Your Comments

This material was a lot more understandable than Gauss's Law.

I understood the prelecture and all the questions through out it... and then I got to the checkpoint questions... In particular the second part of the electric potential energy of a system and the motion of the point charge.

I'm just psyched that there isn't a lab this week. Maybe the professor will read this, probably not, buy a guy can dream.

Be sure to clarify that graph in prelecture question 2 where it asked about the potential energy in case one versus case 2. The graph on the preceding slide showed the MAGNITUDE of the potential energy, whereas the question asked about which was highest in potential energy, at which point sign must be taken into account.

does potential energy increase as distance between them increase?

Please explain the third check point question. Also, fro the first question on the prelecture, I do not see how we were suppose to know the direction gravity was pointing in. Please explain! And one more thing (last one, I promise) so I get that the potential energy in a system is just the sum of all the set pairs of particles, and the triangle example made sense, but what would happen if there were, say, 4 particles in a square. Would you factor in the charges diagonal to each other as well?

Physics 212
Lecture 5

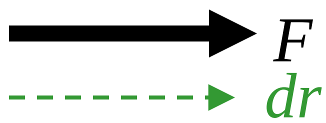
Today's Concept:

Electric Potential Energy

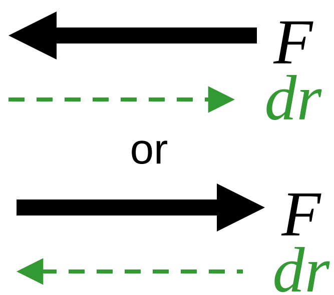
Work (Mechanics Review)

Recall from physics 211:

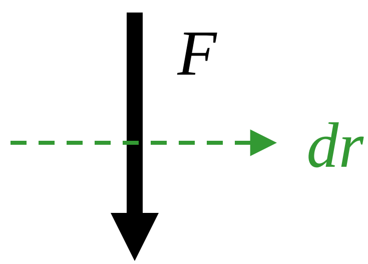
$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \qquad W_{TOT} = \Delta K$$



$W > 0$ (e.g. $W_{gravity}$ on object dropped)



$W < 0$ (e.g. $W_{gravity}$ on ball going up)



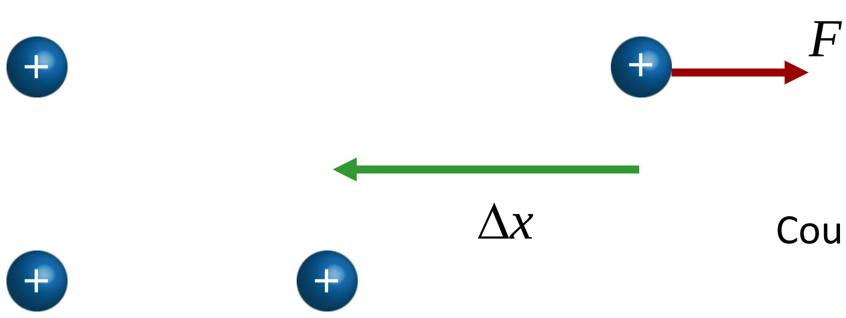
$W = 0$ (e.g. $W_{gravity}$ on moving horizontally)

Potential Energy

$$\Delta U \equiv -W_{conservative}$$

If gravity does negative work, potential energy increases!

Same idea for Coulomb force… if Coulomb force does negative work, potential energy increases.

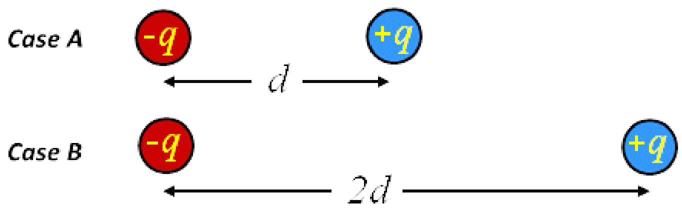


Coulomb force does negative work
Potential energy increases

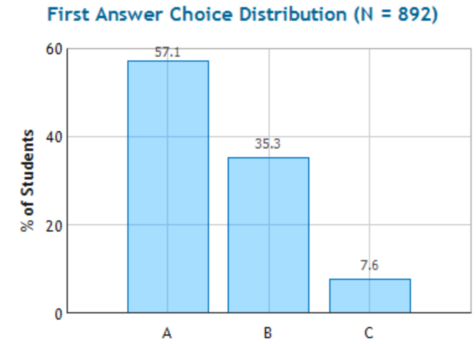
Prelecture Question 2

In Case A two charges which are equal in magnitude but opposite in charge are separated by a distance *d*. In Case B the same charges are separated by a distance 2*d*.

