

# *Your comments*

Having taken ece 110 made it easy for me to grasp most of the concepts.

God I can go to bed now. Oh wait I can't cuz EXAM??!?!111

I know that this course has to move at a fast pace and that there is no time to dilly dally and putts around, but I really really really don't like the idea of having a prelecture and lecture over a topic not covered on an upcoming exam. I find that I can't give the prelecture or the lecture the time they deserve to solidify my understanding, so this prelecture, though important, was gone through very quickly.

This stuff is tough, I think this is mostly because we were thrown a ton of unfamiliar variables. Could we go over what all they exactly mean?

Current density should be covered a little more thoroughly.

PLEASE go over how to calculate voltage drops across a capacitor!

Please help. I'm stressing out because the test is next Wednesday and we have to know circuits and all these different values ( $Q$ ,  $I$ ,  $R$ ,  $V$ ) with resistors and capacitors. For someone who has never done this before, this is hard!!!

# Electric Current

## *Physics 212* *Lecture 9*

Today's Concept:

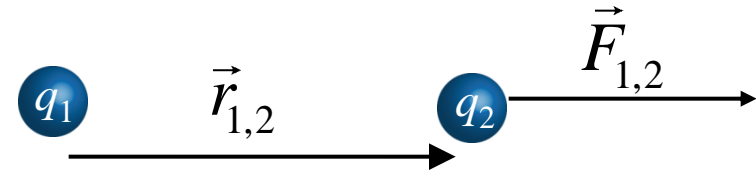
Ohm's Law, Resistors in circuits

# A Big Idea Review

## Coulomb's Law

Force law between point charges

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2} \hat{r}_{1,2}$$



## Electric Field

Force per unit charge

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

## Electric Field

Property of Space  
Created by Charges  
Superposition

## Gauss' Law

Flux through closed surface is always proportional to charge enclosed

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

## Gauss' Law

Can be used to determine E field



Spheres  
Cylinders  
Infinite Planes

## Electric Potential

Potential energy per unit charge

$$\Delta V_{a \rightarrow b} \equiv \frac{\Delta U_{a \rightarrow b}}{q} = - \int_a^b \vec{E} \cdot d\vec{l}$$

## Capacitance

Relates charge and potential for two conductor system

$$C \equiv \frac{Q}{V}$$

## Electric Potential

Scalar Function that can be used to determine E

$$\vec{E} = -\vec{\nabla} V$$

# Applications of Big Ideas

Conductors  
Charges free to move



What Determines  
How They Move?

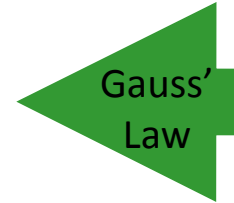


They move until  
 $E = 0$  !

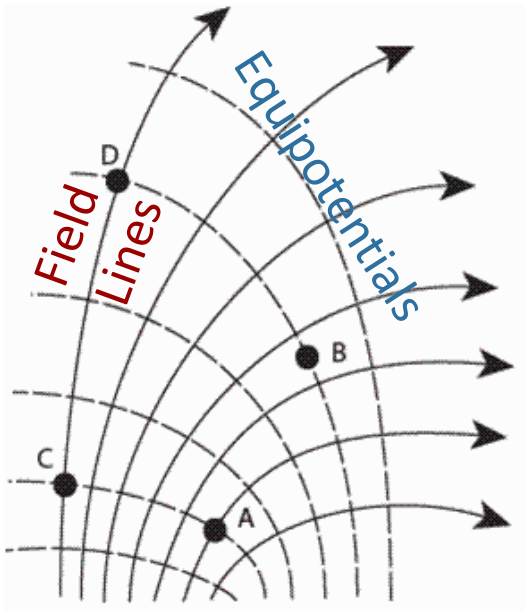


$E = 0$  in conductor  
determines charge  
densities on surfaces

Spheres  
Cylinders  
Infinite Planes



Field Lines &  
Equipotentials



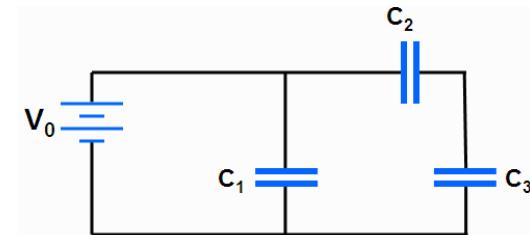
Work Done By E Field

$$W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q\vec{E} \cdot d\vec{l}$$

Change in Potential Energy

$$\Delta U_{a \rightarrow b} = -W_{a \rightarrow b} = -\int_a^b q\vec{E} \cdot d\vec{l}$$

Capacitor Networks



Series:

$$(1/C_{23}) = (1/C_2) + (1/C_3)$$

Parallel

$$C_{123} = C_1 + C_{23}$$

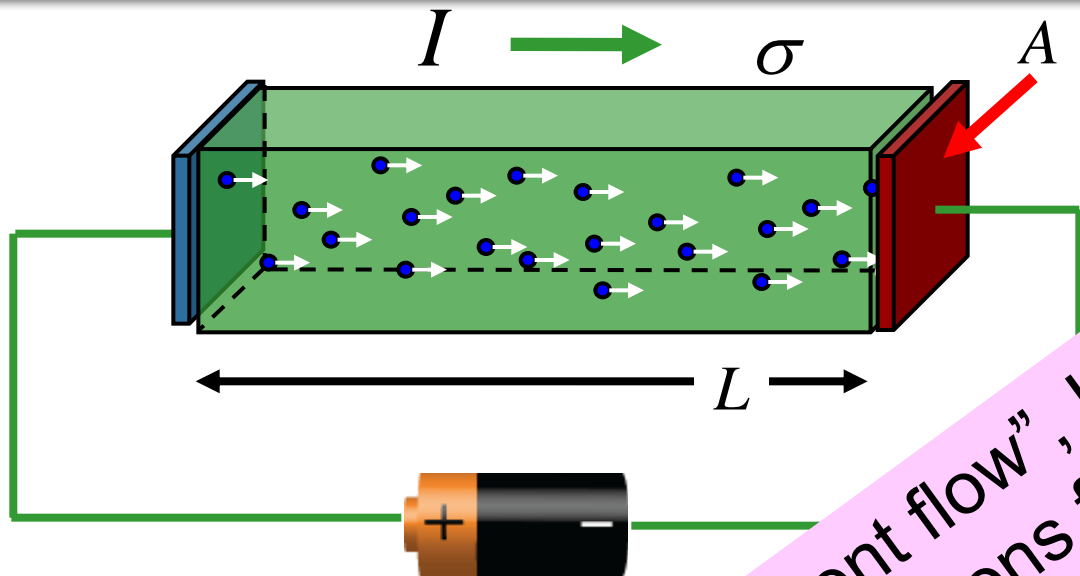
# *Current and Resistance*

## Key Concepts:

- 1) How resistance depends on  $A$ ,  $L$ ,  $s$ ,  $r$
- 2) How to combine resistors in series and parallel
- 3) Understanding resistors in circuits

