Your Comments

I really liked how he said "Woah"

I saw someone else's frivolous comment on the screen last lecture. I guess it's true what they say, the first cut is the deepest. :(

I'm really confused about all of these concepts. I don't understand in words (much less the formulas) what electric potential and electric potential difference are. I think it would be a good idea to try to explain what these are first before going into a lot of examples.

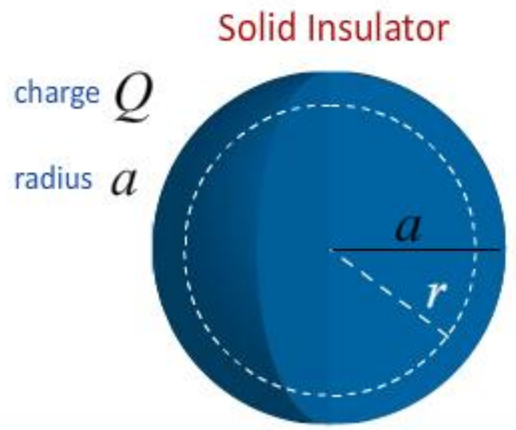
I feel really confused about the difference between electric potential energy and electric potential, and how these relate to the electric field.

Electric potential energy and electric potential?? what?! For the most part I understood everything, some topics were a little difficult to grasp at first.

Just unclear on one point, in the example: Charged Spherical Insulator when integrating the field from infinity to a you integrate with respect to 1/r^2, but for the field from a to r you integrate with respect to 1/a^3 and I'm just a little confused about why...

Please go over the change in potential in a uniformly charged insulating sphere slowly in lecture. Personally, I found the integral for the change for r<a rather confusing.

Charged Spherical Insulator



$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$$

For $r < a$

$$V(r) = -\int_{\infty}^{a} E\, dr \; - \int_{a}^{r} E\, dr$$

$$V(r) = -\int_{\infty}^{a} k \frac{Q}{r^2}\, dr \; - \int_{a}^{r} k \frac{Q}{a^3}\, r\, dr$$

Physics 212
Lecture 6

Today's Concept:

Electric Potential

(Defined in terms of Path Integral of Electric Field)

Big Idea

Last time we defined the electric potential energy of charge q in an electric field:

$$\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

The only mention of the particle was through its charge q.

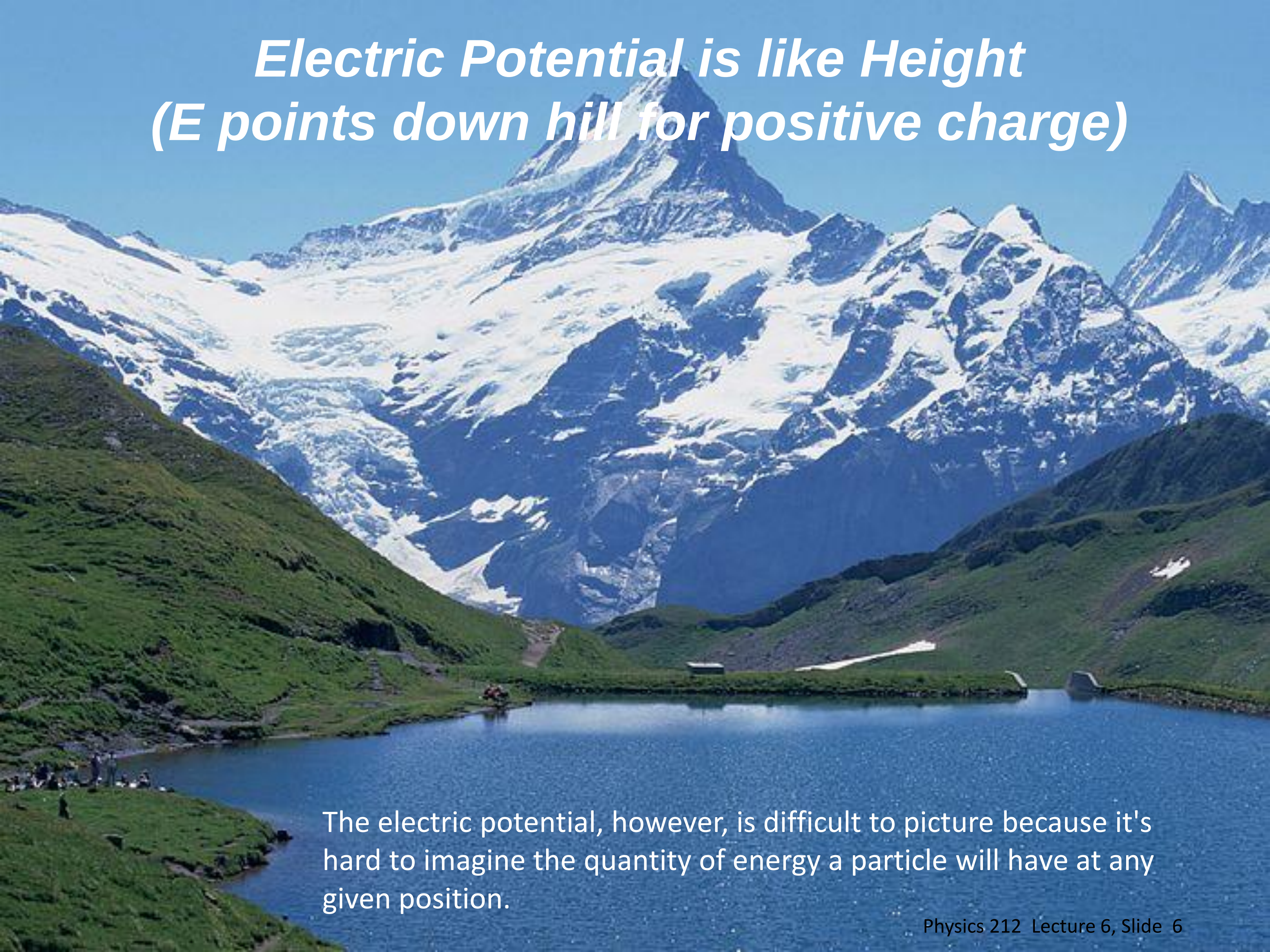
We can obtain a new quantity, the electric potential, which is a PROPERTY OF THE SPACE, as the potential energy per unit charge.

