Your Comments

How did they get those three equations for U with the different combinations of Q, V, and C? Or more specifically, how did they get Q = CV?

Seems like a lot of information, the last two slides about energy tripped me up so I'm not sure how much of that information was expected of us to retain. I could use some explanation on the question three from the prelecture and the second checkpoint as well. Finally, only the TA's office hours are posted online, when are the professor's?

Last weeks lectures are still a little foggy to me, so I'm more concerned about them then this lecture. It seems like whenever something was clarified in class, it was done in an extremely complicated way with very unclear calculations, so I'm still a bit lost.

I found the idea of capacitance to be the most difficult to understand.

Could we go over the derivations of the equations? I think I understand this, but I was really tired while doing the prelecture, so I'm not entirely sure.

I still don't understand how the uncharged conducting plate affects the capacitor

Physics 212
Lecture 7

Today's Concept: (Applications of Gauss, E and V)

A) Conductors

B) Capacitance

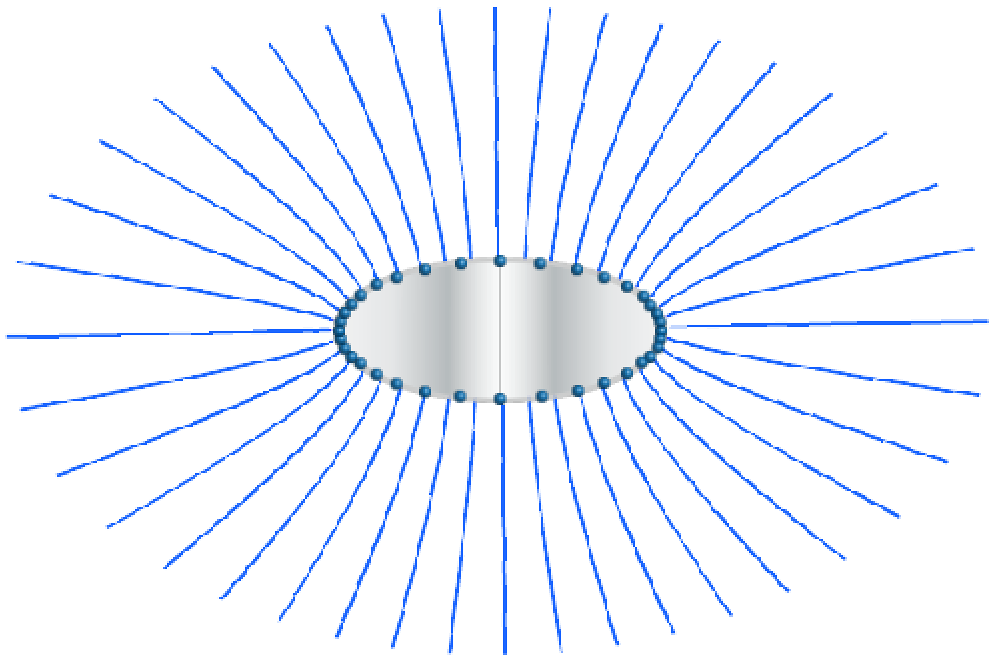
Exam Logistics

1) EXAM 1: WED February 19th at 7pm
   - Sign Up in Gradebook for Conflict Exam at 5:15pm if desired
   - If you have double conflict please email Dr. Wan Kyu Park
   - MATERIAL: Lectures 1 - 8

2) EXAM 1 PREPARATION
   - Old Exams are a good way to assess what you need to know
   - Prelecture of Fall 2010 solutions available

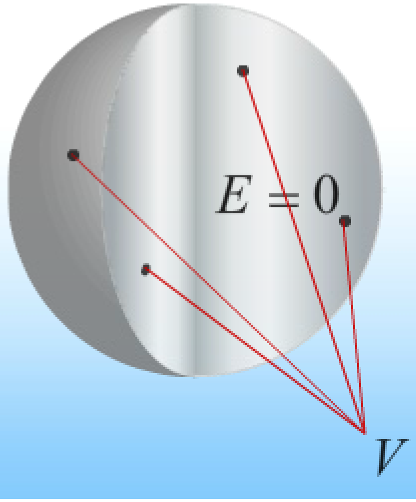
3) Extra Office Hours (Tuesday/ Wednesday next week in 234/279 Loomis)

Main Point 1: (Conductors)