Which configuration has the highest potential energy *U*?

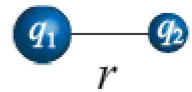


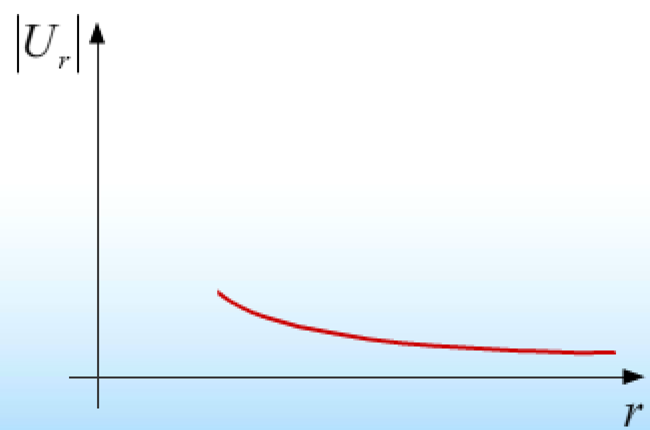
- Case A has the highest potential energy
- Case B has the highest potential energy
- Both cases have the same potential energy



Electric Potential Energy

$$U_r \equiv \Delta U_{\infty r} = \frac{q_1 q_2}{4\pi\varepsilon_o r}$$





Could you explain why in the prelecture question #2 the increased distance also increased the electric potential energy? Right before that the prelecture had specified that the closer the charged particles are to each other, the larger the magnitude of the electric potential energy.

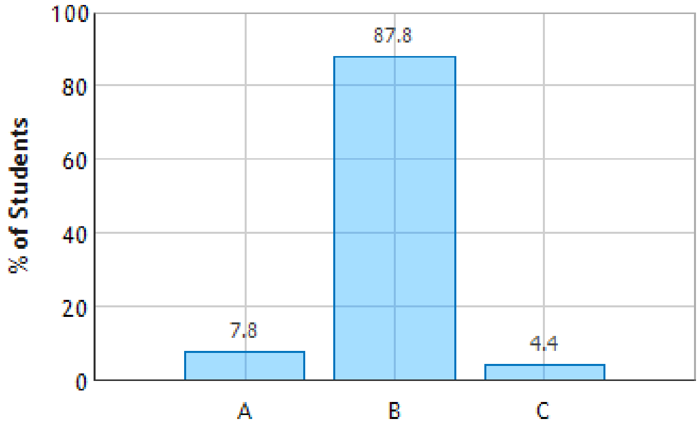
CheckPoint 4

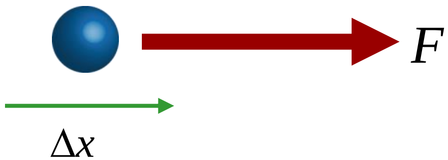
A charge is released from rest in a region of electric field. The charge will start to move

A) In a direction that makes its potential energy increase.

B) In a direction that makes its potential energy decrease

C) Along a path of constant potential energy.





"Potential energy is always minimized in systems. Balls roll downhill, not uphill."

It will move in the same direction as F

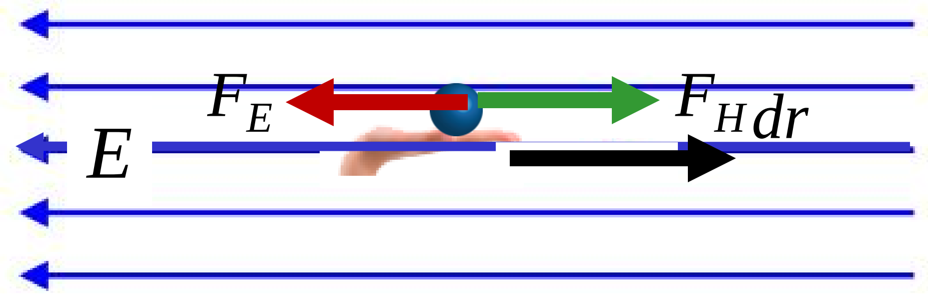
Work done by force is positive

$\Delta U = -$Work, so is is negative

Nature wants things to move in such a way that PE decreases

Clicker Question

You hold a positively charged ball and walk due east in a region that contains an electric field directed due west.