## Today's Plan:

- 1) Review of resistance & preflights
- 2) Work out a circuit problem in detail



conductivity – high for good conductors.

$$V = \text{Ohm} = \sigma E$$

Observables:

$$V = EL$$

$$I = JA$$

$$J = \sigma V/L$$

$$I = V/(L/\sigma A)$$

resistance  
 $\rho = 1/\sigma$

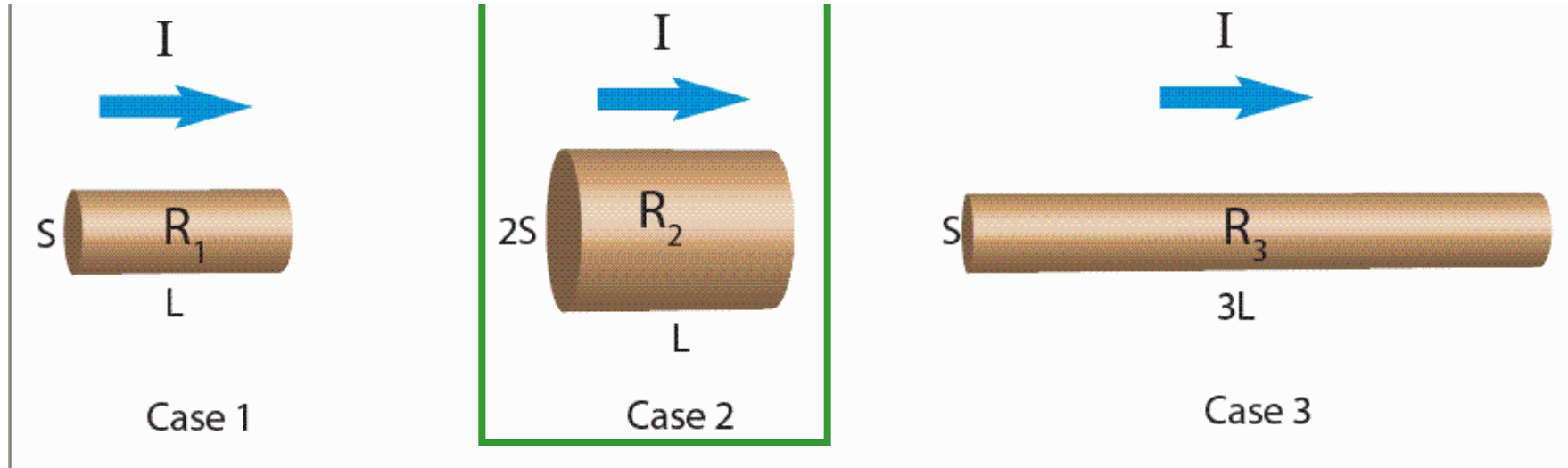
$$I = V/R$$

$$R = \frac{L}{\sigma A}$$

Note: "Conventional current flow",  $I$ , is opposite to direction electrons flow

# CheckPoint 3

The SAME amount of current  $I$  passes through three different resistors.  $R_2$  has twice the cross-sectional area and the same length as  $R_1$ , and  $R_3$  is three times as long as  $R_1$  but has the same cross-sectional area as  $R_1$ .



In which case is the CURRENT DENSITY through the resistor the smallest?

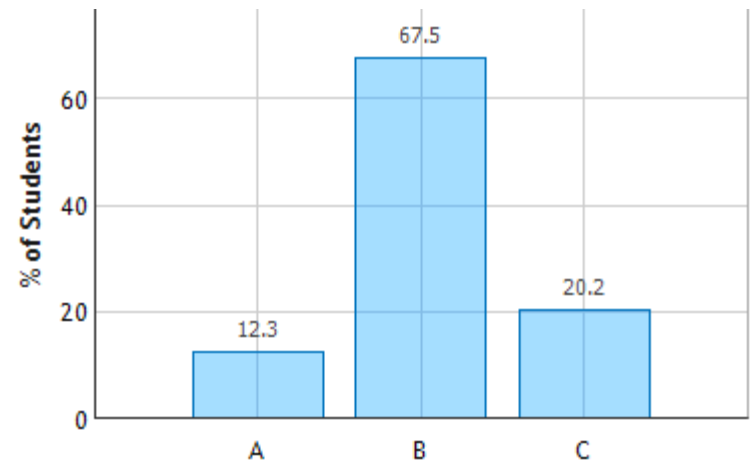
**A.** Case 1

**B.** Case 2

**C.** Case 3

$$J \equiv \frac{I}{A} \quad \longrightarrow \quad J_1 = J_3 = 2J_2$$

Same Current  $\longrightarrow J \propto \frac{1}{A}$



# *This is just like Plumbing!*

$I$  is like flow rate of water

$V$  is like pressure

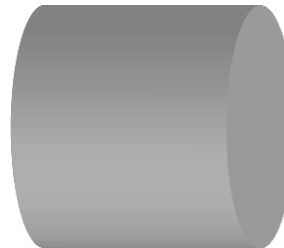
$R$  is how hard it is for water to flow in a pipe

$$R = \frac{L}{\sigma A}$$

To make  $R$  big, make  $L$  long or  $A$  small



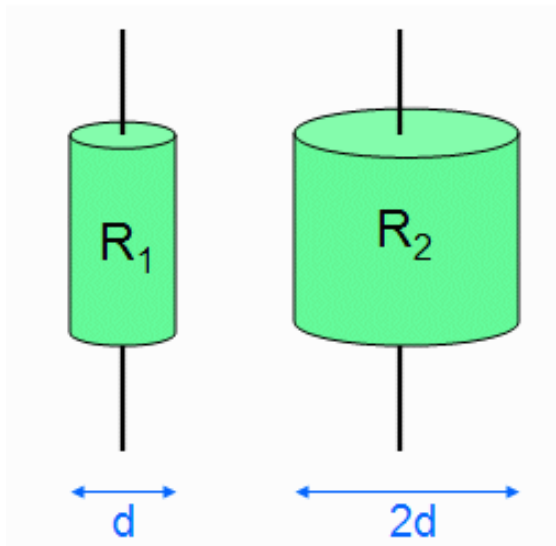
To make  $R$  small, make  $L$  short or  $A$  big





# CheckPoint 1a

# CheckPoint 1b



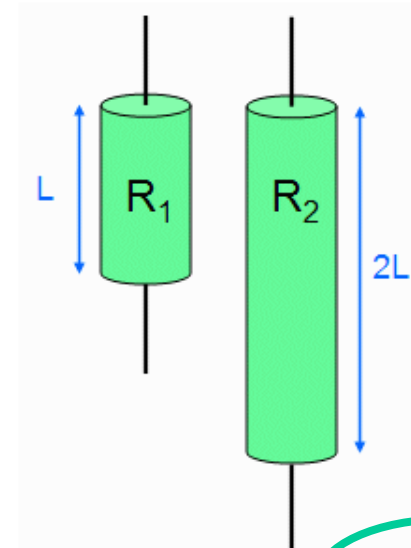
☒  $V_1 > V_2$    ☐  $V_1 = V_2$    ☐  $V_1 < V_2$

