This material is a little abstract. Can you explain these phase diagrams and how to draw the right triangles to find phi in more detail?

Is there ever going to be more than one loop/ more than one of each piece of the circuit? If so, can we please go through an example?

So we just get done with a decently difficult exam and this is what you throw at us??!?!?!?!?

Between this and the exam, I kinda sorta really, really hate E&M now. And did we really set up a differential equation, examine three separate complex instances, introduce phasers (which confuse the int(B * dA) out of me), and graph phasers all to arrive at ... a problem solved by simple geometry and algebra? Not only do I hate E&M, E&M constantly confuses me with how it can be so complex and yet so simple.

I really like the pretty diagrams... I just wish I understood them haha.

Sorry, i didn't really do this prelecture/checkpoint. but i'll come back to it in a few days! studying for a CS exam :0

This pre-lecture went way over my head. Going over everything in class will definitely help because I need some general explanation of what things are in order to understand the details of the example.
Today’s Concept:

AC Circuits
Maximum currents & voltages
Phasors: A Simple Tool
Maximum Values (easy  \( V=IR \))

\[
V_{R_{max}} = I_{max} R \\
I_{max} = \frac{\epsilon_{max}}{Z} \\
V_{L_{max}} = I_{max} X_L \\
V_{C_{max}} = I_{max} X_C
\]

Value at specific time (phasors)

- \( y \) component gives voltage
- \( V_{\text{Inductor}} \) Leads current
- \( V_{\text{Capacitor}} \) Lags current
$$E = V_{\text{max}} \sin(\omega t)$$

$$I = \frac{V_R}{R} = \frac{V_{\text{max}}}{R} \sin(\omega t)$$

Amplitude = $V_{\text{max}}/R$

$V_R(t)$ is in Phase with $I_R(t)$
Capacitors

\[ E = V_{\text{max}} \sin(\omega t) \]

\[ Q = CV = CV_{\text{max}} \sin(\omega t) \]

\[ I = \frac{dQ}{dt} \]

\[ I = V_{\text{max}} \omega C \cos(\omega t) \]

Amplitude = \[ \frac{V_{\text{max}}}{X_C} \]

where \( X_C = \frac{1}{\omega C} \) is like the “resistance” of the capacitor.

\( X_C \) depends on \( \omega \)
**Inductors**

\[ L \frac{dI}{dt} = V_L = V_{\text{max}} \sin(\omega t) \]

\[ I = -\frac{V_{\text{max}}}{\omega L} \cos(\omega t) \]

**Amplitude** = \[ \frac{V_{\text{max}}}{X_L} \]

where \( X_L = \omega L \)

is like the "resistance" of the inductor

\( X_L \) depends on \( \omega \)
An RL circuit is driven by an AC generator as shown in the figure.

For what driving frequency \( \omega \) of the generator will the current through the resistor be largest

A) \( \omega \) large

B) Current through \( R \) doesn’t depend on \( \omega \)

C) \( \omega \) small
Summary

\[ I_{\text{max}} = \frac{V_{\text{max}}}{R} \quad (V_R \text{ in phase with } I) \]

Because resistors are simple

\[ I_{\text{max}} = \frac{V_{\text{max}}}{X_C} \]

\[ X_C = \frac{1}{\omega C} \quad (V_C \text{ 90° behind } I) \]

Current comes first since it charges capacitor

Like a wire at high \( \omega \)

\[ I_{\text{max}} = \frac{V_{\text{max}}}{X_L} \]

\[ X_L = \omega L \quad (V_L \text{ 90° ahead of } I) \]

Opposite of capacitor

Like a wire at low \( \omega \)
A RL circuit is driven by an AC generator as shown in the figure.