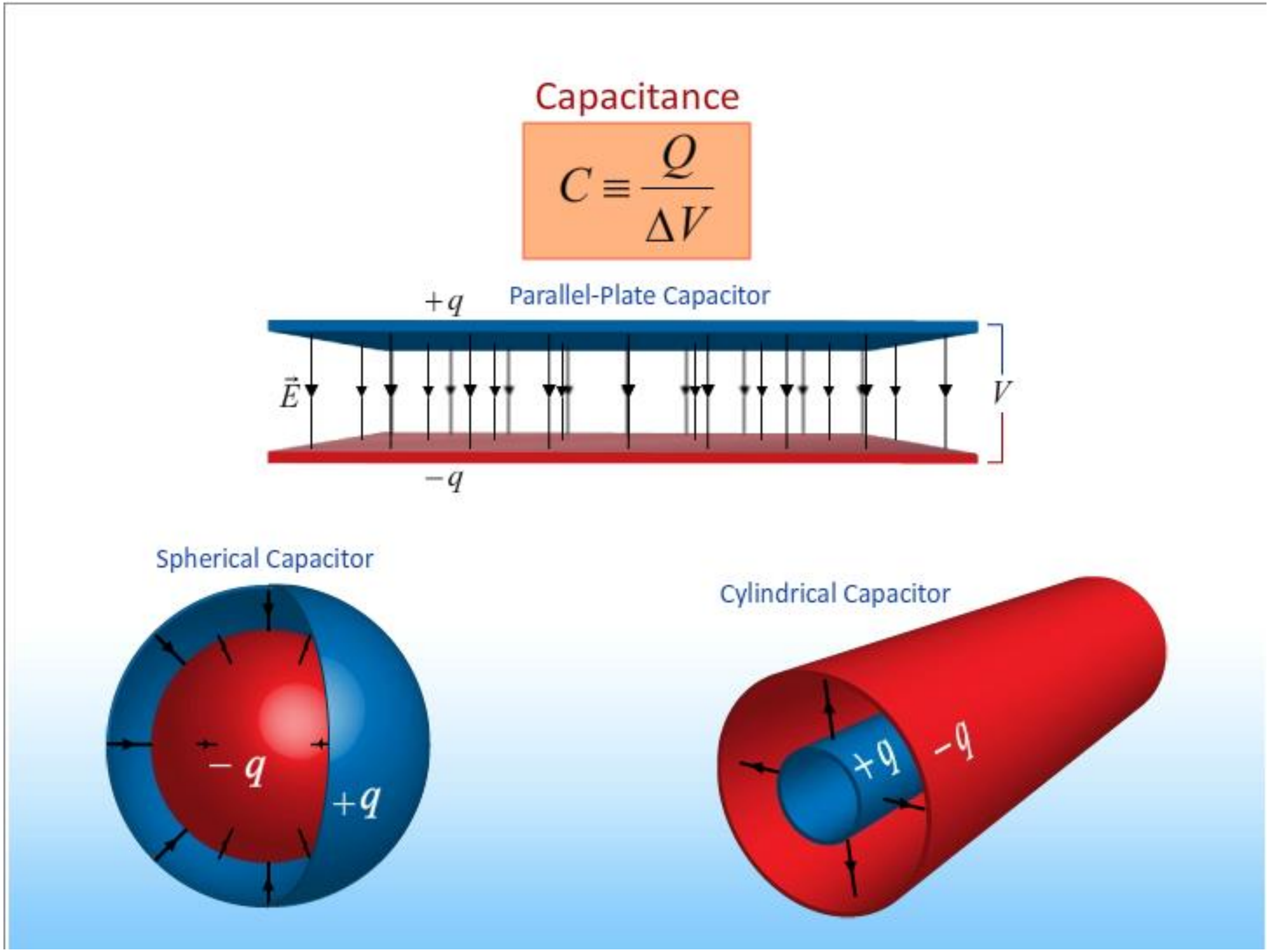


- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

Conducting Sphere

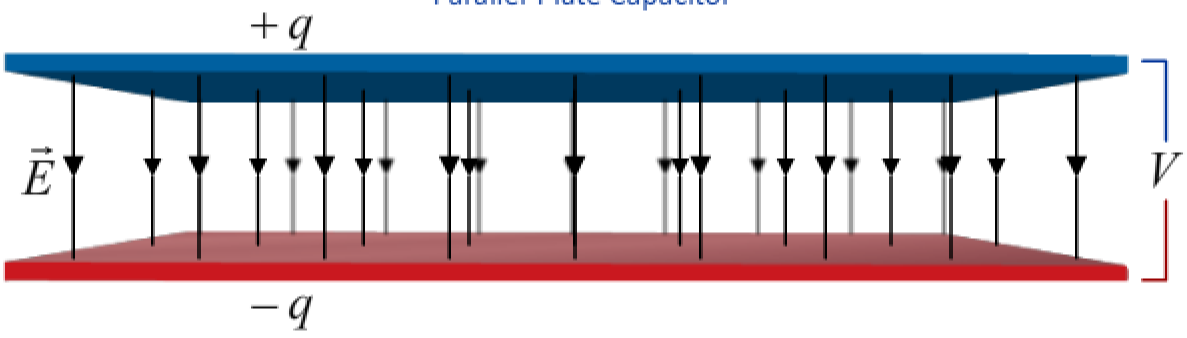


Main Point 2: Capacitance = Q/V



Main Point 3: Capacitors Store Energy in E

Parallel-Plate Capacitor



$$u = \frac{1}{2}\varepsilon_o E^2$$ Energy Density

Energy Stored in Capacitors

$$U = \frac{1}{2}QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2}CV^2$$

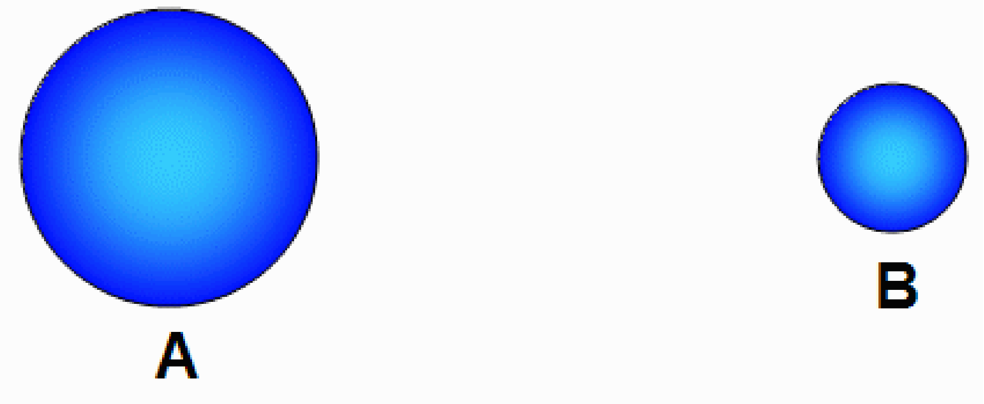
Conductors

The Main Points

- Charges free to move
- $E = 0$ in a conductor
- Surface $=$ Equipotential
- E at surface perpendicular to surface

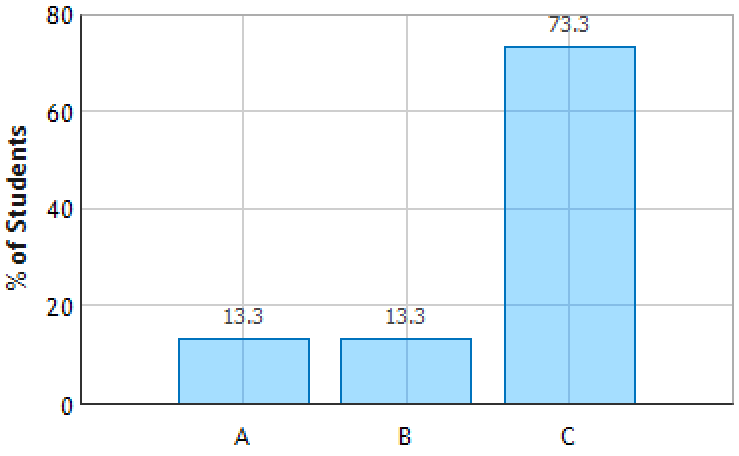
CheckPoint 1a

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B



Compare the potential on surface A with the potential on surface B

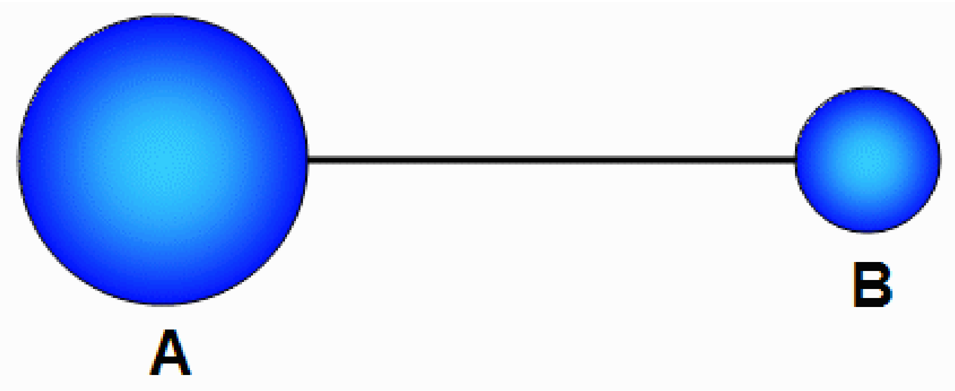
A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$



"potential is kQ/r"