W_H is the work done by the hand on the ball

W_E is the work done by the electric field on the ball

Which of the following statements is true:

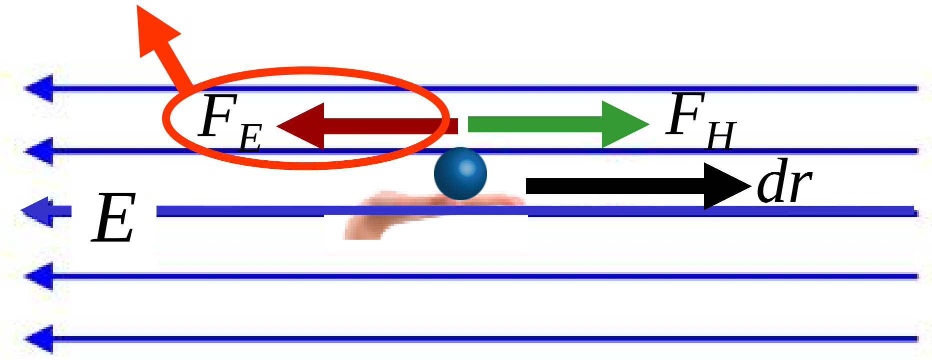
A) $W_H > 0$ and $W_E > 0$

B) $W_H > 0$ and $W_E < 0$

C) $W_H < 0$ and $W_E < 0$

D) $W_H < 0$ and $W_E > 0$

Conservative force: $\Delta U = -W_E$



B) $W_H > 0$ and $W_E < 0$

Is ΔU positive, negative or zero?

A) Positive

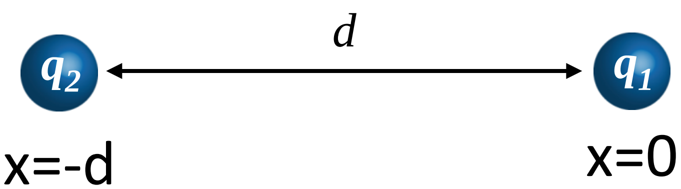
B) Negative

C) Zero

Example: Two Point Charges

Calculate the change in potential energy for two point charges originally very far apart moved to a separation of "d"

$$\Delta U \equiv -\int_i^f \vec{F} \cdot d\vec{r}$$



$$= \int_{-\infty}^{-d} F \cdot dx$$

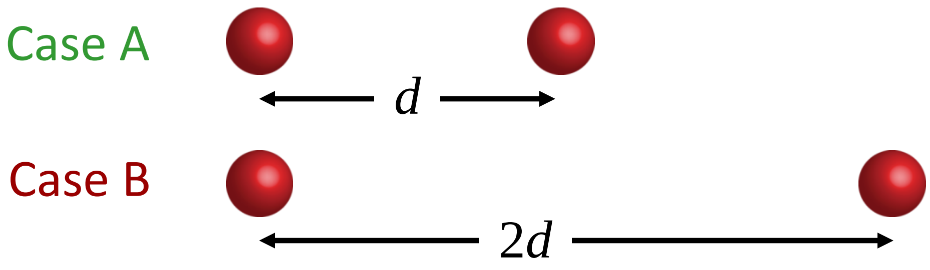
$$= \int_{-\infty}^{-d} k \frac{q_1 q_2}{x_{12}^2} dx$$

$$= -kq_1 q_2 \left[-\frac{1}{d} - \left(-\frac{1}{\infty} \right) \right] = k \frac{q_1 q_2}{d} \quad = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d}$$

Charged particles with the same sign have an increase in potential energy when brought closer together.

For point charges often choose $r =$ infinity as "zero" potential energy.