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Note the similarity to the definition of another quantity which is also a PROPERTY OF THE SPACE, the electric field.

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

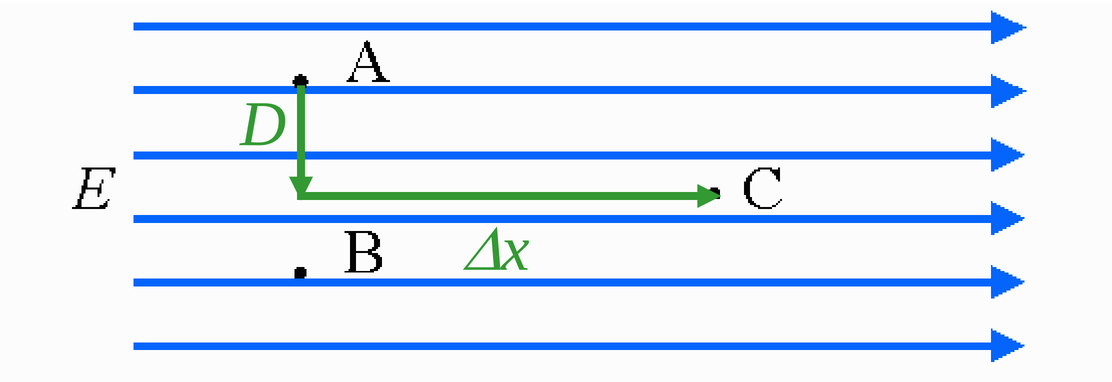
Electric Potential is like Height (E points down hill for positive charge)



The electric potential, however, is difficult to picture because it's hard to imagine the quantity of energy a particle will have at any given position.

Electric Potential from E field

Consider the three points A, B, and C located in a region of constant electric field as shown.



What is the sign of $\Delta V_{AC} = V_C - V_A$?

A) $\Delta V_{AC} < 0$ B) $\Delta V_{AC} = 0$ C) $\Delta V_{AC} > 0$

E points down hill

Remember the definition: $\Delta V_{A \to C} = -\int_A^C \vec{E} \cdot d\vec{l}$

Choose a path (any will do!)

$$\Delta V_{A \to C} = -\int_A^D \vec{E} \cdot d\vec{l} - \int_D^C \vec{E} \cdot d\vec{l} \quad \longrightarrow \quad \Delta V_{A \to C} = 0 - \int_D^C \vec{E} \cdot d\vec{l} = -E\Delta x < 0$$

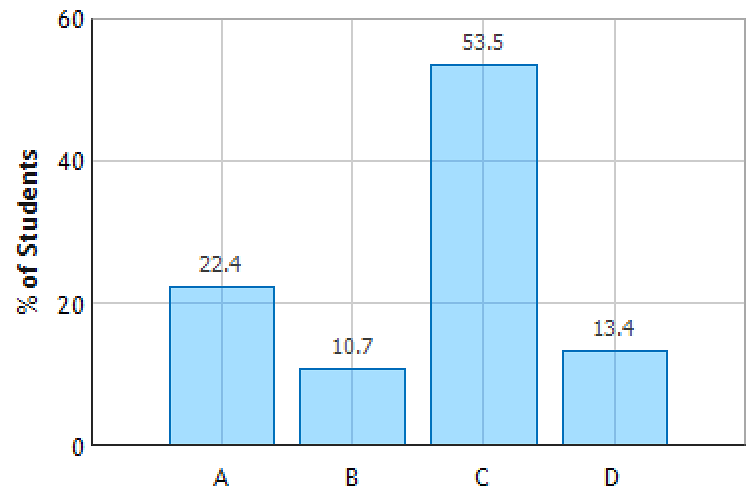
CheckPoint 2

If the electric field is zero in a region of space, what does that tell you about the electric potential in that region?

A) The electric potential is zero everywhere in this region.
B) The electric potential is zero at at least one point in this region.
C) The electric potential is constant everywhere in this region.
D) There is not enough information given to distingush which of the above answers is correct.

Remember the definition

$$\Delta V_{A\to B} = -\int_A^B \vec{E}\cdot d\vec{l}$$



$$\vec{E} = 0 \longrightarrow \Delta V_{A\to B} = 0 \longrightarrow V \text{ is constant!}$$

E from V

If we can get the potential by integrating the electric field:

$$\Delta V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{l}$$

We should be able to get the electric field by differentiating the potential?