Same current through both resistors

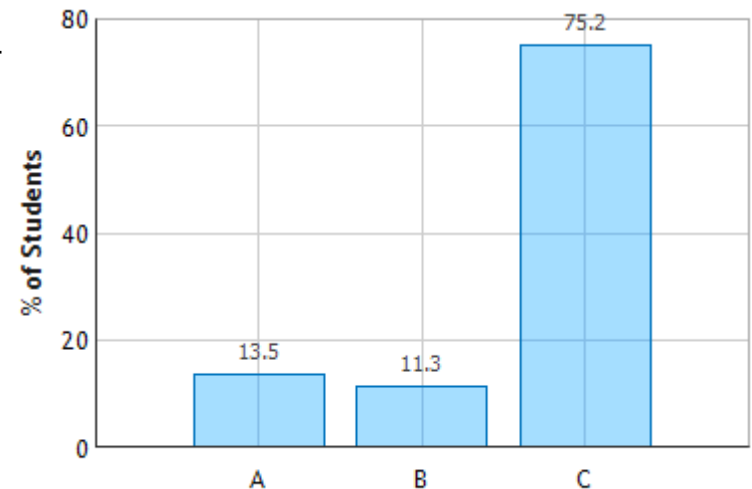
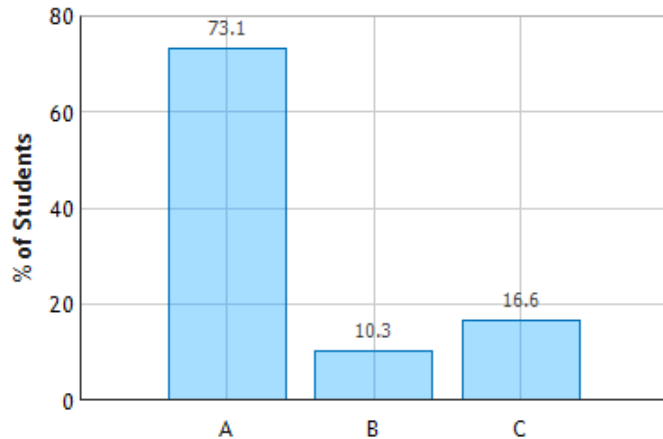
Compare voltages across resistors

$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$



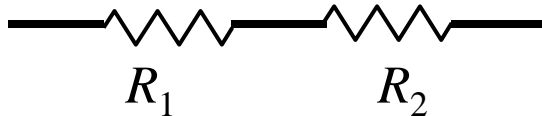
☐  $V_1 > V_2$    ☐  $V_1 = V_2$    ☒  $V_1 < V_2$



# Resistor Summary

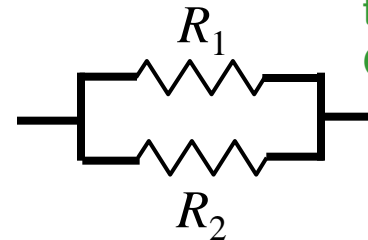
## Series

Every loop with  $R_1$  also has  $R_2$



## Parallel

There is a loop that contains ONLY  $R_1$  and  $R_2$



Wiring

Each resistor on the same wire.

Each resistor on a different wire.

Voltage

Different for each resistor.

$$V_{total} = V_1 + V_2$$

Same for each resistor.

$$V_{total} = V_1 = V_2$$

Current

Same for each resistor

$$I_{total} = I_1 = I_2$$

Different for each resistor

$$I_{total} = I_1 + I_2$$

Resistance

Increases

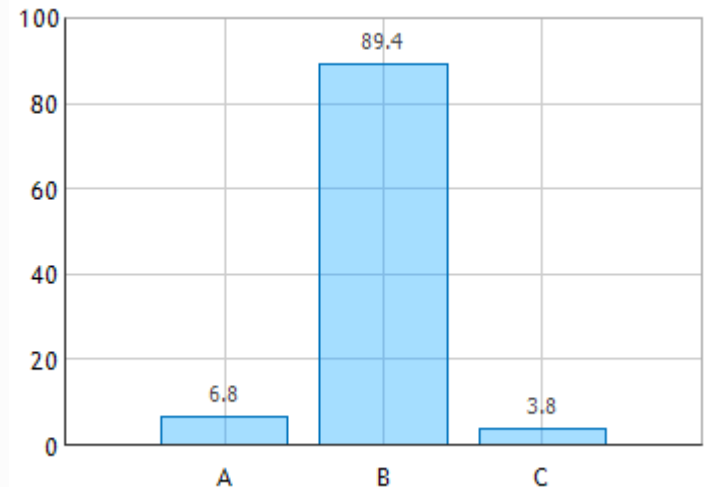
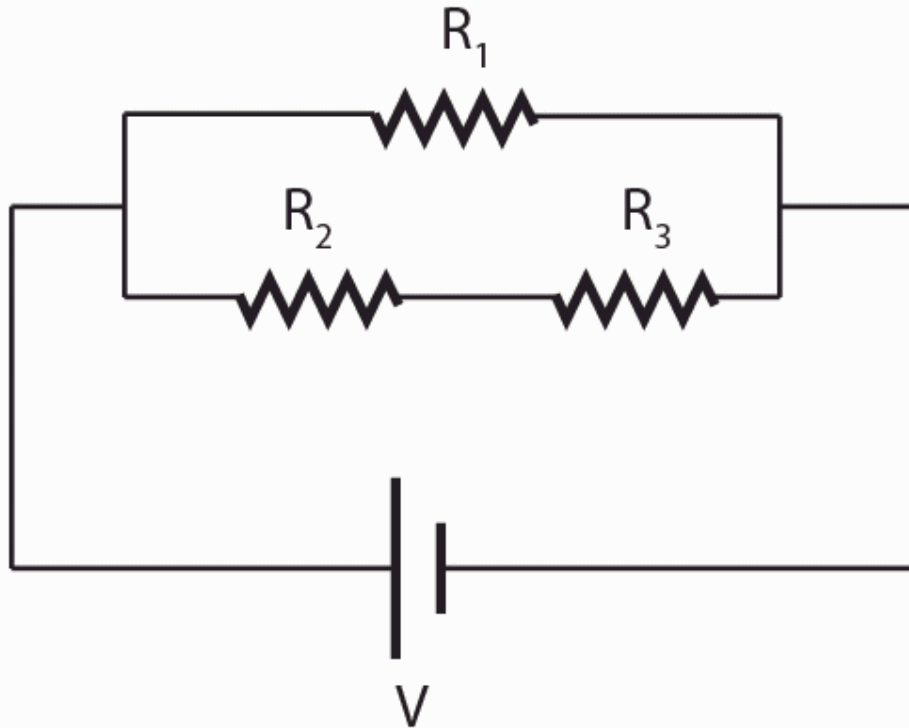
$$R_{eq} = R_1 + R_2$$

Decreases

$$1/R_{eq} = 1/R_1 + 1/R_2$$

# CheckPoint 2a

Three resistors are connected to a battery with emf  $V$  as shown. The resistances of the resistors are all the same, i.e.  $R_1 = R_2 = R_3 = R$ .