The phase difference between the CURRENT through the resistor and the CURRENT through the inductor is

A) Is always zero  
B) Is always 90°  
C) Depends on the value of L and R  
D) Depends on L, R and the generator voltage

The CURRENT is THE CURRENT
There is only 1 current in this circuit
Same everywhere in circuit
Makes sense to write everything in terms of $I$ since this is the same everywhere in a one-loop circuit:

$$V_{\text{max}} = I_{\text{max}} \times X_C$$

$V$ 90° behind $I$

$$V_{\text{max}} = I_{\text{max}} \times X_L$$

$V$ 90° ahead of $I$

$$V_{\text{max}} = I_{\text{max}} \times R$$

$V$ in phase with $I$

Phasors make this simple to see.

Always looks the same. Only the lengths will change.
The Voltages still Add Up

But now we are adding vectors:

\[ I_{\text{max}} X_L \quad I_{\text{max}} R \]

\[ I_{\text{max}} X_C \]

\[ \varepsilon_{\text{max}} \]

\[ I_{\text{max}} X_L \quad I_{\text{max}} R \]

\[ I_{\text{max}} X_C \]

\[ \varepsilon_{\text{max}} \]
Make this Simpler

Electricity & Magnetism Lecture 20, Slide 12
$I_{\text{max}} X_L$

$\mathcal{E}_{\text{max}} = I_{\text{max}} Z$

$I_{\text{max}} (X_L - X_C)$

Electricity & Magnetism  Lecture 20, Slide 13
\( \varepsilon_{\text{max}} = I_{\text{max}} Z \)

\[ I_{\text{max}} R \]

\[ I_{\text{max}} (X_L - X_C) \]
\[ E_{\text{max}} = I_{\text{max}} Z \]

\[ I_{\text{max}} (X_L - X_C) \]

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

\[ \tan(\phi) = \frac{X_L - X_C}{R} \]
\[
V_{C_{\text{max}}} = I_{\text{max}} X_C
\]
\[
V_{L_{\text{max}}} = I_{\text{max}} X_L
\]
\[
V_{R_{\text{max}}} = I_{\text{max}} R
\]
\[
\varepsilon_{\text{max}} = I_{\text{max}} Z
\]

\[
I_{\text{max}} = \varepsilon_{\text{max}} / Z
\]

\[
Z = \sqrt{R^2 + (X_L - X_C)^2}
\]

\[
\tan (\phi) = \frac{X_L - X_C}{R}
\]
Example: RL Circuit $X_c = 0$
2) A RL circuit is driven by an AC generator as shown in the figure.

The voltages across the resistor and generator are

A) Always out of phase

B) Always in phase

C) Sometimes in and sometimes out of phase

Draw Voltage Phasors

$I_{\text{max}} X_L$

$\mathcal{E}_{\text{max}}$

$I_{\text{max}} R$

AC Circuit 1: Question 1 (N = 729)
A RL circuit is driven by an AC generator as shown in the figure.

The voltages across the resistor and inductor are

A) Always out of phase  
B) Always in phase  
C) Sometimes in and sometimes out of phase
A driven RLC circuit is represented by the phasor diagram below.

The vertical axis of the phasor diagram represents voltage. When the current through the circuit is maximum, what is the potential difference across the inductor?

A) $V_L = 0$

B) $V_L = V_{L,max}/2$

C) $V_L = V_{L,max}$

What does the voltage phasor diagram look like when the current is a maximum?
A driven RLC circuit is represented by the phasor diagram below.

When the capacitor is fully charged, what is the magnitude of the voltage across the inductor?

A) \( V_L = 0 \)  

B) \( V_L = \frac{V_{L,\text{max}}}{2} \)  

C) \( V_L = V_{L,\text{max}} \)

What does the voltage phasor diagram look like when the capacitor is fully charged?
A driven RLC circuit is represented by the phasor diagram below.

When the voltage across the capacitor is at its positive maximum, $V_C = +V_{C,max}$, what is the voltage across the inductor?

A) $V_L = 0$
B) $V_L = V_{L,max}$
C) $V_L = -V_{L,max}$

What does the voltage phasor diagram look like when the voltage across capacitor is at its positive maximum?
Consider the harmonically driven series \( LCR \) circuit shown.

\[ V_{\text{max}} = 100 \text{ V} \]
\[ I_{\text{max}} = 2 \text{ mA} \]
\[ V_{C_{\text{max}}} = 113 \text{ V} \]

The current leads generator voltage by \( 45^\circ \)

\( L \) and \( R \) are unknown.