Clicker Question



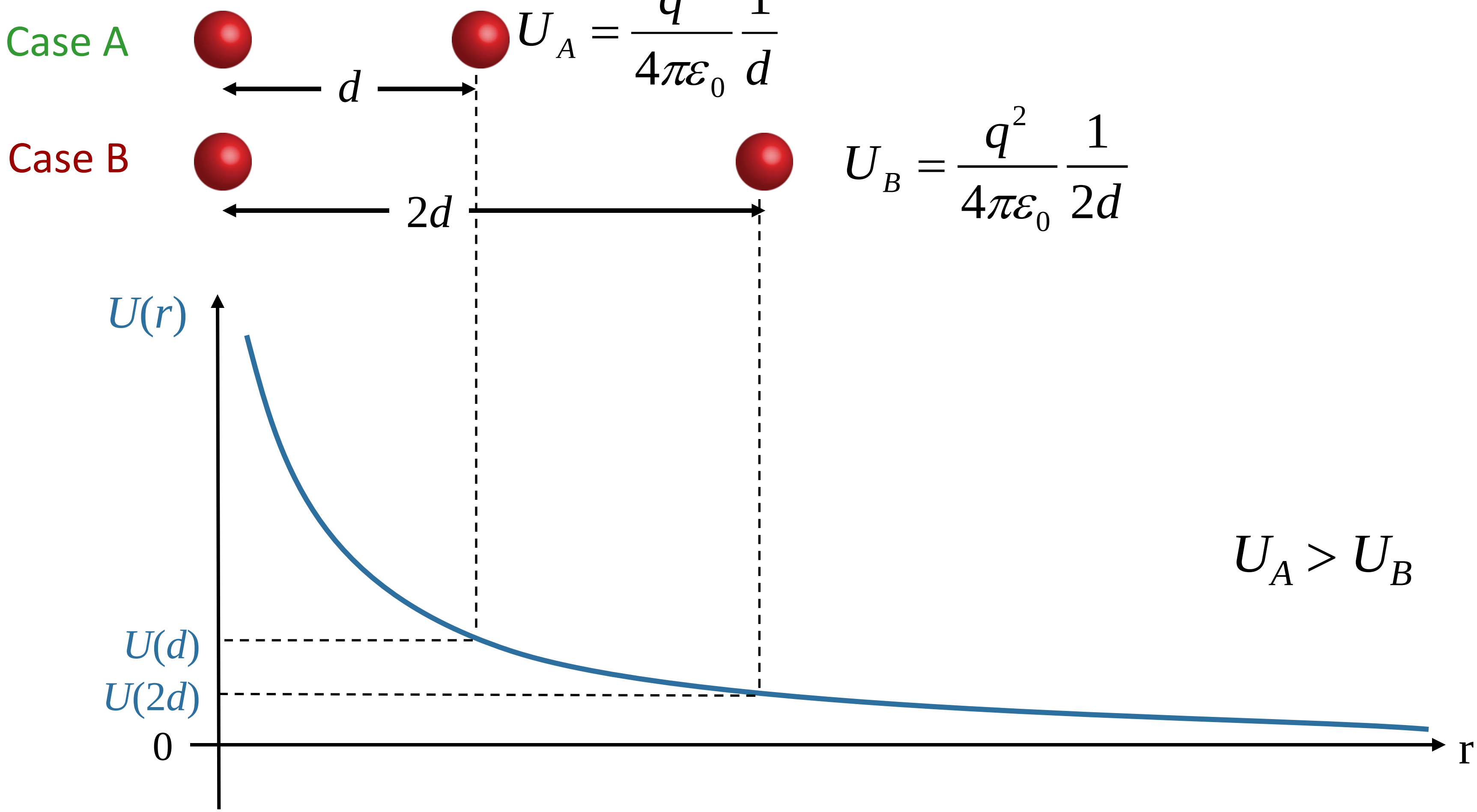
In case A two negative charges which are equal in magnitude are separated by a distance d. In case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

A) Case A

B) Case B

As usual, choose $U = 0$ to be at infinity:

$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}$$



$$U_A = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{d}$$

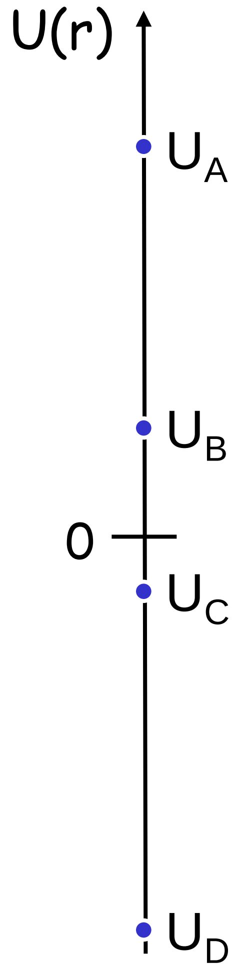
$$U_B = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{2d}$$

$$U_A > U_B$$

And Remember

U is just a number (not a vector)