Compare the current through  $R_2$  with the current through  $R_3$ :

**A.**  $I_2 > I_3$

**B.**  $I_2 = I_3$

**C.**  $I_2 < I_3$

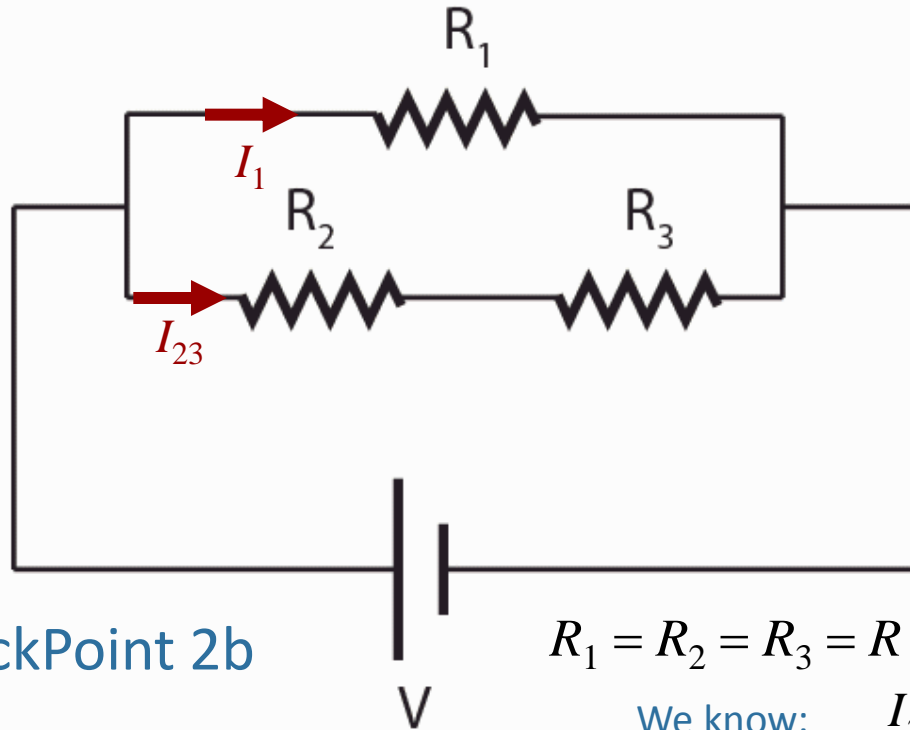
$R_2$  in series with  $R_3$



Current through  $R_2$  and  $R_3$  is the same

$$I_{23} = \frac{V}{R_2 + R_3}$$

# Checkpoint 2b



## CheckPoint 2b

Compare the current through  $R_1$  with the current through  $R_2$

- A  $I_1/I_2 = 1/2$
- B  $I_1/I_2 = 1/3$
- C  $I_1/I_2 = 1$
- D  $I_1/I_2 = 2$**
- E  $I_1/I_2 = 3$

$$R_1 = R_2 = R_3 = R$$

We know:

$$I_{23} = \frac{V}{R_2 + R_3}$$

Similarly:

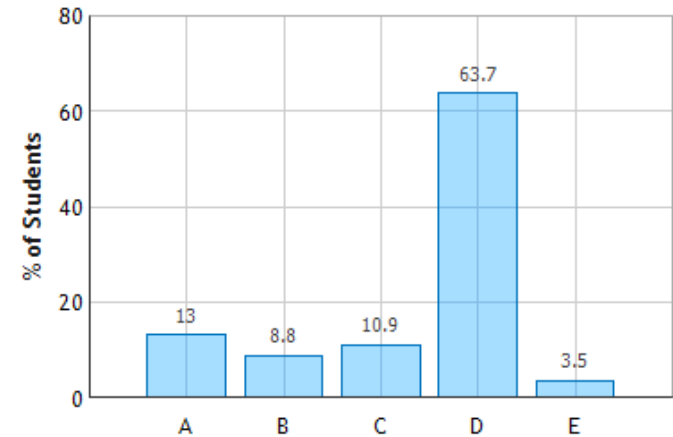
$$I_1 = \frac{V}{R_1}$$

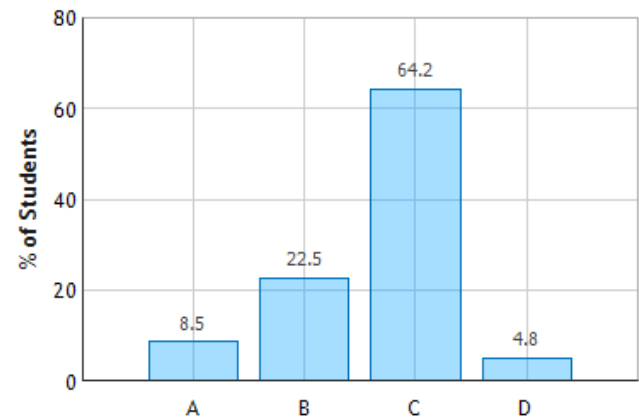
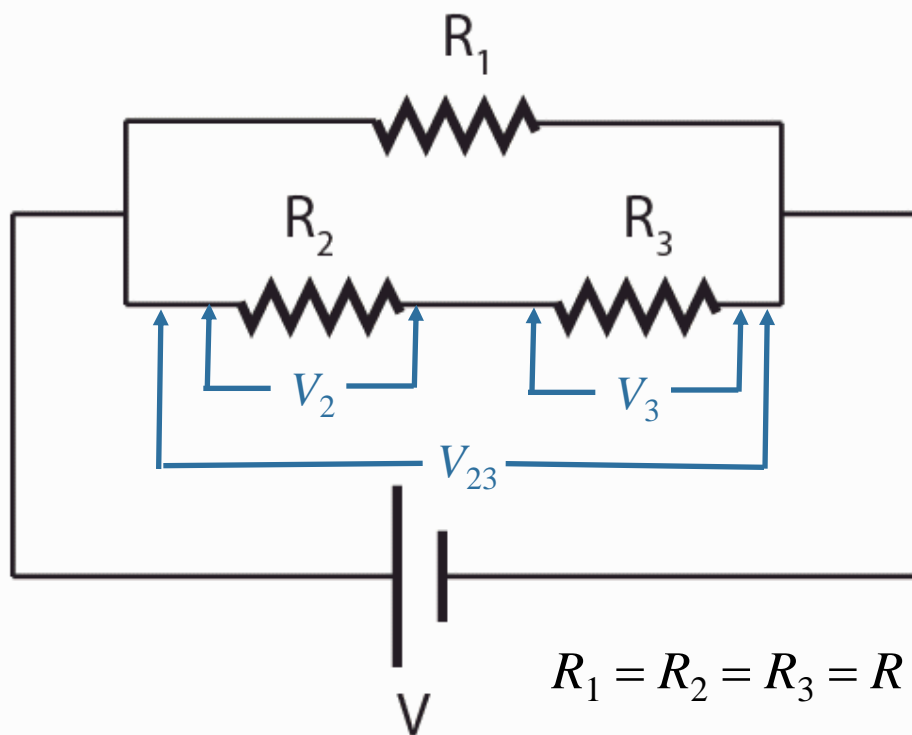