What is \( X_L \), the reactance of the inductor, at this frequency?

**Conceptual Analysis**

The maximum voltage for each component is related to its reactance and to the maximum current.

The impedance triangle determines the relationship between the maximum voltages for the components.

**Strategic Analysis**

Use \( V_{\text{max}} \) and \( I_{\text{max}} \) to determine \( Z \)

Use impedance triangle to determine \( R \)

Use \( V_{C_{\text{max}}} \) and impedance triangle to determine \( X_L \)
Consider the harmonically driven series $LCR$ circuit shown.

$V_{max} = 100 \text{ V}$

$I_{max} = 2 \text{ mA}$

$V_{Cmax} = 113 \text{ V}$

The current leads generator voltage by $45^\circ$

$L$ and $R$ are unknown.

What is $X_L$, the reactance of the inductor, at this frequency?

Compare $X_L$ and $X_C$ at this frequency:

A) $X_L < X_C$  
B) $X_L = X_C$  
C) $X_L > X_C$  
D) Not enough information

This information is determined from the phase

Current leads voltage

$V_L = I_{max}X_L$

$V_C = I_{max}X_C$
Consider the harmonically driven series \( LCR \) circuit shown.

\[
\begin{align*}
V_{\text{max}} &= 100 \text{ V} \\
I_{\text{max}} &= 2 \text{ mA} \\
V_{C_{\text{max}}} &= 113 \text{ V}
\end{align*}
\]

The current leads generator voltage by \( 45^\circ \)

\( L \) and \( R \) are unknown.

What is \( X_L \), the reactance of the inductor, at this frequency?

What is \( Z \), the total impedance of the circuit?

A) 70.7 k\( \Omega \)  \hspace{1cm} B) 50 k\( \Omega \)  \hspace{1cm} C) 35.4 k\( \Omega \)  \hspace{1cm} D) 21.1 k\( \Omega \)

\[
Z = \frac{V_{\text{max}}}{I_{\text{max}}} = \frac{100\text{V}}{2\text{mA}} = 50\text{k}\Omega
\]
Consider the harmonically driven series LCR circuit shown.

\[ V_{\text{max}} = 100 \text{ V} \]
\[ I_{\text{max}} = 2 \text{ mA} \]
\[ V_{C,\text{max}} = 113 \text{ V} \]

The current leads generator voltage by 45°.

\( L \) and \( R \) are unknown.

What is \( X_L \), the reactance of the inductor, at this frequency?

**What is \( R \)?**

A) 70.7 kΩ  
B) 50 kΩ  
C) 35.4 kΩ  
D) 21.1 kΩ

Determined from impedance triangle

\[ \cos(45) = \frac{R}{Z} \]

\[ R = Z \cos(45°) \]

\[ = 50 \text{ kΩ} \times 0.707 \]

\[ = 35.4 \text{ kΩ} \]
Consider the harmonically driven series LCR circuit shown.

\[ V_{\text{max}} = 100 \text{ V} \]
\[ I_{\text{max}} = 2 \text{ mA} \]
\[ V_{C_{\text{max}}} = 113 \text{ V} \]

The current leads generator voltage by 45°.

\( L \) and \( R \) are unknown.

What is \( X_L \), the reactance of the inductor, at this frequency?

\[ X_L = X_C - R \]

A) 70.7 kΩ  B) 50 kΩ  C) 35.4 kΩ  D) 21.1 kΩ

We start with the impedance triangle:

\[ \frac{X_C - X_L}{R} = \tan 45° = 1 \]

What is \( X_C \)?

\[ V_{C_{\text{max}}} = I_{\text{max}}X_C \]

\[ X_C = \frac{113}{2} = 56.5 \text{ kΩ} \]

\[ X_L = 56.5 \text{ kΩ} - 35.4 \text{ kΩ} \]