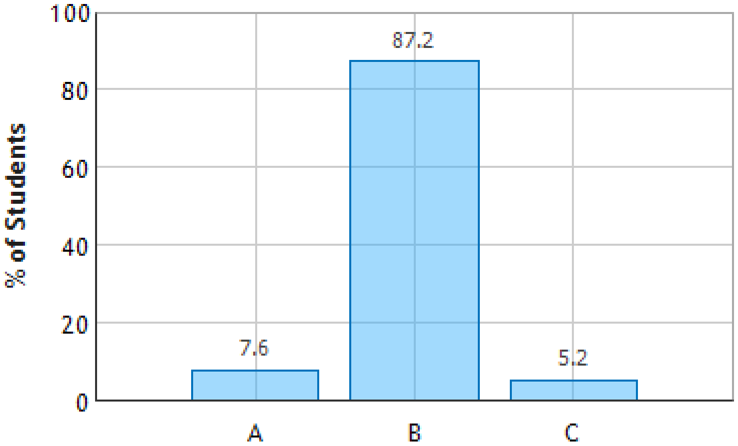
CheckPoint 1b

The two conductors are now attached by a conducting wire.



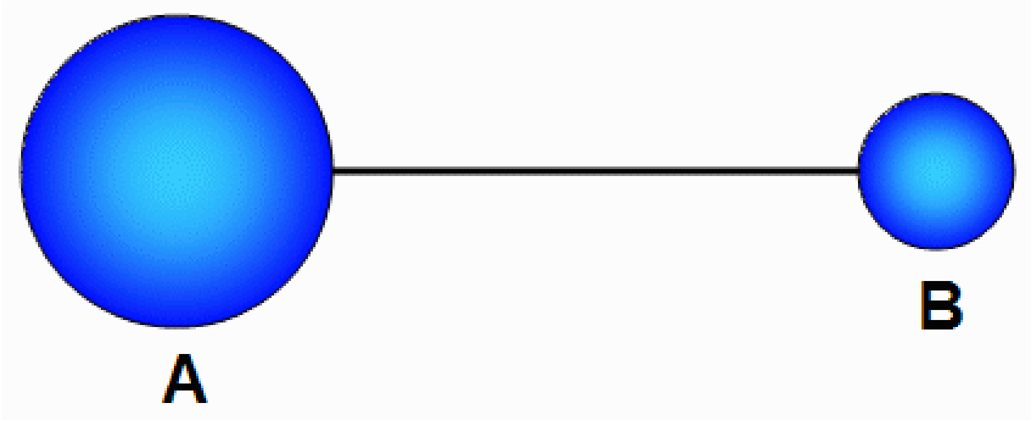
Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$



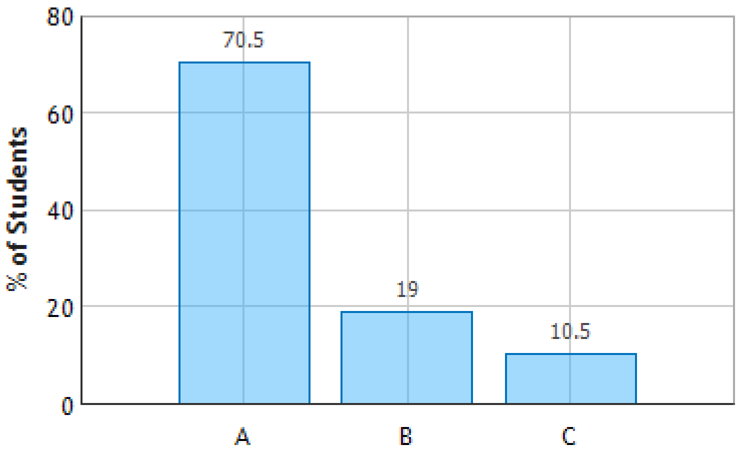
"By connecting the two conductors, we are effectively making them one conductor, and the potential on a conductor is the same everywhere, so the potential of A = the potential of B."

CheckPoint 1c



What happens to the charge on sphere A when the wire is attached

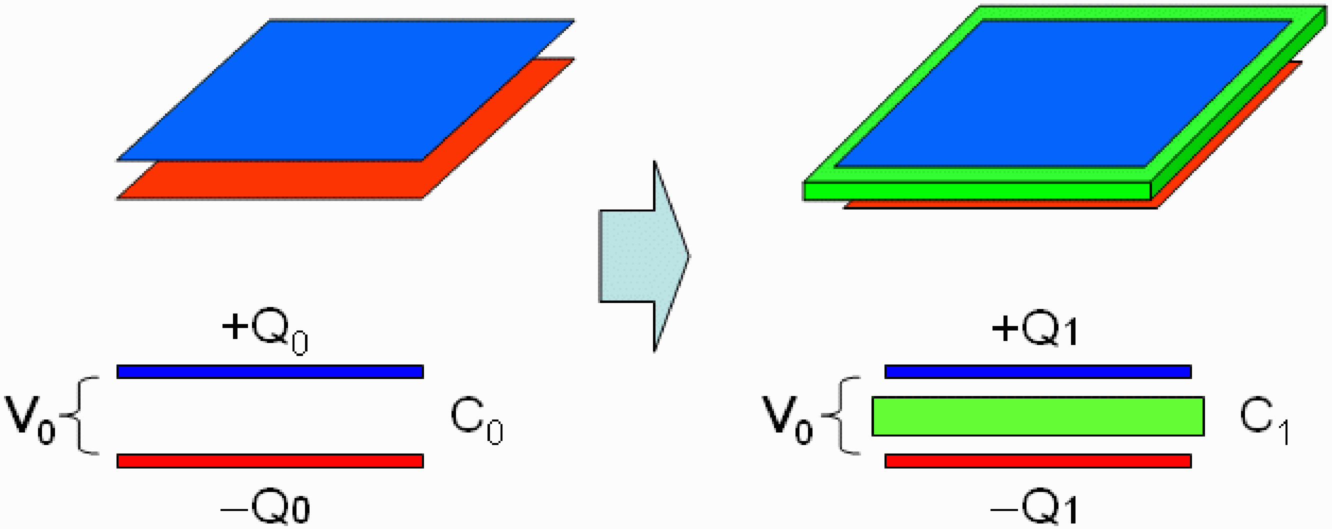
A) Q_A increases B) Q_A decreases C) Q_A does not change



"Because the spheres will have the same potential, and the potential is given as kq/R, the radius of the sphere, then the charge will redistribute so that the potentials are equal. This means that the larger sphere will accumulate charge, while the small charge will give it up."

Parallel Plate Capacitor

Two parallel plates of area carry equal and opposite charge Q_0. The potential difference between the two plates is measured to be V_0. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value Q_1 such that the potential difference between the plates remains the same as before.



THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We'll work through the example in the prelecture and then do the preflight questions.