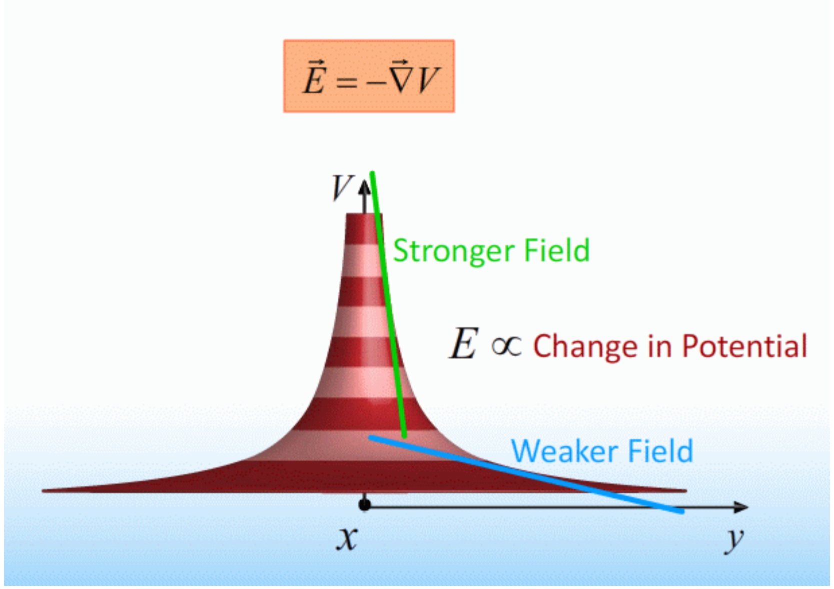
$$\vec{E} = -\vec{\nabla}V$$

In Cartesian coordinates:

$$E_x = -\frac{\partial V}{dx}$$

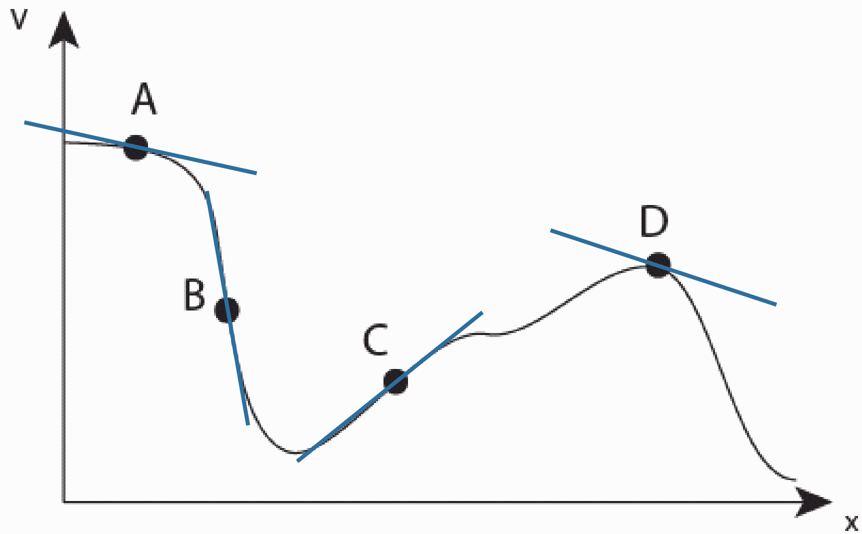
$$E_y = -\frac{\partial V}{dy}$$

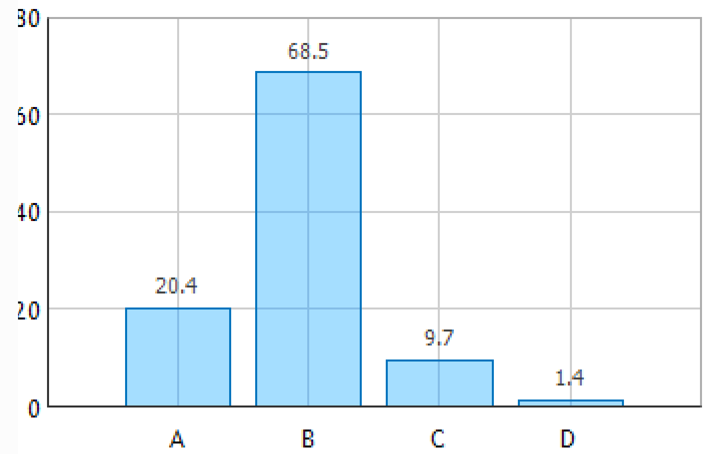
$$E_z = -\frac{\partial V}{dz}$$



CheckPoint 1a

2) The electric potential in a certain region is plotted in the following graph





At which point is the magnitude of the electric field greatest?

"A) higher electric potential indicates higher magnitude electric field"

"B) B has the steepest slope so the electrical potential is decreasing the fastest."

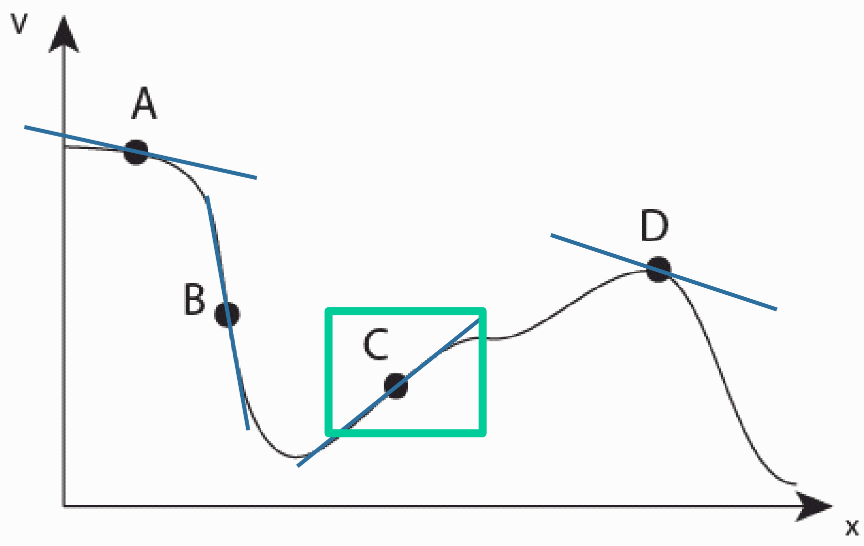
"C) Since voltage is the result of integrating the electric field, E will be the greatest when the slope has the largest positive slope, which is point c"

