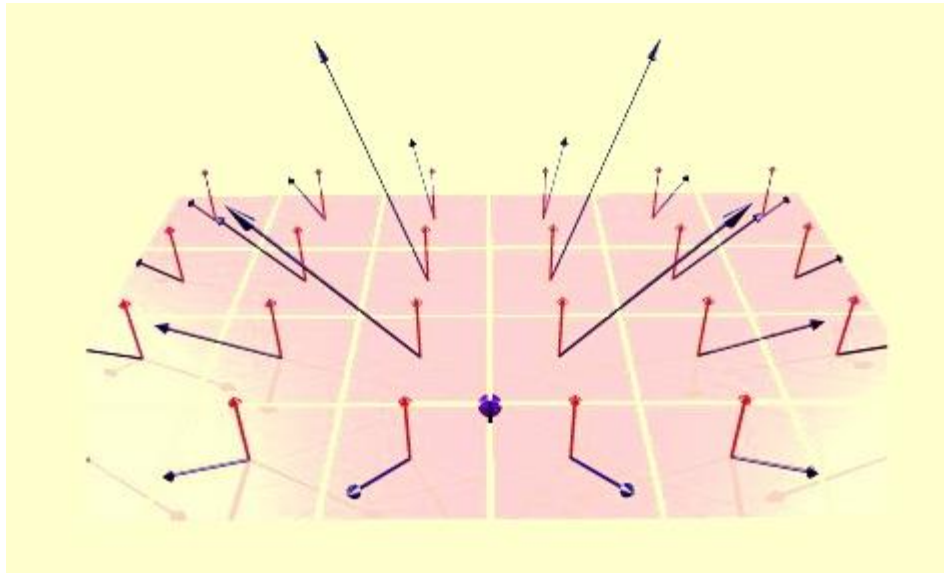
Can you give us a clear, simple definition of what flux is?

Flux through surface  $S$

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A}$$

Integral of  $\vec{E} \cdot d\vec{A}$  on surface  $S$

Representing the area of a surface as a vector in order to take the dot product.



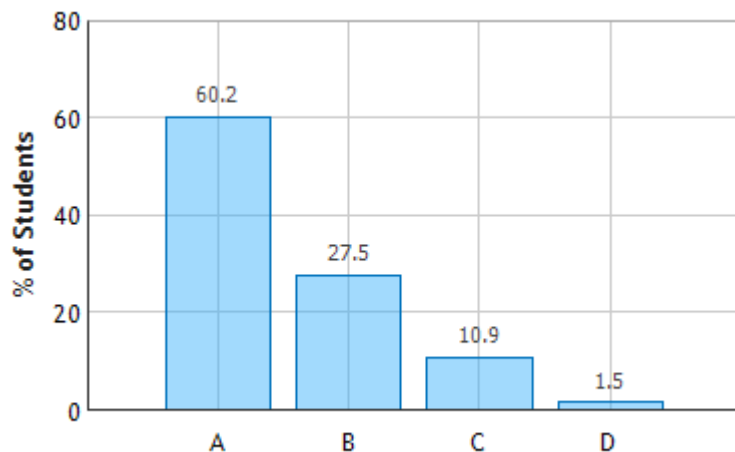
# Checkpoint 1



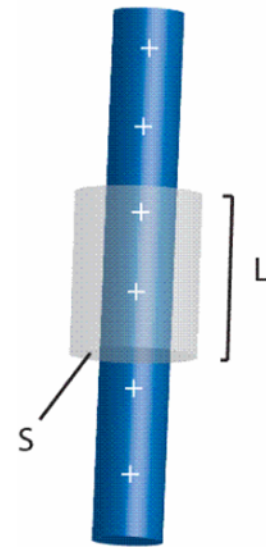
“Because the charge is proportional to length of the cylinder enclosing the charge, and the flux is proportional to the charge, this means that the flux is proportional to the length and an increase in length of  $2l$  yields in an increase in flux of  $2\Phi$ .”

“The flux is just  $E \cdot A$ , and the area is  $2(\pi)RL$ , so in both cases the flux is  $(\lambda)(s)(L)/(\epsilon_0)$ . The fluxes are the same.”

