Pretty straightforward! The analogy to spring motion really helped. Is there any false comparison with it that we should watch out for?

I think I got a lot of this, the pre-lecture confused me a little bit though with the phase angle and when it called omega the oscillation frequency= 1/sqrt(LC) and then later it called that same omega the natural frequency and gave us a different equation for the oscillation frequency

Seems interesting, but my mind is on the test right now.

As soon as I heard the word "damping" my mind switched to Diff Equ. I love it when the things that I learn in one class matches up to what I learn in another class :)

Can we please have a review session tomorrow???? Spring break took a toll on all of us and it would mean so much to the student body if you guys pushed back lecture which you guys can and do a review session. Obviously you guys want everyone to do good, this review session will help everyone so much AND I can guarantee it will improve the test average. It's just one lecture and we would love you guys for doing it. If you say no to this, post this comment so you guys can see how upset the student body will be if you choose not to. Once more plesase!!!!!!!!!!

Come on, teacher!! Give me a break!!! Oscillations just after my amazing vacation? My brain hurts ever since the oscillatory frequency started popping into and out of the screen.
Exam Wed. night (April 2\textsuperscript{nd}) at 7:00

- Covers material in Lectures 9 – 18
- Bring your ID: Rooms determined by discussion section (see link)

Don’t forget:

- Worked examples in homeworks (the optional questions)
- Other old exams

For most people, taking old exams is most beneficial

» Take them like real exam (calculator and formula sheet)
» Complete full exam, then grade (harshly)
» Review problems got wrong (why did you get it wrong)
» Repeat
Kirchoff’s Rules

- Sum of voltages around a loop is zero
- Sum of currents into a node is zero
- Kirchoff’s rules with capacitors and inductors
  - In RC and RL circuits: charge and current involve exponential functions with time constant: “charging and discharging”
  - E.g. \[ IR + \frac{Q}{C} = V \]
- Capacitors and inductors store energy

Magnetic fields

- Generated by electric currents (no magnetic charges)
- Magnetic forces only on charges in motion \[ \vec{F}_{mag} = q\vec{v} \times \vec{B} \]
- Easiest to calculate with Ampere’s Law \[ \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}} \]
- Changing magnetic fields can generate electric fields! \[ \int \vec{E} \cdot d\vec{\ell} = \text{EMF} = \Delta V = -\frac{d}{dt} \int B \cdot dA = -\frac{d \phi_{mag}}{dt} \]
Today’s Concepts:

A) Oscillation Frequency
B) Energy
C) Damping
\[ V_L = L \frac{dI}{dt} \]

\[ V_C = \frac{Q}{C} \]

\[ \frac{Q}{C} + L \frac{dI}{dt} = 0 \]

\[ I = \frac{dQ}{dt} \quad \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \quad \frac{d^2Q}{dt^2} = -\omega^2 Q \]

where

\[ \omega = \frac{1}{\sqrt{LC}} \]
At time $t = 0$ the capacitor is fully charged with $Q_{\text{max}}$ and the current through the circuit is 0.

What is the potential difference across the inductor at $t = 0$?

A) $V_L = 0$
B) $V_L = Q_{\text{max}} / C$
C) $V_L = Q_{\text{max}} / 2C$

The two elements are in parallel, so $V_L = V_C = Q / C$.
LC Circuits analogous to mass on spring

\[ \frac{d^2 Q}{dt^2} = -\omega^2 Q \quad \omega = \frac{1}{\sqrt{LC}} \]

\[ \frac{d^2 x}{dt^2} = -\omega^2 x \quad \omega = \sqrt{\frac{k}{m}} \]

Same thing if we notice that \[ k \leftrightarrow \frac{1}{C} \quad \text{and} \quad m \leftrightarrow L \]
Time Dependence

\[ Q(t) = Q_0 \cos(\omega t + \phi) \]

\[ I(t) = \frac{dQ}{dt} = -\omega Q_0 \sin(\omega t + \phi) \]
At time $t = 0$ the capacitor is fully charged with $Q_{\text{max}}$ and the current through the circuit is 0.

What is the potential difference across the inductor at when the current is maximum?

A) $V_L = 0$

B) $V_L = Q_{\text{max}}/C$

C) $V_L = Q_{\text{max}}/2C$

$dI/dt$ is zero when current is maximum
At time $t = 0$ the capacitor is fully charged with $Q_{\text{max}}$ and the current through the circuit is 0.

How much energy is stored in the capacitor when the current is a maximum?

A) $U = \frac{Q_{\text{max}}^2}{2C}$
B) $U = \frac{Q_{\text{max}}^2}{4C}$
C) $U = 0$

Total Energy is constant!

$U_{L_{\text{max}}} = \frac{1}{2} LI_{\text{max}}^2$
$U_{C_{\text{max}}} = \frac{Q_{\text{max}}^2}{2C}$
$I = \text{max when } Q = 0$
The capacitor is charged such that the top plate has a charge $+Q_0$ and the bottom plate $-Q_0$. At time $t = 0$, the switch is closed and the circuit oscillates with frequency $\omega = 500$ radians/s.

What is the value of the capacitor $C$?