- U DOES have a sign



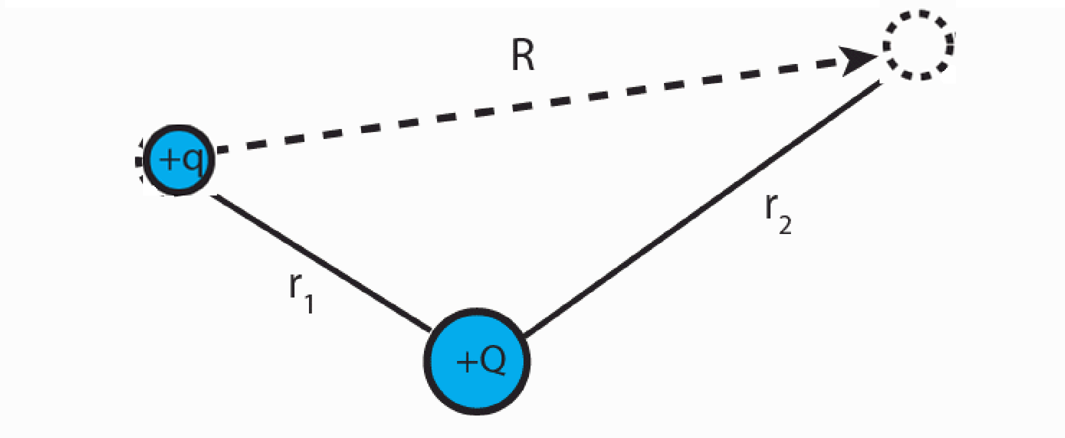
$U_A > U_B$

$U_B > U_C$

$U_C > U_D$

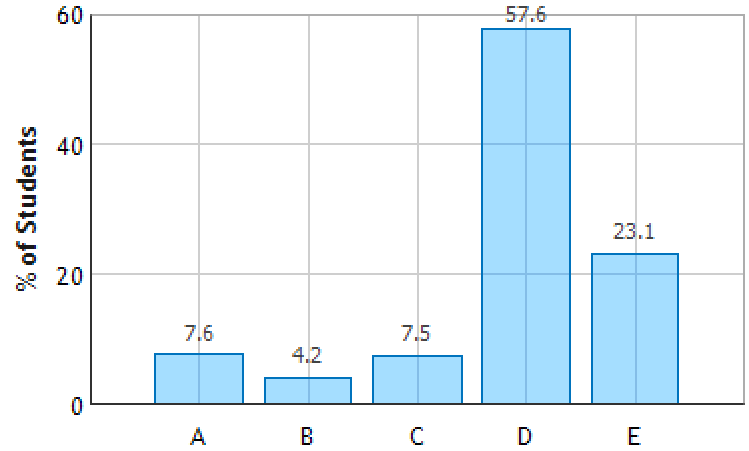
CheckPoint 1

A charge +q is moved from position 1 to position 2,
What is the change in potential energy?



A $\frac{kQq}{R}$ B $\frac{kQqR}{r_1^2}$ C $\frac{kQqR}{r_2^2}$

D $kQq\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$ E $kQq\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$



$$U_1 = \frac{kQq}{r_1} \qquad U_2 = \frac{kQq}{r_2}$$

$$\Delta U \equiv U_2 - U_1 = kQq\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

The initial potential energy is represented by kQq/r1, and the final is represented by kQq/r2. The difference is then kQq(1/r2-1/r1).

Note: $+q$ moves AWAY from $+Q$.
Its Potential energy MUST DECREASE

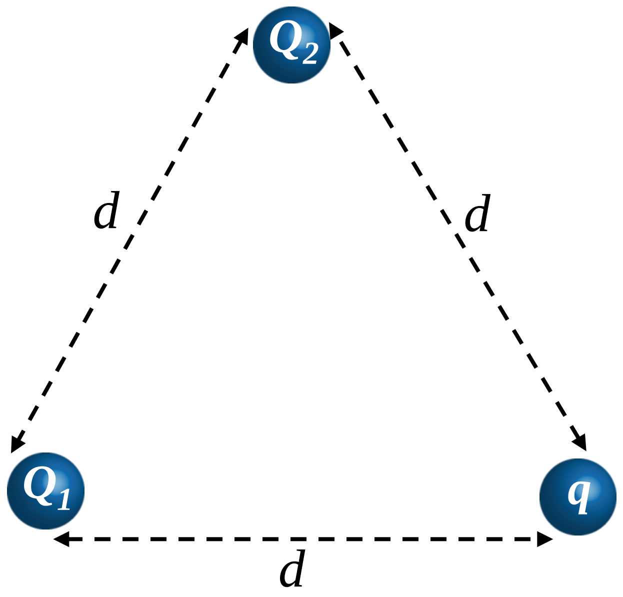
$$\Delta U < 0$$

Potential Energy of Many Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi\varepsilon_0}\frac{1}{d} + \frac{qQ_2}{4\pi\varepsilon_0}\frac{1}{d}$$

(superposition)



Potential Energy of Many Charges

What is the total energy required to bring in three identical charges, from infinitely far away to the points on an equilateral triangle shown.