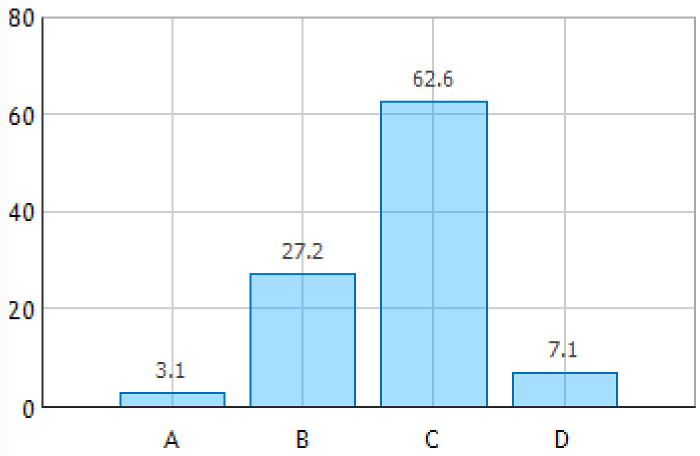
How do we get E from V?

$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow \quad \text{Look at slopes!}$$

CheckPoint 1b

2) The electric potential in a certain region is plotted in the following graph





At which point is the electric field pointing in the negative x direction?

"B) negative slope"

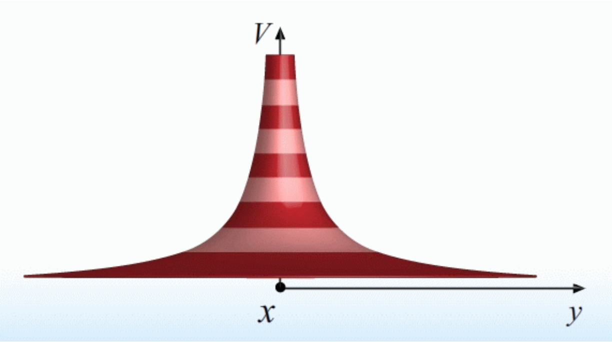
"C) The graph has a positive slope"

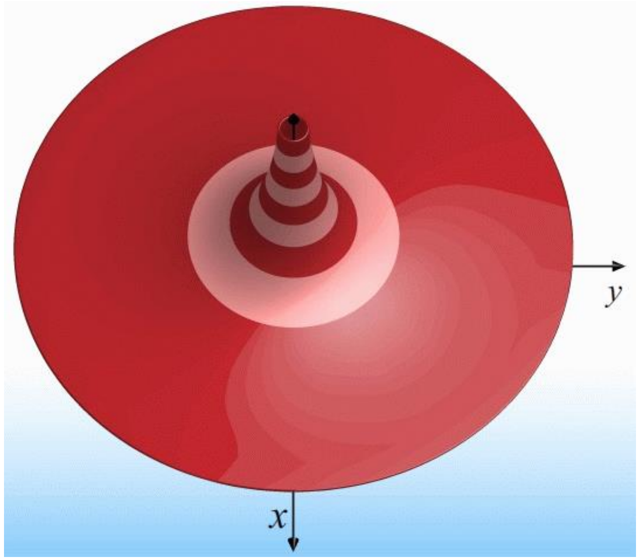
How do we get E from V?

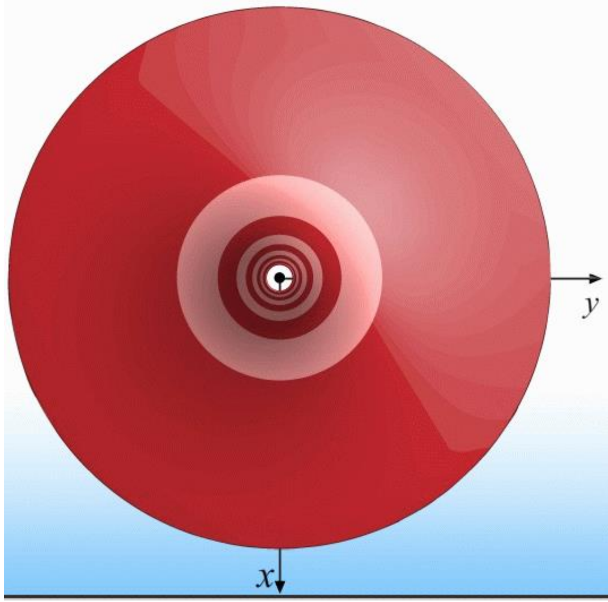
$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow \quad \text{Look at slopes!}$$

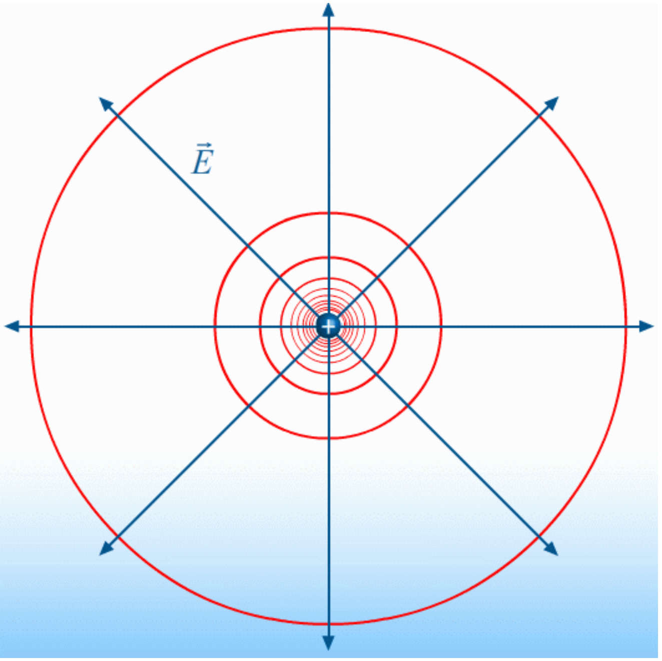
Equipotentials

Equipotentials are the locus of points having the same potential.