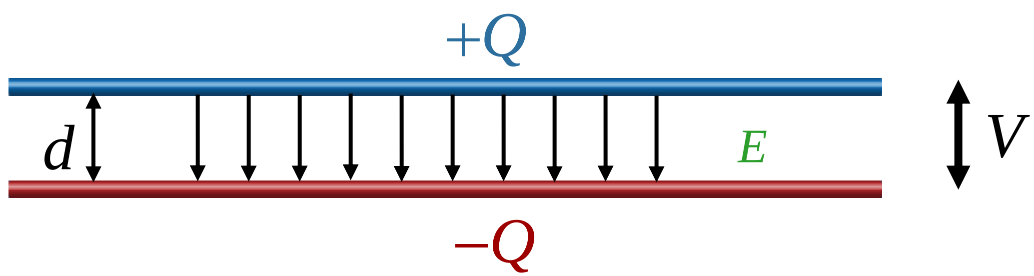
Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

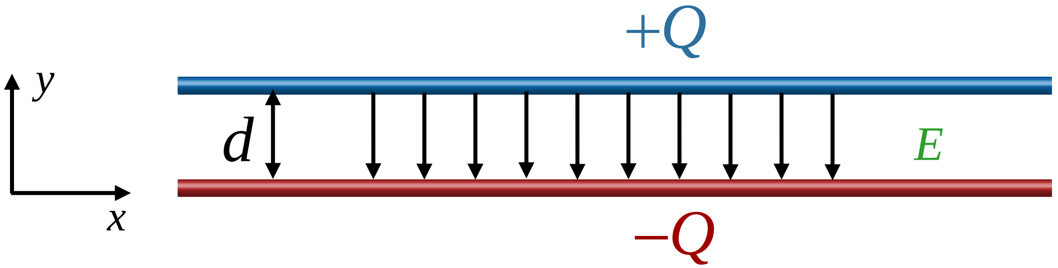
How do we understand this definition ?

- ➢ Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- ➢ These charges create an electric field in the space between them
- ➢ We can integrate the electric field between them to find the potential difference between the conductor
- ➢ This potential difference should be proportional to Q !
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Example (done in Prelecture 7)



What is σ ?

$$E = \frac{\sigma}{\varepsilon_o} \qquad \sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

$$V = -\int_0^d \vec{E} \cdot d\vec{y} \;\longrightarrow\; V = -\int_0^d (-E dy) = E\int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

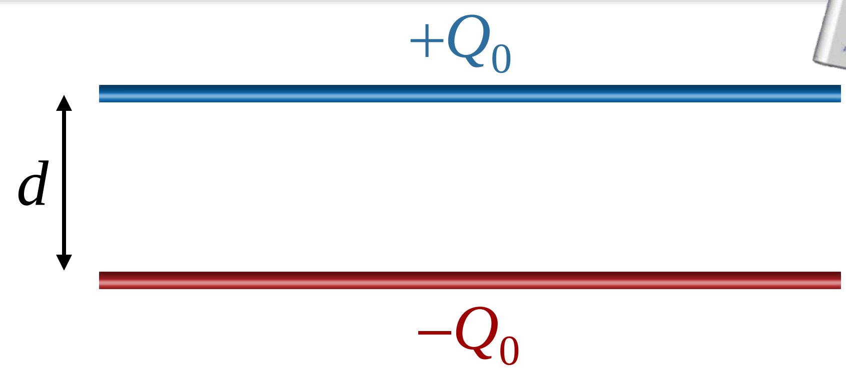
As promised, V is proportional to Q !

$$C \equiv \frac{Q}{V} = \frac{\cancel{Q}}{\cancel{Q}d/\varepsilon_o A} \;\longrightarrow\; C = \frac{\varepsilon_0 A}{d}$$

C determined by geometry !

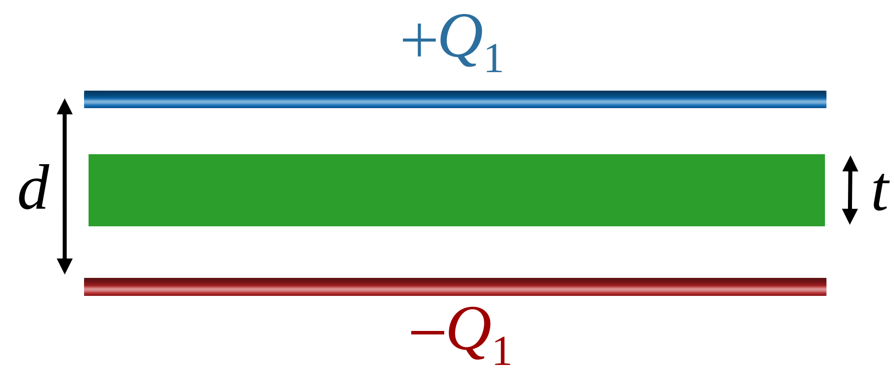
Question Related to CheckPoint

Initial charge on capacitor $= Q_0$



Insert uncharged conductor

Charge on capacitor now $= Q_1$



How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$

B) $Q_1 = Q_0$

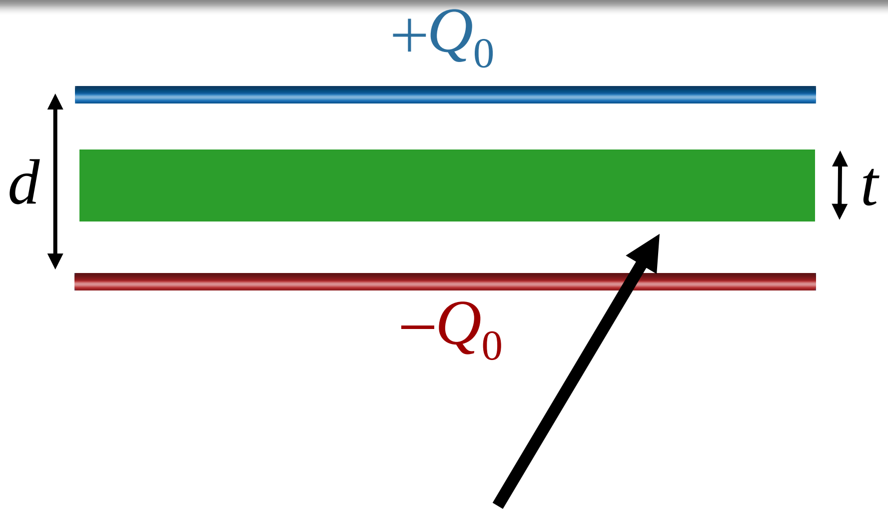
C) $Q_1 > Q_0$

Plates not connected to anything

↓

CHARGE CANNOT CHANGE !