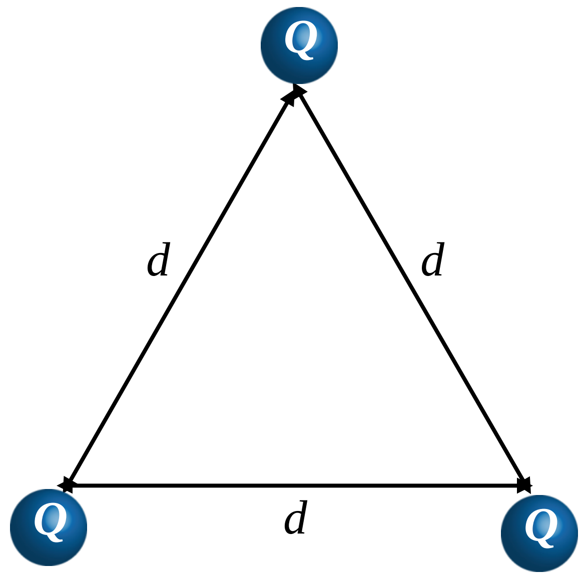
A) 0

B) $\Delta U = \frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

C) $\Delta U = 2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

D) $\Delta U = 3\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

E) $\Delta U = 6\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



$$W = \sum W_i = -\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$\Delta U = +\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

Work by E to bring in first charge: $W_1 = 0$

Work by E to bring in second charge : $W_2 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Work by E to bring in third charge : $W_3 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} = -\frac{2}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Potential Energy of Many Charges

Suppose one of the charges is negative. Now what is the total energy required to bring the three charges in from infinitely far away?

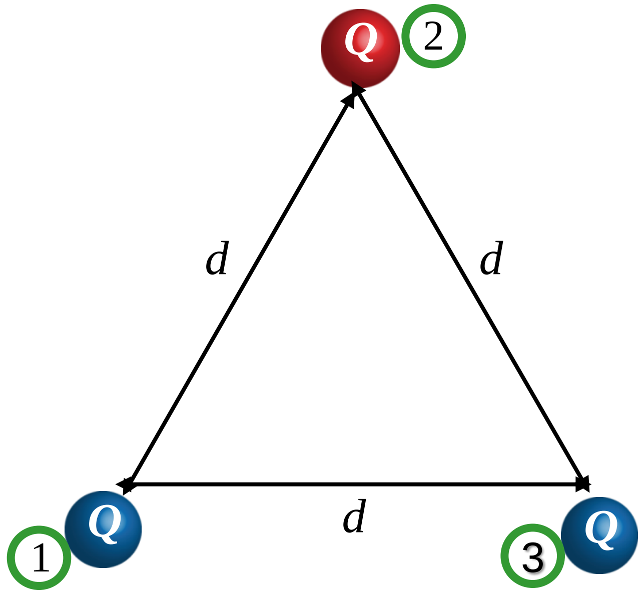
A) 0

B) $\Delta U = +1\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

C) $\Delta U = -1\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

D) $\Delta U = +2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

E) $\Delta U = -2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



$$W = \sum W_i = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$\Delta U = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

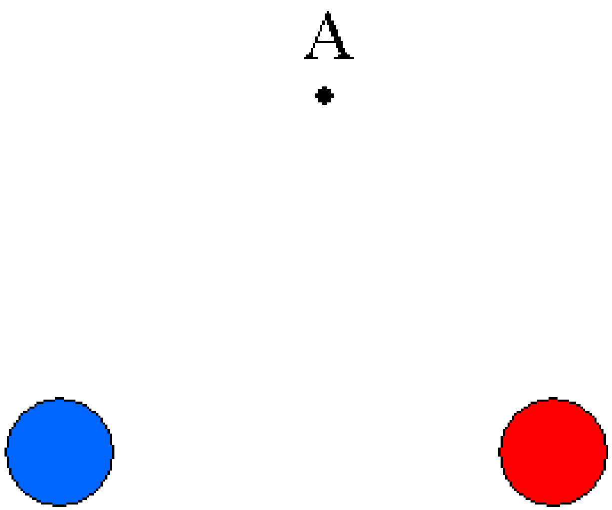
Work by E to bring in first charge: $W_1 = 0$