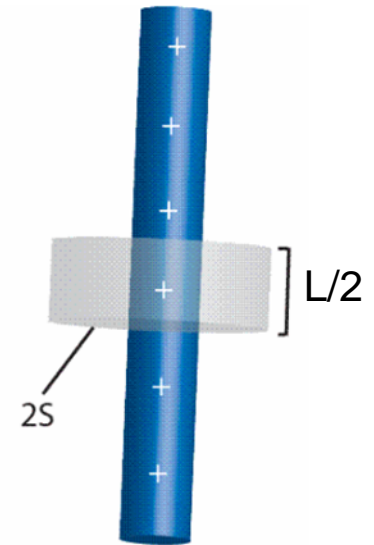
“The first cylinder has double the area, and double the charge so the flux should be quadrupled.”



An infinitely long charged rod has uniform charge density  $\lambda$  and passes through a cylinder (gray). The cylinder in Case 2 has twice the radius and half the length compared with the cylinder in Case 1.



Case 1



Case 2

$$\Phi_1 = 2\Phi_2$$

(A)

$$\Phi_1 = \Phi_2$$

(B)

$$\Phi_1 = 1/2\Phi_2$$

(C)

none  
(D)

# CheckPoint 2

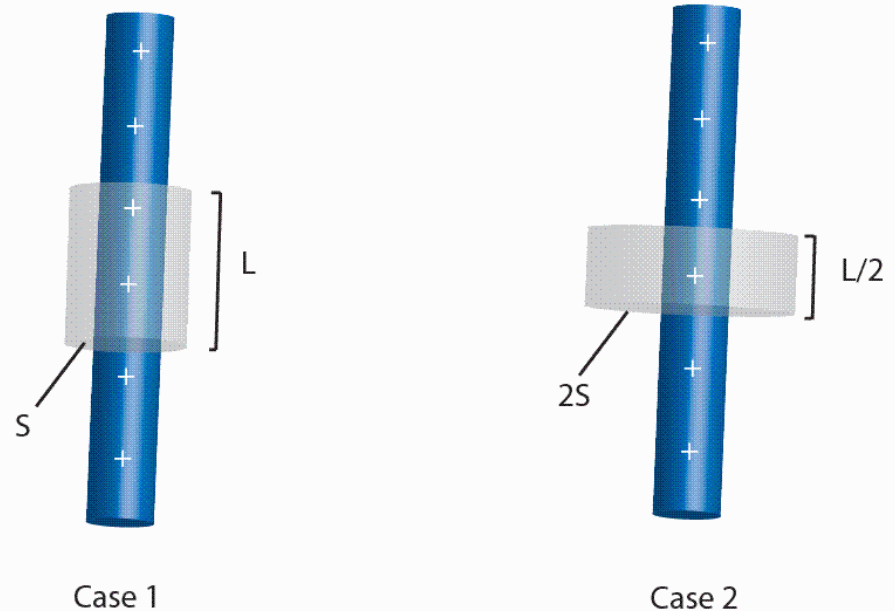
## Definition of Flux:

$$\Phi \equiv \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

$E$  constant on barrel of cylinder  
 $E$  perpendicular to barrel surface  
 ( $E$  parallel to  $dA$ )

$$\begin{aligned} \Phi &= E \int_{\text{barrel}} d\vec{A} \\ &= EA_{\text{barrel}} \end{aligned}$$

2) An infinitely long charged rod has uniform charge density of  $\lambda$ , and passes through a cylinder (gray). The cylinder in case 2 has twice the cross sectional area and half the length compared to the cylinder in case 1.



$\Phi_1 = 2\Phi_2$ (A)	$\Phi_1 = \Phi_2$ (B)	$\Phi_1 = 1/2\Phi_2$ (C)	none (D)
---------------------------	--------------------------	-----------------------------	-------------

Case 1

$$\begin{aligned} A_{\text{barrel}} &= 2\pi sL \\ E_1 &= \frac{\lambda}{2\pi\epsilon_0 s} \end{aligned} \rightarrow \boxed{\Phi_1 = \frac{\lambda L}{\epsilon_0}}$$