$$I_1 = I_{23} \frac{R_2 + R_3}{R_1}$$



$$\frac{I_1}{I_{23}} = \frac{R_2 + R_3}{R_1} = 2$$





## CheckPoint 2c

Compare the voltage across  $R_2$  with the voltage across  $R_3$

A  $V_2 > V_3$

B  $V_2 = V_3 = V$

C  $V_2 = V_3 < V$

D  $V_2 < V_3$

$$V_2 = I_2 R_2$$

$$V_3 = I_3 R_3$$

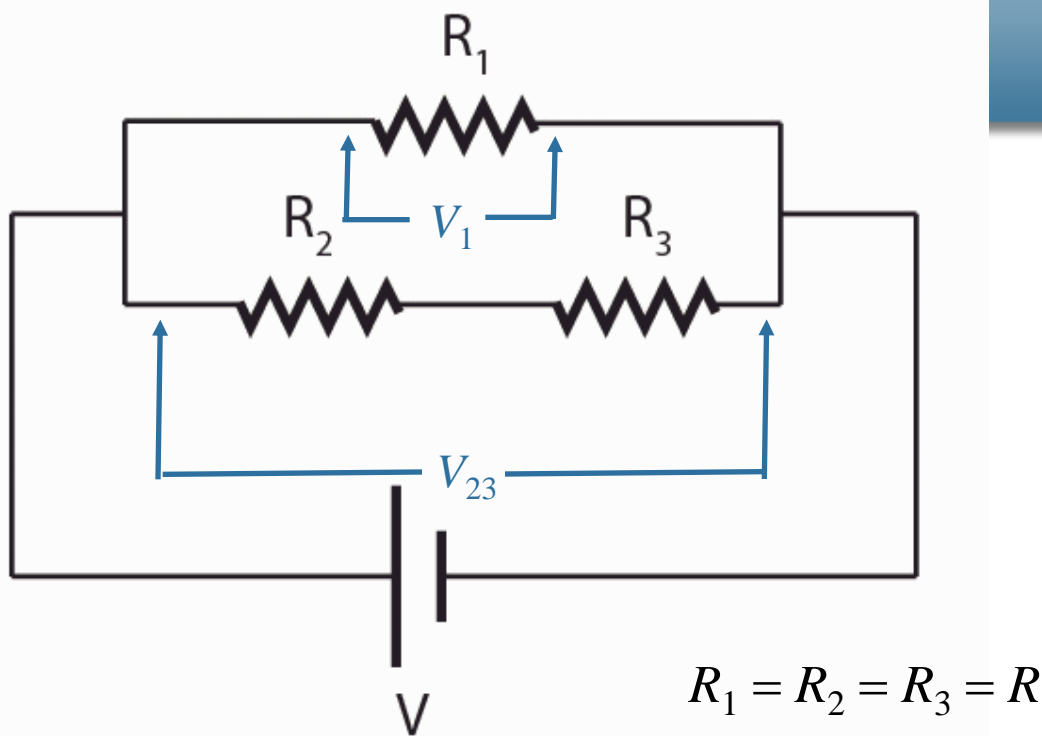
$I_2 = I_3$  (Series)  
 $R_2 = R_3$  (Problem statement)

$$V_2 = V_3$$

$$V_{23} = V$$

$$V_{23} = V_2 + V_3$$

$$V_2 = V_3 = \frac{V}{2}$$



## CheckPoint 2d

Compare the voltage across  $R_1$  with the voltage across  $R_2$

A  $V_1 = V_2 = V$

B  $V_1 = \frac{1}{2} V_2 = V$

**C  $V_1 = 2V_2 = V$**

D  $V_1 = \frac{1}{2} V_2 = \frac{1}{5} V$

E  $V_1 = \frac{1}{2} V_2 = \frac{1}{2} V$

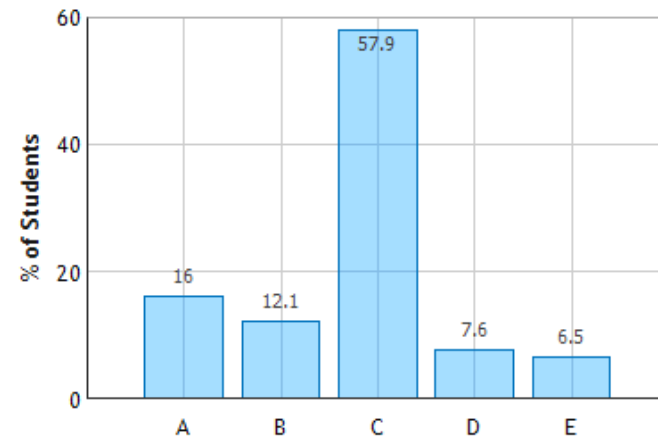
$R_1$  in parallel with series combination of  $R_2$  and  $R_3$

$$V_1 = V_{23}$$

$$R_2 = R_3 \Rightarrow V_2 = V_3$$

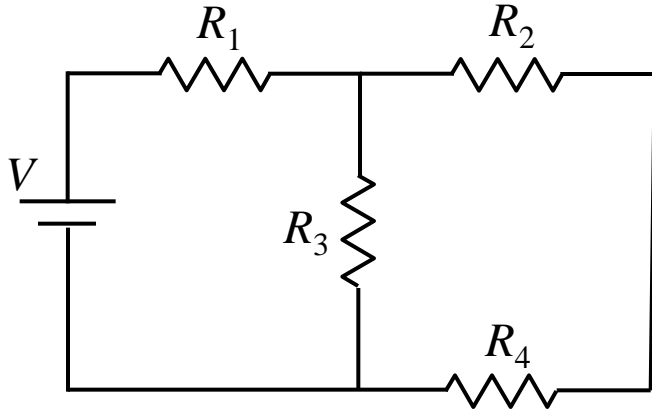
$$V_{23} = V_2 + V_3 = 2V_2$$

Resistor Network: Question 5 (N = 820)



→  $V_1 = 2V_2 = V$

# Calculation



In the circuit shown:  $V = 18V$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  $R_4 = 4\Omega$ .

What is  $V_2$ , the voltage across  $R_2$ ?

## Conceptual Analysis:

Ohm's Law: when current  $I$  flows through resistance  $R$ , the potential drop  $V$  is given by:  
 $V = IR$ .