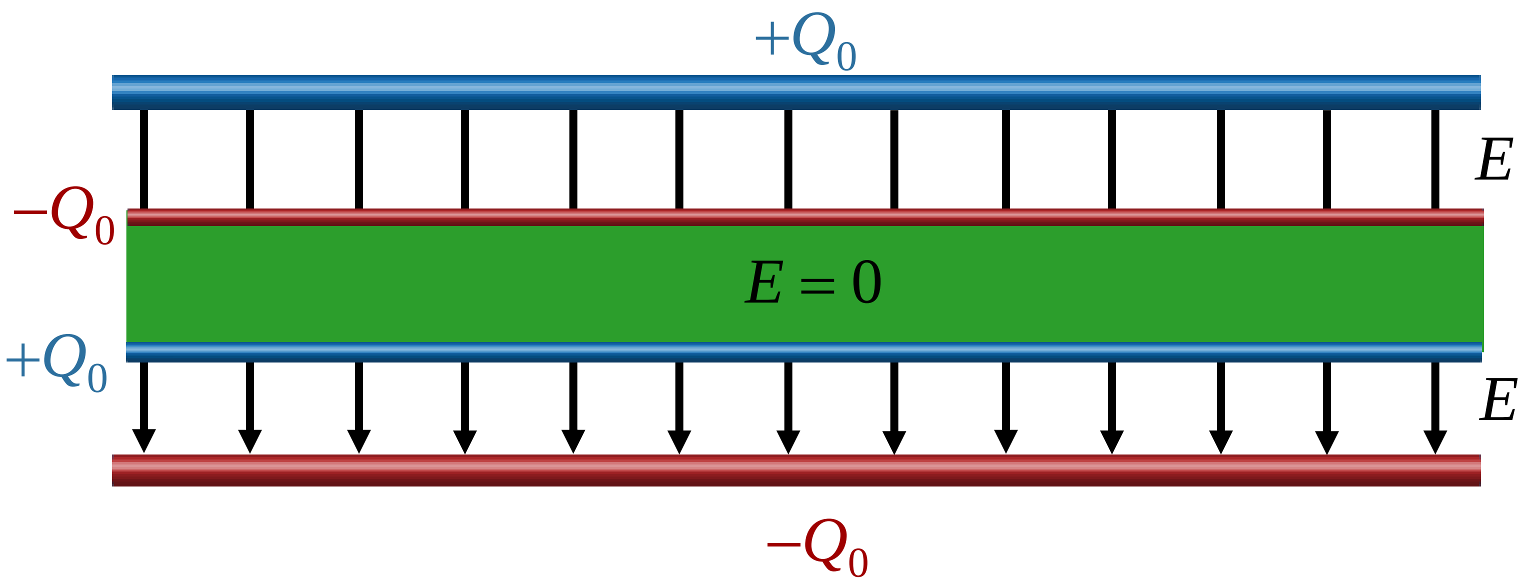
Where to Start ?



What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$
B) $+Q_0/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$

Why ?



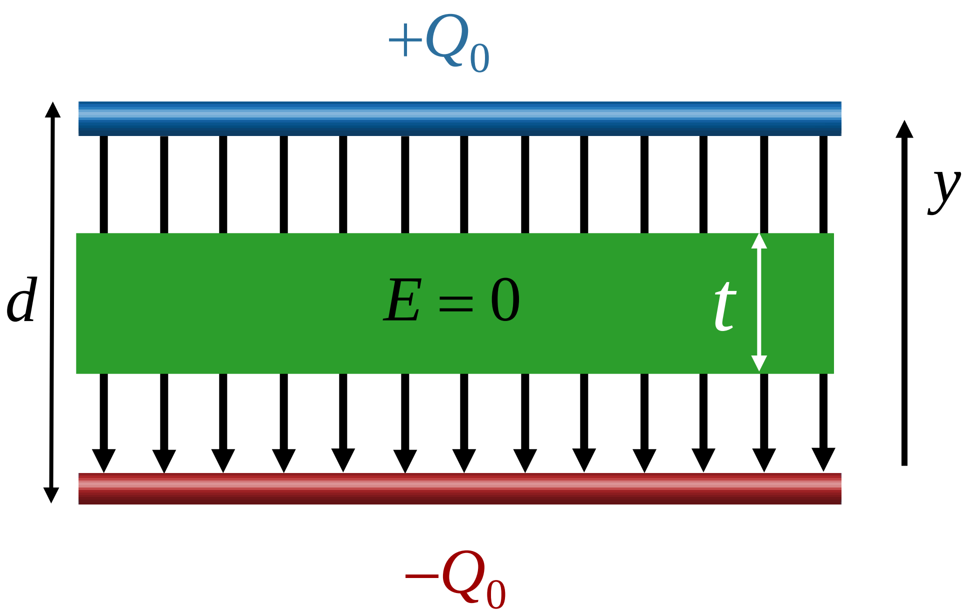
WHAT DO WE KNOW ?

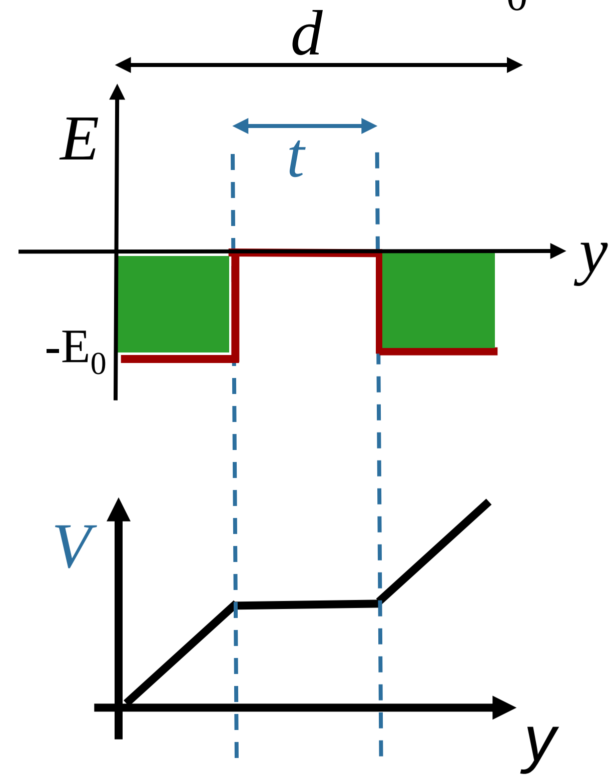
E must be $= 0$ in conductor !

Charges inside conductor move to cancel E field from top & bottom plates.

Now calculate V as a function of distance from the bottom conductor.