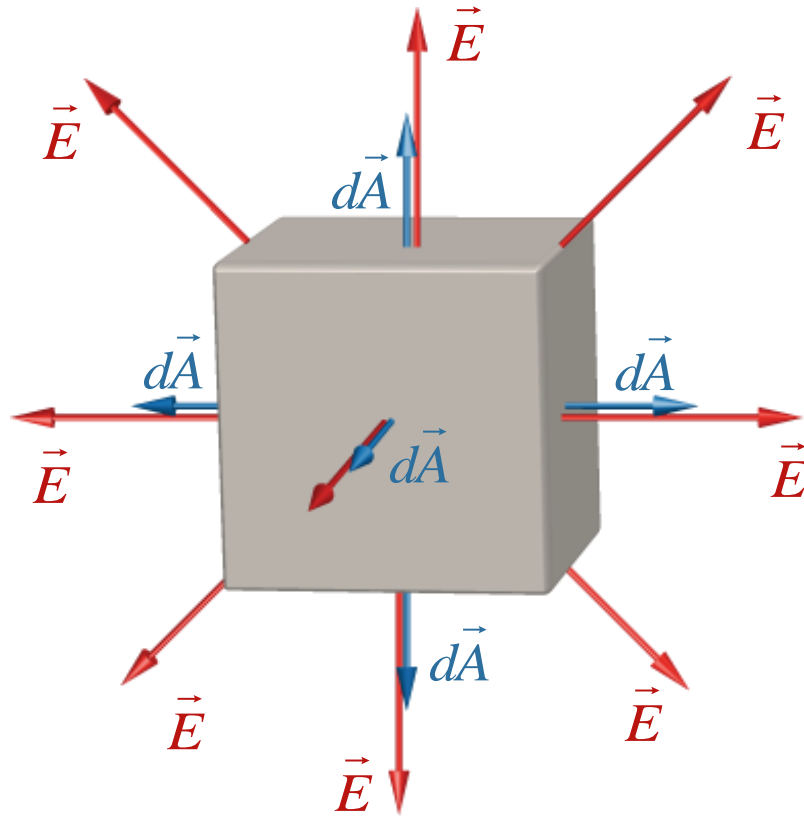
Case 2

$$\begin{aligned} A_2 &= (2\pi(2s))L/2 = 2\pi sL \\ E_2 &= \frac{\lambda}{2\pi\epsilon_0(2s)} \end{aligned} \rightarrow \boxed{\Phi_2 = \frac{\lambda(L/2)}{\epsilon_0}}$$

RESULT: GAUSS' LAW

$\Phi$  proportional to charge enclosed !

# Direction Matters:

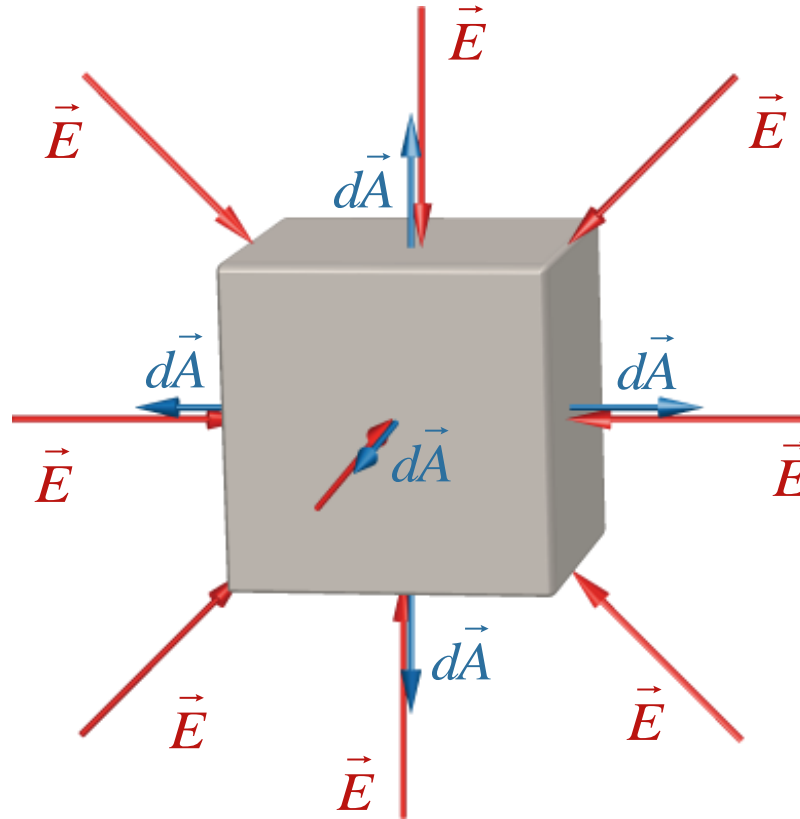


For a closed surface,  
 $d\vec{A}$  points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

# Direction Matters:

Can flux through a flat plane be negative? What would that represent?