Resistances are combined in series and parallel combinations

$$R_{series} = R_a + R_b$$

$$(1/R_{parallel}) = (1/R_a) + (1/R_b)$$

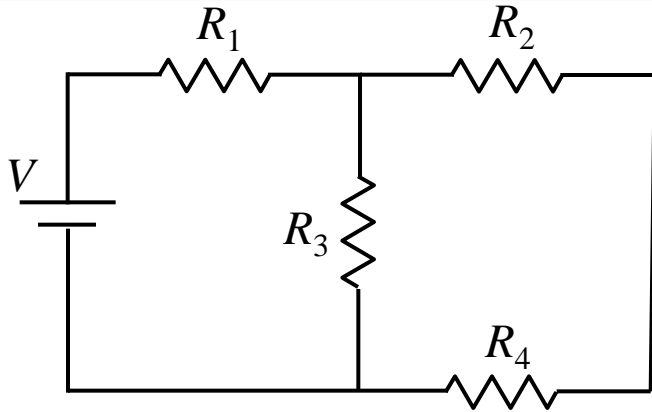
## Strategic Analysis:

Combine resistances to form equivalent resistances

Evaluate voltages or currents from Ohm's Law

Expand circuit back using knowledge of voltages and currents

# Calculation



In the circuit shown:  $V = 18V$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  $R_4 = 4\Omega$ .

What is  $V_2$ , the voltage across  $R_2$ ?

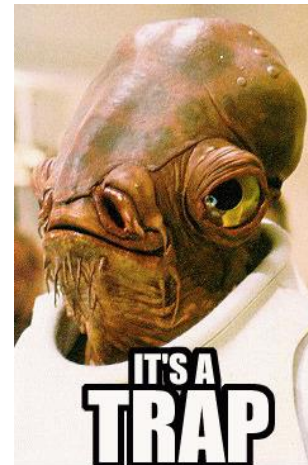
Combine Resistances:

$R_1$  and  $R_2$  are connected:

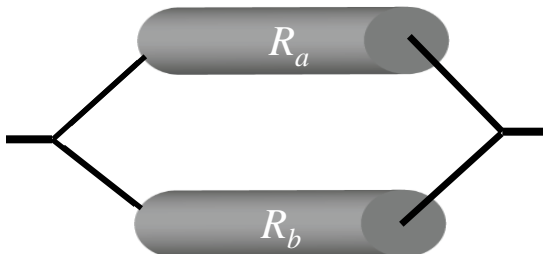
A) in series

B) in parallel

C) neither in series nor in parallel



Parallel Combination



**Parallel:** Can make a loop that contains only those two resistors

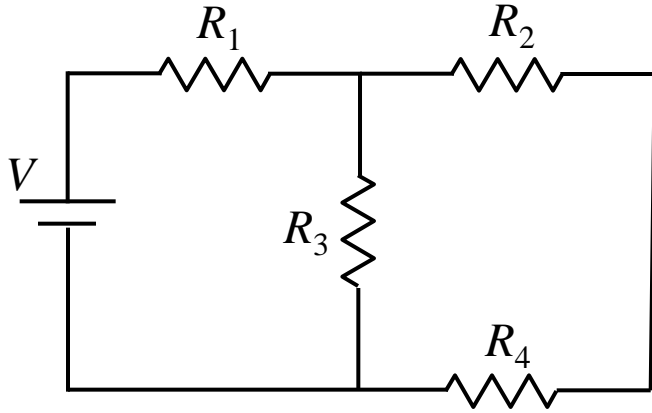
Series Combination



**Series :** Every loop with resistor 1 also has resistor 2.



# Calculation



In the circuit shown:  $V = 18V$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  $R_4 = 4\Omega$ .

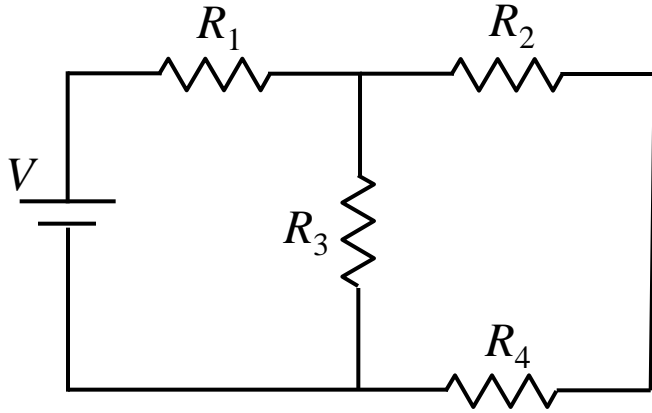
What is  $V_2$ , the voltage across  $R_2$ ?

We first will combine resistances  $R_2$ ,  $R_3$ ,  $R_4$ :

Which of the following is true?

- A)  $R_2$ ,  $R_3$  and  $R_4$  are connected in series
- B)  $R_2$ ,  $R_3$ , and  $R_4$  are connected in parallel
- C)  $R_3$  and  $R_4$  are connected in series ( $R_{34}$ ) which is connected in parallel with  $R_2$
- D)  $R_2$  and  $R_4$  are connected in series ( $R_{24}$ ) which is connected in parallel with  $R_3$
- E)  $R_2$  and  $R_4$  are connected in parallel ( $R_{24}$ ) which is connected in parallel with  $R_3$

# Calculation

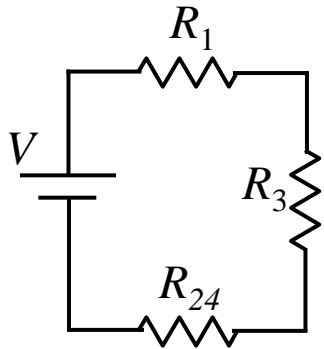


In the circuit shown:  $V = 18V$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  $R_4 = 4\Omega$ .

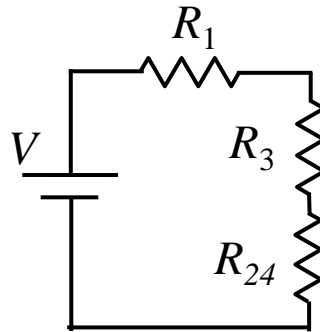
What is  $V_2$ , the voltage across  $R_2$ ?

$R_2$  and  $R_4$  are connected in series ( $R_{24}$ ) which is connected in parallel with  $R_3$

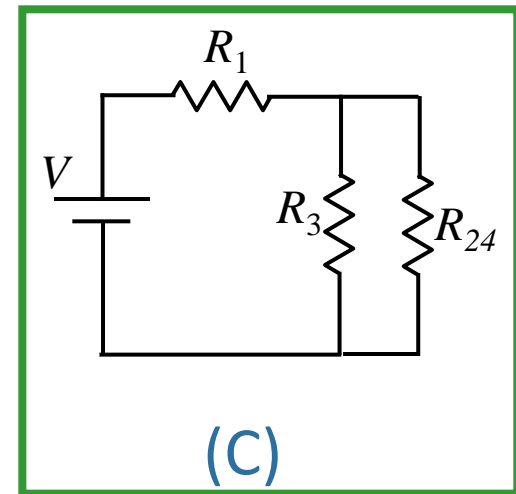
Redraw the circuit using the equivalent resistor  $R_{24} =$  series combination of  $R_2$  and  $R_4$ .