$$V(y) = -\int_0^y \vec{E} \cdot d\vec{y}$$





What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

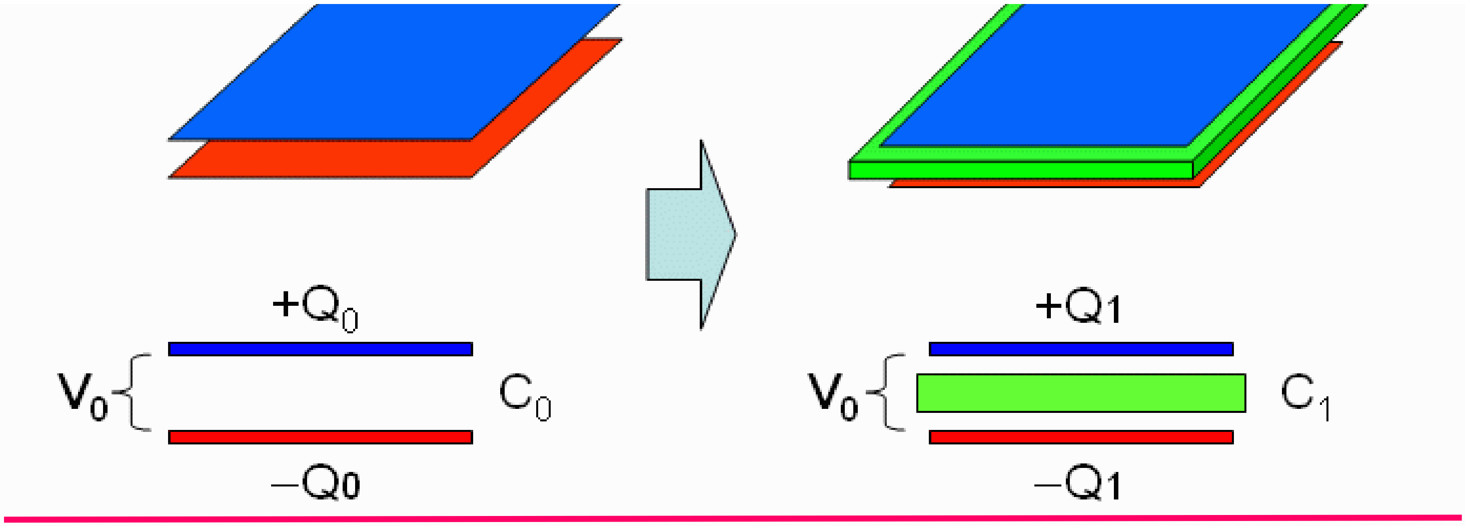
B) $\Delta V = E_0(d - t)$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

Back to CheckPoint 2a

Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, how would charge need to be adjusted to keep same potential difference?

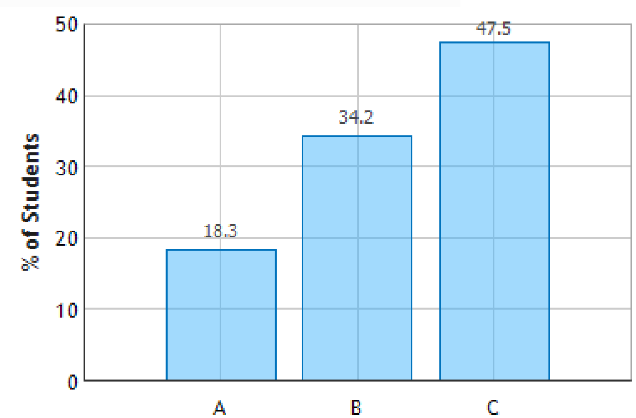


A) $Q_1 < Q_o$ B) $Q_1 = Q_o$ C) $Q_1 > Q_o$

"Since we learned in pre-lecture that the voltage decreases in this configuration, since voltage is directly proportional to charge, charge will decrease as well. "

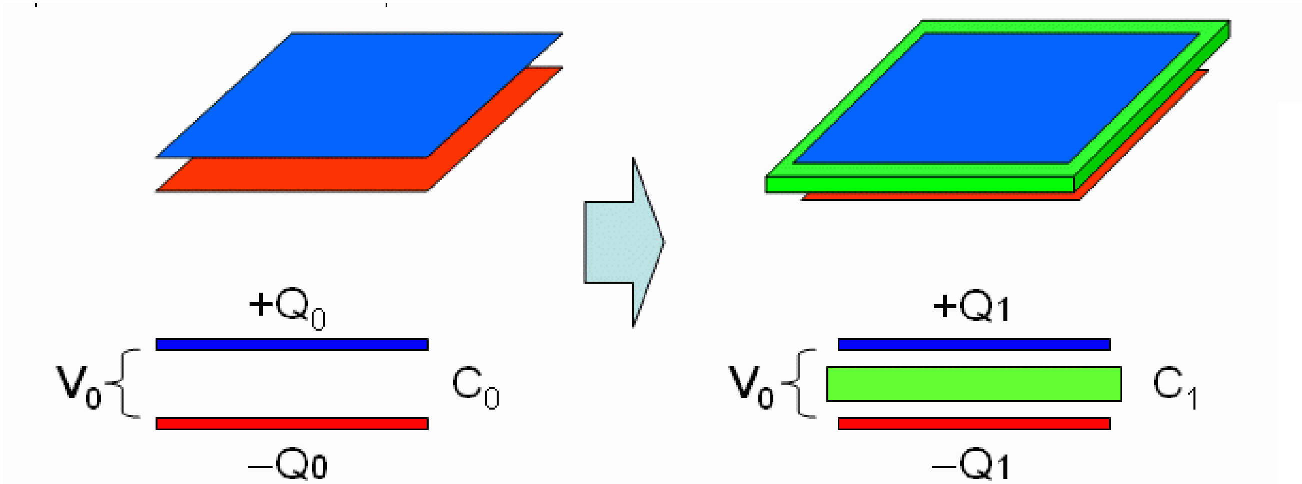
"The green body inserted has no charge, so the potential should not change. hence, the value of Q1 should be equal to that of Q0."

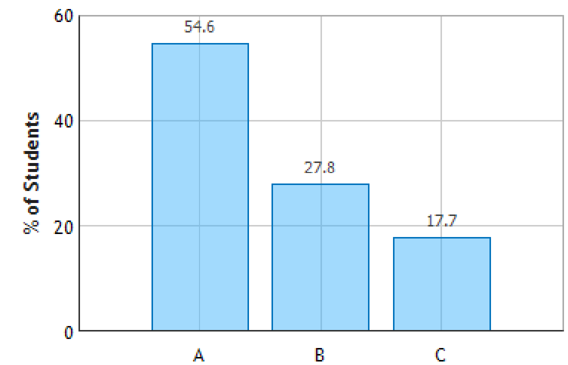
"The voltage of the second capacitor will be smaller because of the material in the middle, so more charge has to be put on it in order for it to have the same voltage."



CheckPoint 2b

Two parallel plates are given a charge Q_0 such that the potential difference between the plates is V_0. If a conductor is slid between plates, does C change?