For a closed surface,  
 $d\vec{A}$  points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} < 0$$

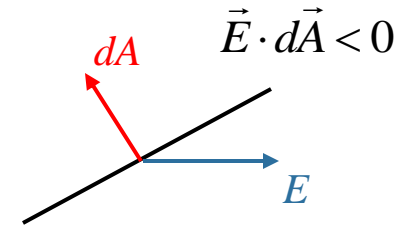
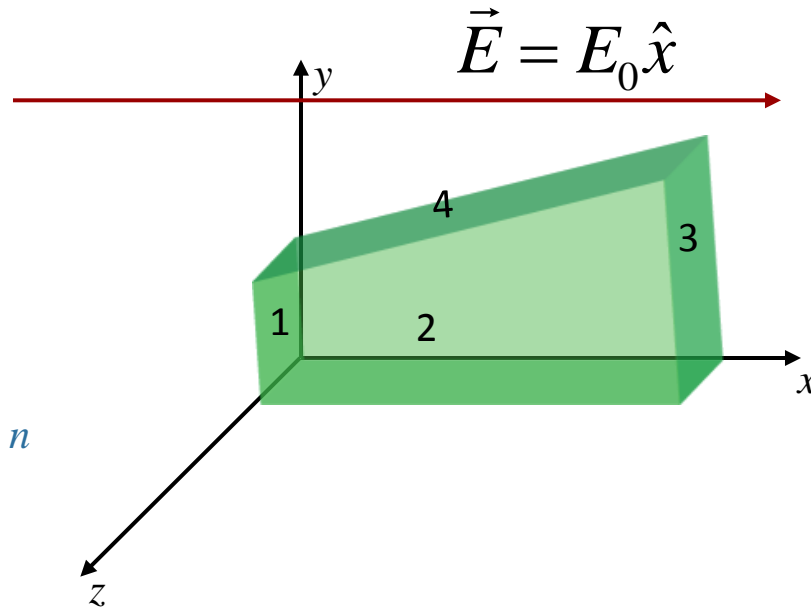
# Trapezoid in Constant Field



Label faces:

- 1:  $x = 0$
- 2:  $z = +a$
- 3:  $x = +a$
- 4: slanted

Define  $\Phi_n$  = Flux through Face  $n$



A)  $\Phi_1 < 0$

B)  $\Phi_1 = 0$

C)  $\Phi_1 > 0$

A)  $\Phi_2 < 0$

B)  $\Phi_2 = 0$

C)  $\Phi_2 > 0$

A)  $\Phi_3 < 0$

B)  $\Phi_3 = 0$

C)  $\Phi_3 > 0$

A)  $\Phi_4 < 0$

B)  $\Phi_4 = 0$

C)  $\Phi_4 > 0$

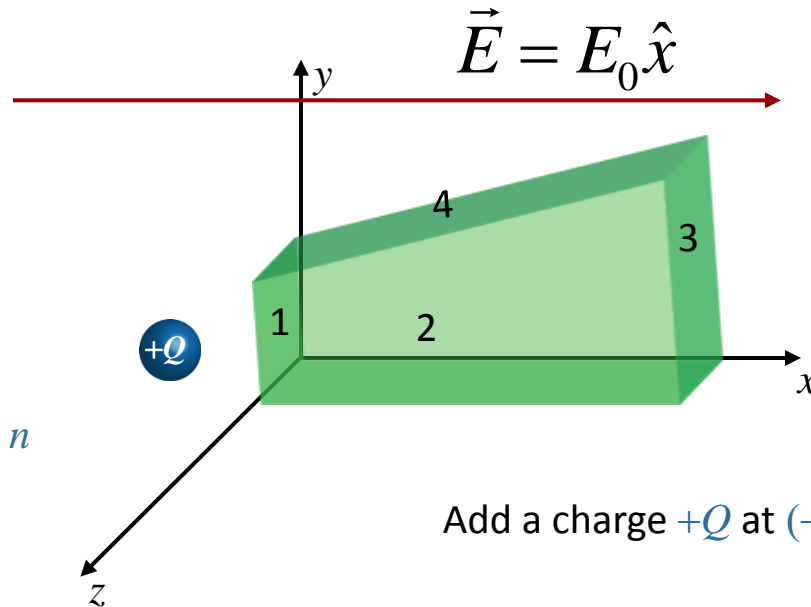
# Trapezoid in Constant Field + $Q$



Label faces:

- 1:  $x = 0$
- 2:  $z = +a$
- 3:  $x = +a$
- 4: slanted

Define  $\Phi_n$  = Flux through Face  $n$   
 $\Phi$  = Flux through Trapezoid



Add a charge  $+Q$  at  $(-a, a/2, a/2)$

How does Flux change?  
Note  $(-6 < -4)$  sign matters

A)  $\Phi_1$  increases

B)  $\Phi_1$  decreases

C)  $\Phi_1$  remains same

A)  $\Phi_3$  increases

B)  $\Phi_3$  decreases

C)  $\Phi_3$  remains same

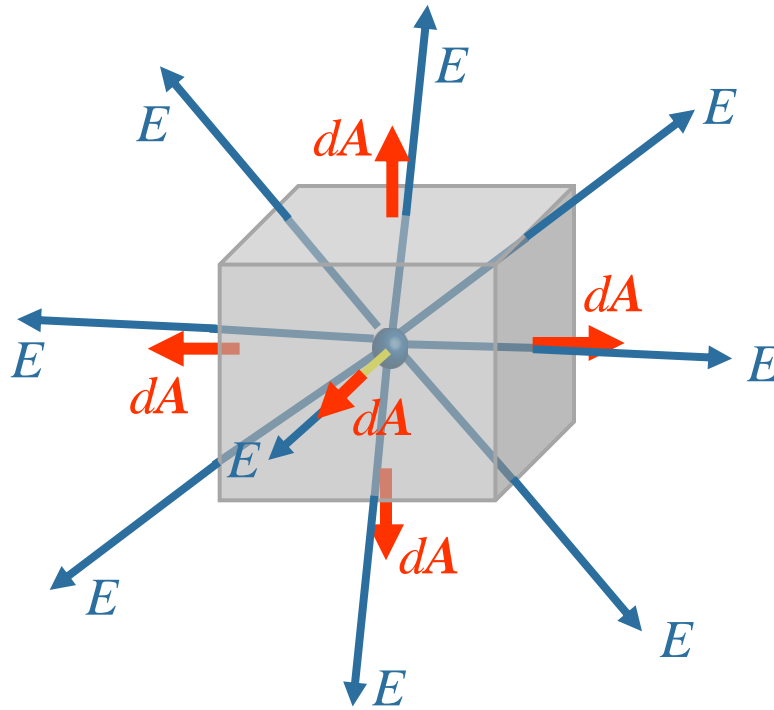
A)  $\Phi$  increases

B)  $\Phi$  decreases