(A)

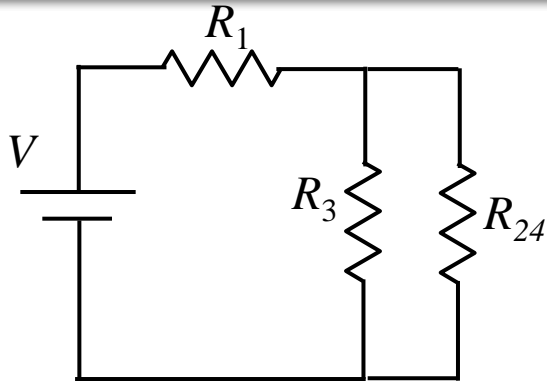


(B)



(C)

# Calculation



In the circuit shown:  $V = 18V$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  $R_4 = 4\Omega$ .

What is  $V_2$ , the voltage across  $R_2$ ?

Combine Resistances:

$R_2$  and  $R_4$  are connected in series  $= R_{24}$

$R_3$  and  $R_{24}$  are connected in parallel  $= R_{234}$

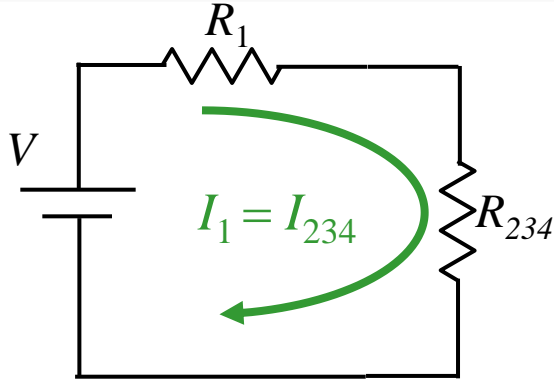
What is the value of  $R_{234}$ ?

A)  $R_{234} = 1\ \Omega$    B)  $R_{234} = 2\ \Omega$    C)  $R_{234} = 4\ \Omega$    D)  $R_{234} = 6\ \Omega$

$R_2$  and  $R_4$  in series  $\rightarrow R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$

$(1/R_{\text{parallel}}) = (1/R_a) + (1/R_b) \rightarrow 1/R_{234} = (1/3) + (1/6) = (3/6)\ \Omega^{-1} \rightarrow R_{234} = 2\ \Omega$

# Calculation



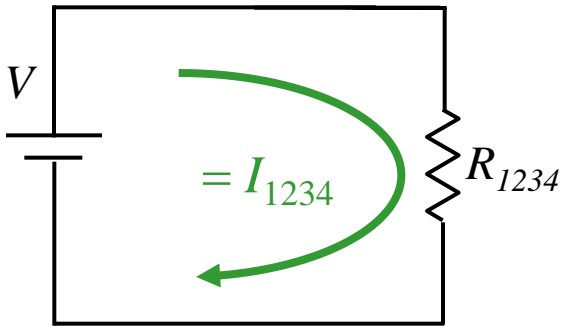
In the circuit shown:  $V = 18V$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  $R_4 = 4\Omega$ .

$$R_{24} = 6\Omega \quad R_{234} = 2\Omega$$

What is  $V_2$ , the voltage across  $R_2$ ?

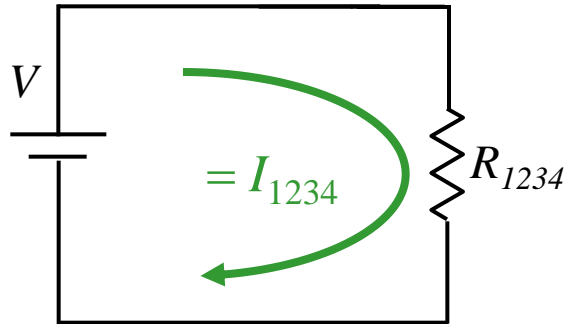
$R_1$  and  $R_{234}$  are in series.  $R_{1234} = 1 + 2 = 3\Omega$

Our next task is to calculate the total current in the circuit



Ohm's Law tells us: 
$$\begin{aligned} I_{1234} &= V/R_{1234} \\ &= 18 / 3 \\ &= 6 \text{ Amps} \end{aligned}$$

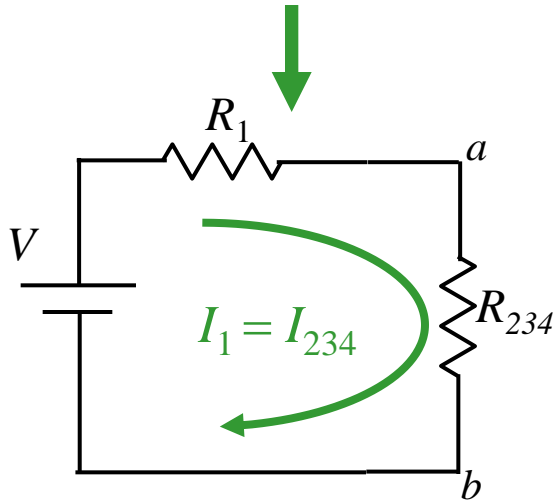
# Calculation



In the circuit shown:  $V = 18V$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  $R_4 = 4\Omega$ .

$$R_{24} = 6\Omega \quad R_{234} = 2\Omega \quad I_{1234} = 6A$$

What is  $V_2$ , the voltage across  $R_2$ ?



$$I_{234} = I_{1234} \quad \text{Since } R_1 \text{ in series with } R_{234}$$

$$\begin{aligned} V_{234} &= I_{234} R_{234} \\ &= 6 \times 2 \\ &= 12 \text{ Volts} \end{aligned}$$

What is  $V_{ab}$ , the voltage across  $R_{234}$  ?

A)  $V_{ab} = 1V$

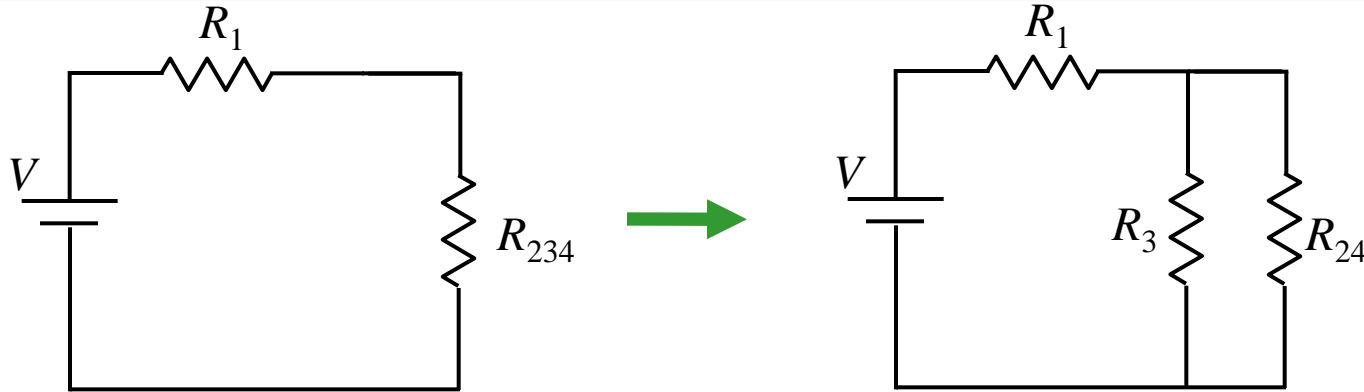
B)  $V_{ab} = 2V$

C)  $V_{ab} = 9V$

**D)  $V_{ab} = 12V$**

E)  $V_{ab} = 16V$

# Calculation



Which of the following are true?