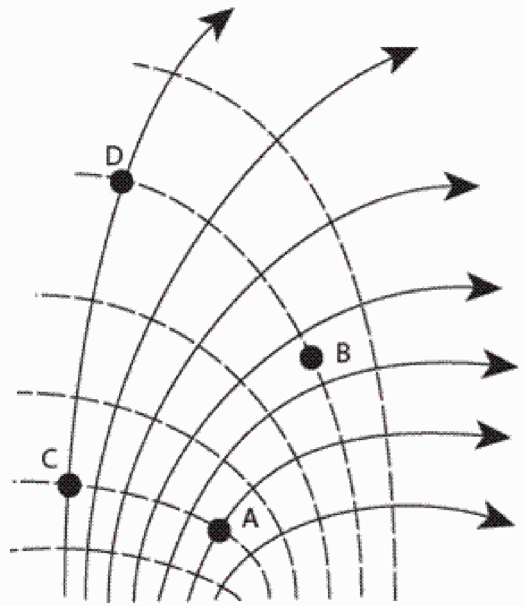


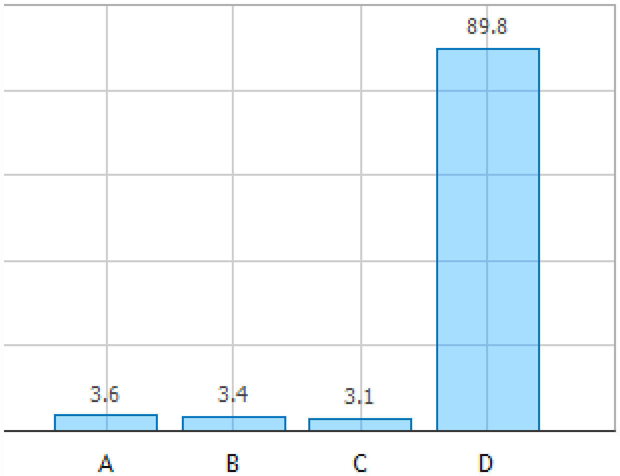
Equipotentials are
ALWAYS
perpendicular to the electric field lines.

The SPACING of the equipotentials indicates
The STRENGTH of the electric field.

CheckPoint 3a

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



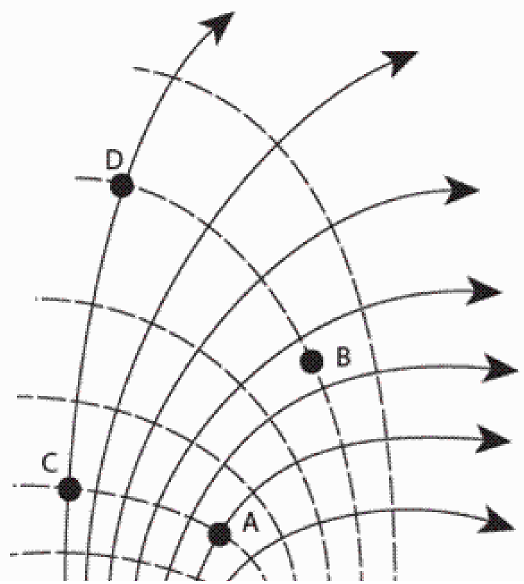


At which point is the magnitude of the electric field the smallest?

"This is where the density of the lines is the smallest."

CheckPoint 3b

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.