C)  $\Phi$  remains same

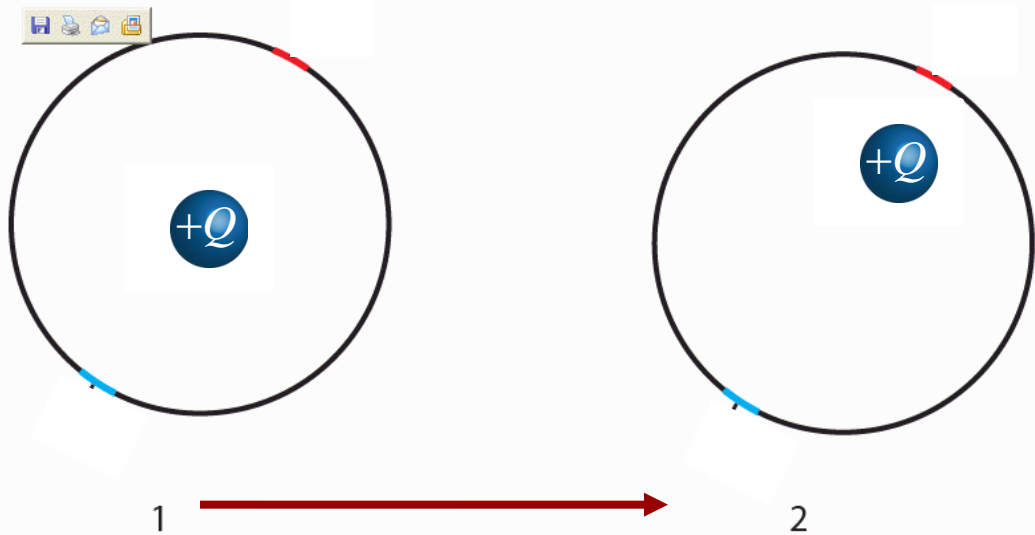


# Gauss Law



$$\int_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

# Checkpoint 2.3

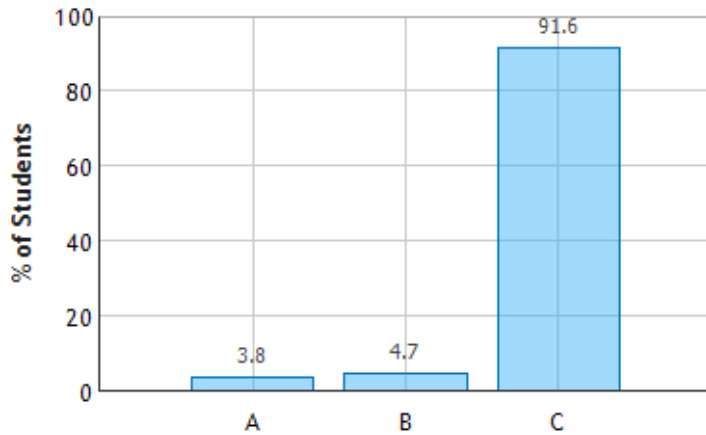


A  
 $\Phi_E$  increases

B  
 $\Phi_E$  decreases

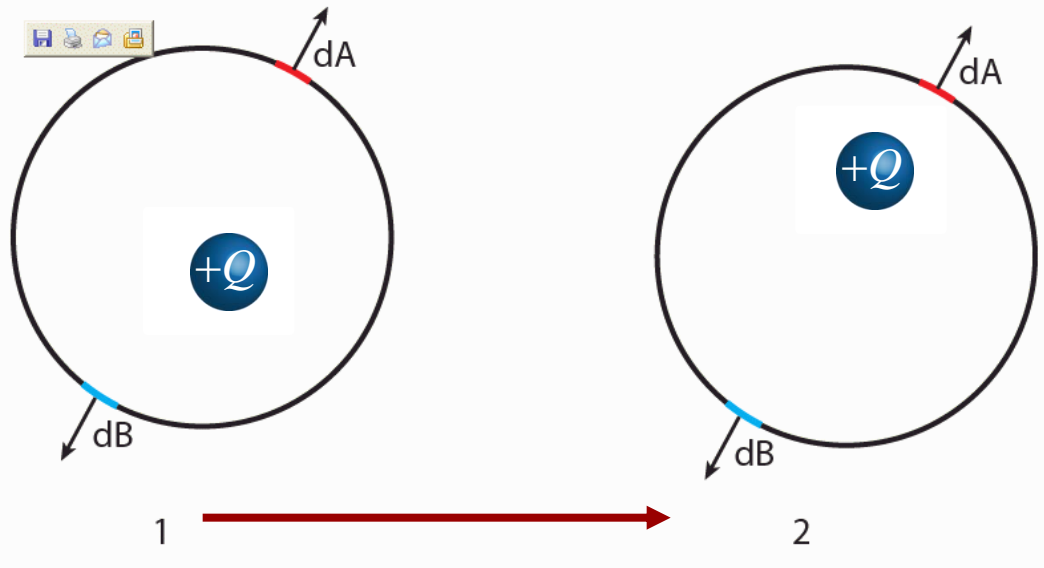
C  
 $\Phi_E$  stays same

Flux from Point Charge Through Surfaces of Sphere: Question 3 (N = 879)



“Because  $+Q$  is still in the shell, no matter where it is, the total flux is the same.”