A) $C_1 > C_o$ B) $C_1 = C_o$ C) $C_1 < C_o$

We can determine C from either case

same V (preflight)

same Q (lecture)

C depends only on geometry !

$$E_0 = Q_0 / \varepsilon_0 A$$

Same Q:

$$V_0 = E_0 d \quad \rightarrow \quad C_0 = Q_0 / E_0 d \quad \rightarrow \quad C_0 = \varepsilon_0 A / d$$

$$V_1 = E_0 (d - t) \quad \rightarrow \quad C_1 = Q_0 / (E_0 (d - t)) \quad \rightarrow \quad C_1 = \varepsilon_0 A / (d - t)$$

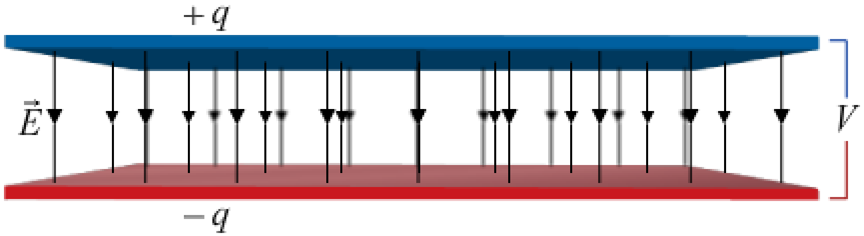
Energy in Capacitors

Energy Stored in Capacitors

$$U = \frac{1}{2}QV \quad \text{or} \quad U = \frac{1}{2}\frac{Q^2}{C} \quad \text{or} \quad U = \frac{1}{2}CV^2$$

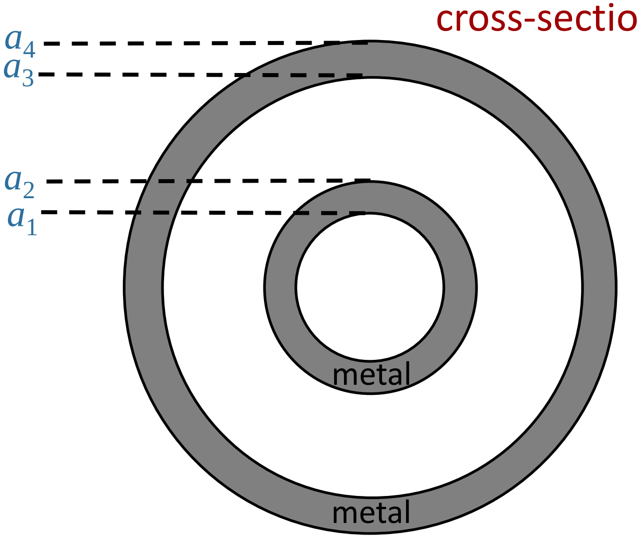
Energy Density

$$u = \frac{1}{2}\varepsilon_o E^2$$



BANG

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

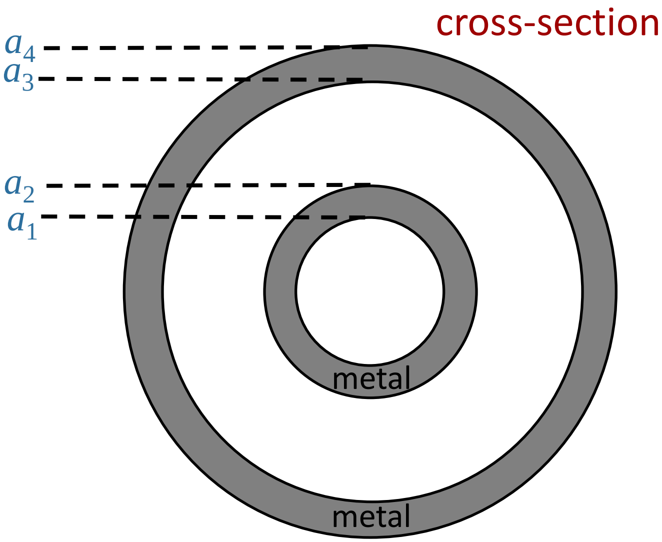
- Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

But what is Q and what is V ? They are not given?

- Important Point: C is a property of the object! (concentric cylinders here)
  - Assume some Q (i.e., $+Q$ on one conductor and $-Q$ on the other)
  - These charges create E field in region between conductors
  - This E field determines a potential difference V between the conductors
  - V should be proportional to Q; the ratio Q/V is the capacitance.

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

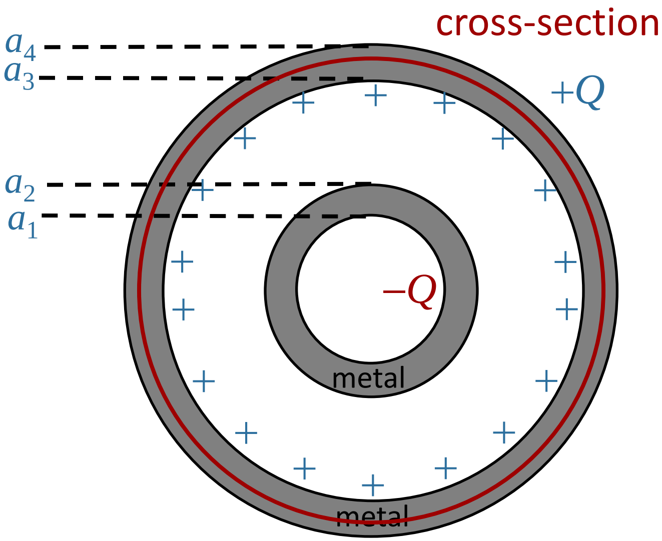
What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

➢ Strategic Analysis:

- Put $+Q$ on outer shell and $-Q$ on inner shell
- Cylindrical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V
- Take ratio Q/V: should get expression only using geometric parameters (a_i, L)

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

A) at $r = a_4$ B) at $r = a_3$ C) both surfaces D) throughout shell

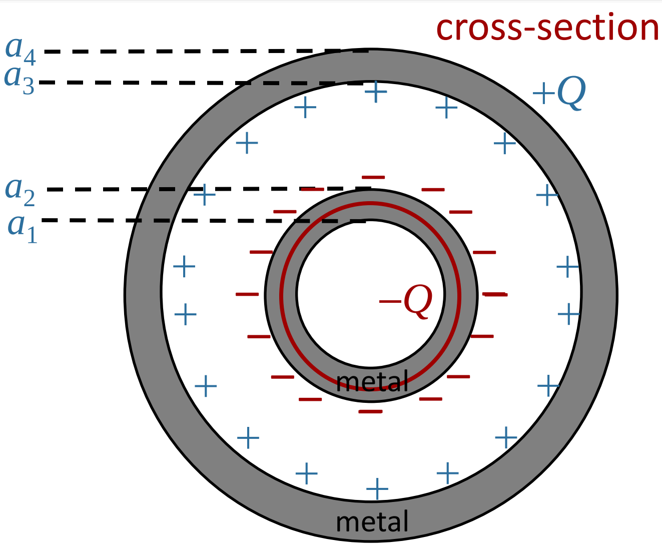
Why?

Gauss' law: $$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

We know that $E = 0$ in conductor (between a_3 and a_4) → $Q_{enclosed} = 0$

$$Q_{enclosed} = 0$$ → $+Q$ must be on inside surface (a_3), so that $Q_{enclosed} = +Q - Q = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $-Q$ on inner conductor located?