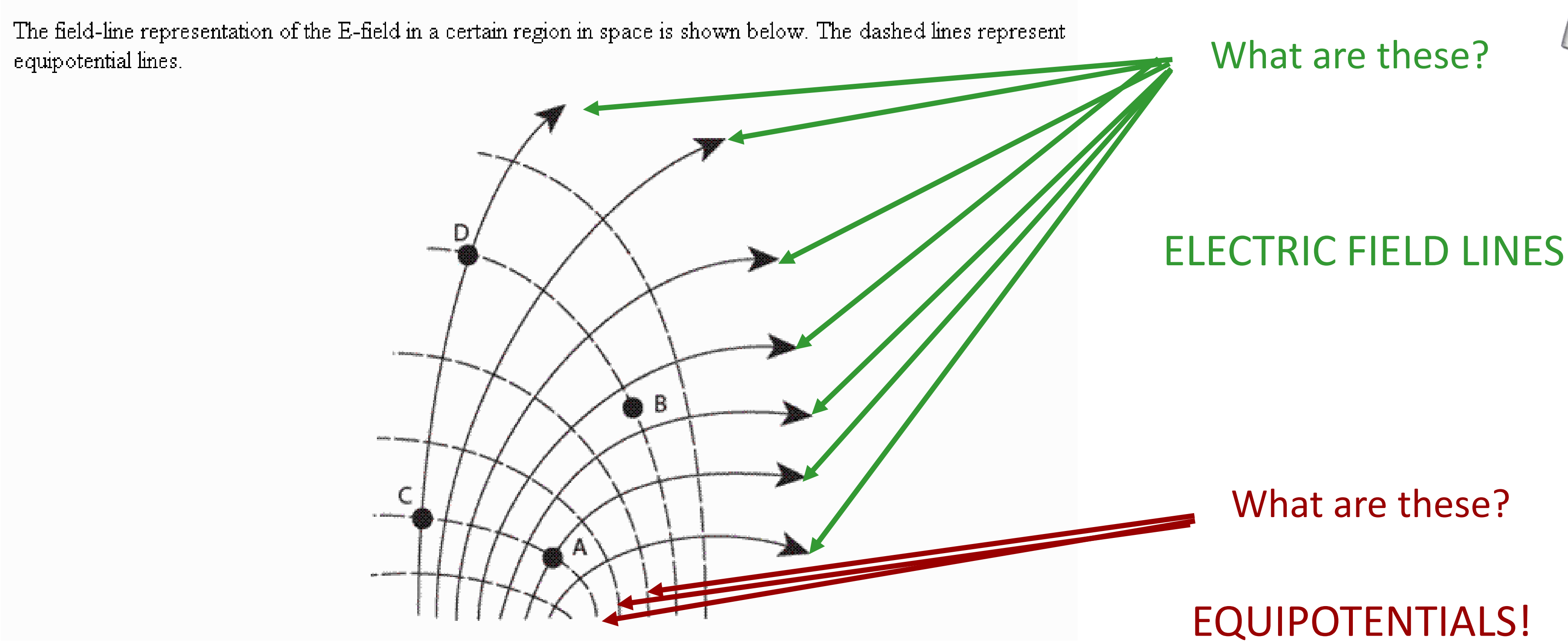


Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

Less than ½ got this correct!

Hint



What is the sign of $W_{AC} =$ work done by E field to move negative charge from A to C ?

A) $W_{AC} < 0$

B) $\boxed{W_{AC} = 0}$

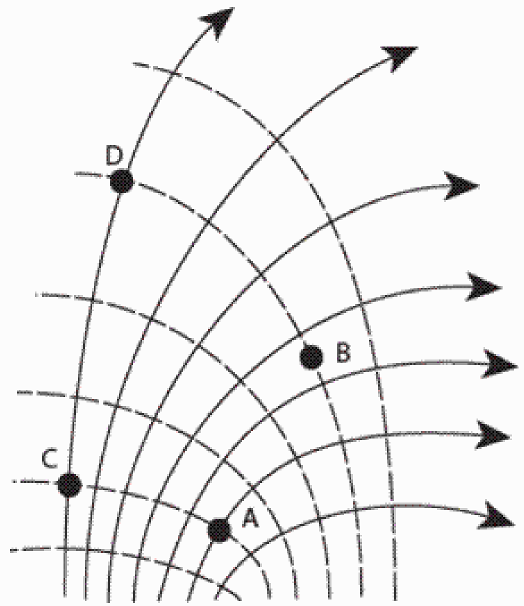
C) $W_{AC} > 0$

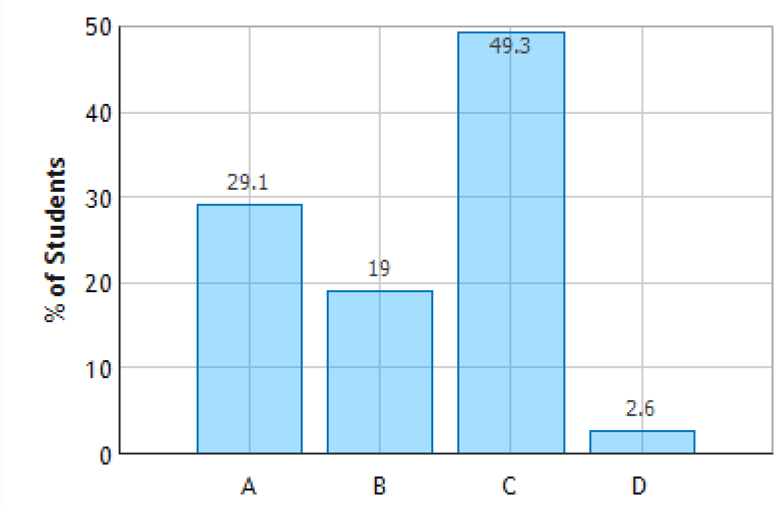
A and C are on the same equipotential → $W_{AC} = 0$

Equipotentials are perpendicular to the E field: No work is done along an equipotential

CheckPoint 3b Again?

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.





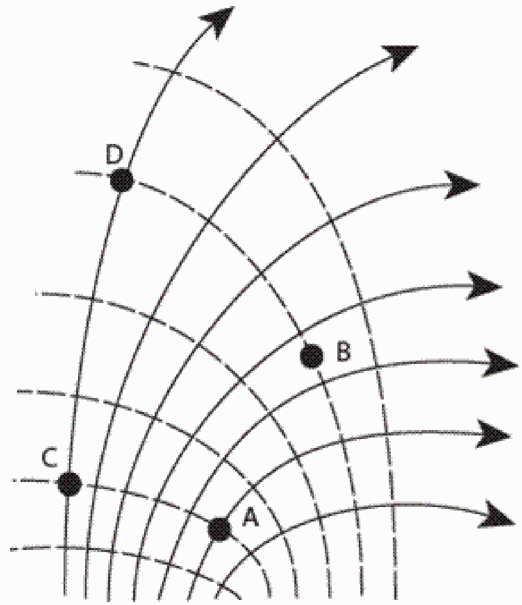
Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

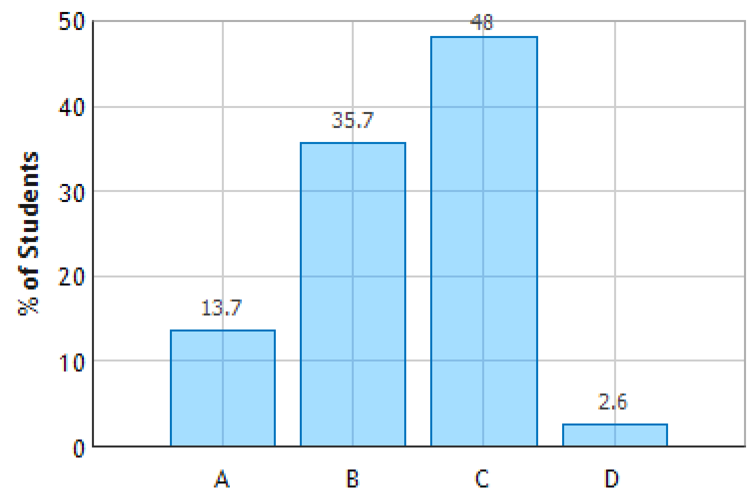
A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