A) $C = 1 \times 10^{-3} F$
B) $C = 2 \times 10^{-3} F$
C) $C = 4 \times 10^{-3} F$

\[ \omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{(25 \times 10^4)(4 \times 10^{-3})} = 10^{-3} \]
Which plot best represents the energy in the inductor as a function of time starting just after the switch is closed?

\[ U_L = \frac{1}{2} LI^2 \]

Energy proportional to \( I^2 \) ⇒ \( U_L \) cannot be negative

Current is changing ⇒ \( U_L \) is not constant

Initial current is zero
When the energy stored in the capacitor reaches its maximum again for the first time after $t = 0$, how much charge is stored on the top plate of the capacitor?

A) $+Q_0$

B) $+Q_0 / 2$

C) 0

D) $-Q_0 / 2$

E) $-Q_0$

$Q$ is maximum when current goes to zero

\[ I = \frac{dQ}{dt} \]

Current goes to zero twice during one cycle
Just like \( LC \) circuit but energy but the oscillations get smaller because of \( R \)

Concept makes sense... ...but answer looks kind of complicated
Physics Truth #1:

Even though the answer sometimes looks complicated...

\[ Q(t) = Q_o \cos(\omega t - \phi) \]

...the physics under the hood is still very simple!

\[ \frac{d^2 Q}{dt^2} = -\omega^2 Q \]
The elements of a circuit are very simple:

\[ V_L = L \frac{dI}{dt} \]

\[ V = V_L + V_C + V_R \]

\[ V_R = IR \]

\[ V_C = \frac{Q}{C} \]

\[ I = \frac{dQ}{dt} \]

This is all we need to know to solve for anything!
The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.

What is $Q_{\text{MAX}}$, the maximum charge on the capacitor?

**Conceptual Analysis**
Once switch is opened, we have an $LC$ circuit
Current will oscillate with natural frequency $\omega_0$

**Strategic Analysis**
Determine initial current
Determine oscillation frequency $\omega_0$
Find maximum charge on capacitor
The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.

What is $I_L$, the current in the inductor, immediately after the switch is opened? Take positive direction as shown.

A) $I_L < 0$  
B) $I_L = 0$  
C) $I_L > 0$

Current through inductor immediately after switch is opened is the same as the current through inductor immediately before switch is opened

before switch is opened:
all current goes through inductor in direction shown
The switch in the circuit shown has been closed for a long time. At \( t = 0 \), the switch is opened.

The energy stored in the capacitor immediately after the switch is opened is zero.

**A) TRUE**

**B) FALSE**

before switch is opened:

\[
dI_L/dt \sim 0 \Rightarrow V_L = 0
\]

BUT: \( V_L = V_C \) since they are in parallel

\[
V_C = 0
\]

after switch is opened:

\( V_C \) cannot change abruptly

\[
V_C = 0
\]

\[
U_C = \frac{1}{2} CV_C^2 = 0 !
\]

**IMPORTANT: NOTE DIFFERENT CONSTRAINTS AFTER SWITCH OPENED**

CURRENT through INDUCTOR cannot change abruptly

VOLTAGE across CAPACITOR cannot change abruptly
The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.

Current through inductor immediately after switch is opened is the same as the current through inductor immediately before switch is opened.

Before switch is opened: Current moves down through $L$

After switch is opened: Current continues to move down through $L$
The switch in the circuit shown has been closed for a long time. At \( t = 0 \), the switch is opened.

What is the magnitude of the current right after the switch is opened?

A) \( I_o = V \sqrt{\frac{C}{L}} \)  
B) \( I_o = \frac{V}{R^2} \sqrt{\frac{L}{C}} \)  
C) \( I_o = \frac{V}{R} \)  
D) \( I_o = \frac{V}{2R} \)

Current through inductor immediately after switch is opened is the same as the current through inductor immediately before switch is opened.
The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.

**Hint:** Energy is conserved

What is $Q_{\text{max}}$, the maximum charge on the capacitor during the oscillations?

\[ Q_{\text{max}} = \frac{V}{R} \sqrt{LC} \]  

When $I$ is max (and $Q$ is 0) 

\[ U = \frac{1}{2} LI_{\text{max}}^2 \]

When $Q$ is max (and $I$ is 0) 

\[ U = \frac{1}{2} \frac{Q_{\text{max}}^2}{C} \]

\[ \frac{1}{2} LI_{\text{max}}^2 = \frac{1}{2} \frac{Q_{\text{max}}^2}{C} \]

\[ Q_{\text{max}} = I_{\text{max}} \sqrt{LC} \]

\[ = \frac{V}{R} \sqrt{LC} \]
Is it possible for the maximum voltage on the capacitor to be greater than $V$?

A) YES  B) NO

$$Q_{\text{max}} = \frac{V}{R} \sqrt{LC} \quad \Rightarrow \quad V_{\text{max}} = \frac{V}{R} \sqrt{\frac{L}{C}}$$

$V_{\text{max}}$ can be greater than $V$ IF:

$$\frac{\sqrt{L}}{\sqrt{C}} > R$$

We can rewrite this condition in terms of the resonant frequency:

$$\omega_0 L > R \quad \text{OR} \quad \frac{1}{\omega_0 C} > R$$

We will see these forms again when we study AC circuits!