A) at $r = a_2$ B) at $r = a_1$ C) both surfaces D) throughout shell

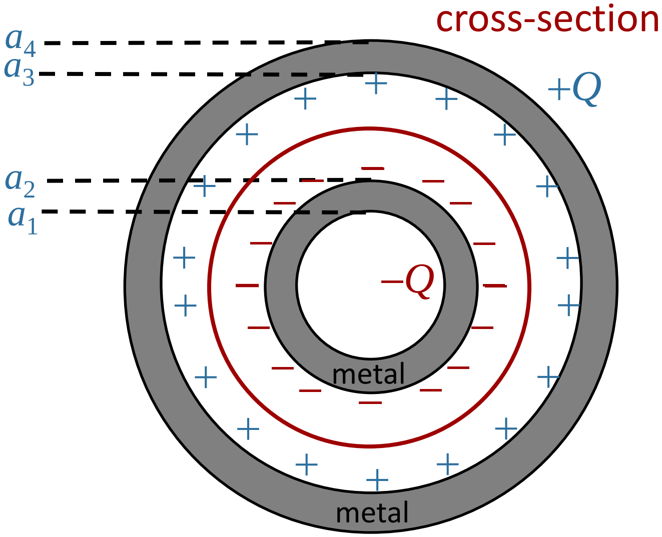
Why?

Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ → $Q_{enclosed} = 0$

We know that $E = 0$ in conductor (between a_1 and a_2)

$Q_{enclosed} = 0$ → $+Q$ must be on outer surface (a_2), so that $Q_{enclosed} = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is $E(r)$?

A) 0

B) $\frac{1}{4\pi\varepsilon_o} \frac{Q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_o} \frac{Q}{Lr}$

D) $\frac{1}{2\pi\varepsilon_o} \frac{2Q}{Lr}$

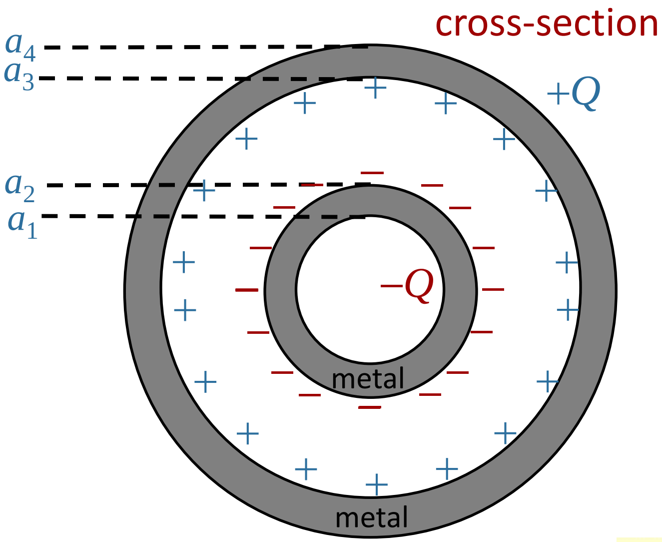
E) $\frac{1}{4\pi\varepsilon_o} \frac{2Q}{r^2}$

Why?

Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ → $E \cdot 2\pi r L = \frac{Q}{\varepsilon_o}$ → $E = \frac{1}{2\pi\varepsilon_o} \frac{Q}{Lr}$

Direction: Radially In

Calculation



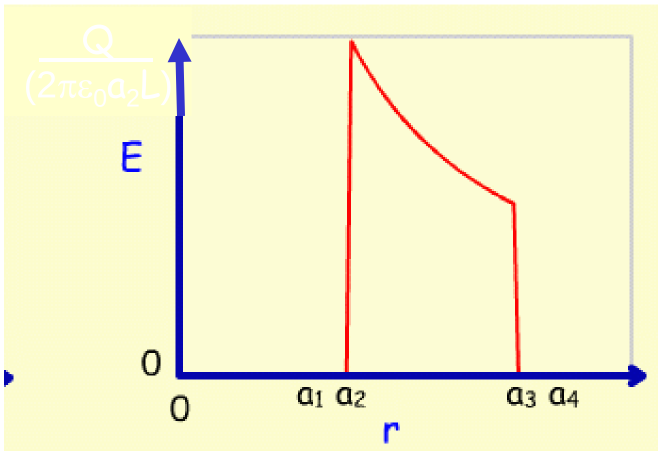
A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

$r < a_2$: $E(r) = 0$

since $Q_{enclosed} = 0$



What is V ?

The potential difference between the conductors.

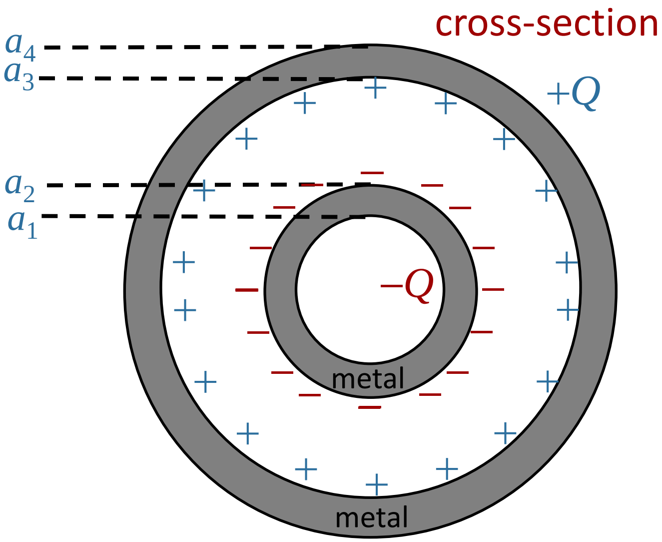
What is the sign of $V = V_{outer} - V_{inner}$?

A) $V_{outer} - V_{inner} < 0$

B) $V_{outer} - V_{inner} = 0$

C) $V_{outer} - V_{inner} > 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

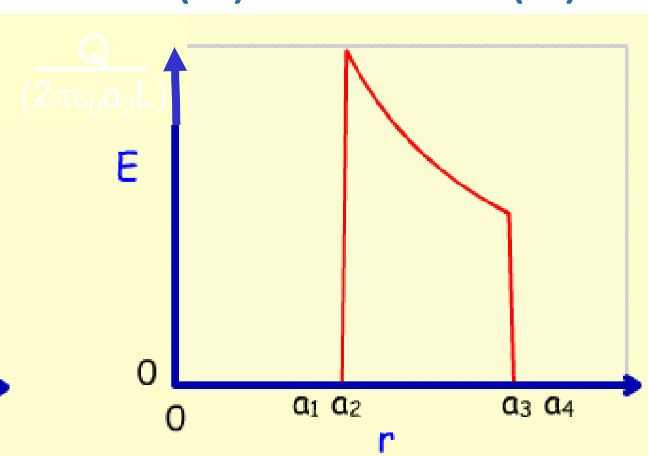
What is $V \equiv V_{outer} - V_{inner}$?

(A) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_1}{a_4}$

(B) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_4}{a_1}$

(C) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_3}{a_2}$

(D) $\frac{Q}{2\pi\varepsilon_o L}\ln\frac{a_2}{a_3}$



$$V = -\int_{a_2}^{a_3} \frac{-Q}{2\pi\varepsilon_o L}\frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\varepsilon_o L}\int_{a_2}^{a_3}\frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{a_3}{a_2}$$

V proportional to Q, as promised

$$\rightarrow \quad C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(a_3/a_2)}$$