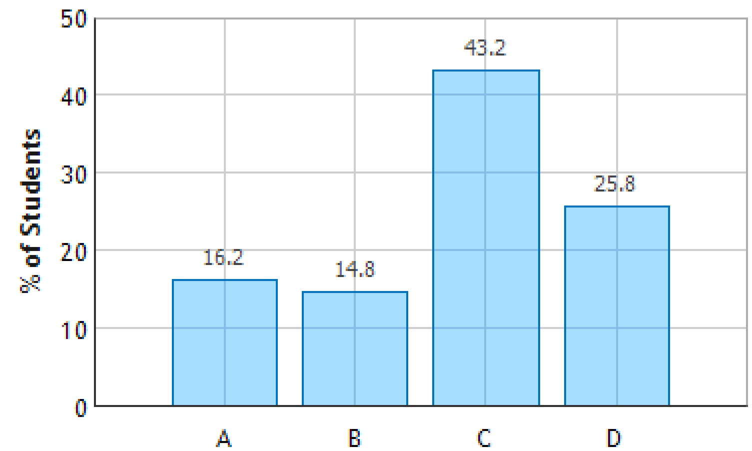
Work by E to bring in second charge : $W_2 = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Work by E to bring in third charge : $W_3 = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} = 0$

CheckPoint 2

Two charges with equal magnitude but opposite sign are located at equal distances from the point labeled A





If a third charge is brought in from far away to point A, how does the potential energy of the collection of charges change?

Increases	Decreases	Same	Depends on sign of charge
A	B	C	D

C) "The potential energy will be positive for the charge of the same sign and negative for the charge of the opposite sign, yielding a potential that is the same relative to the first."

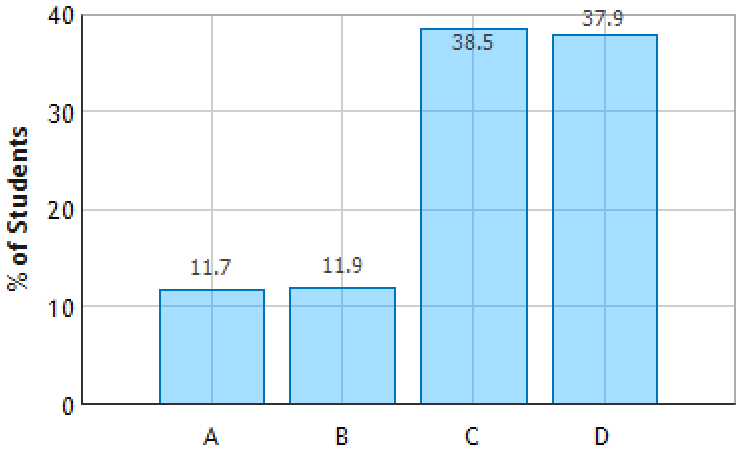
D) "The sign will determine if the sum of the potential energies will be a positive or a negative number."

CheckPoint 3

A positive charge is placed on the left side of a negative charge. The magnitude of the negative charge is twice that of the positive charge.

Is there any (finite) location that a third charge can be placed such that the total potential energy of the system does not change?

- YES, as long as the third charge is positive
- YES, as long as the third charge is negative
- YES, no matter what the third charge is
- NO



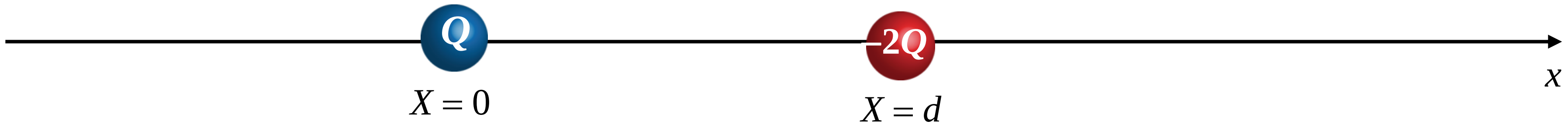
C) "Placing a charge twice the distance from the plus to the minus will keep the potential energy the same no matter the sign."

D) "It is impossible because the potential energy between the new charge and each of the original charges will be different from one another and will be unable to completely cancel."

LET'S DO THE CALCULATION!