- A and C are on the same equipotential
- B and D are on the same equipotential
- Therefore the potential difference between A and B is the SAME as the potential between C and D

CheckPoint 3c

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



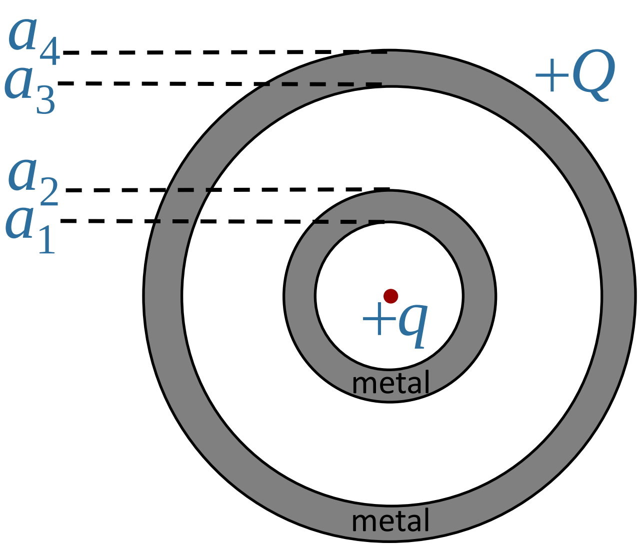


Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from A to D

A) More work from A to B
B) More work from D to D
C) Same
D) Can not determine w/o performing calculation

Calculation for Potential

cross-section



Point charge q at center of concentric conducting spherical shells of radii a_1, a_2, a_3, and a_4. The inner shell is uncharged, but the outer shell carries charge Q.

What is V as a function of r?

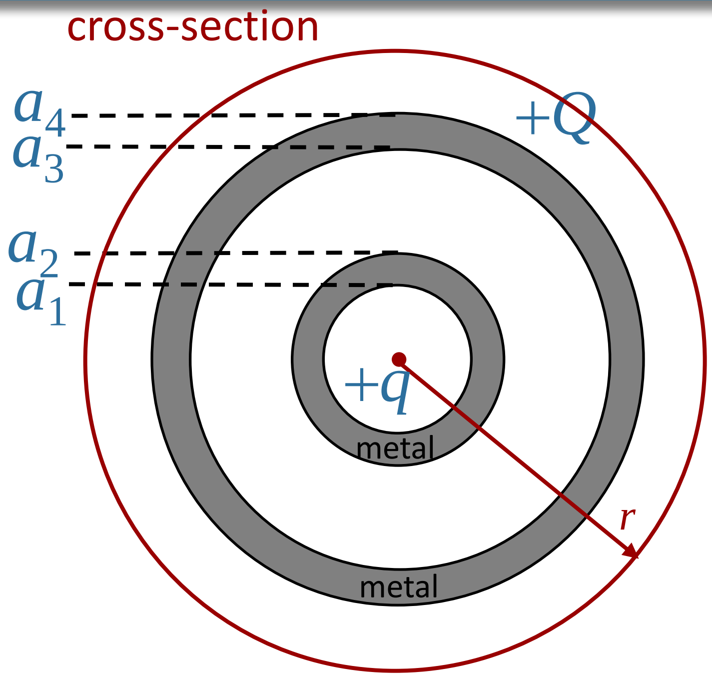
Conceptual Analysis:

- Charges q and Q will create an E field throughout space
- $V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$

Strategic Analysis:

- Spherical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V

Calculation: Quantitative Analysis



$r > a_4$: What is $E(r)$ outside spheres?

A) 0

B) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_0}\frac{Q+q}{r}$

D) $\boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}}$

E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

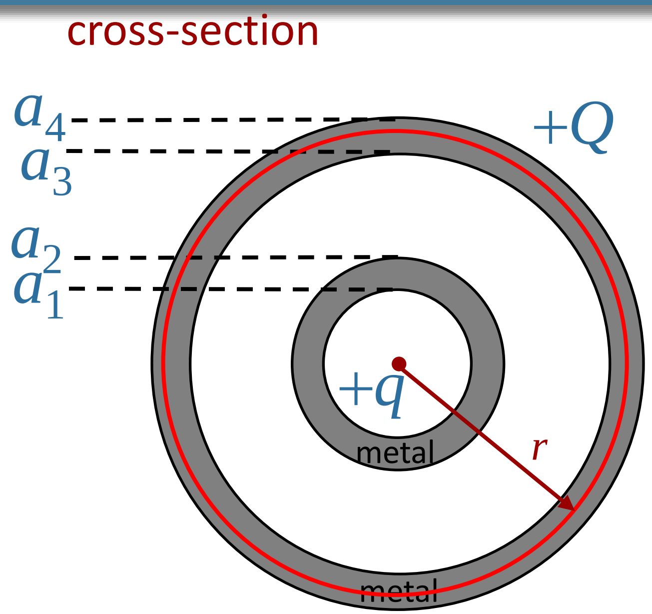
Why?

Gauss' law: $\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

$$E 4\pi r^2 = \frac{Q+q}{\varepsilon_0}$$

$$\rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}$$

Calculation: Quantitative Analysis



$a_3 < r < a_4$: What is $E(r)$ Inside outer metal sphere?

A) 0

B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_0}\frac{q}{r}$

D) $\frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}$

E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Applying Gauss' law, what is $Q_{enclosed}$ for red sphere shown?