# CheckPoint 2.1



A

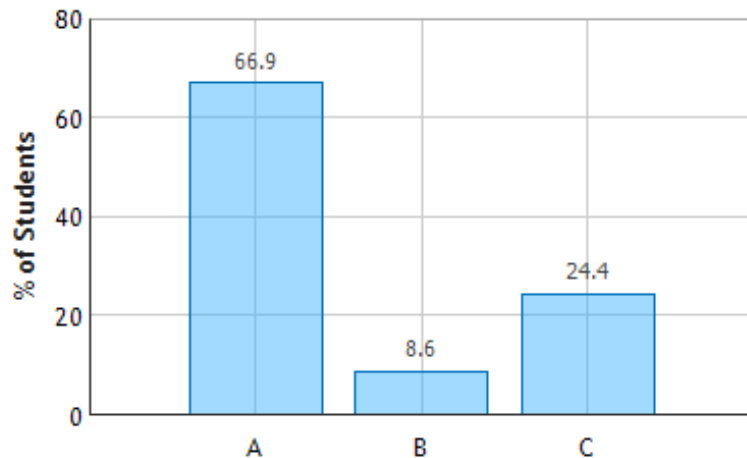
B

C

$d\Phi_A$  increases  
 $d\Phi_B$  decreases

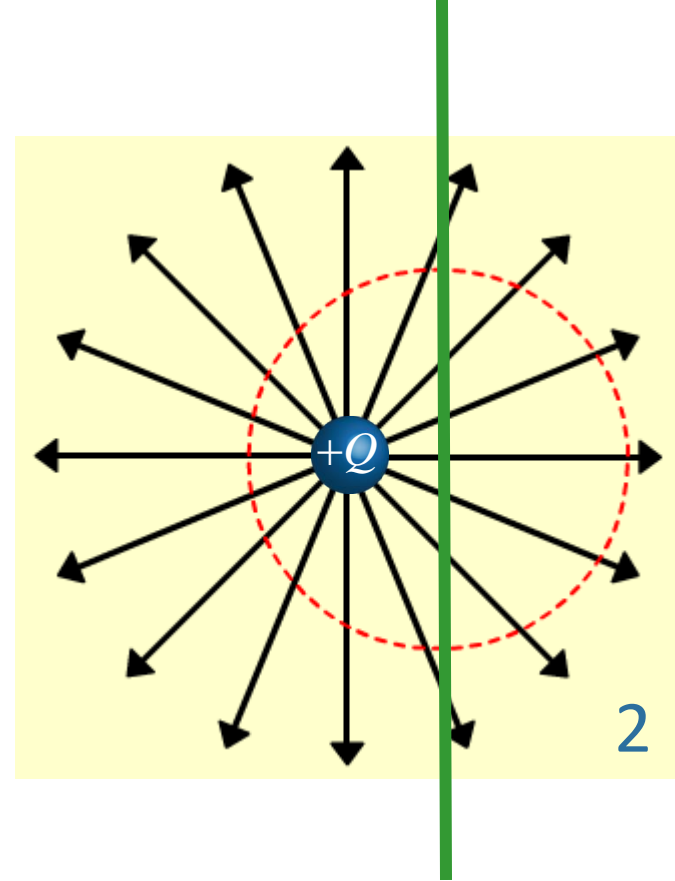
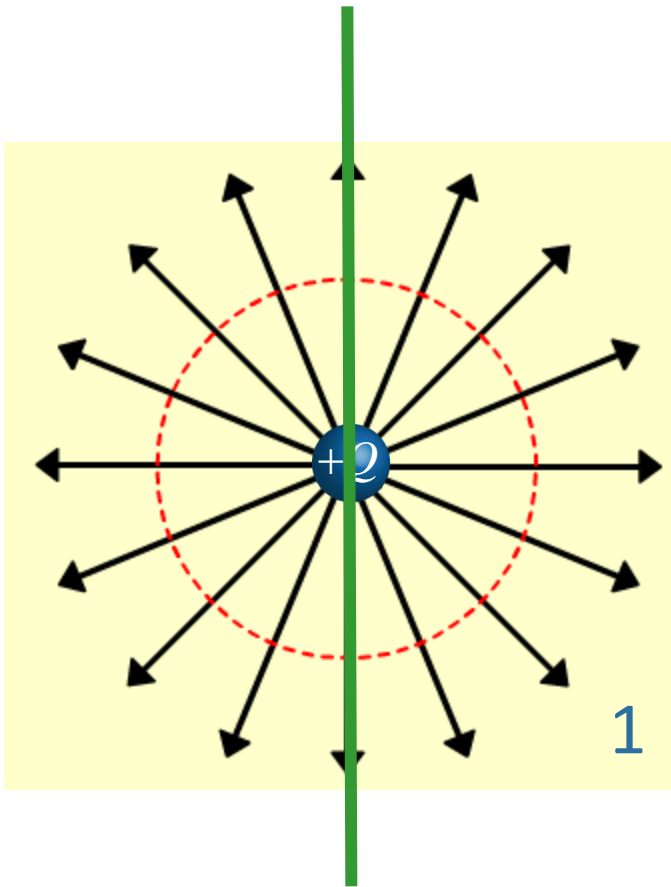
$d\Phi_A$  decreases  
 $d\Phi_B$  increases