Example

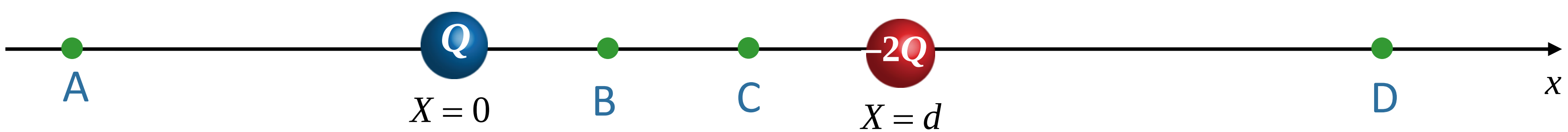
A positive charge q is placed at $x = 0$ and a negative charge $-2q$ is placed at $x = d$. At how many different places along the x axis could another positive charge be placed without changing the total potential energy of the system?



A) 0
B) 1
C) 2
D) 3

Example

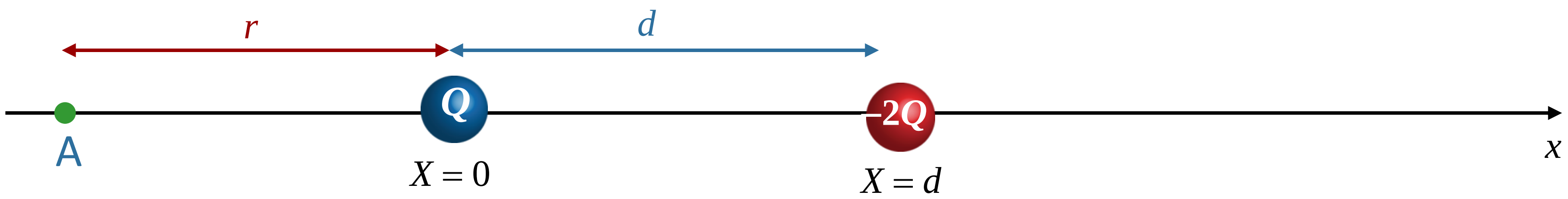
At which two places can a positive charge be placed without changing the total potential energy of the system?



A) A & B
B) A & C
C) B & C
D) B & D
E) A & D

Let's calculate the positions of A and B

Lets work out where A is



$$\Delta U = +\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{r+d}$$

Set $\Delta U = 0$

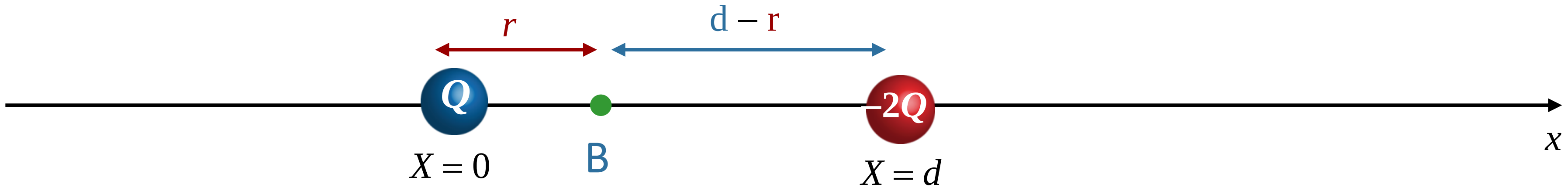
$$\frac{1}{r} = \frac{2}{r+d}$$

$$\boxed{r = d}$$

Makes Sense!

Q is twice as far from $-2q$ as it is from $+q$

Lets work out where B is



Setting $\Delta U = 0$ → $$\frac{1}{r} = \frac{2}{d-r}$$

$$2r = d - r$$

$$r = \frac{d}{3}$$

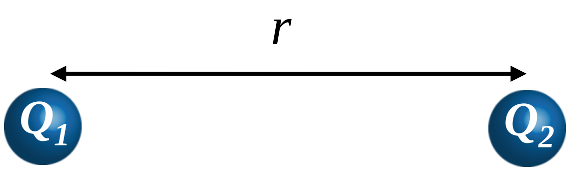
Makes Sense!

Q is twice as far from $-2q$ as it is from $+q$

Summary

For a pair of charges:

Just evaluate $$U = k\frac{q_1 q_2}{r}$$



(We usually choose $U = 0$ to be where the charges are far apart)

For a collection of charges:

Sum up $$U = k\frac{q_1 q_2}{r}$$ for all pairs