A) q B) $-q$ C) 0

How is this possible?

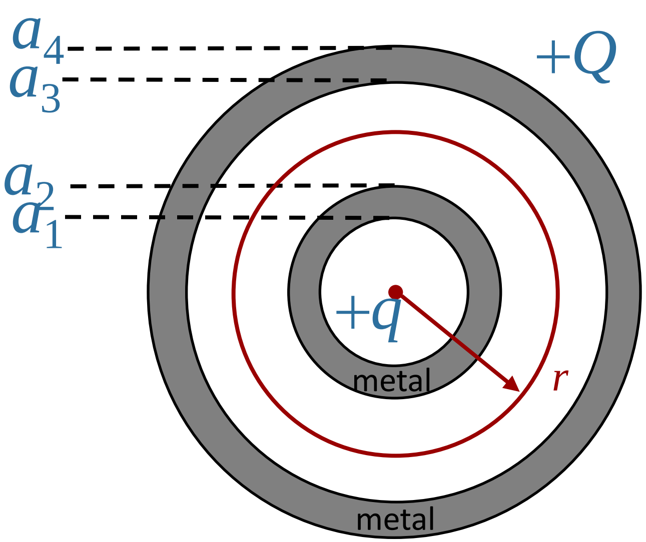
$-q$ must be induced at $r = a_3$ surface → charge at $r = a_4$ surface $= Q + q$

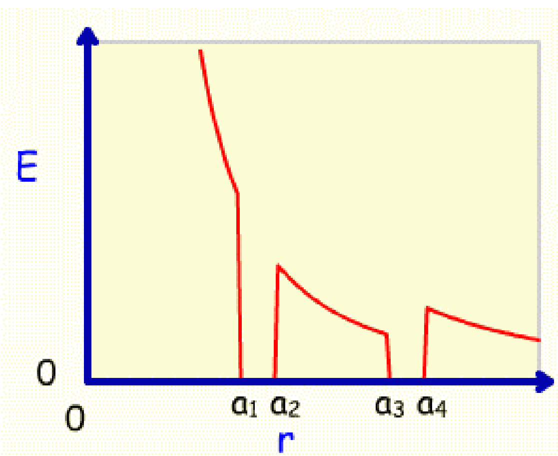
$$\sigma_3 = \frac{-q}{4\pi a_3^2}$$

$$\sigma_4 = \frac{Q+q}{4\pi a_4^2}$$

Calculation: Quantitative Analysis

cross-section





Continue on in…

$a_2 < r < a_3:$ $\boxed{E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}}$

$a_1 < r < a_2:$ $\boxed{E = 0}$

$r < a_1:$ $\boxed{E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}}$

To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: $r_0 =$ infinity)
2) Integrate!

$r > a_4:$ $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

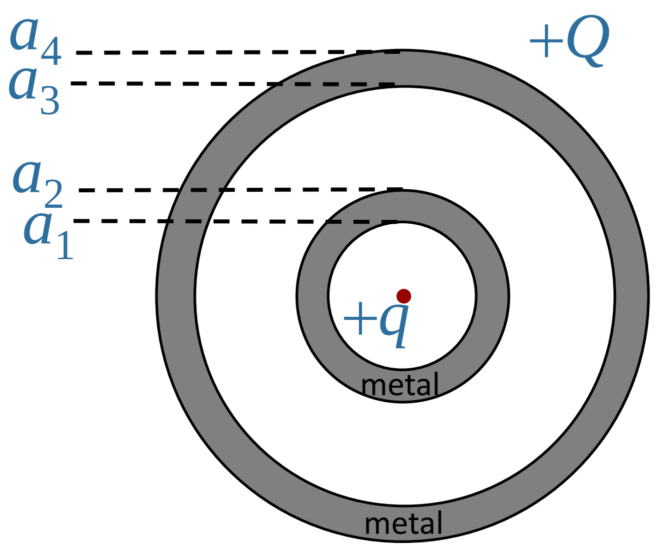
$a_3 < r < a_4:$

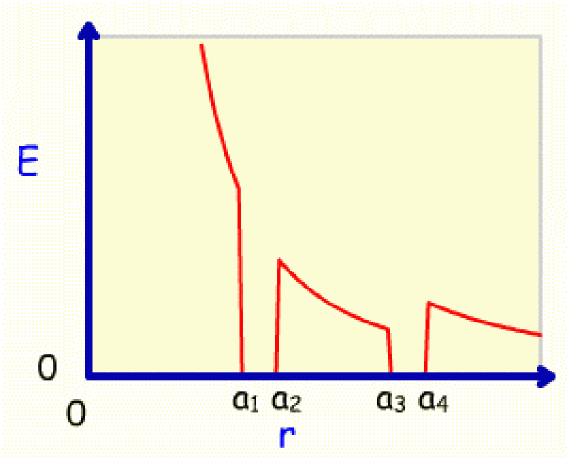
A) $V = 0$

B) $\boxed{V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_4}}$

C) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_3}$

Calculation: Quantitative Analysis





$r > a_4:$ $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

$a_3 < r < a_4:$ $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_4}$

$a_2 < r < a_3:$ $V(r) = \Delta V(\infty \to a_4) + 0 + \Delta V(a_3 \to r)$

$$V(r) = \frac{Q+q}{4\pi\varepsilon_0 a_4} + 0 + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a_3}\right) \rightarrow \boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{r} - \frac{q}{a_3}\right)}$$

$a_1 < r < a_2:$ $V(r) = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q+q}{a_4} + \dfrac{q}{a_2} - \dfrac{q}{a_3}\right)$

$0 < r < a_1:$ $V(r) = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q+q}{a_4} + \dfrac{q}{a_2} - \dfrac{q}{a_3} + \dfrac{q}{r} - \dfrac{q}{a_1}\right)$