$d\Phi_A$  stays same  
 $d\Phi_B$  stays same



“Electric field is now stronger at point A. More field line passing through means a greater value for electric flux”

# *Think of it this way:*

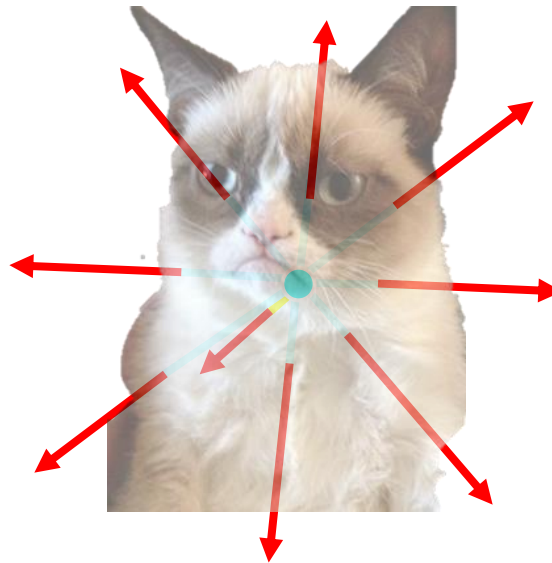
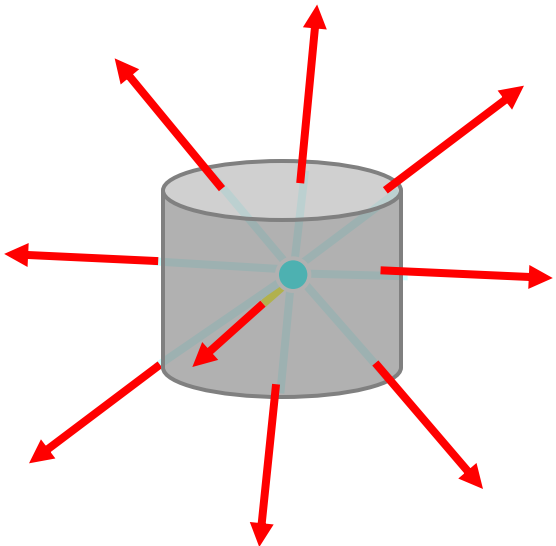
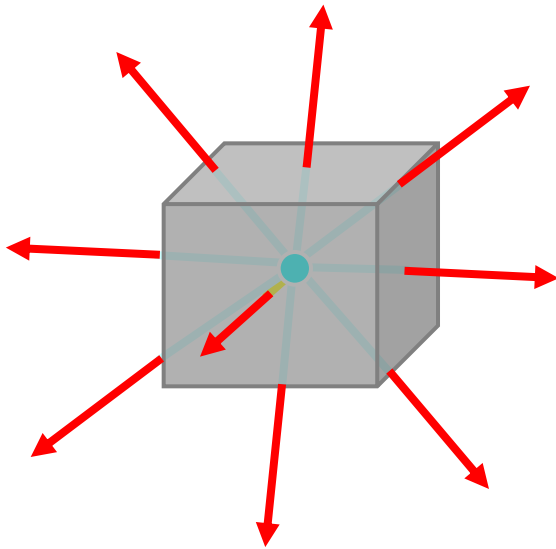


The total flux is the same in both cases (just the total number of lines)  
The flux through the right (left) hemisphere is smaller (bigger) for case 2.

# Things to notice about Gauss Law

$$\Phi_S = \int_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

If  $Q_{\text{enclosed}}$  is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.



# Things to notice about Gauss Law

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

In cases of high symmetry it may be possible to bring  $E$  outside the integral. In these cases we can solve Gauss Law for  $E$

$$E \int dA = \frac{Q_{enclosed}}{\epsilon_0}$$

$$E = \frac{Q_{enclosed}}{A \epsilon_0}$$

So - if we can figure out  $Q_{enclosed}$  and the area of the surface  $A$ , then we know  $E$ !

This is the topic of the next lecture.