A)  $V_{234} = V_{24}$

B)  $I_{234} = I_{24}$

C) Both A+B

D) None

$$V = 18V$$

$$R_1 = 1\Omega$$

$$R_2 = 2\Omega$$

$$R_3 = 3\Omega$$

$$R_4 = 4\Omega$$

$$R_{24} = 6\Omega$$

$$R_{234} = 2\Omega$$

$$I_{1234} = 6 \text{ Amps}$$

$$I_{234} = 6 \text{ Amps}$$

$$V_{234} = 12V$$

What is  $V_2$ ?

$R_3$  and  $R_{24}$  were combined in parallel to get  $R_{234}$  → Voltages are same!

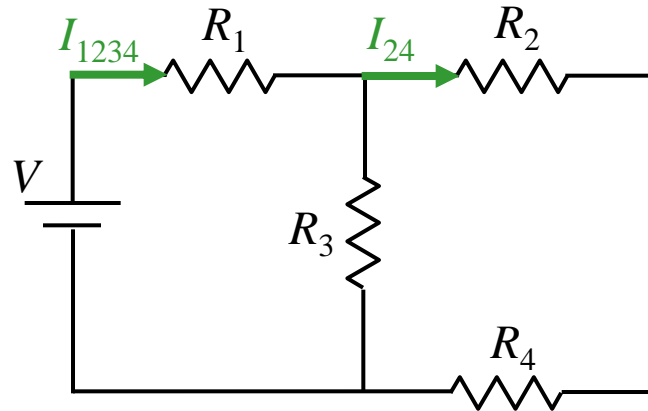
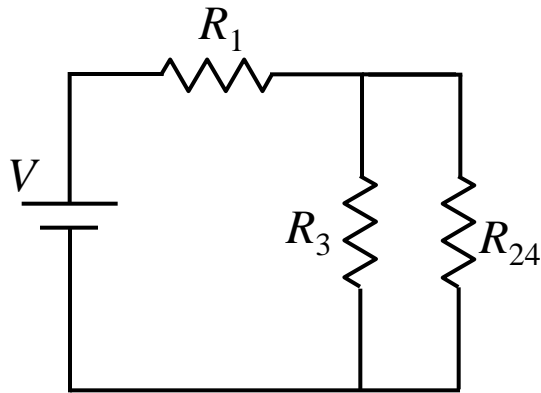
Ohm's Law

$$I_{24} = V_{24} / R_{24}$$

$$= 12 / 6$$

$$= 2 \text{ Amps}$$

# Calculation



$$V = 18V$$

$$R_1 = 1\Omega$$

$$R_2 = 2\Omega$$

$$R_3 = 3\Omega$$

$$R_4 = 4\Omega$$

$$R_{24} = 6\Omega$$

$$R_{234} = 2\Omega$$

$$I_{1234} = 6 \text{ Amps}$$

$$I_{234} = 6 \text{ Amps}$$

$$V_{234} = 12V$$

$$V_{24} = 12V$$

$$I_{24} = 2 \text{ Amps}$$

What is  $V_2$ ?

Which of the following are true?

- A)  $V_{24} = V_2$    B)  $I_{24} = I_2$    C) Both A+B   D) None

$R_2$  and  $R_4$  where combined in series to get  $R_{24}$  → Currents are same!

Ohm's Law

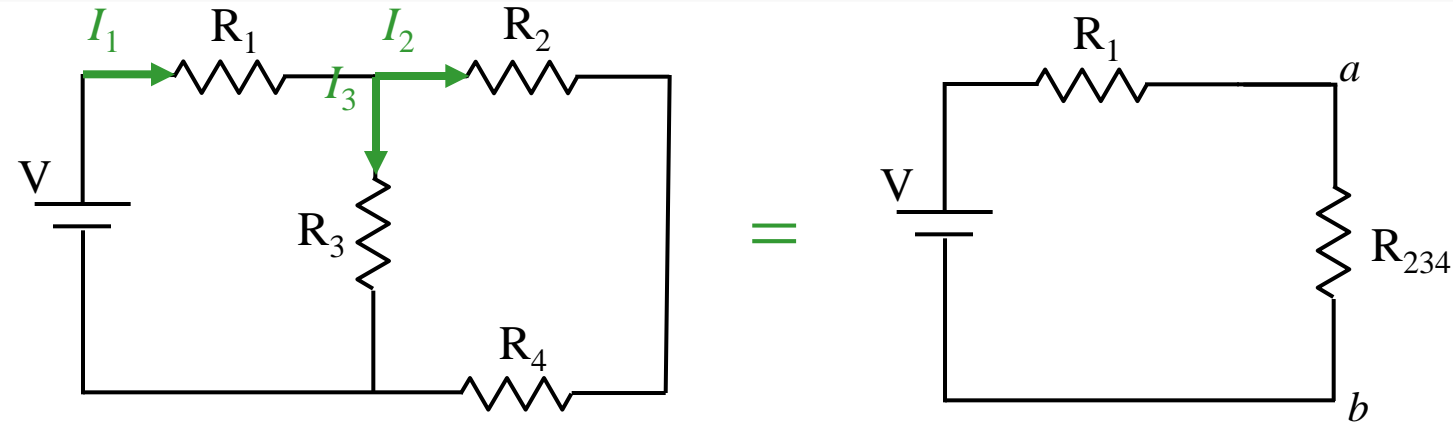
$$V_2 = I_2 R_2$$

$$= 2 \times 2$$

$$= 4 \text{ Volts!}$$

The Problem Can Now Be Solved!

# Quick Follow-Ups



$$V = 18V$$

$$R_1 = 1\Omega$$

$$R_2 = 2\Omega$$

$$R_3 = 3\Omega$$

$$R_4 = 4\Omega$$

$$R_{24} = 6\Omega$$

$$R_{234} = 2\Omega$$

$$V_{234} = 12V$$

$$V_2 = 4V$$

$$I_{1234} = 6 \text{ Amps}$$

What is  $I_3$  ?

A)  $I_3 = 2 A$

B)  $I_3 = 3 A$

C)  $I_3 = 4 A$

$$V_3 = V_{234} = 12V \rightarrow I_3 = V_3/R_3 = 12V/3\Omega = 4A$$

What is  $I_1$  ?

$$\text{We know } I_1 = I_{1234} = 6 A$$

NOTE:  $I_2 = V_2/R_2 = 4/2 = 2 A$

$$\rightarrow I_1 = I_2 + I_3$$

Make Sense?